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Measuring neutron separation energies far from stability

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A method is proposed for the experimental measurement of neutron separation energies for nuclei far from stability. The procedure is based on determining cross sections for the production of nuclei, by projectile fragmentation, for which only protons are removed but for which the number of neutrons is left unchanged. A simple abrasion-ablation analysis leads to a cross section prediction that is sensitive to the neutron separation energy after a single parameter is adjusted in comparison with data. Examples that illustrate the method are presented.

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Few experimentally measured values of neutron separation energies have been determined for nuclei near the neutron drip line [1–4]. However, it is precisely in this region that such information is particularly necessary for investigating nucleosynthesis, and for testing models of nuclear structure at the greatest distance from the valley of stability [5–7]. As the values of N and Z move away from the valley of stability on the neutron-rich side, the neutron separation energy is systematically reduced [5–7], due primarily to the asymmetry in the proton-neutron composition. This asymmetry is reflected by the value of the parameter δ, which is defined as $(N-Z)/(N+Z)$. The separation energies for nuclei with large asymmetry are often extrapolated from information available close to the valley of stability [8]. This approach may be uncertain in describing how their values go to zero with increasing δ. However, this region tests the role of the symmetry energy most stringently in theoretical models, and hence provides information about the symmetry energy in the more general context of the nuclear equation of state [9]. Thus, the systematic behavior of the separation energy with increasing values of δ offers direct evidence for such effects.

One of the most successful methods for the production of neutron-rich rare nuclides has been the fast projectile fragmentation process [10,11]. This method has been used to extend the list of particle stable nuclei to the extremes [12,13]. In most of the cases where the values of the neutron separation energy have been measured, the values are in excess of 5 MeV. In the case of nuclides very close to the drip line the pairing energy may cause two-neutron separation energies to be less than the one-neutron separation energies. For those cases the lowest separation energy, referred to as S in this Rapid Communication, be it for one or two neutrons, is the one of interest. In this paper we suggest that, under certain circumstances, the same measurement that provides the verification for the existence of a rare nucleus may also be used to estimate S.

One avenue to this information lies in the recent suggestion that ‘‘cold’’ fast fragmentation [10,11] seems an efficient method of producing these extremely rare nuclei. The simple scenario for this process follows the concepts of the abrasion-ablation (AA) models [10,11,14]. From that point of view, a direct reaction (abrasion) removes a number of nucleons, leaving the residue excited, and free to lose more nucleons by evaporation (ablation).

To minimize the uncertainties associated with the abrasion and ablation processes, we focus on the production of fragments where only protons and no neutrons are removed from the projectile, i.e., the abrasion process removes the protons, and leaves the residue with too little excitation energy to permit further loss of particles (neutrons). Such a production mechanism is referred to as a ‘‘p-removal chain’’ in this Rapid Communication. If the residue is neutron rich, it decays by neutron emission. Thus, the upper limit on the excitation energy is the neutron separation energy S. It is this feature that provides the production cross section with the sensitivity to the separation energy. By limiting our attention to the nuclei produced in p-removal chains, we avoid the ambiguities related to specific evaporation models.

Using the framework of the AA model [11], the cross section to produce a nucleus with $(Z-x)$ protons and N neutrons from the fragmentation of a projectile with Z proton and N neutrons can be written as $\sigma_x = \mathrm{Abr}_x \times \mathrm{Abl}_x$. Here, the factor Abr_x gives the cross section for removing x protons (and no neutrons) by abrasion, and Abl_x is the dimensionless probability that the residue will not further decay following the removal of those x protons.

The factor Abr_x can be estimated by the geometric overlap [15] of projectile with the target. This can provide the cross section for the removal of x particles [10,14]. This cross section must then be multiplied by the probability that all the abraded particles are protons. By assuming that the positions of neutrons and protons in the projectile are uncorrelated, the probability that all x of the abraded particles are protons can be written as $[Z!/(Z-x)!]/[(N+Z)!/(N+Z-x)!]$. Both of these assumptions (geometric overlap and uncorrelated positions) are simplistic but their validity can be calibrated by comparison with a measured set of cross sections for projectile fragmentation where the separation energies of the resulting nuclei are known.

The crucial factor is Abl_x, which depends on both the distribution of excitation energy following the removal of x protons, $F_x(E^*)$, and the separation energy S_x for the nucleus produced by this removal. Specifically, Abl_x is the integral of the excitation function from zero to the separation energy.

Clearly, the form of the excitation function is a critical component of this procedure. The literature of AA models suggests that this distribution function may be quite uncertain [10,14]. Recent approaches [10,14] suggest that the distribution function $F_x(E^*)$ is a convolution of x distribution functions, $f_1(e^*)$, where $f_1(e^*)$ is the function for the removal of a single nucleon,

$$F_x(E^*)=\int\prod_{i=1}^{x}\,[de_i^* f_1(e_i^*)]\delta\left(\sum_{i=1}^{x}e_i^*-E^*\right). \qquad (1)$$

The functional form of $f_1(e^*)$ is, however, not well determined, and there is great uncertainty as to the mean value of the excitation energy $\langle e^*\rangle$ it provides. Some of this uncertainty can be removed by fitting calculated cross sections to sets of measured cross sections. The fitting would be accomplished by the adjustment of $\langle e^*\rangle$.

One form of the suggested single-particle excitation distribution, $f_1(e^*)$, which is widely used [14], is the "triangle" distribution. This has the form $f_1(e^*)=2/E_m(1-e^*/E_m)$ for $e^*<E_m$ and the average excitation energy $\langle e^*\rangle$ is $E_m/3$. A wide range of values of E_m have been suggested in different models [10,14]. Convolution of this (triangle) single-particle distribution leads to a value for Abl_x which is approximately $[2S_x/(3\langle e^*\rangle)]^x/x!$ for $S_x\ll 3\langle e^*\rangle$. Exact values for Abl_x can be obtained with

$$\mathrm{Abl}_x=C_{tri}(x)[2S_x/(3\langle e^*\rangle)]^x/x!, \qquad (2)$$

where

$$C_{tri}(x)=\sum_{s=0}^{x-s}\,[-S_x/(3\langle e^*\rangle)]^s\{x!^2/[s!(x+s)!(x-s)!]\}. \qquad (3)$$

The value of $C_{tri}(x)$ goes to 1.0 for small values of $[S_x/(3\langle e^*\rangle)]$, and Abl_x is seen to be a function of the parameter $[2S_x/(3\langle e^*\rangle)]$.

We have also considered a different form for the single-particle excitation function, i.e., the exponential function $f_1(e^*)=1/\langle e^*\rangle\exp(-e^*/\langle e^*\rangle)$ with an average excitation energy $\langle e^*\rangle$. A convolution of this function provides an x-particle distribution function of the form

$$F_x(E^*)=(E^*/\langle e^*\rangle)^{x-1}/(x-1)!\exp(-E^*/\langle e^*\rangle). \qquad (4)$$

When this function is integrated from zero to the separation energy S_x, one obtains a value for Abl_x which is approximately $(S_x/\langle e^*\rangle)^x/x!$, and an exact expression can be calculated as a function of $(S_x/\langle e^*\rangle)$,

$$\mathrm{Abl}_x=C_{exp}(x)(S_x/\langle e^*\rangle)^x/x!, \qquad (5)$$

with

$$C_{exp}(x)=\sum_{s=0}\,(-S_x/\langle e^*\rangle)^s\{x/[(x+s)s!]\}. \qquad (6)$$

The value of $C_{exp}(x)$ goes to 1.0 for small values of $(S_x/\langle e^*\rangle)$.

For small values of S_x, the functional form of the Abl_x (as a function of different parameters) is the same for the two single-particle distribution functions. For the "triangle" distribution the parameter is $[2S_x/(3\langle e^*\rangle)]$, while for the exponential distribution the parameter is $(S_x/\langle e^*\rangle)$. Hence one might expect that a choice of $\langle e^*\rangle$ in the "triangle" distribution which is 2/3 the choice of$\langle e^*\rangle$ in the exponential distribution would predict similar cross sections. When the full x-particle excitation distributions are used, however, this scaling is only approximate.

For the nuclei in a given p-removal chain, estimates of unknown values of S_x can be extracted from measured values of cross sections, σ_x, after a value of $\langle e^*\rangle$ has been adjusted to fit the data for nuclei with known separation energies.

In illustrative examples below we have found that we can, generally, well represent the data by first calculating Abr_x using simple assumptions, and then adjusting the parameter $\langle e^*\rangle$ to give Abl_x. The assumptions for Abr_x include both the estimation of the removal cross section from the geometric overlap of target and projectile, and also the calculation of probability for obtaining pure proton removal by assuming uncorrelated positions for the nucleons. Any systematic correction required in Abr_x may possibly be accommodated by adjusting $\langle e^*\rangle$ in Abl_x. The ambiguity in the choice of the single-particle distribution function f_1 prevents a unique determination of the mean excitation energy through the fitting. However, we do find interesting systematic changes in the required values of $\langle e^*\rangle$, which appear to depend on the mass and isospin values of the fragmenting projectile. These features will be explored more fully in the future when more systematic data are available.

We have considered, for illustration, the p-removal chains given in Ref. [10] for ^{208}Pb, ^{197}Au, and ^{136}Xe. We have also examined data in the literature for ^{86}Kr [16] and ^{48}Ca [12], and, in addition, preliminary results for ^{58}Ni, which is under current investigation [17]. We first tested the form for the cross section suggested in the expressions of Eqs. (2) and (5), by fitting to data for the fragmentation production of nuclei where the separation energies are known. Both the "triangle" and the exponential forms for the excitation functions were used, and fits to the data were achieved by adjustment of the respective values of $\langle e^*\rangle$.

In Fig. 1, we plot the fitted cross sections for the fragmentation of ^{86}Kr. The separation energies [4] are known for the first five members the p-removal chain (the data only cover 2–5). There are three degrees of freedom for this fit, providing a χ^2 per degree of freedom of 0.99 for the exponential distribution. The respective fitting values of $\langle e^*\rangle$ are 11.7 MeV for the "triangle" distribution and 16.6 MeV for the exponential distribution.

In Table I we list the values of the χ^2/degree of freedom for fits to other datasets including only nuclei where the separation energies are known [4]. Excellent fits can be achieved. We have also listed values of the fitting parameters $\langle e^*\rangle$ with estimates of deviations (both plus and minus) for 70% confidence. There are variations in the values of these fitting parameters from one reaction to another. In each case an approximate ratio of approximately 2:3 is found for the values related to the "triangle" and the exponential $f_1(e^*)$ distributions with practically no difference in the resulting

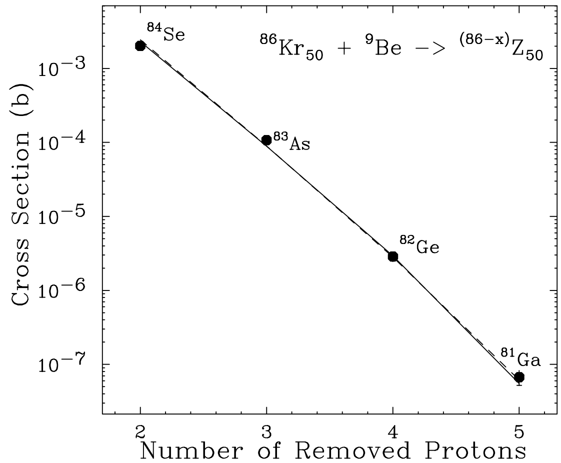

FIG. 1. Measured cross sections (solid circles) for the production of p-removal nuclides with $N=50$, ^{86-x}Z from the fragmentation of ^{86}Kr with a target of ^{9}Be [16]. Lines are predictions described in text using two different excitation distributions with the adjustment of a single parameter, given in Table I.

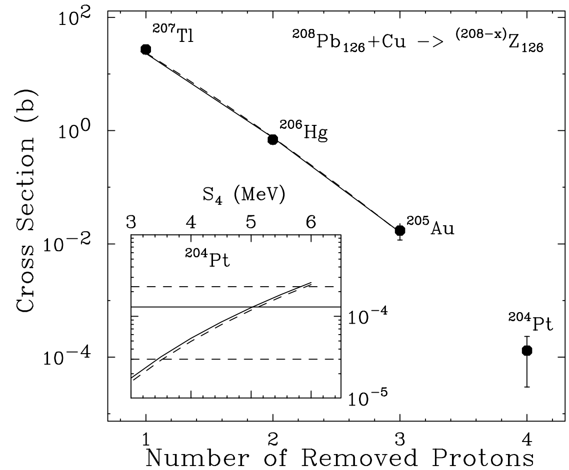

FIG. 2. Measured cross sections for the production of p-removal nuclides with $N=126$, $^{208-x}Z$ from the fragmentation of ^{208}Pb with a target of ^{63}Cu [19]. Lines are predictions described in text using two different excitation distributions with the adjustment of a single parameter, given in Table I. The inset shows the predicted cross sections as a function of the separation energy for ^{204}Pt nuclei. The horizontal solid and dashed lines are measured cross sections.

fits from the two distributions. Except for the Xe fragmentation, the data and calculated values, based on the excitation energies that provide the best fit [18], are plotted in Figs. 1 to 4. Preliminary results show similar behavior for the p-removal chain of ^{58}Ni for which the chain has been measured through eight p removals [17].

While the current data are quite limited, we also examined the predictive power of the method for nuclei where the separation energy is unknown, even lacking estimation by extrapolation. For this we looked at ^{204}Pt which was measured in the chain from ^{208}Pb projectile [19]. In Fig. 2 we show the best fit to the data for the first three nuclei in the chain, where separation energies are known or estimated [4]. In the insetted graph we show the sensitivity to the assumed binding energies of ^{204}Pt using the values of $\langle e^* \rangle$ which best fit the first three members of the chain given in Table I. The apparent estimate for the separation energy is about 5 MeV with large uncertainty due to the experimental uncertainty in the measured cross section for ^{204}Pt. This value is well in line with systematic decrease of the separation energy with increased $(N-Z)$.

TABLE I. Values of $\langle e^* \rangle$ are obtained by fitting the measured cross sections of nuclides with know separation energies. The columns labeled + and − indicate the deviations of the the best fit values with 70% confidence. The second and sixth columns reflect the goodness of the fit for the "triangle" (tri.) and exponential (exp.) distributions. "dof" denotes degree of freedom.

Reaction	χ^2/dof tri.	$\langle e^* \rangle$ tri.	−	+	χ^2/dof exp.	$\langle e^* \rangle$ exp.	−	+
^{208}Pb+Cu [19]	0.38	18.4	1.1	1.5	0.42	26.6	1.8	2.1
^{197}Au+^{27}Al [22]	0.87	22.4	1.6	3.6	0.88	32.2	3.8	5.2
^{197}Au+^{9}Be [10]	1.87	25.0	1.4	1.8	1.58	36.3	2.2	2.6
^{136}Xe+^{9}Be [22]	0.36	23.8	2.8	2.6	0.36	34.2	3.8	5.6
^{86}Kr+^{9}Be [16]	1.45	11.7	0.3	0.25	0.99	16.6	0.4	0.45
^{48}Ca+^{9}Be [20]	1.24	7.70	0.35	0.4	1.81	10.80	0.45	0.60

We have also attempted to estimate the separation energy of ^{41}Al, which has recently been observed [12] for the first time. Even though the fragmentation of neutron-rich ^{48}Ca [12,13,20] has been studied intensely in the past few years, there is no systematic measurement of cross sections for the p-removal chain. The separation energies are not known for the nuclei with more than four protons removed. Some of the extrapolated values have uncertainties of much more than 1 MeV [4,13]. Thus, in principle, ^{48}Ca would be a good candidate for the full examination of our method. We have examined the existing data to make a rough estimate. We found that the fragmentation cross sections have been measured from previous studies [20] for two nuclei, ^{45}Cl and ^{44}Si, which have, respectively, three and four protons removed from ^{48}Ca. This experiment used a target of ^{9}Be. Using the separation energies 6.241 MeV and 5.21 MeV [4,13], respectively, for ^{45}Cl and ^{44}Si the fit parameters listed in Table I are obtained. We next examined the results from the experiment [12] which first observed the ^{41}Al nucleus corresponding to the removal of seven protons from the projectile. Unfortunately, these data were obtained with a ^{181}Ta target. To connect this point with the other two points (obtained with a Be target), we used abrasion calculations which suggest that the difference in targets provides a cross section from Be which is 0.545 times the value obtained with a Ta target. The reduction arises primarily from the difference in the respective sizes of the impact parameters for the two targets. We plot in Fig. 3 a point for $x=7$ (^{41}Al) at a cross section of 4.4 pb, which is the value of 8 pb [12,21], reported for the Ta target, scaled down by the estimated ratio of cross sections. (We have also scaled down the error bar.) The inset in Fig. 3 shows the dependence of the calculated cross section for $x=7$ as a function of the separation energy. The experimental uncertainty is high since only three events were observed [12,21]. Even so, extraction of a value of 3.5 ± 0.5 MeV

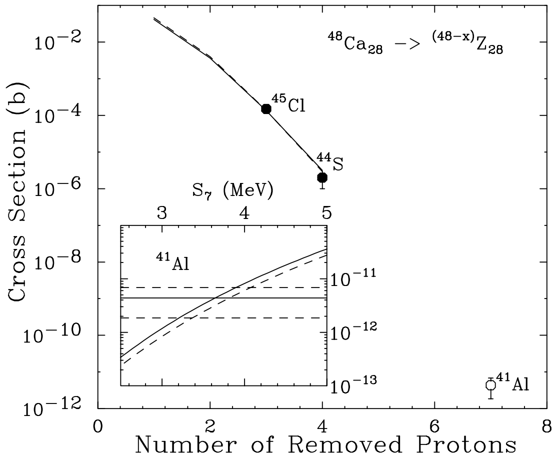

FIG. 3. Measured cross sections for the production of *p*-removal nuclides with $N=28$, ^{48-x}Z from the fragmentation of ^{48}Ca with a ^{9}Be target [20] (solid points). Lines are predictions for $x=1,2,3,4$ described in text using two different excitation distributions with the adjustment of a single parameter, given in Table I. The open point for $x=7$ (^{41}Al) is obtained from a separate experiment with ^{181}Ta target [12] and adjusted as described in the text. The inset shows the predicted cross sections as a function of the separation energy for ^{41}Al nuclei. The horizontal solid and dashed lines are measured cross sections.

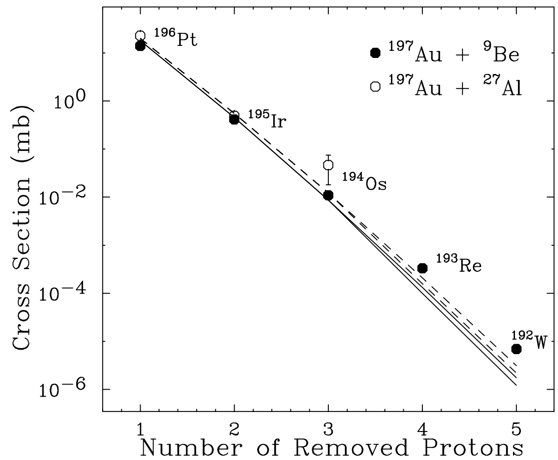

FIG. 4. Measured cross sections for the production of *p*-removal nuclides with $N=118$, $^{197-x}Z$ from a projectile of ^{197}Au with a ^{9}Be target [10] (solid points). The open circle are data [22] for ^{197}Au $+^{27}$Al scaled down by 0.95. Lines are predictions described in the text.

would be consistent with the information in the inserted graph. We cannot claim that this value is, indeed, the separation energy of ^{41}Al due to the fact that two different targets were used, and there is a scarcity of information in the *p*-removal chain. [For example, there are no measured cross sections and no accurate separation energies for for $x=5$ and 6 (^{43}P and ^{42}Si) isotopes.] However, this exercise shows the potential for extracting the separation energies for ^{44}S, ^{43}P, ^{42}Si, as well as ^{41}Al, the nuclei with four, five, six, and seven protons removed from ^{48}Ca. This might be accomplished with careful measurements of the complete *p*-removal chain from $x=1$ to 7 with one target and one beam energy. Specifically, additional data in the $x=1$–3 region where the S_x values are known by observation will provide greater constraints on the values of $\langle e^* \rangle$.

Finally, we have examined the situation for the fragmentation of ^{197}Au where a chain of five proton removal is reported in Ref. [10] for a target of ^{9}Be (solid points in Fig. 4). For the fragmentation of ^{197}Au using ^{27}Al target nuclei, there are *p*-removal cross sections up to $x=3$ [10,22]. For the case of ^{197}Au projectiles with ^{27}Al and ^{9}Be targets, abrasion estimates suggest a 5% reduction in going from the larger to the smaller target [23]. The three open points in Fig. 4 are the ^{27}Al data scaled down by 5%. They are consistently higher than the corresponding ^{9}Be data (solid points). If we apply the fitting procedures to this set of data, using the values of $\langle e^* \rangle$ listed in Table I, we obtain a separation energy greater than 7 MeV for both ^{193}Re and ^{192}W nuclei. These values are clearly inconsistent with systematic trends and expectations, both of which would have led to values below 7.0 Mev. For comparisons, the solid and dashed lines are calculations using the best fit $\langle e^* \rangle$ listed in Table I for ^{9}Be (36.3 MeV) and for ^{27}Al (32.2 MeV) targets, with the assumption of an exponential energy distribution. The upper and lower curves in each pair of lines use the separation energies of 7.0 and 6.5 MeV, respectively, as the separation energies for both ^{193}Re and ^{192}W nuclei. The calculated cross sections are lower than the experimental values. In brief, the reported cross sections for the ^{197}Au$+^{9}$Be reaction do not lead to reasonable separation energies for the last two members of the proton chain. The reasons for this failure are not clear at this time.

In summary, the illustrated calculations show that excellent agreement with fragment cross sections can be obtained when simple estimates are used with the abrasion-ablation model. One assumption made in these estimates is that, for chains involving a given projectile and target, the single-particle removal energy distributions can all be characterized by the same parameter $\langle e^* \rangle$, which then can be adjusted for each reaction chain when separation energies are known. The quality of agreement is equally good for both the ‘‘triangle’’ and the exponential single-particle excitation distribution functions. The two distributions require values of the mean energy, which are approximately in the ratio of 2:3. The smaller the parameter, the slower the fall of cross section with the number of protons removed. The quality of the fit inspires confidence in the use of the AA model for calculating the *p*-removal chains. Once the parameter is determined for each chain the only remaining input is the set of separation energies. From some of the data in the literature we were able to suggest the power of the *p*-removal method for observing unknown separation energies in ^{204}Pt and ^{41}Al. A puzzling disagreement was found for the unknown separation energies of ^{193}Re and ^{192}W in the chain reported for the fragmentation of ^{197}Au when the target was ^{9}Be. Clearly, more data and more understanding of the uncertainties in the cross-section measurements are needed to confirm the utility of the method. Since the procedure may be generally applied to all *p*-removal chains, it opens an avenue for measuring separation energies for neutron rich nuclei near the drip line as illustrated by the fragmentation of extremely neutron-rich projectiles such as ^{48}Ca. While the method cannot compete with dedicated mass measurements where masses can be measured to uncertainties better than 10^{-7} [5–7], the simplicity of cross section measurements with fragment separators may allow the wide use of this method to measure the separation energies for extremely neutron rich nuclei to an uncertainty of a few hundred keV as this energy decreases toward zero at the drip line.

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