Quasielastic electron scattering at high energy from ^{12}C, ^{56}Fe, and ^{197}Au

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A relativistic mean-field single particle knockout model is compared to inclusive electron scattering (e,e') from a range of nuclei (^{12}C, ^{56}Fe, and ^{197}Au) at high electron energy, but in kinematics where the cross section is dominated by the quasielastic response. These experiments were done with incident electron energies of approximately 2 GeV at SLAC with four-momentum transfer squared of approximately 0.20–0.30 $(\mathrm{GeV}/c)^2$. We include the effects of electron Coulomb distortion in the calculation, and propose a simple way of including Coulomb distortion at high energies which can be used to analyze newer (e,e') experiments at Jefferson Lab. The effects of the predicted weakening of the strong scalar and vector potentials of the σ-ω model at high nucleon kinetic energy are included in our model.

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Medium and high energy electron scattering is one of the most useful tools to study nucleon properties inside nuclei, especially in the quasielastic region where the cross sections are dominated by the knocking out of individual nucleons. In this paper we concentrate on the inclusive reaction (e,e') that probes all of the nucleons in the nucleus and is not so sensitive to individual orbits and energy levels. That is, we are investigating a more global response of the nuclear system. There have been many experiments [1–8] on medium and heavy nuclei at incident electron energies less than 1 GeV, and a number of theoretical attempts [9–16] to fit the measured cross section and to separate the longitudinal and transverse structure functions. The Fermi gas model in the impulse approximation provides a rough description of the inclusive (e,e') cross sections but fails in describing the details. This is not too surprising since the Fermi gas model does not include the nuclear spatial geometry correctly. In some cases, there appeared to be large suppression (about 30–40 %) of the longitudinal structure functions [as compared to theoretical calculations of (e,e') using nonrelativistic wave functions and current operators] which implied missing strength in the Coulomb sum rule [1]. There was also disagreement between the transverse structure functions extracted from the experimental data and the predictions of the Fermi gas model, but this was expected since exchange currents, pion production, and other processes induced by transverse photons were not included in the model. Many attempts were made to explain this purported missing strength of the longitudinal structure function by improving the nuclear bound states, modifying the nucleon form factors in the nuclear medium, including final state interactions, and relativistic dynamics effects.

Two ingredients enter the comparison of experimental (e,e') data from medium and heavy nuclei to theory. One of these is the inclusion of electron Coulomb distortion effects and the second is the model used to calculate the nuclear transition current. In the early 1990s, Coulomb distortion for the reactions (e,e') and $(e,e'p)$ in the quasielastic region was treated exactly by the Ohio University group [7,9,17–19] using partial wave expansions of the electron wave functions. While such distorted wave Born approximation (DWBA) calculations permit the comparison of different nuclear models against measured cross sections and provide an invaluable check on various approximate techniques of including Coulomb distortion effects, they are numerically challenging and computation time increases rapidly with higher incident electron energies. Jourdan [20] used the Ohio group's calculation of Coulomb distortion effects to investigate the Coulomb sum rule. His conclusion was that the sum rule appeared to be obeyed and thus there was no "longitudinal suppression."

In order to avoid the difficulties associated with DWBA analyses at higher electron energies and to look for a way to still define structure functions, Kim and Wright [15,16,21,22] developed an approximate treatment of the Coulomb distortion. The essence of the approximation is to include Coulomb distortion in the four-potential arising from the electron current by letting the magnitude of the electron momentum include the effect of the static Coulomb potential. This leads to an r-dependent momentum transfer. This approximation allows the separation of the cross section into a "longitudinal" term and a "transverse" term, which is not formally possible in a full DWBA calculation. For medium and heavy nuclei at moderate incident electron energies, a good treatment of Coulomb distortion effects is necessary in order to extract the "longitudinal" and "transverse" structure functions.

The relativistic "single-particle" model requires bound state and continuum nucleon wave functions and a current operator. The bound nucleon wave functions are solutions to the Dirac equation in the presence of the strong scalar and vector potentials of the σ-ω model [23]. When investigating the exclusive reaction $(e,e'p)$, the continuum protons are taken to be the solutions to the relativistic optical model potentials of the Ohio State group [24]. Using these wave functions and the relativistic current operator, this model was extremely successful in describing $(e,e'p)$ reactions of

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many nuclei [17,19,21,22]. However, when investigating the inclusive reaction (e,e') where the knocked out nucleons are not observed, the Ohio University group used the same real potentials as used for the bound state nucleons. This ansatz is a way of accounting for all the nonmeasured processes that take place in the reaction and, in addition, guarantees current conservation. Calculations using this ''single-particle'' model coupled with the relativistic current operator and a good description of Coulomb distortion provided very good agreement with (e,e') Bates data on ^{40}Ca with no free parameters and found no evidence of longitudinal suppression [7].

This same model, coupled with our approximate treatment of Coulomb distortion, was compared to the Saclay quasielastic data on ^{208}Pb taken with both electrons and positrons. The DWBA and approximate calculations of Coulomb distortion by positrons and electrons were not consistent with the data quoted by Saclay, so it was not possible to extract a ''longitudinal'' structure function [15,16]. In addition, we investigated the approximation [12] used by the Saclay group for Coulomb corrections and did not find it a good approximation.

The absence or presence of ''longitudinal suppression'' has been argued vigorously at various conferences—partially because of different theoretical treatments, but also because of some experimental discrepancies among various laboratories. With our success in including Coulomb distortion effects in a ''plane-wave-like'' approach, we decided to analyze the available (e,e') data from nuclei at higher energies and momentum transfer for cases where the quasielastic process should dominate. Our original goal was to examine recent data from Jefferson Lab on a number of nuclei including ^{56}Fe [25], but we realized that the energy and momentum transfers are such that significant contributions from pion electroproduction are present. However, we did find some older (e,e') data from SLAC [26] on ^{12}C, ^{56}Fe, and ^{197}Au where the quasielastic contribution is kinematically isolated from pion electroproduction, which is referred to as an inelastic process. To our knowledge, no one has attempted to calculate the quasielastic contributions to these cross sections, probably because of the numerical difficulties in calculating the (e,e') process at such high energies. In this paper we will compare our single-particle relativistic mean-field model of quasielastic scattering to the SLAC experimental data. Should our model describe the quasielastic process well at this higher energy and momentum transfer region, it might be useful in scaling studies [25,26] of (e,e') at large Q^2 and in separating the quasielastic process (which does not scale) from inelastic contributions.

In the plane wave Born approximation, where electrons or positrons are described as Dirac plane waves, the cross section for the inclusive quasielastic (e,e') processes can be written as

$$\frac{d^2\sigma}{d\Omega d\omega}=\sigma_M\left\{\frac{q_\mu^4}{q^4}S_L(q,\omega)+\left[\tan^2\frac{\theta}{2}-\frac{q_\mu^2}{2q^2}\right]S_T(q,\omega)\right\}, \tag{1}$$

where $q_\mu^2=\omega^2-\mathbf{q}^2=-Q^2$ is the four-momentum transfer, σ_M is the Mott cross section, and S_L and S_T are the longitudinal and transverse structure functions that depend only on the three-momentum transfer q and the energy transfer ω. Explicitly, the structure functions for a nucleus are given in terms of the Fourier transform N^μ of the nuclear current by

$$S_L(q,\omega)=\sum_\alpha \frac{S_\alpha\rho_p}{s_p}\int |N_0|^2 d\Omega_p, \tag{2}$$

$$S_T(q,\omega)=\sum_\alpha \frac{S_\alpha\rho_p}{s_p}\int (|N_x|^2+|N_y|^2)d\Omega_p \tag{3}$$

with the outgoing nucleon density of states $\rho_p=pE_p/(2\pi)^2$. The $\hat{\mathbf{z}}$ axis is taken to be along the momentum transfer $\mathbf{q}$ and s_p is the z component of the angular momentum of the continuum state nucleons, while α labels the bound states of the nucleus and S_α contains occupation numbers and spin average factors for these states. The Fourier transform of the nuclear current is simply

$$N^\mu=\int J^\mu(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}d^3r, \tag{4}$$

where $J^\mu(\mathbf{r})$ denotes the nucleon transition current. The continuity equation has been used to eliminate the z component (N_z) via the equation $N_z=-(\omega/q)N_0$, which is valid if current is conserved. Note that if we do not have nuclear current conservation (i.e., a different Hamiltonian in the initial and final states), we need to calculate N_z directly.

The *ad hoc* expressions for the longitudinal and transverse structure functions with inclusion of the electron Coulomb distortion (see Ref. [16] for details) are similar to above, but the Fourier operators are modified by Coulomb distortion. We include the Coulomb distortion effects in our results, but unlike the medium energy cases, Coulomb effects on the cross section at these high electron energies are quite small.

The nucleon transition current in the relativistic single particle model is given by

$$J^\mu(\mathbf{r})=e\overline{\psi}_p(\mathbf{r})\hat{J}^\mu\psi_b(\mathbf{r}), \tag{5}$$

where $\hat{J}^\mu$ is a free nucleon current operator, and ψ_p and ψ_b are the wave functions of the knocked out nucleon and the bound state, respectively. For a free nucleon, the operator consists of the Dirac contribution and the contribution of an anomalous magnetic moment μ_T given by $\hat{J}^\mu=F_1(q_\mu^2)\gamma^\mu+F_2(q_\mu^2)(i\mu_T/2M)\sigma^{\mu\nu}q_\nu$. The form factors F_1 and F_2 are related to the electric and magnetic Sachs form factors G_E and G_M by $G_E=F_1+(\mu_T q_\mu^2/4M^2)F_2$ and $G_M=F_1+\mu_T F_2$, which are assumed to take the following standard form [27]:

$$G_E=\frac{1}{\left(1-\frac{q_\mu^2}{\Lambda^2}\right)^2}=\frac{G_M}{(\mu_T+1)}, \tag{6}$$

where the standard value for Λ^2 is 0.71 $(\mathrm{GeV}/c)^2$.

Our standard calculation at lower energies for the inclusive reaction (e,e') is to use a current conserving model where the bound and continuum nucleons move in the real scalar $S(r)$ and vector $V(r)$ potentials generated by the TIMORA code [23]. The constant $S(r)$ and $V(r)$ potentials fail as the momentum transfer increases, although they are good at low momentum transfers. Calculations using these potentials will be labeled "Const." However, the Ohio State group [24] found in their global fits to proton-nucleus scattering from a range of nuclei that the strengths of the real and imaginary parts of the scalar and vector potentials in the Dirac equation decreased as the proton energy increased. We choose to investigate this effect, also considered in an approximate way in Ref. [28], by using a parametrization of the real part of the S and V strengths as a function of proton kinetic energy that is consistent up to $T_p = 1.5$ GeV with the results of Cooper *et al.* [24]. In particular, we calculated (e,e') with $S(r)$ and $V(r)$ for protons and neutrons scaled by the functions $f_S = 0.97 - 0.66x + 0.28x^2$ and $f_V = 0.97 - 0.91x + 0.30x^2$, respectively, where x is the outgoing nucleon kinetic energy divided by the nucleon mass ($x = T_p/M$). The results using these weakened potentials for the outgoing nucleons are labeled "T_p-dep." Of course, as noted earlier, having different potentials for the bound and continuum nucleons results in current nonconservation. In our calculations we do not use current conservation to eliminate N_z, but rather work in the Lorentz gauge and calculate N_z directly. For the kinematics we examined, the kinetic energy of the outgoing nucleons is not very large, so the violation of current conservation is rather small and would only change the results by a few percent if we were to use $N_z = -(\omega/q)N_0$.

In reviewing the SLAC [26] datasets we found experiments on three nuclei (^{12}C, ^{56}Fe, and ^{197}Au) at kinematics so that the quasielastic peak is well separated from inelastic processes. The incident electron energy was $E_i = 2.02$ GeV, and the electron scattering angle was $\theta = 15°$. We choose to compare our calculations to these three data sets.

In Figs. 1–3 we compare four calculations to the experimental data. The calculations are separated into two classes labeled Dw and Pw. The Dw curves include electron Coulomb distortion and the Pw curves use Dirac plane waves for the electrons. As noted earlier, at these high electron energies, Coulomb distortion is quite small but not insignificant for a heavy nucleus like Au. The solid curves labeled "Dw. Tp-dep." use weakened scalar and vector potentials for the outgoing nucleons and at the peak of the quasielastic peak exceed the results using the bound state energy potentials by about 5–8 %. More significantly, on the high energy side of the quasielastic peak, the weakened scalar and vector potentials result in a much sharper dropoff of the quasielastic peak than the bound state potentials. This suggests that most of the cross section above the quasielastic peak is due to inelastic processes. Note further that we show the longitudinal and transverse contributions predicted by our model for each cross section curve, and that while the transverse portion of the cross section is larger than the longitudinal portion, the longitudinal contribution is quite significant, being approximately 45% of the total.

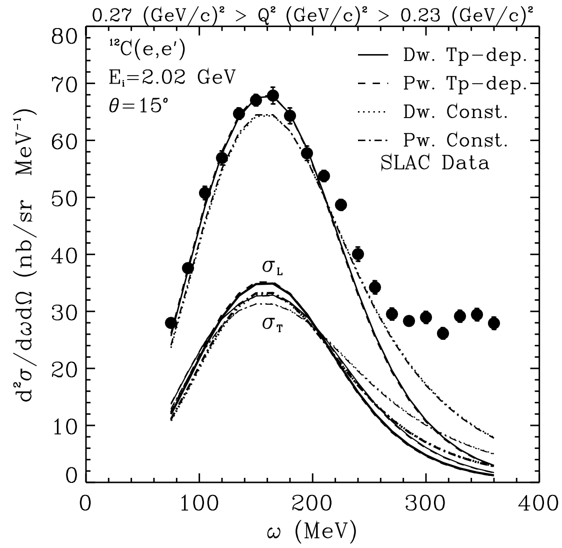

FIG. 1. Theoretical quasielastic scattering cross sections for ^{12}C with incident electron energy of $E_i = 2.02$ GeV and electron scattering angle $\theta = 15°$ as a function of energy transfer compared to experimental data from SLAC. The cross sections labeled σ_T and σ_L add up to the various total cross sections. See text for details of the models.

The calculations with Coulomb distortion (relevant only to Au) and the energy dependent scalar and vector potentials agree very well with the quasielastic peak data for ^{12}C, ^{56}Fe, and ^{197}Au. Clearly, our "single-particle" relativistic model provides an excellent description of the quasielastic peak over a range of nuclei from carbon to gold. A number of experiments have investigated scaling of the inelastic contributions to the inclusive cross sections' high large momentum

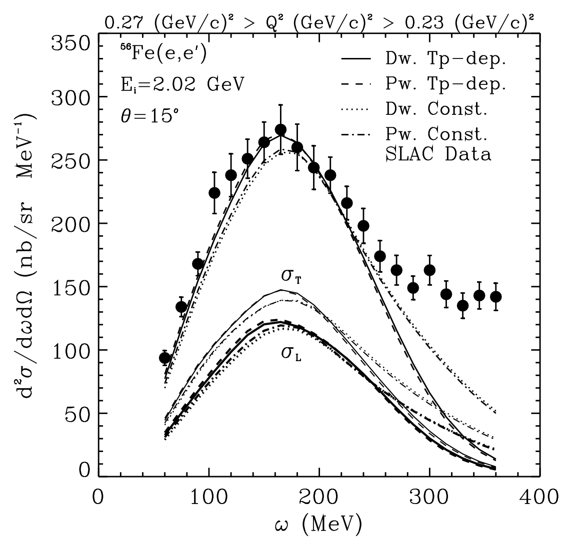

FIG. 2. Theoretical quasielastic scattering cross sections for ^{56}Fe with incident electron energy of $E_i = 2.02$ GeV and electron scattering angle $\theta = 15°$ as a function of energy transfer compared to experimental data from SLAC. The cross sections labeled σ_T and σ_L add up to the various total cross sections. See text for details of the models.

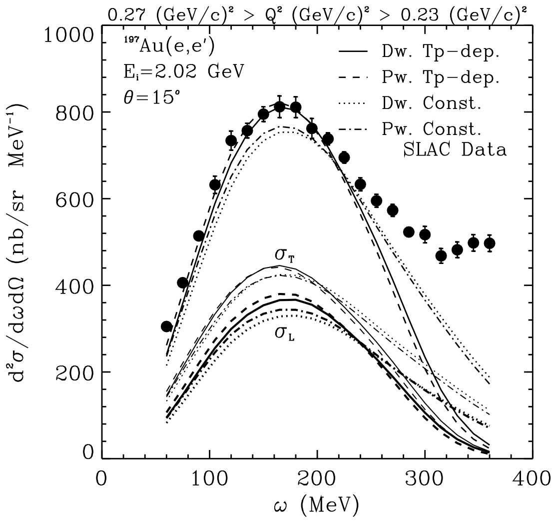

FIG. 3. Theoretical quasielastic scattering cross sections for ^{179}Au with incident electron energy of $E_i = 2.02$ GeV and electron scattering angle $\theta = 15°$ as a function of energy transfer compared to experimental data from SLAC. The cross sections labeled σ_T and σ_L add up to the various total cross sections. See text for details of the models.

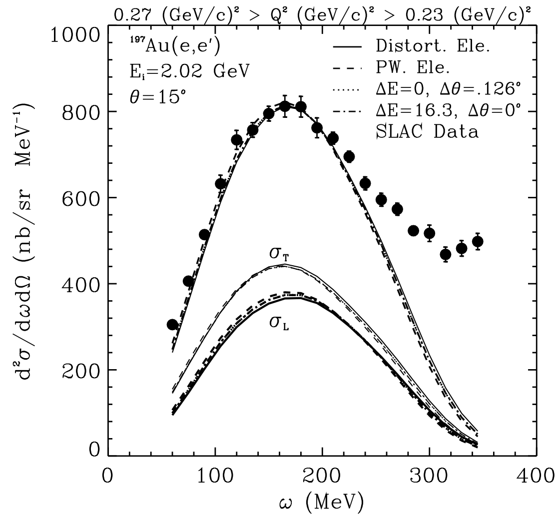

FIG. 4. The quasielastic cross section and longitudinal and transverse contributions for ^{179}Au with incident electron energy of $E_i = 2.02$ GeV and electron scattering angle $\theta = 15°$ are calculated with our treatment of Coulomb effects and compared to a simple energy or angle shift in the structure functions as discussed in the text.

transfer, but the quasielastic response does not obey simple scaling laws. So our excellent description of the quasielastic response which contains no free parameters suggests that one could use this model to construct pseudodata by subtracting out the quasielastic peak contributions at kinematics where the inelastic contributions are much larger, e.g., at forward electron angles at higher electron energies as measured at JLAB [25]. This would allow one to examine the approach to scaling of the inelastic contributions more cleanly.

Unfortunately, the SLAC experiments did not have a sufficient range of kinematic data to extract the longitudinal and transverse cross sections using a Rosenbluth separation, so we cannot separately compare our calculations to the experimentally extracted results. However, as noted above, the longitudinal and transverse contributions at these kinematics are comparable in our model. It would appear that if something were to suppress the longitudinal response in this Q^2 range $[0.23\text{–}0.29\ (\mathrm{GeV}/c)^2]$, there would need to be an approximately equal enhancement of the transverse response.

As noted earlier, at these high electron energies, Coulomb distortion effects are quite small. It is tempting to look for a simple shift of energy or angle that would incorporate the Coulomb distortion effects. We know that this does not work for the case of large distortion effects, but it might work at these energies. Since the most important element of our approximate treatment of Coulomb distortion is the radial dependence of the three momentum transfer since this enters into the Fourier transform of the current, one could try to fix the three-momentum transfer at a value corresponding to the nuclear surface and examine the consequences. The three-momentum transfer squared is given by $\mathbf{q}^2 = \omega^2 + 4E_iE_f\sin^2(\theta_e/2)$, where we have neglected the electron mass. The Coulomb potential energy for an electron at $r = R$ for a uniformly charged sphere of radius R with net charge Ze is given by $E_c = \alpha Z/R$, and the effective energy for electrons near the nucleus is increased by this amount. For convenience, we use the simple formula $R = 1.2A^{1/3}$ fm to calculate the Coulomb energy. Using this, one can do a plane wave calculation of the structure functions with energies $E' = E + E_c$ in the structure functions. Of course, one could also shift the scattering angle and not change the effective energies in the expression for the three momentum squared. Since ω is not affected by the shift in the initial and final electron energies, for small Coulomb effects $\Delta\theta_e \cong 2(\sqrt{E_i'E_f'/E_iE_f} - 1)\tan(\theta_e/2)$. In Fig. 4 we show the quasielastic peak for ^{197}Au calculated with the Coulomb distorted electron waves, electron plane waves, and plane-wave calculations with the energy shifted or the angle shifted.

Clearly, either of these shifts (but not both at the same time) result in a "plane-wave" result that approximates quite well the Coulomb distorted result, although there are small discrepancies in the wings of the quasielastic peak and the longitudinal and transverse responses are not equally well described. However, at these energies the percentage of these effects is small and should be even smaller at higher energies. Note that we have made these shifts in calculating the Fourier transforms of the transition current components and have not modified the Mott cross section in Eq. (1). Thus, to apply either of these shifts to experimental data, the extracted cross sections should be divided by σ_M before the shift is applied.

In conclusion, we find that our "single-particle" relativistic model where the bound and continuum nucleons move in real scalar and vector potentials of the σ-ω model provides an excellent description of the (e,e') data in the quasielastic peak from light to heavy nuclei (^{12}C, ^{56}Fe, and ^{197}Au). We suggest that our model can be used to subtract out the quasielastic contributions in (e,e') reactions at higher momentum transfers in order to investigate scaling of the inelastic response. In addition, we have confirmed that a rather simple energy or scattering angle shift can be used at these energies for (e,e') reactions to correct for Coulomb effects.

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