

Isomer bands, $E0$ transitions, and mixing due to shape coexistence in $^{188}_{82}\text{Pb}_{106}$

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Structures above the two-particle isomeric states in ^{188}Pb have been identified. Their properties imply the presence of three minima in the nuclear potential well, associated with spherical, oblate, and prolate deformations. The fragmented decay of the $K^\pi=8^-, 1 \mu\text{s}$ prolate isomer leads to the identification of the yrare collective band presumably based on the excited 0^+ state. Analysis of the excitation energies, branching ratios, and $E2$ and $E0$ decay widths through a three-band-mixing model provides evidence for shape differences and the hybridization of the wave functions at low spin.

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Nuclei have the ability to exhibit distinct coexisting shapes, a phenomenon controlled by inhomogeneities in the single-particle level densities near the Fermi surface. Prediction of such multiple shapes is a sensitive test of nuclear models and the nuclear potentials in common use, and thus bears on the general validity of mean-field theories and methods for estimating the stability of heavy nuclei.

The observation of shape coexistence in the neutron-deficient (i.e., $N \leq 126$) mercury isotopes was at first a surprise, since these nuclei are only two protons away from the magic number at $Z=82$. The manifestations included unexpected values of charge radii and magnetic moments in the odd-neutron isotopes [1,2], and anomalies in the spacing of the yrast (lowest energy state for a given spin) sequences in the even-even isotopes at low spin, attributed to the crossing of collective bands arising from each subminimum. Studies continue into the nature of the phenomenon, taken to be indicative of a secondary prolate-deformed minimum in the potential well, lying only a few hundred keV above the oblate mercury ground states [3,4].

In microscopic terms, these so-called “intruder” configurations are related to the occupation of orbitals that lie above the $Z=82$ closed shell at sphericity, but which can be occupied at a low cost in energy at large prolate or oblate deformations. A more complicated and spectroscopically rich situation will occur at $Z=82$ where similar shapes are expected, but with the intrusion of a deep spherical minimum. Spherical configurations naturally dominate the ground and excited state structures in lead nuclei near the $N=126$ closed neutron shell, but if a substantial number of neutrons are removed, *three* potential minima (spherical, oblate, and prolate) are calculated to occur, all at low energies. The early Nilsson-Strutinsky calculations of potential energy surfaces (PES) by May, Pashkevich, and Frauendorf [5], and more recently those of Bengtsson and Nazarewicz [6,7] predicted that the prolate minimum would have a parabolic energy dependence on neutron number, becoming lowest near midshell, $N \sim 104$ (^{186}Pb).

However, because of the complexity of the energy surface, isolation of the different minima in PES and other calculations is not straightforward, particularly for the 0^+ states whose orthogonality is difficult to establish [8]. Clearly,

these nuclei present a theoretical challenge. While the recent systematics [9–11] suggest that excited states from each deformation might compete near $N=106, 108$, the nuclei are also a challenge experimentally being very neutron deficient, with key excited states that will be nonyrast and therefore difficult to populate.

Nevertheless, evidence for a triple shape coexistence has been claimed [12] for $^{186}\text{Pb}_{104}$ in the form of three 0^+ states observed at low energy, while in $^{188}\text{Pb}_{106}$ the presence of $8^-, 11^-$, and 12^+ isomeric two-quasiparticle states has been proposed as a different manifestation of the same phenomenon [10]. In an ideal situation, one would hope to characterize the minima through the properties of the bands associated with such intrinsic states. The isomers in ^{188}Pb were associated [10] with a 12^+ state from the $i_{13/2}^{-2}$ neutron-hole configuration characteristic of *sphericity*; a $K^\pi=11^-$ intrinsic state from the excitation of two protons into the $\Omega=j$ Nilsson orbitals, $9/2^-$ [505] and $13/2^+$ [606], which are near the Fermi surface at *oblate* deformation; and a two-quasineutron state with $K^\pi=8^-$, from the $9/2^+$ [624] $\otimes 7/2^-$ [514] configuration that is the lowest two-quasiparticle high- K isomer in *prolate* nuclei with $N=106$.

The first aim of the new experiments was to identify the structures above each of these (nonyrast) intrinsic excitations by exploiting time correlations and multiple gating to gain sensitivity. The second aim was the study of low-lying, low-spin states and possible connections to the proposed excited 0^+ states. The previous work [10] did not fully resolve the decay of the 8^- isomer itself, but showed that its decay path proceeded through low spin states not otherwise accessible.

Gammasphere with 101 Compton suppressed detectors was used to measure three-fold and higher-fold γ - γ time coincidences. The nuclei were produced in the $^{156}\text{Gd}(^{36}\text{Ar},4n)^{188}\text{Pb}$ reaction with 174-MeV beams provided by the 88-Inch Cyclotron operated by the Lawrence Berkeley National Laboratory. The nuclei were stopped in a gold backing at the target position, and particular attention was paid to the measurement and analysis of the time information. The ± 700 ns acceptance allowed correlation across the isomers whose mean lives range from 38 ns to 1.2 μs . A variety of time conditions relating both the time difference

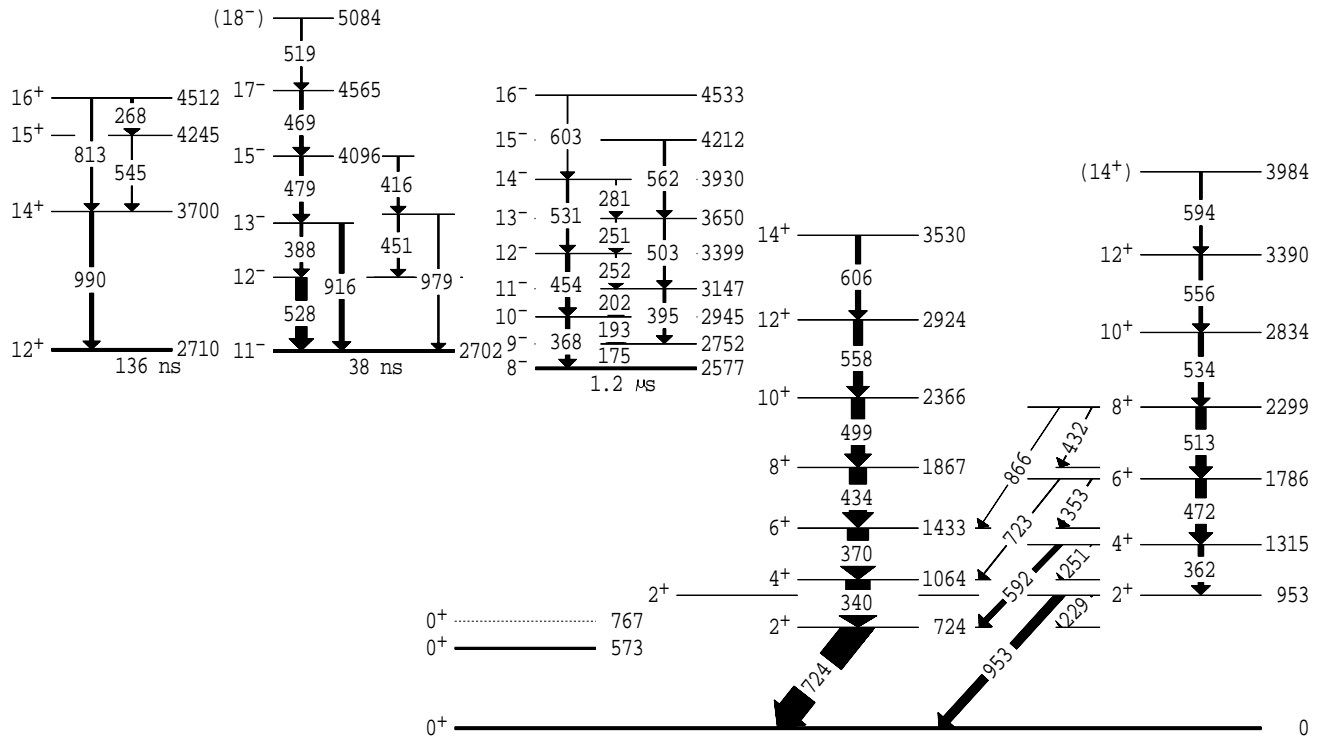


FIG. 1. Partial level scheme showing the yrast and yrare positive parity sequences in ^{188}Pb and the main sequences above the isomers. Note that the (many) connecting paths from the isomers are not shown, and that the relative branches are approximated by the widths of the transitions within a particular band. For illustrative purposes, intensities of the yrast sequence have been scaled down by a factor of about 20. The 0^+ state energies are uncertain [13].

between specific γ rays, and their time relative to the natural pulsing of the cyclotron beam, was used to provide high sensitivity for γ rays, which otherwise would be lost in the background of competing reactions. New developments in software [14] allowed the fast production of matrices with different time and gating γ -ray conditions, chosen to select and separate different parts of the level scheme.

The extensive level scheme thus established for ^{188}Pb cannot be covered in detail here, but about 50 new states have been identified including the structures above the 8^- , 11^- , and 12^+ isomers. The scheme largely confirms the earlier one [10], with the qualification that the character of the 11^- and 12^+ isomers has been inverted. That is, the 2702-keV, 38-ns mean life isomer is now associated with the 11^- state and the 2710-keV, 136-ns isomer with the 12^+ . This rearrangement is supported experimentally by the new angular distribution information for the deexciting 335- and 344-keV transitions, and is consistent with the previous total conversion coefficient constraints [10] that favored a combination of $E1$ and $E2$ multipolarities, but in either order. (Intensity arguments were used to favor one of the alternatives previously [10], but that was in the absence of a detailed knowledge of the feeding states above, which will be discussed elsewhere [16].) The new experimental assignment is supported by the character of the structures above.

The lower part of the sequence above the 11^- isomer, part of which is shown in Fig. 1, is very similar to the structures observed in ^{190}Pb and ^{192}Pb [9,15]. The higher states which form a $\Delta I=2$ sequence arise from an aligned band that

crosses the 11^- band at spin 13, as will be discussed elsewhere [16]. The in-band γ -ray branching ratios and the mixing ratios δ of the cascade transitions depend on the configuration through the magnetic properties (g factors) and the magnitude and sign of the deformation expressed in the quadrupole moment Q_0 . Both the magnitude of the $(g_K - g_R)/Q_0$ value of 0.042(2) deduced from the branching at the 13^- state and the negative sign of δ (associated with $\text{sgn}[(g_K - g_R)/Q_0]$) required by the large negative anisotropies of the 528- and 388-keV cascade transitions [$A_2/A_0 = -0.63(5)$ and $-0.42(9)$, respectively] are consistent with a two-proton configuration and a weakly oblate deformation (with a negative value of Q_0), although the value extracted for g_K of $\sim +0.5$ is smaller than the theoretical Nilsson value of $+1.0$ expected for the $9/2^- [505] \otimes 13/2^+ [606]$ configuration. The structure above the 12^+ isomer is similar to that observed in ^{190}Pb [9].

The alignment of the 8^- band which reflects the component orbitals through its sensitivity to Coriolis mixing is essentially the same as that in the 8^- bands of the (prolate-deformed) $N=106$ isotones. The magnitude of the alignment is consistent with the presence of the $9/2^+ [624]$ orbital in the configuration, while the $(g_K - g_R)$ value of $-0.182(18)$ extracted from the in-band branching ratios assuming $Q_0 = 6.5 e b$ agrees with those in the isotones and with the value expected for the two-quasineutron configuration [$-0.3(1)$ assuming $g_R = +0.3(1)$]. The sign of δ is given by the large negative anisotropy observed for the cascade transitions

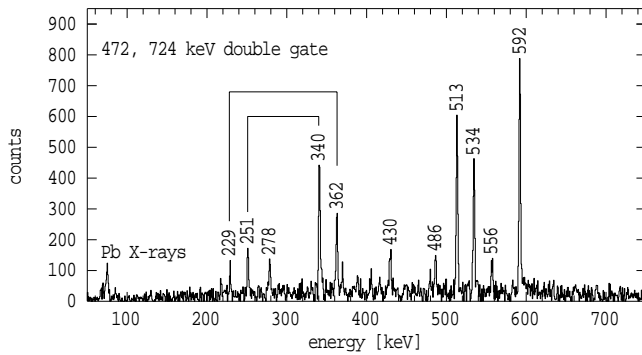


FIG. 2. Triple-coincidence spectrum with gates on the 472- and 723-keV transitions in ^{188}Pb , selecting transitions connecting the yrare and yrast positive-parity bands.

[$A_2/A_0 = -0.63(18)$ for the 175-keV transition], thus constraining the sign of the quadrupole moment to be positive, and implying prolate deformation. These results agree with the configuration-constrained PES calculations of Xu *et al.* [17], which predict prolate deformations of similar magnitude for the 8^- configurations in ^{186}Hg and ^{188}Pb .

Complementing the results on states above the isomers is the identification of the next-to-lowest energy (yrare) positive-parity band, as summarized in Fig. 1. Band members up to the 8^{+} state at 2299 keV are populated by a complicated but (now) unambiguous path from the fragmented decay of the 8^- isomer, while the higher states are clear in prompt double gating. (Primes and double primes are used in the text to indicate the second and third states of a given spin and parity, and for clarity, the 8^- decay is not shown in the figure.) A key element in the spin assignments is the observation of $E0$ components in the transitions connecting the yrare states to the yrast band. The evidence for these is contained in coincidence spectra, such as that obtained by gating on both the 472-keV $6^{+} - 4^{+}$ in-band transition and the 724-keV $2^{+} - 0^{+}$ transition shown in Fig. 2. The requirement that the intensity of the 251-keV transition must balance the 340-keV $4^{+} - 2^{+}$ $E2$ transition, and similarly, that the 229-keV connecting transition must balance the 362-keV in-band transition, can only be satisfied if both connecting transitions have large total conversion coefficients. Further, in the absence of significant lifetimes for these states, only $E1$, $M1$, or $E2$ γ -ray multiplicities need to be considered. Given that the hierarchy of total conversion is $\alpha_T(M1) > \alpha_T(E2) > \alpha_T(E1)$, and that the observed values exceed the $M1$ value, a significant $E0$ conversion component is implied. This is the case for each of the connecting transitions, resulting in the unambiguous $J^\pi - J^\pi$ assignments. When both bands have $K=0$, a further simplification is possible since the relevant Clebsch-Gordan coefficient for $J-J$ $M1$ transitions vanishes. The observed total conversion coefficient $\alpha_T(\text{expt.})$ and the theoretical value for $E2$ multipolarity are then related to the $E2$ and $E0$ decay widths for the $J-J$ transition simply by

$$\alpha_T(\text{expt.}) = \alpha_T(E2) + \Gamma_e(E0)/\Gamma_\gamma(E2),$$

allowing the extraction of width ratios for the $J-J$ transitions. The values obtained are $\Gamma_e(E0)/\Gamma_\gamma(E2) \sim 0.3, 1.2(3), 2.2(3),$ and $2.6(5)$ for the 432-, 353-, 251-, and 229-keV $J-J$ transitions, respectively. (Note that the population of the yrare states is relatively low and $E0$ conversion electron lines from the 353-keV transition, for example, would have been below the sensitivity of the electron experiments of Ref. [18].)

The importance of the $E0$ components is not just that they restrict the spin and parity assignment, but that such $E0$ and $E2$ widths only arise in the case of significant mixing and deformation difference. As is well known (see, for example, Ref. [19]) for simple two-state mixing the $J-J$ transition, $E0$ and $E2$ transition strengths depend on the mixing amplitude a and deformation β as

$$\rho(E0, J \rightarrow J) \sim a \sqrt{1-a^2} [(\beta_1)^2 - (\beta_2)^2],$$

$$\sqrt{B(E2, J \rightarrow J)} \sim a \sqrt{1-a^2} [\beta_1 - \beta_2].$$

Both strengths, whose squares are directly related to the decay widths $\Gamma(E0)$ and $\Gamma(E2)$, are only nonzero if a is neither zero nor unity, and if the deformations β_1 and β_2 are not equal. To put these experimental results into context, band-mixing calculations were carried out to reproduce the energies of the yrast and nonyrast sequences, using a formalism similar to that reported before [20] and commonly used in this region (see, for example, Ref. [21]).

The fits to both yrast and nonyrast states yield moment-of-inertia parameters for the unperturbed bands, and limits on the interaction matrix elements and the values of the mixing amplitudes. (The energies of the excited 0^+ states have been disputed [13], hence a range of values has been considered.) The unperturbed moment-of-inertia parameters give the deformation/quadrupole moment of the unperturbed bands and these, together with the amplitudes, can be used to calculate the $B(E2)$ and $B(E0)$ transitions using the general formulation given by Kibedi *et al.* [22].

The calculated widths and total decay branches predicted from a representative calculation are compared with the experimental branching ratios in Table I and Fig. 3. Considering first the $E0$ branches and the $\Gamma_e(E0)/\Gamma_\gamma(E2)$ ratios for the $J-J$ transitions, as can be seen from Fig. 3, the $E0$ branch is reasonably well predicted for each of the states but the ratio calculated from Table I is overestimated. That discrepancy can be traced directly to an underestimate of the $\Gamma_\gamma(E2)$ value. Overall, the branching ratios for all transitions are reasonably well reproduced, except for the two strongest transitions from the 4^{+} state where the out-of-band $E2$ transition to the yrast 2^{+} state is underestimated.

Other transitions of specific interest are the possible branches from the 2^{+} state to the excited 0^+ states, which would provide a confirmation of the 0^+ state energies. We give experimental limits of a few percent of the main transition to ground, which are roughly consistent with the predicted values (remembering the uncertainty in the experimental energy), indicating that significant population of the 0^+ states through this path is unlikely.

TABLE I. Predicted decay widths and corresponding branching ratios of nonyrast states in ^{188}Pb from a representative band-mixing calculation, compared with experiment.

Initial state	I_i^π	I_f^π	$M\lambda$	E_γ (keV)	Γ_γ (10^{-5} eV)	Γ_e (10^{-5} eV)	Theor. (%)	Expt. (%)
2299	$8^{+'}$	6^+	$E2$	866	1.89	0.018	15.9	6(2)
		$6^{+'}$	$E2$	513	9.50	0.290	81.3	75(5)
		8^+	$E2$	432	0.063	0.003	0.5	15(2)
		8^+	$E0$	432		0.277	2.3	3.7(6)
1786	$6^{+'}$	4^+	$E2$	723	0.63	0.008	9.6	10(3)
		$4^{+'}$	$E2$	472	5.41	0.213	84.5	79(4)
		6^+	$E2$	353	0.04	0.003	0.7	5(1)
		6^+	$E0$	353		0.344	5.2	6(1)
1315	$4^{+'}$	2^+	$E2$	592	0.22	0.004	7.0	40(3)
		$2^{+'}$	$E2$	362	2.28	0.131	75.8	43(2)
		$2^{+''}$	$E2$	206	0.020	0.006	0.8	
		4^+	$E2$	251	0.020	0.003	0.7	6(1)
		4^+	$E0$	251		0.498	15.6	10(1)
953	$2^{+'}$	0^+	$E2$	953	6.59	0.049	88.2	68(2)
		$0^{+'}$	$E2$	380 ^a	0.59	0.037	8.3	≤ 3
		$0^{+''}$	$E2$	186 ^a	0.02	0.013	0.5	≤ 5
		2^+	$E2$	229	0.003	0.001	0.1	8(1)
		2^+	$E0$	229		0.225	3.0	16(2)

^aExpected energies assuming $0^{+'}$ and $0^{+''}$ states at 573 and 767 keV.

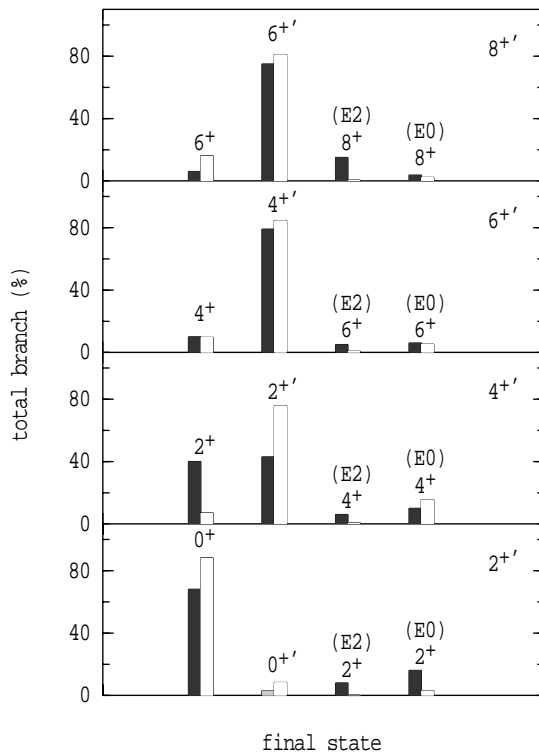


FIG. 3. Calculated (open columns) and experimental (filled) total branching ratios for the decay of yrare, positive-parity states in ^{188}Pb . The initial state is indicated in each panel. The $E2$ and $E0$ components are shown separately for J - J transitions, and the shaded column for the $2^{+'}-0^+$ decay represents the experimental limit.

We wish to draw several conclusions from these results. First, considering the simplicity of the band-mixing model, relatively good reproduction of $E2$ and $E0$ branching ratios is found for the whole sequence. Since each of the branches depends on the amplitudes and relative phases of the admixtures, we can be confident of the implications that significant admixtures are present, and there must be a difference in quadrupole moment, and therefore deformation. Precise amplitudes in the low spin states could only be obtained if the experimental energies of the $0^{+''}$ and $2^{+''}$ states were known. With that qualification, the squares of the amplitudes of the wave function components correspond to mixtures of spherical, prolate, and oblate shapes of approximately 95%, 4%, and 1% in the 0^+ (ground) state; 2%, 15%, and 83% in the $0^{+'}$ state; and 3%, 81%, and 16% in the $0^{+''}$ state; the yrast 2^+ state is described as a 18%, 69%, and 13% admixture while the $2^{+'}$ state is given as 7%, 7%, and 86%, being dominated by the oblate shape. The higher spin yrast states are mainly prolate with about a 9% oblate admixture at spin 6, falling to about 3% at spin 12, with complementary admixtures in the yrare band.

The unperturbed moments used were 6.2 eb , 3.2 eb , and (equivalent to) 1.5 eb for the sequences corresponding notionally to the prolate, oblate, and spherical configurations. The first two correspond to quadrupole deformations of $|\beta| = 0.20$ and $|\beta| = 0.13$, in agreement with predicted values. These results taken together with the new information on the bands based on the three isomers sketched out previously, comprise comprehensive evidence for the underlying triple-shape coexistence, complicated as foreshadowed, by signifi-

cant mixing at low spin.

There is, however, a qualification in the band mixing, in that the agreement is only obtained if the same sign is used for the intrinsic quadrupole moment of the unperturbed oblate and prolate bands. This problem was identified many years ago [23] in studies of the even-even Hg isotopes, and again in analyses of results for the odd-A Hg isotopes [24] and remains, as far as we know, unresolved. One alternative is to take this result at face value and require both bands to be prolate, or both to be oblate, a possibility that cannot be dismissed on the basis of direct experimental evidence, but it is the one that would have implications for the Pb and Hg nuclei, which are difficult to reconcile with both indirect experimental evidence and with theory.

A common limitation in theoretical studies, which may well be related to this problem is their neglect of asymmetric distortions (the γ degree of freedom), which is acknowledged to be important for determining the barrier between minima [5] but is usually not included, although it is likely to be important for determining transition rates. Its inclusion is

outside the scope of the present band-mixing model, but the $E2$ and $E0$ branching ratios and band structures observed here should provide a stringent test for future calculations. The calculated decay widths given in Table I can be translated to predicted lifetimes, which may also be accessible to the experimental test.

Considering further theoretical analyses, the mixing matrix elements required to reproduce experiment are 70–120 keV, implying relatively low potential barriers. The question arises as to whether these (static model) matrix elements are consistent with more sophisticated calculations that incorporate dynamical effects.

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