Glauber model and the heavy ion reaction cross section

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We reexamine the Glauber model and calculate the total reaction cross section as a function of energy in the low and intermediate energy range, where many of the corrections in the model are effective. The most significant effect in this energy range is by the modification of the trajectory due to the Coulomb field. The modification in the trajectory due to nuclear field is also taken into account in a self-consistent way. The energy ranges in which particular corrections are effective are quantified, and it is found that when the center of mass energy of the system becomes 30 times the Coulomb barrier, none of the trajectory modification to the Glauber model is really required. The reaction cross sections for light and heavy systems, from the Coulomb barrier to intermediate energies, have been calculated. The exact nuclear densities and free nucleon-nucleon (*NN*) cross sections have been used in the calculations. The center of mass correction, which is important for light systems, has also been taken into account. There is an excellent agreement between the calculations with the modified Glauber model and the experimental data. This suggests that the heavy ion reactions in this energy range can be explained by the Glauber model in terms of free *NN* cross sections without incorporating any medium modification.

DOI: 10.1103/PhysRevC.67.054607

PACS number(s): 24.10.Ht, 25.70.-z, 25.60.Dz

I. INTRODUCTION

The Glauber model (GM) [1] has been employed for describing heavy ion reactions at high energies. It is a semiclassical model picturing the nuclei moving in a straight path along the collision direction, and gives the nucleus-nucleus interaction [2] in terms of interaction between the constituent nucleons (*NN* cross section) and nuclear density distributions. It is a well-established model for high energies and has been applied to heavy ion collision for describing a number of reaction processes (see, e.g., Refs. [3–5]). One of the most important physical quantities characterizing the nuclear reactions is the total reaction cross section. It is very useful for extracting information about the nuclear sizes. The Glauber model has been successively used to get the radii of radioactive nuclei from measured total reaction cross sections [6].

At low energies, the straight-line trajectory is modified due to the Coulomb field between two nuclei. The Coulomb modified Glauber model (CMGM) [7,8] consists of replacing the eikonal trajectory at an impact parameter b with the eikonal trajectory at the corresponding distance r_c of closest approach in the presence of the Coulomb field. Later, the noneikonal nature of the trajectory around r_c has also been taken into account [9]. Replacing r_c by the distance r_{cn} of closest approach in the presence of both the Coulomb and the nuclear field gives the Coulomb plus nuclear modified Glauber model (CNMGM) [9,10]. This model (CMGM/ CNMGM) has been widely used in recent literature [11-14]. Warner et al. [11] have shown in their calculations for light projectiles on various targets that trajectory modifications are minor. Let us remark that the energies considered by them are much above the Coulomb barrier. The Coulomb modified Glauber model has also been used at very high energies [12] where trajectory modifications may be ineffective. There have been various prescriptions [13,14] to modify the NN cross sections due to nuclear density.

Most of the work reported earlier [7,8,10,9,13,14] has

been done using Gaussian densities. The reaction cross section is sensitive to surface density of the colliding nuclei. Depending on how well one has produced the surface shape with the Gaussian form, it will make a 5-10 % change in the reaction cross section over that done with realistic densities [15].

In light of this, we reexamine the various trajectory corrections in the Glauber model and calculate the total reaction cross section as a function of energy, up to 50 times the Coulomb barrier. We quantify the energy range in which a particular correction is effective and find that when the center of mass energy of the system becomes 30 times the Coulomb barrier, the above modifications to the GM are insignificant. Thus, the energy at and above which the results of the CNMGM and the GM coincide depends on the Coulomb barrier between the two nuclei and will be different for a light and a heavy system. In the present work, we use exact nuclear densities and free NN cross sections. The center of mass correction that is important for light systems has also been taken into account. Comparison of the experimental data with our calculations is presented in the plots of $\sigma_R(E)$ vs E.

II. THE GLAUBER MODEL

Consider the collision of a projectile nucleus A on a target nucleus B. The probability for occurrence of a nucleonnucleon collision, when the nuclei A and B collide at an impact parameter **b** relative to each other, is given [4,5] by

$$T(b)\bar{\sigma}_{NN} = \int \rho_A^z(\mathbf{b}_A) d\mathbf{b}_A \rho_B^z(\mathbf{b}_B) d\mathbf{b}_B t(\mathbf{b} - \mathbf{b}_A + \mathbf{b}_B) \bar{\sigma}_{NN}.$$
(1)

Here $\rho_A^z(\mathbf{b}_A)$ and $\rho_B^z(\mathbf{b}_B)$ are the *z*-integrated densities of projectile and target nuclei, respectively, while $t(\mathbf{b})d\mathbf{b}$ is the probability for having a nucleon-nucleon collision within the

transverse area element $d\mathbf{b}$, when one nucleon approaches at an impact parameter \mathbf{b} relative to another nucleon. All these distribution functions are normalized to 1. Here $\bar{\sigma}_{NN}$ is the average total nucleon-nucleon cross section.

There can be up to $A \times B$ collisions. The probability of occurrence of *n* collisions will be

$$P(n,b) = \binom{AB}{n} (1-s)^n (s)^{AB-n}.$$
 (2)

Here $s = 1 - T(b)\overline{\sigma}_{NN}$. The total probability for the occurrence of at least one *NN* collision in the collision of *A* and *B* at an impact parameter **b** is

$$\frac{d\sigma_R}{db} = \sum_{n=1}^{AB} P(n,b) = 1 - s^{AB}.$$
(3)

The total reaction cross section σ_R can be written as

$$\sigma_R = 2\pi \int bdb(1 - s^{AB}). \tag{4}$$

From this, one can read the scattering matrix as

$$|S(b)|^{2} = s^{AB} = [1 - T(b)\bar{\sigma}_{NN}]^{AB}.$$
 (5)

In the optical limit, where a nucleon of the projectile undergoes only one collision in the target nucleus, Eq. (5) can be written as

$$|S(b)|^2 \simeq \exp[-T(b)\bar{\sigma}_{NN}AB].$$
(6)

The scattering matrix can be defined in terms of eikonal phase shift $\chi(b)$ as

$$S(b) = \exp(-i\chi(b)).$$
(7)

If we assume $\bar{\alpha}_{NN}$ to be the ratio of real to imaginary part of *NN* scattering amplitude, the eikonal phase shift can be obtained as

$$\chi(b) = \frac{1}{2} \bar{\sigma}_{NN} (\bar{\alpha}_{NN} + i) AB T(b).$$
(8)

Here $\bar{\alpha}_{NN}$ will not be directly used for calculating the reaction cross section but will come through the correction in the trajectory due to the nuclear field. Once we know the phase shift and thus the scattering matrix, we can calculate the reaction cross section and also the elastic scattering angular distribution. In the present work, we restrict ourselves to the calculations of reaction cross section only.

In momentum space, T(b) is derived as

$$T(b) = \frac{1}{2\pi} \int J_0(qb) S_A(\mathbf{q}) S_B(-\mathbf{q}) f_{NN}(q) q dq.$$
(9)

Here $S_A(\mathbf{q})$ and $S_B(-\mathbf{q})$ are the Fourier transforms of the nuclear densities and $J_0(qb)$ is the cylindrical Bessel function of zeroth order. The function $f_{NN}(q)$ is the Fourier transform of the profile function $t(\mathbf{b})$ and gives the q dependence of NN scattering amplitude. The profile function $t(\mathbf{b})$ for the

NN scattering can be taken as a δ function if the nucleons are point particles. In general, it is taken as a Gaussian function of width r_0 as

$$t(\mathbf{b}) = \frac{\exp(-b^2/2r_0^2)}{2\pi r_0^2}.$$
 (10)

Thus,

$$f_{NN}(q) = \exp(-r_0^2 q^2/2).$$
 (11)

Here, r_0 is the range parameter and has a weak dependence on energy (see, for discussions, Ref. [12]). We use r_0 = 0.6 fm [8] in the energy range 2–200 MeV.

The average *NN* scattering parameter $\bar{\sigma}_{NN}$ is obtained in terms of proton-proton (pp) cross section σ_{pp} and neutronneutron (nn) cross section σ_{nn} , the parametrized forms for which are available in Ref. [8]. For projectile energies below 10 MeV/nucleon, we use the prescription given in Ref. [8] for computing $\bar{\alpha}_{NN}$. For energies above 10 MeV/nucleon, we take the parametrized forms from the work of Ref. [20]. These forms produce the elastic scattering data for various systems satisfactorily.

A. Optical potential in the Glauber model

The usual Glauber optical limit phase shift function is given by

$$\chi(b) = -\frac{1}{\hbar v} \int_{-\infty}^{\infty} V_N(\mathbf{r}) dz.$$
 (12)

Comparing Eq. (12) with Eq. (8), the optical potential $V_N(r)$ for nucleus-nucleus scattering in the momentum representation can be identified as

$$V_{N}(\mathbf{r}) = -\hbar v \ \bar{\sigma}_{NN}(\bar{\alpha}_{NN} + i) \frac{AB}{2(2\pi)^{3}}$$

$$\times \int e^{-i\mathbf{q}\cdot\mathbf{r}} S_{A}(\mathbf{q}) S_{B}(-\mathbf{q}) f_{NN}(\mathbf{q}) d^{3}q,$$

$$= -\hbar v \ \bar{\sigma}_{NN}(\bar{\alpha}_{NN} + i) \frac{AB}{(2\pi)^{2}}$$

$$\times \int j_{0}(qr) S_{A}(\mathbf{q}) S_{B}(-\mathbf{q}) f_{NN}(\mathbf{q}) q^{2} dq. \quad (13)$$

Here $j_0(qr)$ is the spherical Bessel function of zeroth order.

B. Trajectory modifications in the Glauber model

The basic assumption in the Glauber model is the description of the relative motion of the two nuclei in terms of straight line trajectory. For low energy heavy ion reactions, the straight line trajectory is assumed at the distance of closest approach r_c , calculated under the influence of the Coulomb potentials for each impact parameter *b* as given by

$$r_c = (\eta + \sqrt{\eta^2 + b^2 k^2})/k, \qquad (14)$$

TABLE I. Density parameters for various nuclei used in the present work.

A	Ζ	Density form	d/lpha (fm)	<i>R</i> (fm)
12	6	НО	1.082	1.692
16	8	НО	1.544	1.833
28	14	2pf	0.537	3.150
40	20	2pf	0.563	3.510
58	28	2pf	0.560	3.823
90	40	2pf	0.550	4.274
208	82	2pf	0.546	5.513

which is a solution of the following equation without the nuclear potential $V_N(r)$:

$$E - \frac{Z_P Z_T e^2}{r} - \frac{\hbar^2 k^2}{2\mu} \frac{b^2}{r^2} - \operatorname{Re} V_N(r) = 0.$$
(15)

Here $\eta = Z_P Z_T e^2/\hbar v$ is the dimensionless Sommerfeld parameter. The CMGM [7,8] consists of replacing the eikonal trajectory at an impact parameter *b*, with the eikonal trajectory at the corresponding distance r_c of closest approach. The distance of closest approach, r_{cn} , in the presence of both the Coulomb and nuclear potential [9], is obtained by solving Eq. (15) numerically, with $V_N(r)$ obtained from Eq. (13). By replacing *b* by r_{cn} , the CNMGM is obtained. The noneikonal trajectory [9] around r_c in the presence of the Coulomb field is represented by $r^2 = r_c^2 + (C+1)z^2$, where the quantity *C* is given by

$$C = \frac{\eta}{kb^2} r_c \,. \tag{16}$$

TABLE II. Reaction cross section data for ${\rm ^{12}C}$ on various targets along with their references.

System	$E_{\rm lab}/A$ (MeV/nucleon)	σ_R (mb)	Ref.
$^{12}C + ^{12}C$	9.33	1444.00	[21]
$V_C / M = 1.258$	15.00	1331.00	[22]
	25.00	1296.00	[22]
	30.00	1315.00	[23]
	35.00	1259.00	[22]
¹² C+ ⁴⁰ Ca	15.0	2165.0	[22]
$V_C / M = 2.186$	25.0	2030.0	[22]
	35.0	2014.0	[22]
$^{12}C + ^{90}Zr$	10.0	2219.0	[24]
$V_C / M = 3.214$	15.0	2297.0	[22]
	25.0	2415.0	[22]
	35.0	2528.0	[22]
$^{12}C + ^{208}Pb$	8.0	1754.0	[22]
$V_C / M = 5.068$	15.0	2873.0	[22]
	25.0	3236.0	[22]
	35.0	3561.0	[22]

TABLE III. Reaction cross section data for ¹⁶O on various targets along with their references.

System	$E_{\rm lab}/A$ (MeV/nucleon)	σ_R (mb)	Ref.
$^{16}O + ^{16}O$	2.0	73.86	[25]
$V_C / M = 1.524$	3.0	1136.00	[25]
-	4.0	1300.00	[25]
	5.0	1395.00	[25]
	9.0625	1650.00	[26]
	15.625	1664.00	[26]
	21.875	1639.00	[26]
	30.000	1655.00	[26]
¹⁶ O+ ²⁸ Si	2.0625	509.0	[27]
$V_C / M = 1.90$	2.3750	765.6	[27]
	3.1250	1341.0	[27]
	3.4375	1451.0	[27]
	4.125	1626.0	[27]
	5.0625	1777.0	[27]
	8.9000	2013.0	[27]
	13.45	2067.0	[27]
	94.00	1757.0	[10]

Thus in the CMGM, T(b) will be simply replaced by $T[r_c(b)]/\sqrt{(C+1)}$.

C. The nuclear densities

The two-parameter Fermi (2pf) density is given by

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-c}{d}\right)},\tag{17}$$

TABLE IV. Reaction cross section data for ¹⁶O on various targets along with their references.

System	$E_{\rm lab}/A$ (MeV/nucleon)	σ_R (mb)	Ref.
¹⁶ O+ ⁵⁸ Ni	2.50	48.2	[28]
$V_C / M = 2.683$	2.75	288.7	[28]
	3.00	539.4	[28]
	3.75	994.5	[28]
	4.375	1282.0	[28]
	5.00	1485.0	[28]
	6.25	1765.0	[28]
	7.50	1953.0	[28]
¹⁶ O+ ²⁰⁸ Pb	5.00	124.1	[29]
$V_C / M = 5.019$	5.50	566.6	[29]
	6.00	939.6	[29]
	6.50	1171.0	[30]
	8.093	2023.0	[24]
	12.00	2847.0	[24]
	19.54	3452.0	[31]
	94.00	3600.0	[32]



FIG. 1. Reaction cross section for ${}^{16}O + {}^{16}O$ system calculated with the CMGM with and without c.m. correction.

where $\rho_0 = 3/\{4\pi c^3[1 + (\pi^2 d^2/c^2)]\}$. Thus, the momentum density can be derived [16] as

$$S(q) = \frac{8\pi\rho_0}{q^3} \frac{ze^{-z}}{1 - e^{-2z}} \left(\sin x \frac{z(1 + e^{-2z})}{1 - e^{-2z}} - x\cos x\right),$$
(18)

where $z = \pi dq$ and x = cq. Here *d* is the diffuseness and *c*, the half value radius in terms of rms radius *R* for the 2pf distribution, is calculated by $c = [(5/3R)^2 - (7/3)\pi^2 d^2]$



FIG. 2. Reaction cross section for ${}^{16}O+{}^{16}O$ system calculated with the GM, the CMGM, and the CNMGM along with the data.



FIG. 3. Same as Fig. 2 but for ${}^{16}O + {}^{28}Si$ system.

 $-(5/3)r_p^2]^{1/2}$. Here r_p is the proton radius. Equation (9) can be evaluated numerically for this density and the overlap integral can be extracted. In the present work, the density parameters have been taken from the compilation of measured charge density distributions [17,18] and are tabulated in Table I.

For lighter nuclei such as ¹²C and ¹⁶O, the densities are given in the form of harmonic oscillator (HO) densities as

$$\rho(r) = \rho_0 \left(1 + \alpha \frac{r^2}{R^2} \right) \exp\left(-\frac{r^2}{R^2} \right), \quad (19)$$



FIG. 4. Same as Fig. 2 but for ${}^{16}O + {}^{58}Ni$ system.



FIG. 5. Same as Fig. 2 but for ${}^{16}\text{O} + {}^{208}\text{Pb}$ system.

where $\rho_0 = (\sqrt{\pi}R)^{-3}/[1+1.5\alpha]$. The momentum density is given by

$$S(q) = \rho_0 (\sqrt{\pi}R)^{-3} (1 + 1.5\alpha - 0.25\alpha q^2 R^2) \exp\left(-\frac{q^2 R^2}{4}\right).$$
(20)

Center of mass correction. For lighter nuclei such as ¹²C and ¹⁶O, we also take into account the corrections due to center of mass motion. Such a correction for harmonic oscillator wave functions is given in Ref. [19]. With this, the corrected density will become







FIG. 7. Same as Fig. 2 but for ${}^{12}C+{}^{40}Ca$ system.

$$S(q) = \rho_0 (1 + 1.5\alpha - 0.25\alpha q^2 R^2) \exp\left(-\frac{q^2 R^2}{4}\right) \exp\left(\frac{q^2 R^2}{4A}\right).$$
(21)

III. RESULTS AND DISCUSSIONS

The reaction cross sections for light and heavy systems from the Coulomb barrier to intermediate energies have been calculated. We calculate the total reaction cross section as a function of energy up to 50 times the Coulomb barrier. The experimental data for various systems and their references



FIG. 8. Same as Fig. 2 but for ${}^{12}C + {}^{90}Zr$ system.



FIG. 9. Same as Fig. 2 but for ${}^{12}C + {}^{208}Pb$ system.

are given in Tables II–IV. The Coulomb barrier is calculated from the expression $V_C = Z_P Z_T 1.44/[1.5(A^{1/3} + B^{1/3})]$. The reduced mass *M* is defined as M = AB/(A+B).

Figure 1 shows the reaction cross section for ${}^{16}O+{}^{16}O$ system calculated with the CMGM with and without center of mass (c.m.) correction. This correction reduces the reaction cross section for lighter systems. Figures 2-5 show the reaction cross section for ¹⁶O on various targets as a function of center of mass energy, divided by the Coulomb barrier, calculated with the Glauber model (GM), the Coulomb modified Glauber model (CMGM), and the Coulomb plus nuclear modified Glauber model (CMNGM) along with the data. Figures 5–9 show the reaction cross section for 12 C on various targets. The center of mass correction has been taken into account in all the calculations. It can be seen from all the figures that the most significant effect in this energy range is by the trajectory modification due to the Coulomb field. It is very significant up to energies $6V_C$. The modification in the trajectory due to the nuclear field tries to bring the results closer to the GM. We universally find that when the center of mass energy of the system becomes 30 times the Coulomb barrier, the calculations with the CNMGM match those of the GM within 2-3%. Thus, the energy at and above which the results of the CNMGM and the GM coincide depends on the Coulomb barrier between the two nuclei and not on the energy per nucleon of the projectile. Further, it is observed that there is an excellent agreement between the calculations from the modified Glauber model and the experimental data. In contrast to Refs. [13,14] the present study suggests that the heavy ion reactions in this energy range can be explained by the Glauber model in terms of free *NN* cross sections without incorporating medium modification.

There are higher order corrections [33,34] to the optical limit phase shift function $\chi(b)$. These corrections are important at smaller *b* but tend to become smaller at larger *b* [34] and are less significant for the total reaction cross section which depend mostly upon peripheral collisions. They may become important for differential cross sections away from the forward direction (which probe collisions at smaller *b*), increasingly at higher energies. The Gaussian densities [34] that produce only the surface shape of the nuclear density are not adequate and realistic densities are to be used (as done in Ref. [12]) to calculate these corrections.

IV. CONCLUSIONS

To summarize, we have reexamined the various trajectory corrections in the Glauber model and calculated the total reaction cross section as a function of energy up to 50 times the Coulomb barrier. The most significant effect in this energy range is by the trajectory modification due to the Coulomb field. The modification in the trajectory due to nuclear field is also taken into account using $V_N(r)$ calculated from Eq. (13). We quantify the energy range in which a particular correction is effective and find that when the center of mass energy of the system is about 30 times the Coulomb barrier, no trajectory modification to the GM is really required. Exact nuclear densities and free NN cross sections have been used in the calculation. The center of mass correction that is important for light systems has also been taken into account. There is an excellent agreement between our calculations including all the modifications discussed in the paper, and the experimental data. In contrast to Refs. [13,14], the present study suggests that the heavy ion reactions in this energy range can be explained by the Glauber model in terms of free NN cross sections without invoking any medium modification. It implies that the density effects on the NN cross sections are either absent or very minor. One possible explanation for this is the fact that for heavy ions, the contribution to the reaction cross section comes from the surface region where the density is very small.

ACKNOWLEDGMENTS

The author acknowledges the stimulating discussions with Z. Ahmed, S. Kailas, and S. V. S Sastry. He is also thankful to V. Jha for providing the reaction cross section data for some of the systems.

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