The reaction $\Delta+N\rightarrow N+N+\phi$ in ion-ion collisions

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(Received 6 May 2002; published 28 April 2003)

We study the threshold ϕ-meson production in the process $\Delta+N\rightarrow N+N+\phi$, which appears as a possible important mechanism in high-energy nucleus-nucleus collisions. The isotopic invariance of the strong interaction and the selection rules due to P parity and total angular momentum result in a general and model independent parametrization of the spin structure of the matrix element in terms of three partial amplitudes. In the framework of one-pion exchange model these amplitudes can be derived in terms of two threshold partial amplitudes for the process $\pi+N\rightarrow N+\phi$ and three amplitudes for the subprocess $\pi+\Delta\rightarrow N+\phi$. We predict the ratio of cross sections for ϕ-meson production in pp and ΔN collisions and the polarization properties of the ϕ meson, in $\Delta+N\rightarrow N+N+\phi$.

DOI: 10.1103/PhysRevC.67.044004 PACS number(s): 21.30.−x, 14.40.−n, 13.90.+i, 25.40.Ve

I. INTRODUCTION

Experimental [1] and theoretical [2–6] studies devoted to ϕ-meson production in nucleon-nucleon collisions have witnessed a growing interest in recent years. In particular, efforts have been focused on the understanding of the reaction mechanism for $N+N\rightarrow N+N+\phi$, which is one of the simplest processes of vector meson production in NN collisions. The standard approach, based on one-boson exchanges in t channel (the peripheral picture) is not unique, in particular in the threshold region. A picture based on central NN collisions, with s-channel excitation of six-quark bags [7], can also be applied to vector meson production in NN collisions [8].

It has been pointed out that the process $N+N\rightarrow N+N+\phi$, and in particular the study of polarization phenomena in $p+p\rightarrow p+p+\phi$ can bring information on the hidden strangeness degrees of freedom in the nucleon [6,9]. This problem is related with the possible violation of the Okubo-Zweig-Iizuka (OZI) rule [10] for various processes of hadronic ω and ϕ production. In the framework of specific models [3–5] in the near- threshold region, it would be possible to determine or to constrain the coupling constants of the $NN\phi$ interaction, presently unknown.

A suggestion was made [11] that a strong enhancement of the ϕ/ω ratio in heavy ion collisions, in comparison with the value measured in $p+p$ collisions, could be interpreted as a signature of quark-gluon plasma formation. Search for such signal has been done in the CERN experiment NA38 [12].

These examples show that the comprehension of the elementary process $N+N\rightarrow N+N+\phi$ is important for nucleon-nucleus collisions and nucleus-nucleus collisions, as well. However, the reaction $\Delta+N\rightarrow N+N+\phi$, where the nucleon participates in its first excited state, may also contribute to ϕ production in nuclei. At bombarding energies of about 1 GeV/nucleon, a particular state of nuclear matter can be formed [13], where about 1/3 of baryons are excited to the Δ-resonance state. It follows then that the ϕ meson in $p+A$ or $A+A$ collisions can be produced in non-negligible amount through the process $\Delta+N\rightarrow N+N+\phi$ [14–16], or even $\Delta+\Delta\rightarrow N+N+\phi$. The importance of ΔN reactions for the η-meson production [17] and for associative strange particle production $\Delta+N\rightarrow N+\Lambda+K$ has been also discussed [18].

The aim of this paper is to study ϕ production in the process $\Delta+N\rightarrow N+N+\phi$, in particular in the near-threshold region. The threshold region is interesting from an experimental point of view and the theoretical approach can be essentially simplified. The presence of the Δ isobar, with 3/2 spin, makes the situation more complicated than in case of the nucleon, but also for the $\Delta+N$ interaction it is possible to develop a formalism, similar to ϕ-meson production in NN interaction [19]. The symmetry properties of the strong interaction, such as the Pauli principle for the nucleons, the P invariance, the isotopic invariance, and the selection rules for the total angular momentum, allow to define the spin structure of the matrix element for $\Delta+N\rightarrow N+N+\phi$ in terms of a small number of partial amplitudes. This approach is especially adapted to the near-threshold region, where all final particles are produced in S state. The existing experimental data about ϕ and ω production in NN collisions [1,20–27] show that this region is quite wide. There is no particular reason to expect a different behavior in case of ϕ production in $\Delta+N$ collisions. Taking into account the hidden strangeness of the ϕ meson, due to the relatively small *strange* radius of the nucleon [28], the S wave region for ϕ production should be larger than for ω production. A similar expectation has been also confirmed by experimental data on associative strange particle production in NN collisions, in the near-threshold region [29–32].

This paper is organized as follows. In the second section we establish the spin structure of the threshold matrix for the process $\Delta+N\rightarrow N+N+\phi$. Due to the isotopic invariance of the strong interaction, this spin structure must be the same

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for all possible combinations of the electric charge of baryons: $\Delta^{++}+n\to p+p+\phi$, $\Delta^{+}+p\to p+p+\phi$, $\Delta^{0}+p\to n+p+\phi$, $\Delta^{-}+p\to n+n+\phi$, $\Delta^{0}+n\to n+n+\phi$, and $\Delta^{+}+n\to n+p+\phi$, because all these processes, in the s channel, have a single value of the total isotopic spin, $I=1$, and a single value of the total isospin of the produced NN system, as well. The isotopic invariance is also very important with respect to the selection rules related to the P parity and the total angular momentum. In Sec. III, using the spin parametrization of the matrix element, we show that any observable for the process $\Delta+N\to N+N+\phi$ can be calculated in terms of three independent partial amplitudes, which characterize this process in the threshold region. From all possible polarization observables, only the density matrix of the ϕ meson has a physical meaning, as this process can, in principle, occur only in nuclear matter. After this model independent analysis, we consider the one-boson model, with π exchange, taking into account the two mechanisms related to the subprocesses $\pi+N\to N+\phi$ and $\pi+\Delta\to N+\phi$. Taking into account the main, i.e., the ρ-exchange contribution to the matrix elements of these subprocesses, we show that the total cross section for $\Delta+N\to N+N+\phi$ is larger than the cross section for $p+p\to p+p+\phi$, at the same value of the energy excess over threshold, Q. The results are summarized in Sec. IV.

II. SPIN STRUCTURE OF THRESHOLD MATRIX ELEMENT

Considering the Δ isobar as a particle with spin 3/2 and positive P parity, it is possible to find the spin structure of the process $\Delta+N\to N+N+\phi$ in threshold regime. The threshold region can be rigorously defined in the center of mass system (c.m.s.) of the considered reaction in terms of orbital angular momenta of the produced particles. If ℓ_1 is the relative orbital momentum of the final nucleons and ℓ_2 the orbital momentum of the ϕ meson (relative to the center of mass of the $2N$ system), the condition of the S wave production of any pair of particles for the $NN\phi$ final state can be written as $\ell_1=\ell_2=0$.

The spin structure of the threshold matrix element for any process $\Delta+N\to N+N+\phi$ can be established in a general form, from the selection rules in isotopic spin, P parity, and the total angular momentum. From the isotopic invariance of the strong interaction, one can find that the pair of final nucleons in all reactions $\Delta+N\to N+N+\phi$ (for any charge combination, pp, nn, or np) has to be in singlet state, following the Pauli principle (for the pp or nn system) or the generalized Pauli principle (for the np system). Due to the conservation of the total isotopic spin I the final NN system must have $I=1$. Therefore, the total angular momentum $\mathcal{J}$ and the P parity of the entrance channel can take only the value $\mathcal{J}^P=1^-$. Taking into account the conservation of $\mathcal{J}$ and P, one can find the following allowed partial transitions for the process $\Delta+N\to N+N+\phi$:

$$
\begin{aligned}
&S_i=1,\ \ell=1\to\mathcal{J}^P=1^-,\\
&S_i=2,\ \ell=1\to\mathcal{J}^P=1^-,\\
&S_i=2,\ \ell=3\to\mathcal{J}^P=1^-,
\end{aligned}
\tag{1}
$$

where S_i is the total spin of the initial $\Delta+N$ system and ℓ is the angular orbital momentum of this system.

The threshold matrix element, in the c.m.s. of the considered reaction, can be parametrized in the following general form:

$$
\begin{aligned}
\mathcal{M}=&\, i f_1(\tilde{\chi}_1\sigma_y\vec{\Delta}\cdot\hat{\vec{k}}\times\vec{U}^*)(\chi_3^\dagger\sigma_y\tilde{\chi}_2^\dagger)+f_2(\tilde{\chi}_1\sigma_y\vec{\sigma}\cdot\vec{U}^*\vec{\Delta}\cdot\hat{\vec{k}})\\
&\times(\chi_3^\dagger\sigma_y\tilde{\chi}_2^\dagger)+f_3(\tilde{\chi}_1\sigma_y\vec{\sigma}\cdot\hat{\vec{k}}\vec{\Delta}\cdot\hat{\vec{k}})(\chi_3^\dagger\sigma_y\tilde{\chi}_2^\dagger)\hat{\vec{k}}\cdot\vec{U}^*,
\end{aligned}
\tag{2}
$$

where χ_1 (χ_2 and χ_3) is the two-component spinor of the initial (final) nucleon; $\vec{\Delta}$ is a particular two-component spinor (each component is a vector) for the description of the polarization properties of the Δ isobar (with spin 3/2), so that $\vec{\sigma}\cdot\vec{\Delta}=0$; $\vec{U}$ is the three-vector of the V-meson polarization; and $\hat{\vec{k}}$ is the unit vector along the three-momentum of the Δ. f_1–f_3 are the threshold partial amplitudes describing the allowed transitions (1).

The presence of the Pauli matrix σ_y in the parametrization (2) ensures the correct transformation properties of the corresponding two-component spinor products, relative to rotation.

Taking into account the conservation of isospin, one obtains

$$
\begin{aligned}
-\mathcal{M}(\Delta^+p\to pp\phi)&=\frac{1}{\sqrt{3}}\mathcal{M}(\Delta^{++}n\to pp\phi)\\
&=\frac{1}{\sqrt{3}}\mathcal{M}(\Delta^{-}n\to nn\phi)\\
&=-\mathcal{M}(\Delta^{+}n\to np\phi)\\
&=-\mathcal{M}(\Delta^{0}p\to np\phi)=\mathcal{M}(\Delta^{0}n\to nn\phi),
\end{aligned}
$$

i.e., the same set of three amplitudes f_1–f_3 describes all these processes and therefore the polarization phenomena are identical for all reactions $\Delta+N\to N+N+\phi$. Moreover, the following relations among the total cross sections hold:

$$
\sigma(\Delta^+p\to pp\phi):\sigma(\Delta^0p\to np\phi):\sigma(\Delta^-p\to nn\phi)=1:2:3
$$

and the isospin averaged cross section $\overline{\sigma}(\Delta N\to NN\phi)$ is $\overline{\sigma}(\Delta N\to NN\phi)=\frac{1}{2}\sigma(\Delta^-p\to nn\phi)$. Note that all these isotopic relations hold for any kinematical condition of the considered process and for any reaction mechanism.

After summing over the polarizations of the final particles and averaging over the polarizations of the colliding baryons, one can find the following formula for the differential cross section of the considered process, in terms of the partial amplitudes f_1–f_3:

$$\frac{d\sigma}{d\omega}=\frac{1}{6}\mathcal{N}(|f_1-f_2|^2+3|f_1+f_2|^2+2|f_2+f_3|^2), \qquad (3)$$

$$\mathcal{N}=4m^2(E_N+m)(E_\Delta+M),$$

where $d\omega$ is the element of phase space for the three-particle final state. E_N and E_Δ are the energies of the colliding N and Δ at threshold, in c.m.s.:

$$E_N=\frac{W_{th}^2+m^2-M^2}{2W_{th}},\quad E_\Delta=\frac{W_{th}^2-m^2+M^2}{2W_{th}},$$

$$W_{th}=2m+m_\phi,$$

where m, M, and m_ϕ are the masses of N, Δ, and ϕ meson, respectively. We used the following expression for the density matrix of the unpolarized Δ isobar:

$$\rho_{ab}^{(\Delta)}=\overline{\Delta_a\Delta_b^\dagger}=\frac{2}{3}\left(\delta_{ab}-\frac{i}{2}\epsilon_{abc}\sigma_c\right),$$

where the overline denotes the sum over the Δ polarizations. The factor $\mathcal{N}$ in Eq. (3) results from the invariant normalization of the four-component spinors for all baryons in $\Delta+N\to N+N+\phi$.

The general parametrization of the threshold matrix element, Eq. (2), allows to calculate the polarization observables for the considered processes. Taking into account the specificity of this reaction, which can only occur as a second step in ion-ion collisions, the most interesting (and measurable) polarization observable is the density matrix of the ϕ meson, which can be written (for S-state final particles) as

$$\rho_{ab}^{(\phi)}=\hat{k}_a\hat{k}_b+\mathcal{A}(\delta_{ab}-3\hat{k}_a\hat{k}_b),$$

where $\mathcal{A}$ is a real coefficient that drives the angular dependence $W(\theta)$ of the decay products of the ϕ meson. For example, in case of $\phi\to K\bar{K}$ decay, one finds

$$W(\theta)\simeq 1+\mathcal{B}\cos^2\theta,$$

$$\mathcal{B}=\frac{1-3\mathcal{A}}{\mathcal{A}}=-1+\frac{4|f_2+f_3|^2}{|f_1-f_2|^2+3|f_1+f_2|^2}, \qquad (4)$$

where θ is the K-meson production angle (in ϕ-rest system) with respect to the direction of the initial Δ momentum. Therefore the numerical value of the coefficient $\mathcal{B}$ (or $\mathcal{A}$, as well) is determined by the mechanism of ϕ production through the amplitudes f_i, $i=1$–3. One can see from Eq. (4) that, in threshold conditions, the ϕ meson can be only tensorially polarized, for any reaction mechanism.

III. THE DYNAMICS FOR THE *t* CHANNEL

The parametrization of the spin structure of the threshold matrix elements given above is based on fundamental symmetry properties of the strong interaction. It is therefore model independent and can be applied to any reaction mechanism. Following the analogy with the process $p+p\to p+p+\phi$, we will consider here different t-channel exchanges (Fig. 1), where only states with $I=1$ are allowed: π, ρ, etc.

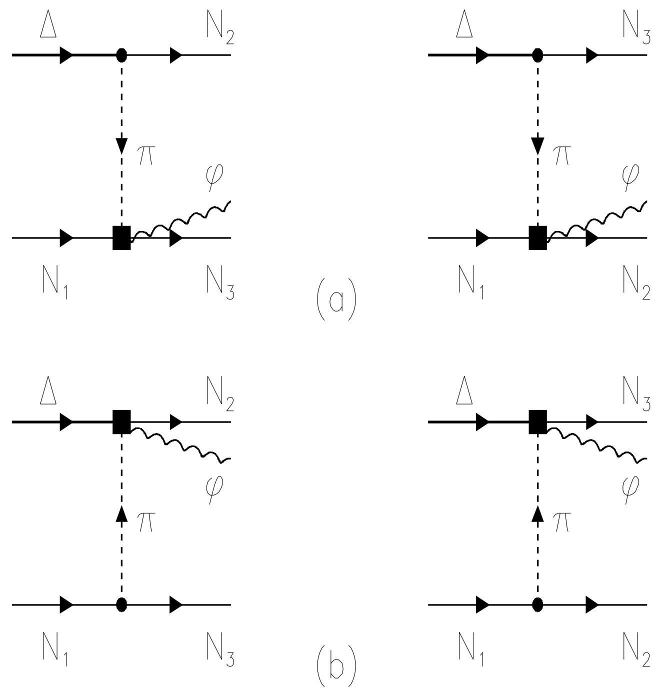

FIG. 1. Feynman diagrams for t-channel π exchanges, corresponding to the subprocess $\pi+N\to N+\phi$ (a), and $\pi+\Delta\to N+\phi$ (b).

The spin structure of the corresponding matrix elements for the subprocesses $\pi+N(\Delta)\to N+\phi$ can be established using the conservation of the total angular momentum and P parity.

A. Subprocess $\pi+N\to N+\phi$

1. Relations between the amplitudes of $\Delta+N\to N+N+\phi$ and $\pi+N\to N+\phi$

For the process $\pi+N\to N+\phi$, at the reaction threshold, where the ϕ meson is produced in the S state, the following partial transitions are allowed: $\ell_\pi=0\to\mathcal{J}^P=1/2^-$ and $\ell_\pi=2\to\mathcal{J}^P=3/2^-$ (ℓ_π is the pion orbital momentum), so the matrix element can be parametrized as follows:

$$\mathcal{M}_{th}(\pi N\to N\phi)=\chi_2^\dagger(h_1\vec{\sigma}\cdot\vec{U}^*+h_2\vec{\sigma}\cdot\hat{\vec{k}}\,\hat{\vec{k}}\cdot\vec{U}^*)\chi_1, \qquad (5)$$

where h_1 and h_2 are the two complex partial amplitudes.

Let us consider the contribution of the diagrams in Fig. 1(a), which are dominated by the process $\pi+N\to N+\phi$. The corresponding matrix element (taking into account the necessary antisymmetrization with respect to the produced nucleons in $\Delta+N\to N+N+\phi$) can be written as

$$\mathcal{M}_a=\mathcal{M}_{1a}-\mathcal{M}_{2a},$$

$$\mathcal{M}_{1a}=\frac{g_{\Delta N\pi}}{t_1-m_\pi^2}(\chi_2^\dagger\vec{\Delta}\cdot\hat{\vec{k}})[\chi_3^\dagger(h_1\vec{\sigma}\cdot\vec{U}^*+h_2\vec{\sigma}\cdot\hat{\vec{k}}\hat{\vec{k}}\cdot\vec{U}^*)\chi_1]\frac{k}{M}F_{\pi N\Delta}(t_1),$$

$$\mathcal{M}_{2a}=\frac{g_{\Delta N\pi}}{t_1-m_\pi^2}(\chi_3^\dagger\vec{\Delta}\cdot\hat{\vec{k}})[\chi_2^\dagger(h_1\vec{\sigma}\cdot\vec{U}^*+h_2\vec{\sigma}\cdot\hat{\vec{k}}\hat{\vec{k}}\cdot\vec{U}^*)\chi_1]\frac{k}{M}F_{\pi N\Delta}(t_1), \quad (6)$$

where $g_{\Delta N\pi}$ is the $\Delta N\pi$ coupling constant, which determines the corresponding width for the decay $\Delta\to N+\pi$, as

$$\Gamma(\Delta\to N\pi)=\frac{g_{\Delta N\pi}^2}{12\pi}\frac{q^3}{M^3}(E+m),\ q^2=E^2-m^2,$$

$$E=\frac{M^2-m_\pi^2+m^2}{2M}.$$

Taking $\Gamma(\Delta\to N\pi)=0.12$ GeV, $M=1.231$ GeV, $m=0.938$ GeV, and $m_\pi=0.139$ GeV, one finds $g_{\Delta^{++}p\pi^+}=19.3$. The function $F_{\pi N\Delta}(t_1)$ is the form factor of the vertex $\pi^+p\to\Delta^{++}$, with a virtual pion.

In the threshold region for $\Delta+N\to N+N+\pi$, the pion propagators in $\mathcal{M}_{1a}$ and $\mathcal{M}_{2a}$ are identical. This approximation is consistent with the previous S-wave considerations. A possible difference in these propagators, which may appear outside the threshold region, generates P and higher partial waves of the produced $NN\phi$ system. Note that $t_1=(p_\Delta-p_2)^2=-mm_\phi+(M^2-m^2)[(m+m_\phi)/(2m+m_\phi)]=-0.527\ \mathrm{GeV}^2$, where p_Δ and p_2 are the four-momenta of the initial Δ and of the final nucleon.

Using the Fierz transformation, in its two-component form, one can transform the matrix element $\mathcal{M}_a$, Eq. (6), to the ''canonical'' (s-channel) parametrization, Eq. (2). For example, the two contributions to $\mathcal{M}_{1a}$ can be written as follows:

$$\begin{aligned}-(\chi_2^\dagger\vec{\Delta}\cdot\hat{\vec{k}})(\chi_3^\dagger\vec{\sigma}\cdot\vec{A}\chi_1)=\tfrac{1}{2}[&-(\tilde{\chi}_1\ \ \sigma_y\vec{\sigma}\cdot\vec{A}\vec{\Delta}\cdot\hat{\vec{k}})(\chi_3^\dagger\sigma_y\tilde{\chi}_2^\dagger)\\&-(\tilde{\chi}_1\sigma_y\vec{\Delta}\cdot\hat{\vec{k}})(\chi_3^\dagger\vec{\sigma}\cdot\vec{A}\sigma_y\tilde{\chi}_2^\dagger)\\&+i\epsilon_{abc}A_a(\tilde{\chi}_1\sigma_y\sigma_b\vec{\Delta}\cdot\hat{\vec{k}})\\&\times(\chi_3^\dagger\sigma_c\sigma_y\tilde{\chi}_2^\dagger)],\end{aligned} \quad (7)$$

for any vector $\vec{A}$. Replacing $\vec{A}=\vec{U}^*$ or $\vec{A}=\hat{\vec{k}}(\hat{\vec{k}}\cdot\vec{U}^*)$ one gets the formulas for the h_1 or the h_2 contribution. The corresponding expression for $\mathcal{M}_{2a}$ can be obtained from Eq. (7), with the substitution $\chi_2\leftrightarrow\chi_3$. So, the partial threshold amplitudes f_{1a}–f_{3a} of the process $\Delta+N\to N+N+\phi$, in the framework of the considered exchange [Fig. 1(a)] can be written as

$$f_{1a}=0,\ f_{2a}=h_1\frac{g_{\Delta N\pi}}{t_1-m_\pi^2}\frac{k}{M}F_{\pi N\Delta}(t_1),$$

$$f_{3a}=h_2\frac{g_{\Delta N\pi}}{t_1-m_\pi^2}\frac{k}{M}F_{\pi N\Delta}(t_1). \quad (8)$$

2. Analysis in terms of the amplitudes h_1 and h_2

Taking into account that in the considered model (with $f_{1a}=0$) the parameter $\mathcal{B}$, Eq. (4), has the form

$$1+\mathcal{B}=\frac{|f_{2a}+f_{3a}|^2}{|f_{2a}|^2},$$

one can find that the simultaneous measurements of the coefficient $\mathcal{B}$ (i.e., the angular distribution of the decay products $\phi\to K^+K^-$) and the differential cross section $d\sigma/d\omega$ allow to find the module of the amplitudes:

$$|f_{2a}|^2=\mathcal{N}^{-1}\frac{3}{\mathcal{B}+3}\frac{d\sigma}{d\omega},$$

$$|f_{2a}+f_{3a}|^2=\mathcal{N}^{-1}\frac{3(\mathcal{B}+1)}{\mathcal{B}+3}\frac{d\sigma}{d\omega}.$$

From Eqs. (2) and (8) one can see that both amplitudes h_1 and h_2 of the subprocess $\pi+N\to N+\phi$ contribute to $\mathcal{M}$ and the differential cross section for $\Delta+N\to N+N+\phi$ can be written (in a particular normalization) as

$$\frac{d\sigma}{d\omega}(\Delta N\to NN\phi)=\frac{\mathcal{N}}{3}\left(\frac{g_{\Delta N\pi}}{t_1-m_\pi^2}\right)^2(2|h_1|^2+|h_1+h_2|^2)\times\frac{k^2}{M^2}F_{\pi N\Delta}^2(t_1). \quad (9)$$

Note that the differential cross section for the subprocess $\pi+N\to N+\phi$ has the following form, in terms of the partial threshold amplitudes h_1 and h_2:

$$\frac{d\sigma}{d\Omega}(\pi N\to N\phi)\propto 2|h_1|^2+|h_1+h_2|^2. \quad (10)$$

Therefore, in the framework of the considered model of π exchange, the differential cross sections for $\Delta+N\to N+N+\phi$ and $\pi+N\to N+\phi$ are proportional in the near-threshold region, and the following relation holds between the cross section of ω and ϕ production in ΔN and πN collisions [33]:

$$\frac{\sigma(\Delta N\to NN\omega)}{\sigma(\Delta N\to NN\phi)}=\frac{\sigma(\pi N\to N\omega)}{\sigma(\pi N\to N\phi)}\mathcal{D}, \quad (11)$$

where

$$\mathcal{D}=\left[\frac{t_\phi-m_\pi^2}{t_\omega-m_\pi^2}\frac{F_{\pi N\Delta}(t_\omega)}{F_{\pi N\Delta}(t_\phi)}\right]^2,$$

$t_V=-mm_V+(M^2-m^2)[(m+m_V)/(2m+m_V)]$, $V=\omega$ or ϕ. One can see that the correction factor $\mathcal{D}$ is very important at threshold, due to the differences in V masses.

But it is not the case for the process $p+p\rightarrow p+p+\phi(\omega)$ or $n+p\rightarrow n+p+\phi(\omega)$, which can also be treated in the framework of one-pion exchange [4]. For the reaction $p+p\rightarrow p+p+\phi$, in the threshold region, the spin structure of the matrix element is characterized by a single contribution [19], corresponding to triplet-singlet transition, of the following form:

$$\mathcal{M}_{th}(pp\rightarrow pp\phi)=f(\tilde{\chi}_2^\dagger\sigma_y\vec{\sigma}\cdot\hat{\vec{k}}\times\vec{U}^*\chi_1)(\chi_4^\dagger\sigma_y\tilde{\chi}_3^\dagger)2m\times(E+m), \quad (12)$$

where f is the threshold amplitude, and E is the energy of the colliding protons in c.m.s.: $E=m+\frac{1}{2}m_\phi$, so that

$$\frac{d\sigma}{d\omega}(pp\rightarrow pp\phi)\propto 2|f|^2. \quad (13)$$

The π exchange for $p+p\rightarrow p+p+\phi$ is described by four Feynman diagrams (to have the correct symmetry properties with respect to the initial and final identical protons). It is possible to prove that

$$f=g_{\pi^0 pp}\frac{2h_1}{t'-m_\pi^2}\frac{k}{E+m}F_{\pi NN}(t'), \quad (14)$$

where $t'=-mm_\phi$ is the threshold value of the momentum transfer squared, for *elastic* ϕ production and $F_{\pi NN}(t')$ is the form factor for the vertex π^*NN with virtual pion. One can see that for the process $p+p\rightarrow p+p+\phi$ only the h_1 amplitude contributes, therefore the proportionality of cross sections for the processes $p+p\rightarrow p+p+\phi$ and $\pi+p\rightarrow p+\phi$, Eq. (11), does not hold (in the framework of one-pion exchange). Another relation holds in this case,

$$\frac{\sigma(pp\rightarrow pp\omega)}{\sigma(pp\rightarrow pp\phi)}=\frac{\sigma(\pi N\rightarrow N\omega)}{\sigma(\pi N\rightarrow N\phi)}\frac{1+r_\phi}{1+r_\omega}\mathcal{D}',$$

where

$$r_V=\left(\frac{|h_1+h_2|^2}{|h_1|^2}\right)_V \quad\text{and}\quad \mathcal{D}'=\left[\frac{t'_\phi-m_\pi^2}{t'_\omega-m_\pi^2}\frac{F_{\pi N\Delta}(t'_\omega)}{F_{\pi N\Delta}(t'_\phi)}\right]^2,$$

which shows that a separation of contributions due to the production of the V mesons with transversal and longitudinal polarization has to be done (for the process $\pi+N\rightarrow N+\phi$).

Therefore the study of polarization phenomena for the process $\pi+N\rightarrow N+\phi$, in the near-threshold region is required. The simplest observable is the tensor polarization of the ϕ meson, determined by the single parameter $\mathcal{B}^{(\pi)}$, which can be expressed in terms of the partial amplitudes h_1 and h_2 as

$$\mathcal{B}^{(\pi)}=-1+\frac{|h_1+h_2|^2}{|h_1|^2}.$$

Therefore the measurement of the unpolarized cross section $(d\sigma/d\Omega)_0$ and of the coefficient $\mathcal{B}^{(\pi)}$ allows to determine both amplitudes, through the following formulas:

$$|h_1|^2=\frac{1}{2(3+\mathcal{B}^{(\pi)})}\left(\frac{d\sigma}{d\Omega}\right)_0,$$

$$|h_1+h_2|^2=\frac{1+\mathcal{B}^{(\pi)}}{2(3+\mathcal{B}^{(\pi)})}\left(\frac{d\sigma}{d\Omega}\right)_0.$$

Using Eqs. (9), (10), (13), and (14), the total threshold cross sections of the processes $\Delta+N\rightarrow N+N+\phi$ and $p+p\rightarrow p+p+\phi$ can be expressed in terms of the coupling constants and of the ratio $x=h_2/h_1$ between the two possible amplitudes for the $\pi+N\rightarrow N+\phi$ process as

$$\mathcal{R}=\frac{\sigma(\Delta N\rightarrow NN\phi)}{\sigma(pp\rightarrow pp\phi)}=\frac{1.7}{12}\frac{g_{\Delta N\pi}^2}{g_{NN\pi}^2}\left(\frac{E+m}{M}\right)^2\left(\frac{t'-m_\pi^2}{t_1-m_\pi^2}\right)^2 g(x),$$

$$g(x)=3+2\,\mathrm{Re}\,x+|x|^2, \quad (15)$$

where in the integration over the final $2p$ system in $p+p\rightarrow p+p+\phi$ we have taken into account the identity of the final protons. The two cross sections have to be compared at the same value of Q, $Q=W_{th}-2m-m_\phi$. We assumed, in Eq. (15),

$$F_{\Delta N\pi}(t)=F_{NN\pi}(t)=\frac{\Lambda_\pi^2-m_\pi^2}{\Lambda_\pi^2-t}\quad\text{with}\quad\Lambda_\pi=1.0\ \text{GeV} \quad (16)$$

[34]. The coefficient 1.7, Eq. (15), is the ratio of form factors squared $[F_{\Delta N\pi}(t_1)/F_{NN\pi}(t')]^2$.

The complex parameter x drives also the coefficient $\mathcal{B}$ for the decay $\phi\rightarrow K^+K^-$, see Eq. (4), so that $3+\mathcal{B}=g(x)$. One can see that these two observables are not independent, in the framework of the considered model.

Figure 2 shows the x dependence of $g(x)$ as a function of the two independent variables, $\mathrm{Re}\,x$ and $|x|^2$, with the evident conditions $|\mathrm{Re}\,x|\leqslant|x|,|x|\geqslant 0$, $\mathrm{Re}\,x$ being positive or negative.

Comparing the threshold values for t_1 and $t'=-mm_\phi$, one can find that the pion pole in $\Delta+N\rightarrow N+N+\phi$ is closer to the physical region, and the ratio of propagators,

$$\left(\frac{mm_\phi+m_\pi^2}{t_1-m_\pi^2}\right)^2\simeq 3.2,$$

will increase the relative value of the ΔN cross section. Due to the instability of Δ, this pion pole, for $\Delta+N\rightarrow N+N$

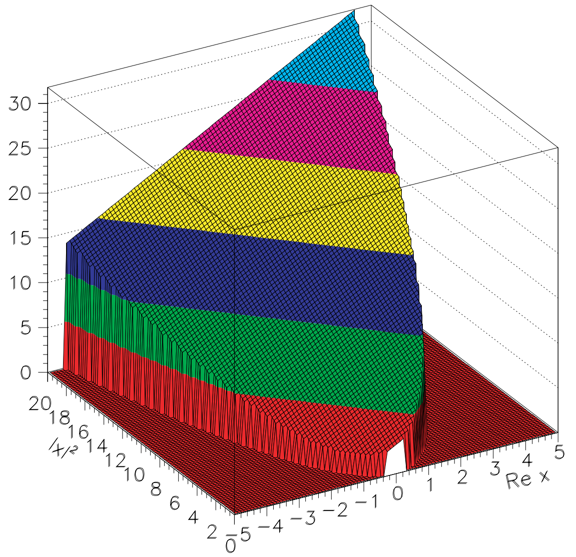

FIG. 2. The x dependence of $g(x)$ as a function of Re x and $|x|^2$.

$+\phi$ could be, in principle, in the physical region, for some kinematical interval [17]. However, as illustrated in Fig. 3, this cannot happen in the near-threshold region, and $t_1 \geqslant m_\pi^2$ in a wide region 3.14 GeV $\leqslant W \leqslant$ 10.00 GeV, W is the total invariant energy of the colliding particles.

Taking $\Gamma(\Delta \to N\pi) \simeq 120$ MeV and $g_{\pi NN}^2/4\pi = 15$ one can find $\mathcal{R}>1$, in the framework of the considered π-exchange mechanism, Fig. 1(a). A more accurate estimation for $d\sigma/d\omega$ and $\mathcal{R}$ could be done, knowing the threshold partial amplitudes h_1 and h_2. In literature [2] it is possible to find an estimation of the energy dependence of the total cross

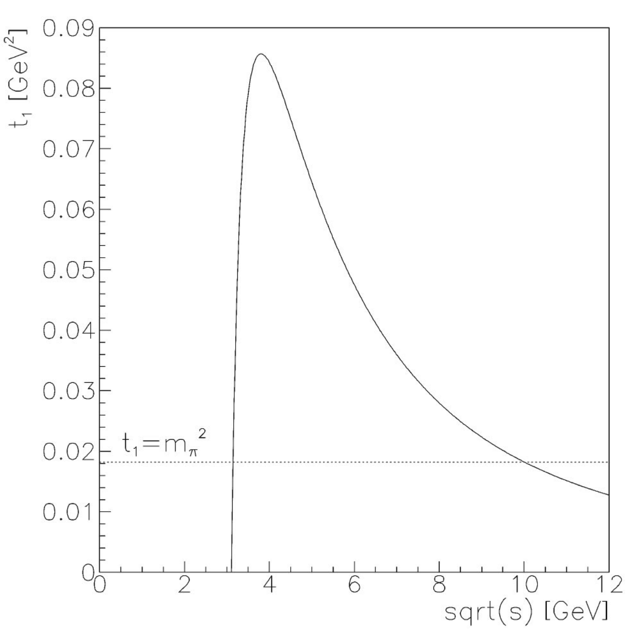

FIG. 3. Dependence of the momentum transfer squared t_1 on $\sqrt{s}$ (in the region $t_1 \geqslant 0$).

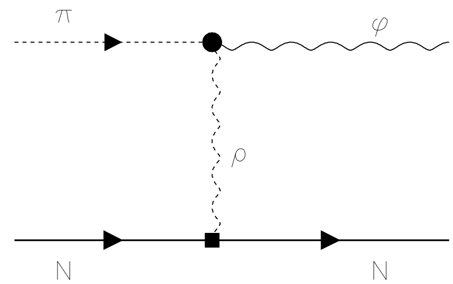

FIG. 4. Feynman diagram for t-channel ρ exchange for the process $\pi + N \to N + \phi$.

section for the subprocess $\pi + N \to N + \phi$ (with unpolarized particles) in the framework of a very specific resonance model where a single nucleon exhibits a ''resonancelike'' contribution. The mass and width of this ''effective'' resonance were determined by fitting the available experimental data [23]. Such model, however, cannot help us to determine the partial amplitudes h_1 and h_2 separately, without additional assumptions, concerning the spin and parity of the resonance R and its coupling constants for the vertex $R \to N + \phi$.

3. *ρ-exchange model for subprocess* $\pi + N \to N + \phi$

In principle, it is possible to develop a more complicated model for the $\pi + N \to N + \phi$ process, which takes into account different contributions: vector meson exchange in t channel, N and N^* contributions in s and u channels [35]. But even near the reaction threshold several nucleon resonances, with $\mathcal{J}^P = 1/2^-$ and $3/2^-$, can contribute, introducing unknown parameters: coupling constants and form factors. Only in the framework of the quark model definite predictions can be done.

Let us apply the ρ exchange as the most probable mechanism for the ''elementary'' subprocess $\pi + N \to N + \phi$, Fig. 4, to estimate the value of the ratio x of the amplitudes h_1 and h_2. One can easily see that, in the near-threshold region, such model gives $x = -1$ and the minimal value for the ratio of cross sections $\mathcal{R}$. In this case one finds $\mathcal{B} = -1$, with a $\sin^2\theta$ angular dependence of the decay products in $\phi \to K + \bar{K}$, which is an evident property of the $\pi\rho\phi$ vertex. This last result does not depend on the different ingredients of the considered reaction mechanism, such as the coupling constants and the phenomenological form factors in the hadronic vertexes, but it depends on the mechanism chosen for the process $\pi + N \to N + \phi$. Note that, following the analysis [3] of the process $p + p \to p + p + \phi$, in the near-threshold region, the ρ exchange is the leading mechanism, which can be justified by a small $g_{NN\phi}$ coupling constant with respect to $g_{\pi\rho\phi}$: $|g_{NN\phi}|/|g_{\pi\rho\phi}| < 1$ [3].

The threshold matrix element for the subprocess $\pi^+ + n \to p + \phi$ (with a virtual pion, with four-momentum squared t_1), corresponding to ρ exchange, has the following form:

$$\mathcal{M}(\pi^* N \to N\phi) = \sqrt{2}(E_N - m)\frac{g_{\pi\rho\phi}}{t_2 - m_\rho^2} g_{\rho NN}(1+\kappa_\rho) \times F_{\rho NN}(t_2) F_{\pi\rho\phi}(t_1, t_2) \times \chi_p^\dagger (\vec{\sigma}\cdot\vec{U}^* - \vec{\sigma}\cdot\hat{\vec{k}}\,\vec{U}^*\cdot\hat{\vec{k}})\chi_n, \quad (17)$$

where $t_2=(p_1-p_2)^2=-mm_\phi+(M^2-m^2)[m_\phi/(2m+m_\phi)]=-0.75\ \mathrm{GeV}^2$; $g_{\rho NN}$ is the vector coupling constant for the vertex ρ^0NN; κ_ρ is the ratio of the tensor and vector coupling constants for this vertex; the coefficient $\sqrt{2}$ takes into account the relation between the couplings in ρ^+np and ρ^0pp vertexes; $F_{\rho NN}(t_2)$ is the phenomenological form factor for the vertex ρNN with virtual ρ meson; and $F_{\pi\rho\phi}(t_1,t_2)$ is the form factor for the $\pi^*\rho^*\phi$ vertex, where the π (ρ) virtuality is characterized by the four-momentum square t_1 (t_2). We take the following normalization of these form factors:

$$F_{\rho NN}(m_\rho^2)=F_{\pi\rho\phi}(m_\pi^2,m_\rho^2)=1. \tag{18}$$

The coupling constant $g_{\pi\rho\phi}$ for the $\pi\rho\phi$ vertex enters in the corresponding Lagrangian:

$$\mathcal{L}_{\pi\rho\phi}=\frac{g_{\pi\rho\phi}}{m_\phi}\epsilon_{\mu\nu\alpha\beta}\partial^\mu\vec{\rho}^{\,\nu}(x)\partial^\alpha\vec{\pi}(x)\phi^\beta(x), \tag{19}$$

where $\epsilon_{\mu\nu\alpha\beta}$ denotes the Levi-Civita antisymmetric tensor, with $\epsilon_{0123}=+1$.

Using Eqs. (17) and (19) one can find the following expressions for the partial amplitudes f_{ia}, $i=1$–3, for the pion exchange mechanism, illustrated in Fig. 1(a):

$$f_{ia}=0,\quad f_{2a}=-f_{3a}=A_Nh_{1N},$$

$$h_{1N}=\sqrt{2}(E_N-m)\frac{g_{\pi\rho\phi}}{t_2-m_\rho^2}g_{\rho NN}(1+\kappa_\rho)\times F_{\rho NN}(t_2)F_{\pi\rho\phi}(t_1,t_2) \tag{20}$$

and

$$A_N=\frac{g_{\Delta^{++}p\pi^+}}{t_1-m_\pi^2}\frac{k}{M}F_{\Delta N\pi}(t_1). \tag{21}$$

B. Subprocess $\pi+\Delta\to N+\phi$

Relations between the amplitudes of $\Delta+N\to N+N+\phi$ and $\pi+\Delta\to N+\phi$

At the reaction threshold, the process $\pi+\Delta\to N+\phi$ is characterized by three possible partial transitions: $\ell_\pi=0\to\mathcal{J}^P=3/2^-$, $\ell_\pi=2\to\mathcal{J}^P=1/2^-$, and $3/2^-$, with the following matrix element:

$$\mathcal{M}_{th}(\pi\Delta\to N\phi)=h_{1\Delta}(\chi^\dagger\vec{\Delta}\cdot\vec{U}^*)+h_{2\Delta}(\chi^\dagger\vec{\Delta}\cdot\hat{\vec{k}})\hat{\vec{k}}\cdot\vec{U}^*+ih_{3\Delta}(\chi^\dagger\vec{\sigma}\cdot\hat{\vec{k}}\vec{\Delta}\cdot\hat{\vec{k}}\times\vec{U}^*), \tag{22}$$

where $h_{1\Delta}$, $h_{2\Delta}$, and $h_{3\Delta}$ are the threshold partial amplitudes for the process $\pi+\Delta\to N+\phi$, and χ is the two-component spinor of the final nucleon.

This allows to write the corresponding contribution $\mathcal{M}_b$ to the matrix element of the process $\Delta^{++}+n\to p+p+\phi$ as follows:

$$\mathcal{M}_b=\mathcal{M}_{1b}-\mathcal{M}_{2b},$$

where $\mathcal{M}_{1b}$ can be expressed in terms of $\mathcal{M}_{th}(\pi\Delta\to N\phi)$ as

$$\mathcal{M}_{1b}=-\sqrt{2}g_{\pi NN}\frac{k}{E_N+m}(\chi_2^\dagger\vec{\sigma}\cdot\hat{\vec{k}}\chi_1)F_{\pi NN}(t_2)\times\frac{\mathcal{M}_{th}(\pi\Delta\to N\phi)}{t_2-m_\pi^2}. \tag{23}$$

The factor $k/(E_N+m)$ results from the transformation of the relativistic vertex πNN into its two-component equivalent.

Taking into account both diagrams for this type of π exchange [Fig. 1(b)], one can find the spin structure for $\mathcal{M}_b$, when the final nucleons are emitted in singlet state, and the following relations between the partial amplitudes $f_{ib}, i=1$–3, for the process $\Delta^{++}+n\to p+p+\phi$ and $h_{i\Delta}, i=1$–3, for the process $\pi+\Delta\to N+\phi$:

$$f_{1b}=A_\Delta(h_{1\Delta}-h_{3\Delta}),\quad f_{2b}=-A_\Delta h_{1\Delta},\quad f_{3b}=-A_\Delta h_{2\Delta} \tag{24}$$

with

$$A_\Delta=-\sqrt{2}g_{\pi NN}F_{\pi NN}(t_2)\frac{1}{t_2-m_\pi^2}\frac{k}{E_N+m},$$

where the coefficient $\sqrt{2}$ results from the vertex $n\to p+\pi^-$.

C. ρ-exchange model for subprocess $\pi+\Delta\to N+\phi$

Again, taking into account that $|g_{NN\phi}|\ll|g_{\pi\rho\phi}|$, we can estimate the amplitudes $h_{i\Delta}, i=1$–3, in frame of ρ exchange only. The corresponding matrix element can be written as follows (for virtual pion, with four-momentum square t_2):

$$\mathcal{M}_\rho(\pi^-\Delta^{++}\to p\phi)=\frac{g_{\pi\rho\phi}}{t_1-m_\rho^2}\frac{g_{\Delta^{++}p\rho^+}}{m+M}k^2F_{\Delta N\rho}(t_1)\times F_{\pi\rho\phi}(t_1,t_2)(\chi^\dagger\vec{\Delta}\times\hat{\vec{k}}\cdot\vec{U}^*\times\hat{\vec{k}}), \tag{25}$$

where $g_{\Delta^{++}p\rho^+}$ is the coupling constant for the $\Delta N\rho$ interaction with production of ρ meson of magnetic type only (i.e., with spin structure: $\chi^\dagger\vec{\Delta}\cdot\hat{\vec{k}}\times\vec{\rho}$, where $\vec{\rho}$ is the ρ-meson polarization vector). Following the vector domain model (VDM) hypothesis, such parametrization implies the dominance of the $M1$ radiation with respect to the $E2$ radiation for the radiative decay $\Delta\to N+\gamma$ [36] and in the Δ electroexcitation on the nucleon, $e^-+p\to e^-+\Delta^+$, up to very large momentum transfer.

Note, in this respect, that the "standard" relativistic parametrization of the $\Delta N\rho$ interaction, through the Lagrangian [17]

$$\mathcal{L}_{\Delta N\rho}=\frac{g_{\Delta N\rho}}{m+M}\overline{\Psi}_\Delta^\mu(\alpha)\vec{T}\gamma^\nu\gamma_5\Psi_N(x)[\partial_\mu\vec{\rho}_\nu(x)-\partial_\nu\vec{\rho}_\mu(x)]$$

produces ρ meson not only of $M1$ type, but also of $E2$ type (with transversal and longitudinal polarizations), in contradiction with VDM.

The matrix element (25) produces the following partial threshold amplitudes $h_{i\Delta}$, $i=1$–3, for $\pi+\Delta\to N+\phi$:

$$h_{1\Delta}=-h_{2\Delta}, \quad h_{3\Delta}=0,$$

i.e., $f_{1b}=-f_{2b}=f_{3b}$, with $\mathcal{B}=-1$. This means that both considered types of π exchange generate only transversally polarized ϕ mesons.

D. Coupling constants and form factors

Finally, the two possible π exchanges for $\Delta^{++}+n\to p+p+\phi$ result in the following amplitudes $f_{i\pi}$ ($f_{i\pi}=f_{ia}+f_{ib}, i=1$–3):

$$f_{1\pi}=A_\Delta h_{1\Delta},$$

$$f_{2\pi}=-f_{3\pi}=A_N h_{1N}-A_\Delta h_{1\Delta}.$$

This gives the following expression for the matrix element squared corresponding to the four different contributions of Fig. 1 (after summing over the polarizations of the produced nucleons and averaging over the polarizations of the colliding Δ and N):

$$\overline{|\mathcal{M}_\pi|^2}=\overline{|\mathcal{M}_a+\mathcal{M}_b|^2}=\frac{2}{3}A_N^2h_{1N}^2(1-r_{N\Delta}+r_{N\Delta}^2). \quad (26)$$

The real parameter $r_{N\Delta}$ characterizes the relative role of the two possible mechanisms for $\Delta+N\to N+N+\phi$, corresponding to the two possible pion exchanges [Figs. 1(a) and 1(b)]:

$$r_{N\Delta}=\frac{A_\Delta h_{1\Delta}}{A_N h_{1N}},$$

and its numerical value is given by three different factors,

$$r_{N\Delta}=r_c r_p(t_1,t_2)r_f(t_1,t_2). \quad (27)$$

Here r_c is the ratio of the corresponding coupling constants,

$$r_c=\frac{\sqrt{2}}{(1+\kappa_\rho)(1+m/M)}\frac{g_{\Delta^+p\rho^0}}{g_{\rho NN}}\frac{g_{\pi NN}}{g_{\Delta^{++}p\pi^+}}. \quad (28)$$

The function $r_p(t_1,t_2)$ characterizes the relative role of the corresponding π and ρ propagators for the considered mechanisms,

$$r_p(t_1,t_2)=\frac{(t_1-m_\pi^2)(t_2-m_\rho^2)}{(t_2-m_\pi^2)(t_1-m_\rho^2)}\simeq 0.86, \quad (29)$$

and the function $r_f(t_1,t_2)$ is the ratio of the hadronic form factors for all these diagrams with pion exchange,

$$r_f(t_1,t_2)=\frac{F_{\Delta N\pi}(t_2)F_{\Delta N\rho}(t_1)F_{\pi\rho\phi}(t_2,t_1)}{F_{NN\pi}(t_1)F_{NN\rho}(t_2)F_{\pi\rho\phi}(t_1,t_2)}, \quad (30)$$

i.e., $r_f(t_1,t_2)$, r_c, and therefore $r_{N\Delta}$ do not depend on the value of the coupling constant $g_{\pi\rho\phi}$.

For numerical estimations, we take, besides Eq. (16), the following parametrizations of the hadronic form factors:

$$F_{NN\rho}(t)=F_{\Delta N\rho}(t)=\left(\frac{\Lambda_\rho^2-m_\rho^2}{\Lambda_\rho^2-t}\right)^2 \text{ with}$$

$$\Lambda_\rho=1.85 \text{ GeV } [34], \quad (31)$$

$$F_{\pi\rho\phi}(t_1,t_2)=\frac{(\Lambda_\phi^2-m_\pi^2)(\Lambda_\phi^2-m_\rho^2)}{(\Lambda_\phi^2-t_1)(\Lambda_\phi^2-t_2)} \text{ with}$$

$$\Lambda_\phi=1.45 \text{ GeV } [3]. \quad (32)$$

This gives $r_f(t_1,t_2)\simeq 0.92$. Taking the coupling constants

$$g_\rho=3.04, \quad \kappa_\rho=6.1, \quad g_{\pi NN}=13.4, \quad g_{\Delta^{++}p\pi^+}=19.3,$$

$$g_{\Delta^+p\rho^0}\simeq 24 \quad (33)$$

we find $r_c=0.62$. Therefore $|r_{N\Delta}|\simeq 0.47$, i.e., the two mechanisms considered above have comparable contributions. As the relative signs of all these coupling constants are not known, we have to consider both possibilities, $r_{N\Delta}=\pm 0.47$, which implies that

$$C(\pm)=1\pm|r_{N\Delta}|+r_{N\Delta}^2=0.75(+) \quad \text{or } 1.69(-)$$

and that the corresponding values for the cross section may differ by a factor of 2.

We can conclude that the ratio $r_{N\Delta}$ is not sensitive to the hadronic form factors and does not depend on the characteristics of the $\pi\rho\phi$ vertex: neither on the absolute value of the $g_{\pi\rho\phi}$ coupling constant nor on the form factor $F(t_1,t_2)$. Varying the constant $g_{\rho NN}$ in the interval 2.64 [37] $\leqslant g_{\rho NN}\leqslant 3.25$ [38] induces a small variation on $r_{N\Delta}$, $0.43\leqslant r_{N\Delta}\leqslant 0.53$, and on $C(\pm)$, $1.61\leqslant C(+)\leqslant 1.81$, and $0.74\leqslant C(-)\leqslant 0.75$.

E. Estimation of the cross section for $\Delta^{++}+n\to p+p+\phi$

In the threshold region, the energy dependence of the total cross section for the process $\Delta^{++}+p\to p+p+\phi$ can be written as [39]

$$\sigma_{tot}(Q)=\overline{|\mathcal{M}|^2}\mathcal{N}_\Delta\mathcal{N}Q^2, \quad Q=\sqrt{s}-2m-m_\phi$$

with

$$\mathcal{N}_\Delta=\frac{1}{\pi^2}\frac{1}{256}\frac{m}{(2m+m_\phi)^2}\sqrt{\frac{m_\phi}{(2m+m_\phi)}}\frac{1}{k}, \quad (34)$$

$$\overline{|\mathcal{M}|^2}=\frac{2}{3}A_N^2h_{1N}^2C(\pm).$$

Taking into account the expressions (20), (21), and (26), we can write

$$\mathcal{N}\overline{|\mathcal{M}|^2}=K_cK_pK_fC(\pm), \quad (35)$$

$$K_c=[\sqrt{2}g_{\Delta^{++}p\pi^+}g_{\pi\rho\phi}g_{\rho NN}(1+\kappa_\rho)]^2, \quad (36)$$

$$K_p=\left[\frac{2mk(E_N-m)\sqrt{(E_N+m)(E_\Delta+M)}}{M(t_2-m_\pi^2)(t_2-m_\rho^2)}\right]^2, \quad (37)$$

$$K_f=[F_{NN\pi}(t_1)F_{NN\rho}(t_2)F_{\pi\rho\phi}(t_1,t_2)]^2. \quad (38)$$

So, with the standard parametrizations of the form factors, see Eqs. (16), (31), and (32), one finds $K_f=1.65\times10^{-2}$, i.e., a very large effect of the form factors on the threshold cross section.

With the values of the coupling constants (33) and $g_{\pi\rho\phi}=1.1$, the threshold behavior of the total cross section for the process $\Delta^{++}+n\to p+p+\phi$ depends on the sign of $r_{N\Delta}$,

$$\sigma_{tot}^{(\pm)}(Q)=\left(\frac{Q}{\mathrm{GeV}}\right)^2\left\{\begin{matrix}161^{(+)}\\ 74^{(-)}\end{matrix}\right\}\quad \mu\mathrm{b}, \quad (39)$$

where $(\pm)$ corresponds to the two possible signs of the ratio $r_{N\Delta}$.

This result can be compared with the experimental result for the total cross section of the process $p+p\to p+p+\phi$ at $Q=83$ MeV [1],

$$\sigma(pp\to pp\phi)=(190\pm14\pm80)\ \ \mathrm{nb}. \quad (40)$$

Assuming the known Q dependence for the cross section for threshold three particle production [39], one can rewrite the result (40) as

$$\sigma(pp\to pp\phi)=27.6(1\pm0.07\pm0.42)\left(\frac{Q}{\mathrm{GeV}}\right)^2\mu\mathrm{b}. \quad (41)$$

This behavior can be quantitatively reproduced using the $\pi\rho\phi$ exchange for the $pp\to pp\phi$ process, also, with the above mentioned coupling constants and form factors.

Comparing Eqs. (39) and (41), one can see that the relative value of the cross section for ϕ-meson production in ΔN and pp collisions depends on the sign of the ratio $r_{N\Delta}$. In any case, for the same value of Q, the cross section for ϕ production in ΔN collisions is three to five times larger than for pp collisions.

The present predictions for the cross section for the process $\Delta^{++}+n\to p+p+\phi$ are about five times larger than in Refs. [14,15], which also underestimate the DISTO result (40) by the same factor.

IV. DISCUSSION AND CONCLUSIONS

It is, in principle, possible to analyze in the same way the ρ exchange for the process $\Delta+N\to N+N+\phi$. One can show that the matrix element for $\rho^*+N\to N+\phi$ contains five independent spin structures, with five corresponding complex threshold amplitudes.

VDM can also be applied to $\rho^*+N\to N+\phi$, to relate this process to ϕ electroproduction. But there are no experimental data about ϕ electroproduction near threshold, therefore, for numerical estimations, a dynamical model has to be built. The process $\rho^*+\Delta\to N+\phi$, even at threshold, is generally

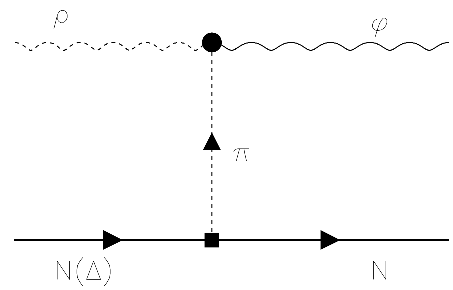

FIG. 5. Feynman diagram for π exchange in the subprocesses $\rho+N(\Delta)\to N+\phi$.

characterized by an even more complicated spin structure, with eight independent partial amplitudes. Sophisticated models (with excitation of nucleonic resonances) contain several unknown parameters, therefore the ρ contribution cannot actually be calculated with sufficient precision. Therefore, the simple π-exchange model for the subprocesses $\rho^*+N(\Delta)\to N+\phi$ seems to be the most reliable.

In any case, the mechanism of π exchange in the subprocesses $\rho^*+N\to N+\phi$ and $\rho^*+\Delta\to N+\phi$ (Fig. 5) will finally generate the same matrix element for $\Delta^{++}+n\to p+p+\phi$ as the ρ exchange in the subprocesses $\pi+N(\Delta)\to N+\phi$, which has been taken into account in the previous considerations. This means that the main part of ρ exchange for $\Delta+N\to N+N+\phi$, which is due to the $\pi\rho\phi$ vertex, is common with the π exchange, considered above. Therefore the difference between the π and ρ exchanges can be induced by nucleonic contributions [of the subprocesses $\pi+N(\Delta)\to N+\phi$ and $\rho+N(\Delta)\to N+\phi$ in s and u channels], which may be neglected, due to the small ratio $|g_{NN\phi}|/|g_{\pi\rho\phi}|$.

Note finally that the process $\Delta+N\to N+N+\phi$ cannot be studied in a direct experiment, as ΔN collisions occur in NA or AA collisions as secondary reactions. As we do not discuss the $\Delta-$production mechanism, we assume the "preexistence" of unpolarized Δ's in the medium, following Ref. [17]. Such hypothesis is not in contradiction with the present calculation of the ΔN cross section, but for the calculation of the polarization properties of the ϕ meson, the knowledge of the $\vec{k}$ direction is important. In case of NA collisions with subsequent ΔN interaction, this direction, with good accuracy, can be identified with the direction of the initial nucleon beam. In case of AA collisions the Δ-production mechanism must be important. For high-energy AA collisions we can expect that Δ production—through binary subprocesses, such as $N+N\to\Delta+N$ and $N+N\to\Delta+\Delta$ [40]—plays a large role, i.e., the vector $\vec{k}$ can be identified with the three-momentum of any of colliding nuclei ($\rho_{ab}^{(\phi)}$ is quadratic in the $\vec{k}$ components). The ϕ polarization due to ΔN collisions has therefore a physical meaning.

In conclusion, we have studied the reactions $\Delta+N\to N+N+\phi$ in the near-threshold region on the basis of the most general symmetry properties of the strong interaction (P parity and angular momentum conservation, as selection rules) and treating the dynamical aspects in the framework of one-boson mechanism. Let us summarize the main results of our analysis.

We established the spin structure of the threshold matrix element for the process $\Delta+N\rightarrow N+N+\phi$ in terms of three partial independent complex amplitudes. In the general case of noncoplanar kinematics, this process is described by 96 independent amplitudes, 48 in the coplanar case and 11 amplitudes for collinear kinematics.

One of these threshold amplitudes vanishes for the pion *t*-channel exchange—with the subprocess $\pi+N\rightarrow N+\phi$ [Fig. 1(a)]. Therefore it is possible to predict the polarization properties of the ϕ mesons, produced in the reaction $\Delta+N\rightarrow N+N+\phi$, and the ratio of the total cross section for the processes $\Delta+N\rightarrow N+N+\phi$ and $p+p\rightarrow p+p+\phi$ in terms of a single parameter x, which characterizes the relative role of production of ϕ mesons with transversal and longitudinal polarizations in the process $\pi+N\rightarrow N+\phi$.

The value of the parameter x depends on the model chosen to describe the process $\pi+N\rightarrow N+\phi$ in the near-threshold region. The ρ-exchange mechanism, for this process, gives $x=-1$.

The threshold cross section for the process $\Delta+N\rightarrow N+N+\phi$ is larger by a factor of 3–5 with respect to $p+p\rightarrow p+p+\phi$.

Two possible pion exchange mechanisms for $\Delta^{++}+n\rightarrow p+p+\phi$ give comparable contributions to the cross section, in the near-threshold region, with large sensitivity to the relative sign of the coupling constants which enter in the calculations.

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