New signatures for octupole deformation in some actinide nuclei

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Energies for three positive and three negative parity bands predicted by the extended coherent state model in ^{218,226}Ra, ^{228}Th, ^{232}Th, in four uranium even-mass isotopes, $^{232-238}$U, and in ^{238}Pu, are calculated and used to point out new signatures for octupole deformation in ground as well as in β and γ bands. A beat pattern is found by using a new displacement energy function, which is more appropriate for a spectrum that exhibits large deviations from a linear $J(J+1)$ dependence.

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The field of negative parity bands became very attractive especially since the first suggestions for octupole deformation were advanced by Chassman [1] and Moler and Nix [2]. Chassman predicted parity doublets for several odd mass isotopes of Ac, Th, and Pa [1] . The lowest doublet constitutes of a degenerate ground state corresponding to an equilibrium shape having a reflection symmetry. If the doublet is not mathematically degenerate but exhibits a small energy splitting, this is a sign of a reflection symmetry breaking. Moler and Nix [2] suggested that some even-even radium isotopes might have an octupole deformed ground state. Indeed, the binding energy of these nuclei gains about 2 MeV when an octupole deformation is assumed in the mean field. The main achievements in this field have been reviewed in several papers [3–5].

Since there is no measurable quantity for the octupole deformation, some indirect information about this variable should be found. Several properties are considered as signatures for octupole deformation: (a) The state 1^-, the head of the $K^\pi=0^-$ band, has a very low position, and this is an indication that the potential energy has a flat minimum, as a function of the octupole deformation. (b) The parity alternating structure in ground and the lowest 0^- bands suggests that the two bands may be viewed as being projected from a sole deformed intrinsic state, exhibiting both quadrupole and octupole deformations. (c) A nuclear surface with quadrupole and octupole deformations might have the center of charge in a different position than the center of mass, which results in having an electric dipole moment that may excite the state 1^- from the ground state, with a large probability. Of course the list is not complete, and thereby any new signature for this new nuclear phase deserves a special attention.

Few years ago our group considered this subject within a phenomenological framework. Thus, in Refs. [6–8] one of us (A.A.R) proposed a phenomenological model to describe simultaneously the ground state band and the $K^\pi=0^-$ band. The model has been applied to nuclei that are proved to have octupole deformation, such as the even-even Ra isotopes [6,7], as well as for Rn isotopes [8] whose negative parity states are interpreted as octupole vibrational states. Both sets of nuclei were described equally well, which leads to the conclusion that the proposed model is able to describe in an unified fashion negative parity spectra of pear shaped and octupole-nondeformed nuclei. In Ref. [9] the formalism was extended to two other pairs of bands, which are conventionally called $\beta^\pm$ and $\gamma^\pm$ by assuming that the intrinsic states associated to β and γ bands, have also an octupole deformation. The resulting phenomenological scheme was named the extended coherent state model (ECSM). By simultaneous projection of the angular momentum and parity from each such state, two bands of opposite parities are obtained. We wanted to see if specific fingerprints of the octupole deformation would also appear in the excited bands $\beta^\pm$ and $\gamma^\pm$.

In this paper, the results for ^{218,226}Ra, ^{228,232}Th, $^{232-238}$U, and ^{238}Pu are presented in a new light by using new interpretation means. Indeed, by studying the dependence of the energies of the three pairs of bands on angular momentum, one notices large deviation from the $J(J+1)$ pattern. In this context it is worth to investigate to what extent an analysis based on a more realistic J dependence may suggest new values for angular momentum where the octupole deformation shows up.

Aiming at a self-consistent presentation, we first sketch the ideas underlying ECSM. ECSM is based on the coherent state model (CSM) proposed by Raduta *et al.* to describe the main properties of the first three collective bands of positive parity, i.e., ground, β and γ bands [10,11]. The name of the formalism comes from the fact that the intrinsic ground state is described by an axial symmetric coherent function for the quadrupole bosons, $b^\dagger_{2\mu}$, while the β and γ intrinsic states are orthogonal polynomial excitations of the intrinsic ground state. ECSM assumes that the intrinsic ground state exhibits both a quadrupole and an octupole deformation. The other bands, β and γ, also have this property because they are excited from the ground state. The octupole deformation is described by means of an axially symmetric coherent state for the octupole boson $b^\dagger_{30}$. Thus, the intrinsic states for ground, β, and γ bands are

$$\Psi_g=e^{f(b^\dagger_{30}-b_{30})}e^{d(b^\dagger_{20}-b_{20})}|0\rangle_{(3)}|0\rangle_{(2)},\quad \Psi_\beta=\Omega^\dagger_\beta\Psi_g,$$
$$\Psi_\gamma=\Omega^\dagger_\gamma\Psi_g, \tag{1}$$

where the excitation operators have the expressions

$$\Omega^\dagger_\beta=(b^\dagger_2b^\dagger_2b^\dagger_2)_0+\frac{3d}{\sqrt{14}}(b^\dagger_2b^\dagger_2)_0-\frac{d^3}{\sqrt{70}},$$
$$\Omega^\dagger_\gamma=(b^\dagger_2b^\dagger_2)_{22}+d\sqrt{\frac{2}{7}}b^\dagger_{22}. \tag{2}$$

The notation $|0\rangle_{(k)}$ stands for the vacuum state of the 2^k-pole boson operators. Because the states (1) are mixtures of positive and negative parity states, they do not have good reflection symmetry. Since in the laboratory frame this symmetry is valid, it should be restored by a projection procedure. By restoring the two symmetries, rotation and reflection, the three orthogonal states defined above generate six sets of mutually orthogonal states:

$$\varphi_{JM}^{(i,k)}=\mathcal{N}_J^{(i,k)}P_{MK_i}^J\Psi_i^{(k)},\quad K_i=2\,\delta_{i,\gamma},\quad k=\pm;i=g,\beta,\gamma, \tag{3}$$

$\mathcal{N}_J^{(i,k)}$ are renormalization factors, while P_{MK}^J is the angular momentum projection operator. The function $\Psi_i^{(k)}$ is the component of parity $k(=\pm)$ of the intrinsic state Ψ_i.

Within the boson space spanned by the projected states, one considers the following effective quadrupole and octupole boson Hamiltonian

$$\begin{aligned}H=&\mathcal{A}_1(22\hat{N}_2+5\Omega_{\beta'}^\dagger\Omega_{\beta'})+\mathcal{A}_2\Omega_\beta^\dagger\Omega_\beta\\&+\mathcal{B}_1\hat{N}_3(22\hat{N}_2+5\Omega_{\beta'}^\dagger\Omega_{\beta'})+\mathcal{B}_2\hat{N}_3\Omega_\beta^\dagger\Omega_\beta+\mathcal{B}_3\hat{N}_3\\&+\mathcal{A}_{(J23)}\vec{J}_2\vec{J}_3+\mathcal{A}_J\vec{J}^2.\end{aligned} \tag{4}$$

If the angular momentum square $\vec{J}^2$ is restricted to the angular momentum carried by the quadrupole bosons $\vec{J}_2^2$, the first two terms and the last one from Eq. (4) define the Hamiltonian used by CSM for ground, β, and γ bands. The pure octupole Hamiltonian is a harmonic operator, i.e., the octupole boson number operator. We assume that the coupling between quadrupole and octupole bosons can be described by a product between the octupole boson number operator, $\hat{N}_3$, and the quadrupole boson anharmonic terms that are involved in the CSM Hamiltonian. Also, two scalar terms depending on the angular momenta carried by the quadrupole ($\vec{J}_2$) and octupole ($\vec{J}_3$) bosons and on total angular momentum $\vec{J}$, respectively, are included. Here $\hat{N}_2$ denotes the quadrupole boson number operator and $\Omega_{\beta'}^\dagger$ stands for the following second-order invariant: $\Omega_{\beta'}^\dagger=(b_2^\dagger b_2^\dagger)_0-d^2/\sqrt{5}$. Arguments supporting this choice for the model Hamiltonian are given in Refs. [6,9].

As shown in Ref. [11], the projected states are linear superposition of states with definite K quantum number. Moreover, in the asymptotic limit of the deformation parameter, a single K component is prevailing for each set. Actually, this property answers the question why one associates the states $\{\phi_{JM}^{(i)}\}$ $(i=g,\beta,\gamma)$ to the ground, β, and γ bands, respectively. A similar analysis could be performed also for the ECSM states. The result is that, preserving the convention mentioned above, the set of projected states given by Eq. (3) comprises two $K^\pi=0^+$, two $K^\pi=0^-$, one $K^\pi=2^+$, and one $K^\pi=2^-$ subsets. The positive parity bands are the ground, β, and γ bands, respectively. Conventionally, these names will be symbolized by g^+, β^+, and γ^+. Correspondingly, the negative parity bands are denoted by g^-, β^-, and γ^-, respectively.

In the boson basis of projected states, the only nonvanishing matrix elements of the effective Hamiltonian are relating g^k and γ^k $(k=\pm)$ states. The eigenvalues of the model Hamiltonian in the boson space generated by the projected state depend on the structure coefficients defining the boson Hamiltonian and two deformation parameters d and f. Because there are no experimental data for the β^- band, we did not include the coefficient $\mathcal{B}_2$ in the fitting procedure. Indeed, the corresponding term from the Hamiltonian affects this band exclusively. The remaining eight parameters were fixed by a least square procedure in order to fit the experimental excitation energies.

For what follows, a short comment on the physical significance of these parameters is necessary. We begin with the deformation parameters d and f. They bear these names since the average values of the harmonic quadrupole and harmonic octupole moments are proportional to d and f, respectively. The quadrupole deformation parameter has been related to the nuclear deformation β in the first paper published on the CSM model [10,11]. Indeed, therein it was shown that in the rotational regime the projected states can be written in a factorized form, one factor being a Wigner function while the other one a function of the dynamic deformations β and γ. For ground band states, the second factor mentioned above depends only on β, and moreover is proportional to

$$F=\frac{1}{\beta}e^{[d-(k\beta/\sqrt{2})]^2}. \tag{5}$$

Here k is a parameter determining the relation between the quadrupole bosons and the quadrupole coordinates:

$$\alpha_{2\mu}=\frac{1}{k\sqrt{2}}[b_{2\mu}^+ +(-)^\mu b_{2-\mu}]. \tag{6}$$

In the harmonic liquid drop model, this parameter has a definite expression in terms of the mass and string parameters. CSM takes k as a free parameter that is consistent with the known fact that the boson operators are related to the collective coordinates and their conjugate momenta by a canonical transformation, determined up to a multiplicative constant. The factor F has two extremes in the dynamic quadrupole deformation β. It can be shown that the maximum value is reached for a deformation β_0, which obeys the equation

$$d=\frac{k\beta_0}{\sqrt{2}}+\frac{1}{2}. \tag{7}$$

To this deformation it corresponds the most likely shape, i.e., the equilibrium one. Thus, our previous analysis suggests that in the large deformation regime, the parameter d and the static quadrupole deformation β_0 are related by a linear equation. This relationship is confirmed by our results for the actinide region shown in Fig. 1.

Now, let us address the question "what is the mechanism reponsible for setting on the static octupole deformation?" Consider first a fourth-order octupole boson Hamiltonian. Since in the laboratory frame this Hamiltonian should be invariant to a spatial inversion operation, the third-order term

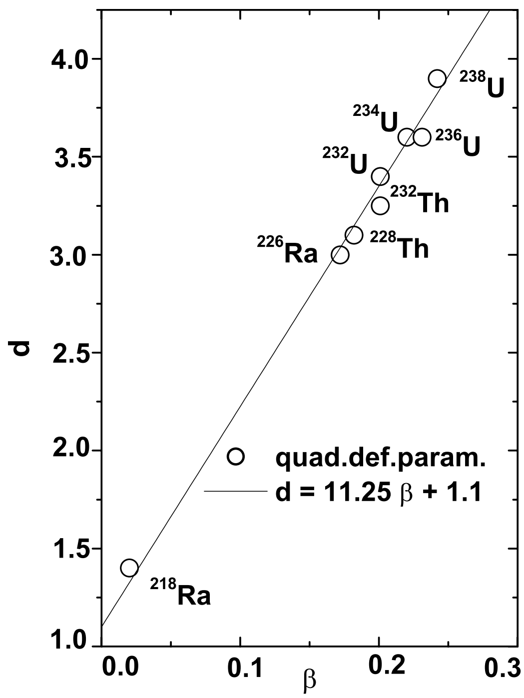

FIG. 1. The quadrupole deformation parameter d is plotted as a function of the nuclear deformation β. The data for β are from Refs. [28,29].

is necessarily missing. Writing the chosen Hamiltonian in the intrinsic frame it is clear that the corresponding potential energy is a biquadratic expression in the octupole deformation β_3. Suppose that this potential has a minimum in the variable β_3^2 for $\beta_3^2 = b$, with b positive. Under these circumstances the potential energy, considered as a function of β_3, has two degenerate minima separated by a maximum reached in $\beta_3 = 0$. Concluding, for a boson Hamiltonian exhibiting a space reflection symmetry in the laboratory frame, the potential energy might have two degenerate minima. The larger the strength of the boson two-body interaction, the higher the intermediate maximum. Had the potential energy a minimum for a vanishing octupole deformation, the nuclear shape would preserve the reflection symmetry. Therefore the eigenstates of the intrinsic Hamiltonian have definite parity, and moreover, the positive and negative parity states are degenerate. If the nuclear shape has an octupole static deformation equal to one of the degenerate minima mentioned above, the corresponding ground state is an admixture of components with different parities. The opposite parity components interact with each other due to the octupole correlations which, as a matter of fact, determine the height of the barrier separating the degenerate minima. Due to this feature, one has been attempting to relate the parity energy split with the height of the potential barrier [12]. This picture for octupole deformation was formulated by several authors of boson description works [4].

Here the situation is essentially different since the effective octupole potential energy is a quadratic polynomial in β_3, which might have a single minimum for a nonvanishing octupole deformation. Indeed, let us consider the octupole boson Hamiltonian H_3, obtained by averaging the model Hamiltonian (4) on the state:

$$\Phi = e^{f(b_{30}^{\dagger} - b_{30})} e^{d(b_{20}^{\dagger} - b_{20})} |0\rangle_{(2)}. \tag{8}$$

The result for H_3 is

$$H_3 = D_0 + f D_1 (b_{30}^{\dagger} + b_{30}) + D_2 \hat{N}_3 + D_3 \hat{J}_3^2, \tag{9}$$

where the following notations have been used:

$$D_0 = (22\mathcal{A}_1 + 6\mathcal{A}_J) d^2 + (\mathcal{B}_3 + 22 d^2 \mathcal{B}_1 + 12\mathcal{A}_J) f^2,$$

$$D_1 = \mathcal{B}_3 + 22 d^2 \mathcal{B}_1 + 12\mathcal{A}_J, \quad D_2 = D_1 - 12\mathcal{A}_J, \quad D_3 = \mathcal{A}_J. \tag{10}$$

Since H_3 contains a linear term in octupole bosons, it is not invariant to rotations. One may say that the operation applied to the model Hamiltonian is equivalent to bringing it to an intrinsic frame, where the conjugate collective variable may be defined as

$$a_{3\mu} = \frac{-1}{k_3\sqrt{2}} [b_{3\mu}^{\dagger} + (-)^{\mu} b_{3-\mu}],$$

$$p_{3\mu} = \frac{-ik_3}{\sqrt{2}} [(-)^{\mu} b_{3-\mu}^{\dagger} - b_{3\mu}]. \tag{11}$$

The collective coordinates $a_{3\mu}$ can be parametrized in terms of octupole deformation β_3 and three angles $\gamma_1, \gamma_2, \gamma_3$, whose static values are equal to zero. Taking for γ variables their static values, one obtains the following expression for the potential energy:

$$V_{oct} = \tfrac{1}{2} D_2 (k_3 \beta_3)^2 - f\sqrt{2} k_3 \beta_3 D_1. \tag{12}$$

If $D_2 > 0$, this function has a minimum for

$$k_3 \beta_3^{(0)} = f\sqrt{2} \left(1 + \frac{12 A_J}{D_2} \right). \tag{13}$$

This equation suggests that the deformation parameter f is proportional to the static octupole deformation. However, as we already mentioned, there is no measurable observable that could confer $\beta_3^{(0)}$ an experimental value. Consequently, a graph for the octupole deformation parameter analogous to that given in Fig. 1 is not possible. The general features of the results supplied by the fitting procedure are as follows: For an isotopic chain, f is an increasing function of A. Thus, for uranium isotopes it varies almost linearily from 0.1 (^{232}U) to 0.6 (^{238}U). For radium isotopes, f has the value 0.3 for ^{218}Ra and 0.8 for ^{226}Ra. For all other nuclei the octupole deformation parameter f aquired a value equal to 0.3.

The structure coefficients involved in the model Hamiltonian (4) have been analyzed *in extenso* in several papers [6,9,11], where the CSM model and its extension have been formulated. Moreover, the accompanying operators have been studied in the restricted collective space in the extreme deformation regimes, vibrational and rotational [11], both analytically and numerically. These extreme regimes confer the quoted coefficients a definite significance.

We note that without exception, the terms entering the boson Hamiltonian have been obtained microscopically

TABLE I. The coefficients defining the model Hamiltonian, determined by a least squares fit are given in units of keV. The deformation parameters are dimensionless.

	^{218}Ra	^{226}Ra	^{228}Th	^{232}Th	^{232}U	^{234}U	^{236}U	^{238}U	^{238}Pu
d	1.40	3.00	3.1	3.25	3.4	3.6	3.6	3.90	3.90
f	0.30	0.80	0.3	0.30	0.1	0.3	0.45	0.60	0.30
$\mathcal{A}_1$	16.86	20.29	17.72	14.26	15.7	17.8	17.8	20.64	18.84
$\mathcal{A}_2$	−23.43	−17.54	−12.66	−8.33	−10.5	−9.58	−8.53	−9.72	−8.63
$\mathcal{A}_J$	1.81	0.49	1.32	2.26	2.08	1.55	1.73	1.55	2.23
$\mathcal{A}_{(J23)}$	18.09	7.17	8.38	11.93	15.9	27.5	19.67	20.45	10.77
$\mathcal{B}_1$	−7.13	−0.74	−2.63	−6.17	−7.35	−13.56	−9.23	−10.77	−8.39
$\mathcal{B}_3$	786.24	362.84	831.25	2 093.34	2 402.6	4 644.	3 279.55	4 239.42	3 308.60

within a boson expansion formalism formulated in connection with a quadrupole-quadrupole plus octupole-octupole two-body interaction [13–15]. The expansion was made in terms of collective and noncollective quasiparticle–random phase approximation bosons. In this context one could say that our phenomenological quadrupole-octupole boson terms have microscopic counterparts, and thereby their strenghts exhibit a microscopic interpretation.

In what follows, we summarize the results concerning the effects of varying these coefficients on the spectrum under consideration. In vibrational ($d\rightarrow 0$) and rotational limit (d is large. It was shown that $d\geqslant 3$ already means that d is large) the ground state energies have the expressions

$$E^{g,vib}=22\mathcal{A}_1\frac{J}{2}+\mathcal{A}_J J(J+1),$$

$$E^{g,rot}=22\mathcal{A}_1\frac{J(J+1)}{6d^2}+\mathcal{A}_J J(J+1). \tag{14}$$

For vibrational nuclei and low angular momentum, the $\mathcal{A}_1$ term is much larger than the $\mathcal{A}_J$ one. Therefore one could say that for such a regime, $22\mathcal{A}_1$ is a good approximation for the energy of the first 2^+ state. In the rotational limit, this coefficient together with $\mathcal{A}_J$ determine the kinematic moment of inertia for the ground band. For transitional nuclei, the result for the ground state energies lies between the limits described above. Actually these energies are obtained by diagonalizing, for a given angular momentum, a 2×2 matrix; the second eigenvalue is the energy of the γ band state. As shown in Ref. [11], the $\mathcal{A}_2$ term contributes only to the energies of β band states. Due to this feature the CSM fixes its strength in order to reproduce the position of the head state of β band relative to the ground state. In the vibrational limit and harmonic regime, the energy of the lowest 3^- state is given by the $\mathcal{B}_3$ term. Departing from this picture, this energy is affected by anharmonicities, i.e., the quadrupole-octupole boson interaction, and by deformation. The two types of collective degrees of freedom are coupled by the $\mathcal{B}_1$ term as well as by those terms depending on the angular momenta carried by the quadrupole and octupole bosons. In Ref. [7] we have shown that for an almost vibrational nucleus, as for example is the case of ^{218}Ra, the low position of the state 1^- is detemined by the $\hat{J}_2\cdot\hat{J}_3$ term that is attractive in the state 1^-, and repulsive in all other negative parity states from the $K^\pi=0^-$ band.

The boson Hamiltonian has novanishing matrix elements only between the pair of states J_g^+, J_γ^+ (J even) and J_g^-, J_γ^- (J odd). The coupling terms contribute to the energy splitting for the states in the parity partner bands but also to the mixtures of the pair of states mentioned before. Also, we have shown that the anharmonic terms of higher order in octupole bosons have the effect of renormalizing the matrix elements of the terms contained by our model Hamiltonian. The coupling terms affect the positive and negative bands in a different manner. For example, for large quadrupole and small octupole deformation, the bands g^-, β^-, and $\gamma-$ are pressed down in energy (otherwise the energy split is small), while for the parity partner bands the effect is about the same as for the ground state, and therefore the excitation energy is left unchanged. In this region for d and f, the average values of the octupole boson number operator exhibit a linear dependence on $J(J+1)$. For small d and f, the average of $\hat{N}_3$ on states from g^-, β^-, and γ^- bands is almost independent of angular momentum, while the averages of $\hat{N}_2\hat{N}_3$ exhibit a strong dependence on J. Concluding, the energies in negative parity bands are obtained as a result of the competition between the octupole harmonic and quadrupole-octupole interacting terms.

The coefficients depend smoothly on the atomic mass number. Comparing the coefficients for ^{238}U and ^{238}Pu as well as for ^{232}U and ^{232}Th, one notices a dependence on the charge asymmetry. Actually, a large number of actinide isotopes have been considered and the corresponding structure coefficients, provided by the fitting procedure, lie on curves given by fourth-order polynomials in $A-(N-Z)/2$. Exception is for $-\mathcal{A}_2$, which is fitted by a second-order polynomial.

Note that the total number of free parameters is about the same as in other phenomenological models (see, for example, Ref. [16]), although none of them treats simultaneously six bands, going up to high and very high spin region. To get an idea about the capability of the proposed model to describe a large number of data, we just mention that in Ref. [9], we obtained a quite accurate description of 55 level energies in ^{232}Th, the deviations being smaller than 20 keV.

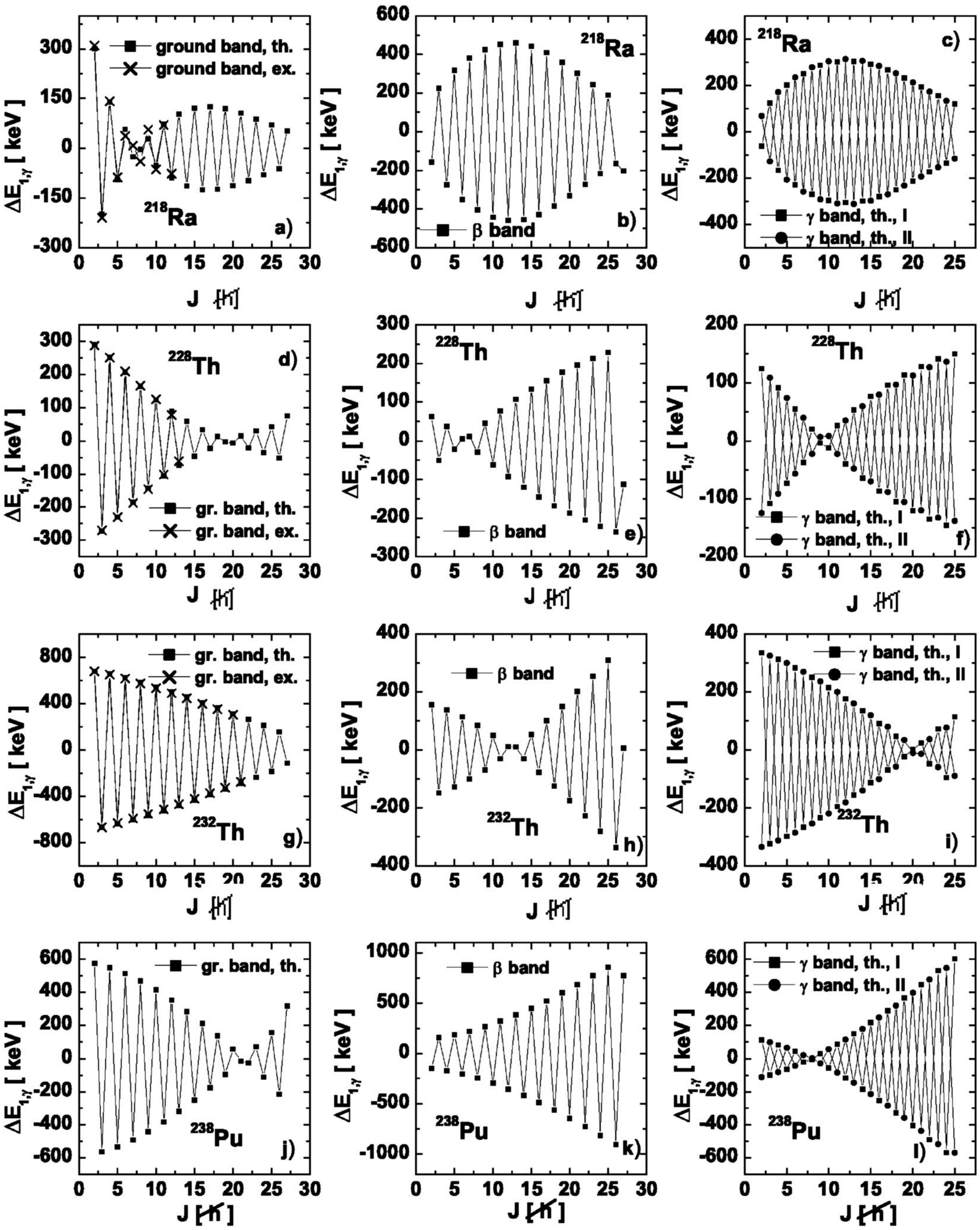

FIG. 2. The displacement function given by Eq. (16) for ground, β, and γ bands in ^{218}Ra, ^{228}Th, ^{232}Th, and ^{238}Pu is represented as a function of angular momentum. For γ bands the label I is used when the state 2^+_γ is the lowest angular momentum state involved in Eq. (16), while II corresponds to the calculation using 2^-_γ as the first state in the chain. Experimental data were taken from Ref. [26] (^{218}Ra), Ref. [21] (^{228}Th), Ref. [19] (^{232}Th), and Ref. [27] (^{238}Pu).

The parameters yielded by the fitting procedure are listed in Table I.

These parameters determine the excitation energies for the six bands. In Ref. [9], the results were presented by giving the dynamic moment of inertia as function of the rotational frequency for some of the nuclei from Table I. Here we want to stress on the fact that the energies in the three pairs of bands exhibit large deviation from a $J(J+1)$ pattern, and therefore the displacement energy function previously used by several authors [4,17] is not appropriate to predict the angular momentum in the partner bands, where the corresponding moments of inertia become equal to each other. Instead, we shall use a new displacement energy function [18], which is suitable for spectra having a quadratic dependence on $J(J+1)$:

$$E(J)=E_0+AJ(J+1)+B[J(J+1)]^2. \qquad (15)$$

This depends on the dipole γ transition energies $E_{1,\gamma}(I)=E(I+1)-E(I)$ and has the expression

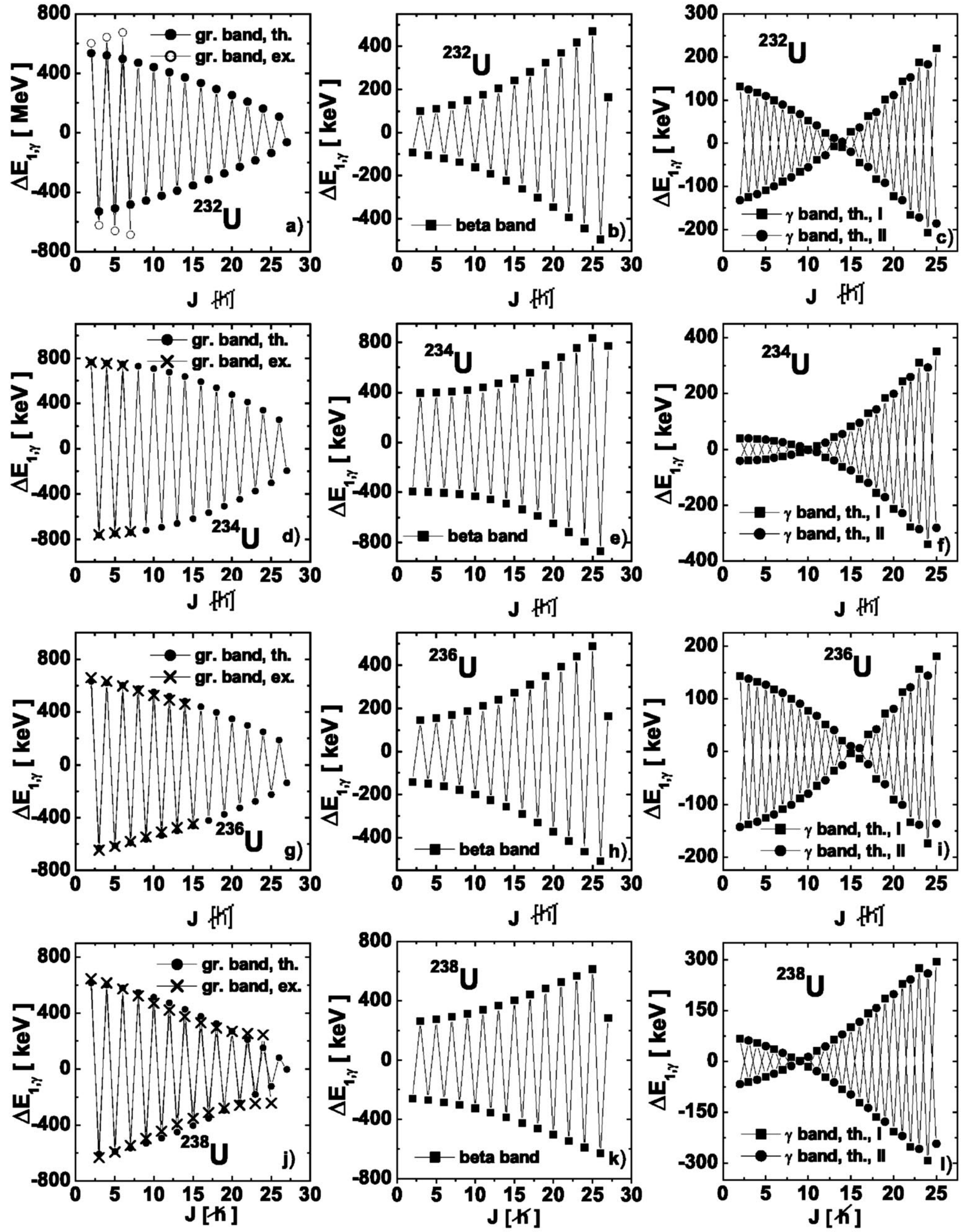

FIG. 3. The same as in Fig. 2 but for ^{232}U, ^{234}U, ^{236}U, and ^{238}U. Experimental data were taken from Ref. [22] (^{232}U), Ref. [23] (^{234}U), Ref. [24] (^{236}U), and Ref. [25] (^{238}U).

$$\Delta E_{1,\gamma}(I)=\frac{1}{16}[6E_{1,\gamma}(I)-4E_{1,\gamma}(I-1)-4E_{1,\gamma}(I+1)+E_{1,\gamma}(I-2)+E_{1,\gamma}(I+2)]. \tag{16}$$

The function $\Delta E_{1,\gamma}(I)$ has been first used by Bonatsos *et al.* [18] for ground and 0^- bands, to analyze the predictions of the $spdf$-interacting boson model.

The energies predicted by our formalism for the three pairs of bands are used in Figs. 2–4 to show the angular momentum dependence of the function $\Delta E_{1,\gamma}$. The parities associated to the angular momenta, involved in Eq. (16), are as follows: for ground and β bands the levels I and $I\pm 2$ have the same parity, while $I\pm 1$ are of opposite parities. In the case of γ bands this rule is preserved, but we have two chains depending on whether the first state $(I-2)$ in Eq. (16) is 2^+ or 2^-. From Figs. 2 and 3 one can see that only ^{218}Ra, ^{228}Th, ^{238}U, and ^{238}Pu have octupole deformation in the ground bands at $J=7$, $J=19{,}21$, $J=27$, and $J=21{,}23$, respectively. The β band shows an octupole deformation in ^{228}Th $(J=9{,}11)$ and ^{232}Th $(J=11{,}13)$. By contradistinction, all γ bands, except that of ^{218}Ra, have octupole deformation for some values of angular momentum. For the bands where the octupole deformation may be noticed, the displacement energy function exhibits a beat pattern. The amplitude of beats is different for ground, β, and γ bands. This remark

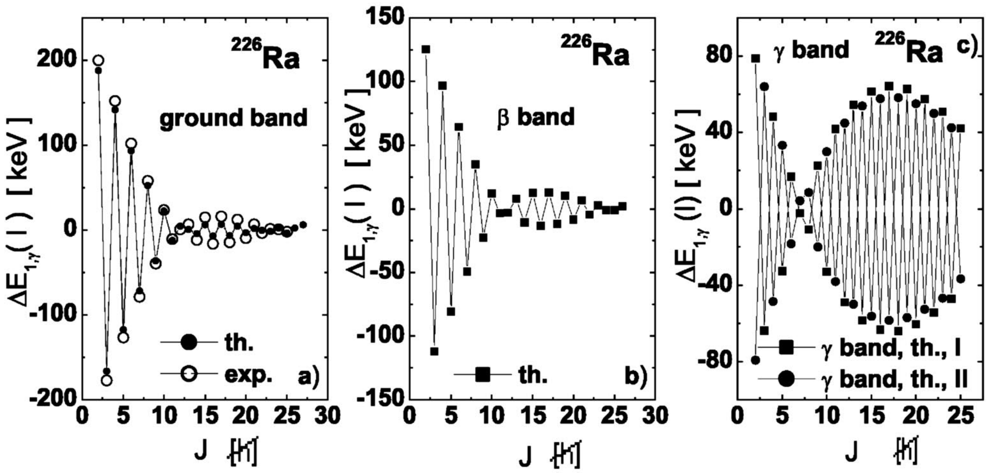

FIG. 4. The same as in Fig. 3 but for ^{226}Ra. Experimental data were taken from Ref. [30].

infers that the octupole deformation may appear at a higher angular momentum. It is clear that the interaction of particles close to the Fermi level plays an important role in stabilizing the system against the octupole deformation [20]. Indeed, comparing the figures for the two isotopes of Th, we see that the last four neutrons in ^{232}Th remove the octupole deformation in the ground band and shifts the angular momentum of β and γ bands, where the octupole deformations appear. The situation for Ra isotopes is different, namely, ^{218}Ra has octupole deformation only in the ground band, while ^{226}Ra, presented in Fig. 4, has octupole deformation in all bands, at different angular momenta. The function ΔE is very sensitive even to small deviations of energies, and due to this fact Figs. 2(j), and 3(d) suggest a very good agreement between computed and experimental energies.

We mention the fact that in order to draw a point corresponding to the angular momentum I, in one of Figs. 2–4, it is necessary to know the energy levels for $I-2,\ I-1,\ I,\ I+1,\ I+2,\ I+3$. Available data for excited bands, and even for ground bands in ^{238}Pu, do not fulfill this condition. However, the existent data for excitation energies are very well described by the results of our calculations.

In the interval of angular momentum, where the parity partner bands have an interleaved structure, the states may be considered as members of a single band. In order for this to be possible, the moments of inertia of the bands of opposite parities must be equal to each other. This restriction is achieved if the discrete derivative of energy in terms of $J(J+1)$ in the composite band is equal to the energy derivative in the positive parity band. Indeed, the standard definition of the energy displacement function can be written in the alternative form:

$$\delta E(J^-)=2J\left[\frac{E(J^-)-E[(J-1)^+]}{2J}-\frac{E[(J+1)^+]-E[(J-1)^+]}{2(2J+1)}\right]. \qquad (17)$$

Plotting the energies of the parity partner bands as a function of angular momenta, one notices that the bands of distinct parities lie on different curves up to a certain angular momentum when these bands intersect each other. From this angular momentum on, the two bands may form a single band for a short interval and then go apart. For the first interval, both derivatives, from the above expression of the energy displacement function, are positive quantities; and moreover, since the negative parity band is higher in energy than the parity partner band, the displacement function is a positive quantity. Within the third interval mentioned before, the negative parity band lies below the positive parity one, and therefore the first derivative is negative while the second term remains positive (since the energy in the positive band is an increasing function of angular momentum). Therefore, the displacement function becomes negative in this interval of angular momentum. Actually, this change of sign for the displacement function has been observed for some isotopes of Ra and Th. To conclude, the change of sign for the energy displacement function is caused by the fact that the slope of the function $E(J^-)$ is smaller than that of $E(J^+)$, which results in determining a moment of inertia for the total band (the sum of the two partner bands) larger than that for the positive parity band. In the present formalism, the moderate slope of energy, as function of J, in the negative parity band is caused by the $\mathcal{B}_1$- coupling term which, according to Table I, is attractive. Indeed, for large values of the quadrupole deformation parameter, the normalized energies in the positive parity bands are almost unchanged by the coupling term, while the negative parity state energies are drastically affected (see Fig. 3 of Ref. [9]).

Were the moments of inertia different from each other and, moreover, independent functions of angular momentum, the intersection of the partner bands would take place in a point and the octupole deformation would not be stabilized. A smooth matching of the energy curves is possible only if one admits that the moments of inertia are angular momentum dependent functions.

As a matter of fact, in this belief underlies the proposal for a new displacement function. Indeed, equating it to zero, one obtains the necessary condition that the two bands are characterized by the same coefficient B defined by Eq. (15), since the new displacement function is nothing else but the difference of the fourth-order derivatives with respect to the angular momentum, of the energies in the partner bands. The beat pattern feature shown in Figs. 2–4 suggests that the coefficient B quoted above is not a constant but a periodic function of angular momentum, which, in fact, is consistent with the complex J dependence of partner band energies revealed by our formalism.

If the moments of inertia in the partner bands converge to a common constant limit when J approaches a certain critical value J_0, the static octupole deformation is set on for J_0 and lasts for any J larger than J_0. It seems that this is the case for most theoretical models, since they predict energies which, for large angular momenta, behave like $aJ(J+1)+b$.

The conclusion of this paper is that studying octupole shapes of nuclei is an appealing subject not only in conjecture of the ground band but also of that of excited bands. Indeed, here we showed examples where the ground band has no octupole deformation whereas the β and especially γ bands may acquire such an equilibrium shape for certain angular momenta.

[1] R.R. Chassman, Phys. Rev. Lett. **42**, 630 (1979); Phys. Lett. **96B**, 7 (1980).
[2] P. Moller and J.R. Nix, Nucl. Phys. **A361**, 117 (1981).
[3] S.G. Rohozinski, Rep. Prog. Phys. **51**, 541 (1988).
[4] P.A. Butler and W. Nazarewicz, Rev. Mod. Phys. **68**, 349 (1996).
[5] S. Frauendorf, Rev. Mod. Phys. **73**, 463 (2001).
[6] A.A. Raduta, Al.H. Raduta, and Amand Faessler, Phys. Rev. C **55**, 1747 (1997).
[7] A.A. Raduta, Al.H. Raduta, and Amand Faessler, J. Phys. G **23**, 149 (1997).
[8] A.A. Raduta, Amand Faessler, and R.K. Sheline, Phys. Rev. C **57**, 1512 (1998).
[9] A.A. Raduta, D. Ionescu, and Amand Faessler, Phys. Rev. C **65**, 064322 (2002).
[10] A.A. Raduta, V. Ceausescu, A. Gheorghe, and R.M. Dreizler, Phys. Lett. **99B**, 444 (1981).
[11] A.A. Raduta, V. Ceausescu, A. Gheorghe, and R.M. Dreizler, Nucl. Phys. **A381**, 253 (1982).
[12] R.V. Jolos and P. von Brentano, Phys. Rev. C **49**, R2301 (1994); Nucl. Phys. **A587**, 377 (1995).
[13] A.A. Raduta, V. Ceausescu, G. Stratan, and A. Sandulescu, Phys. Rev. C **8**, 1525 (1973).
[14] V. Ceausescu and A.A. Raduta, Prog. Theor. Phys. **52**, 903 (1974).
[15] V. Ceausescu and A.A. Raduta, Ann. Phys. (N.Y.) **100**, 94 (1976).
[16] N.V. Zamfir and D. Kusnezov, Phys. Rev. C **63**, 054306 (2001).
[17] W. Nazarewicz and P. Olanders, Nucl. Phys. **A441**, 420 (1985); Phys. Rev. C **32**, 602 (1985).
[18] D. Bonatsos *et al.*, Phys. Rev. C **62**, 024301 (2000).
[19] J.F.C. Cocks *et al.*, Phys. Rev. Lett. **78**, 2920 (1997).
[20] I. Hamamoto, J. Holler, and X.Z. Zhang, Phys. Lett. B **226**, 17 (1989).
[21] From NNDC(BNL) program ENSDAT, Nucl. Data Sheets **80**, 723 (1997).
[22] From NNDC(BNL) program ENSDAT, Nucl. Data Sheets **63**, 139 (1991).
[23] From NNDC(BNL) program ENSDAT, Nucl. Data Sheets **71**, 181 (1994).
[24] From NNDC(BNL) program ENSDAT, Nucl. Data Sheets **63**, 183 (1991).
[25] J.G. Alessi, J.X. Saladin, C. Baktash, and T. Humanic, Phys. Rev. C **23**, 79 (1981).
[26] N. Schulz *et al.*, Phys. Rev. Lett. **63**, 2645 (1989).
[27] E.N. Shurshikov, Nucl. Data Sheets **53**, 601 (1988).
[28] P. Moller, J.R. Nix, W.D. Meyers, and W.J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995).
[29] G.A. Lalazisis, S. Raman, and P. Ring, At. Data Nucl. Data Tables **71**, 1 (1999).
[30] J.F.C. Cocks *et al.*, Nucl. Phys. **A645**, 63 (1999).