Determination of the chiral coupling constants c_3 and c_4 in new pp and np partial-wave analyses

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As a first result of two new partial-wave analyses, one of the pp and another one of the np scattering data below 500 MeV, we report a study of the long-range chiral two-pion exchange interaction which contains the chiral coupling constants c_1 , c_3 , and c_4 . By using as input a theoretical value for c_1 we are able to determine in pp as well as in np scattering accurate values for c_3 and c_4 . The values determined from the pp data and independently from the np data are in very good agreement, indicating the correctness of the chiral two-pion exchange interaction. The weighted averages are $c_3 = -4.78(10)/\text{GeV}$ and $c_4 = 3.96(22)/\text{GeV}$, where the errors are statistical. The value of c_3 is best determined from the pp data and that of c_4 from the np data.

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I. INTRODUCTION

It is beyond doubt that the longest-range strong twonucleon (NN) interaction is the one-pion exchange (OPE) force. Despite more than 50 years of research, the nature of the medium-range NN interaction is not so well understood. What seems clear is that it contains: (i) A strongly attractive central force, (ii) an isospin-dependent tensor force opposite in sign to OPE, and (iii) a rather strongly attractive spin-orbit force. It was discovered in the early 60s that all these features follow naturally from the exchange of scalar and vector mesons, which led to the development of the one-boson exchange (OBE) model of the NN interaction. The role of the two-pion exchange (TPE) interaction and its interplay with the exchange of heavy mesons that decay into two pions has for a long time remained elusive.

In recent years, however, the situation has improved. The derivation of at least the long- and medium-range nuclear forces can be formulated in a model-independent manner by a systematic expansion of the chiral Lagrangian of QCD [1-7]. In particular, the long-range TPE interaction can be derived unambiguously, where the effects of the exchange of broad heavy mesons are incorporated in effective low-energy chiral coupling constants. Most importantly, chiral symmetry and its breaking are correctly implemented in this approach.

In Ref. [7], we studied this long-range chiral two-pion exchange (χ TPE) interaction in an energy-dependent partialwave analysis (PWA) of the proton-proton (pp) scattering data below 350 MeV. The presence of χ TPE in the longrange pp force was demonstrated, and the chiral coupling constants c_3 and c_4 were determined from the pp data. In this paper, we address the question whether the same χ TPE force allows also a good description of the neutron-proton (np) scattering data below 500 MeV. Moreover, we present new, precise determinations of the chiral coupling constants c_3 and c_4 from the pp and np data separately. Accurate values of these chiral coupling constants are an important input in calculations of, for instance, the two-pion exchange three-nucleon force [8].

II. PARTIAL-WAVE ANALYSIS

Because of the high quality of the pp data base all the pp phase shifts with orbital angular momentum $\ell \leq 4$ can be

determined accurately in an energy-dependent PWA of the pp data below 350 MeV. An analysis of the np data, however, is much more difficult, because not only the I=1 phase shifts but also the I=0 phase shifts contribute. Moreover, the np data base, while extensive, is by far not as accurate and varied as the pp data base [9]. In a PWA of only the np data below 350 MeV, it has always been impossible to determine all the important np phase shifts. Therefore, the standard practice has been to take the I=1 phase shifts, with the exception of the ${}^{1}S_{0}$ phase shift, from the pp PWA, with or without corrections for the Coulomb interaction and/or the $\pi^{+}-\pi^{0}$ mass difference in OPE. Since it has long been known that there is a sizable charge-independence breaking (CIB) in the ${}^{1}S_{0}$ phase shifts, the ${}^{1}S_{0}$ np phase shift is always fitted independently of the ${}^{1}S_{0}$ pp phase shift.

This approach to np PWA was also followed in the past by the Nijmegen group. In 1993 the results of the first Nijmegen pp and np PWA's below 350 MeV were published in Ref. [10]. An attempt at that time to extract all the important np phase shifts, both I=0 and I=1, from the np data base below 350 MeV failed, although it was possible to determine the ³P np phase shifts when the I=1 waves for ℓ >1 were taken over from the pp PWA93 [11].

It has been customary to perform NN PWA's without inelasticities up to 350 MeV, although pion production starts already at 280 MeV. It can be shown that the inclusion of inelasticities in the pp PWA below 350 MeV improves the χ^2_{min} slightly. Already some time ago [12], the Nijmegen pp PWA was extended to energies far above the pion-production thresholds, with the inclusion of inelasticities. When, in 1994, the np PWA was extended to 500 MeV, it turned out to be possible, for the first time, to determine uniquely all the important *np* phase shifts, both I=0 and I=1, from the *np* data alone [13]. Such a separate PWA of the *np* data, without input from the pp PWA for the I=1 waves, is, in principle, more model independent. A comparison between the phase shifts from such an independent np PWA and the corresponding phase shifts from the pp PWA provides information about possible CIB in the I=1 waves.

The Nijmegen energy-dependent PWA's can be used as a tool to study the long-range NN interaction [7]. The long-

range forces are included exactly, in order to ensure that the partial-wave amplitudes acquire the proper fast energy dependence from the nearby left-hand singularies due to these long-range forces, while the short-range interactions (more remote left-hand singularities), responsible for a much slower energy dependence, are parametrized. This strategy is implemented by solving the Schrödinger equation with an energy-dependent boundary condition (BC) at some r=band for r > b the long-range NN interaction. This long-range force contains the electromagnetic interaction (i.e., in the pp case the improved Coulomb [14], the magnetic-moment [15], and the vacuum-polarization [16] interactions, and in the npcase the magnetic-moment interaction [15]), the OPE interaction, in the *np* case also the pion-photon $(\pi - \gamma)$ exchange interaction [17], and the long-range part of the χ TPE interaction [7]. The BC is parametrized as an analytic function of energy, and the parameters, representing "short-range physics," are determined from a fit to the data. The option also exists to fit simultaneously some of the parameters in the long-range interactions, viz. the pion-nucleon coupling constants [18–20] and/or the chiral coupling constants $c_i(i)$ = 1,3,4) in χ TPE [7].

The new pp and np PWA's that we discuss here will be referred to as χ PWA03. They differ from the old PWA93, in several aspects.

(i) The energy range is extended from 350 to 500 MeV. Instead of the 1787 pp data and 2514 np data in PWA93, we now have 5109 pp data and 4786 np data [21].

(ii) All the np phase shifts can be determined from the np data alone, instead of taking the I=1 phases from the pp PWA and correcting them.

(iii) Inelasticities are taken into account.

(iv) For r > b a different non-OPE strong interaction is taken. In PWA93 the heavy-meson exchanges of the Nijmegen soft-core OBE potential [22] were used. Motivated by the success of Ref. [7], where an excellent description of the high-quality pp data base below 350 MeV was obtained, we use in χ PWA03 the χ TPE potential.

(v) A minor difference between PWA93 and χ PWA03 is that we take here b = 1.6 fm, while in PWA93 b = 1.4 fm was used; in Ref. [7] we used both b = 1.4 fm and b = 1.8 fm.

Details of χ PWA03 (data, phase shifts, etc.) will be presented elsewhere [23], here we focus on testing the longrange χ TPE interaction in the *pp* and *np* systems below 500 MeV.

III. CHIRAL TWO-PION EXCHANGE POTENTIAL

The χ TPE potential can be derived by a systematic expansion of the effective chiral Lagrangian [1–7]. The form that is appropriate for use in the relativistic Schrödinger equation and that is consistent with our choice of including the minimal-relativity factor M/E in the OPE potential is specified in Ref. [7] (see also Ref. [3]).

The leading-order χ TPE potential consists of the static planar- and crossed-box TPE diagrams, calculated with the derivative (pseudovector) $NN\pi$ Lagrangian, and the triangle and football diagrams with the nonlinear $NN\pi\pi$ WeinbergTomozawa seagull vertices. It contains isospin-independent spin-spin and tensor terms and an isospin-dependent central term.

In subleading order, next to nonstatic corrections to the planar and box diagrams, additional triangle diagrams appear which contain three new $NN\pi\pi$ interactions [2]. The corresponding chiral coupling constants are denoted by $c_i(i = 1,3,4)$. (Unfortunately, they are not scaled to obtain dimensionless numbers and their values are conventionally given in GeV⁻¹.) They are defined by the following terms in the chiral Lagrangian density:

$$\mathcal{L} = -\bar{N} [8c_1 D^{-1} m_{\pi}^2 \vec{\pi}^2 / F_{\pi}^2 + 4c_3 \vec{D}_{\mu} \cdot \vec{D}^{\mu} + 2c_4 \sigma_{\mu\nu} \vec{\tau} \cdot \vec{D}^{\mu} \times \vec{D}^{\nu}] N, \qquad (1)$$

where $F_{\pi} \approx 185$ MeV is the pion decay constant, D=1 $+\vec{\pi}^2/F_{\pi}^2$, and the chiral-covariant derivative of the pion field $\vec{\pi}$ is $\vec{D}^{\mu} = D^{-1} \partial^{\mu} \vec{\pi}/F_{\pi}$. The c_3 and c_4 terms are manifestly chiral invariant. The c_1 term, which is proportional to m_{π}^2 , violates chiral symmetry explicitly and is related to the much-discussed pion-nucleon σ term [24]. Using the rationalized pseudovector pion-nucleon coupling constant f^2 the relation reads [25]

$$c_1 = -\left[\frac{\sigma}{4m_\pi^2} + \frac{9}{16}\frac{f^2}{m_s^2}m_\pi\right],$$
 (2)

where $m_{\pi} = 138.04$ MeV is the average pion mass, and $m_s \equiv m_{\pi^+}$ is the scaling mass conventionally introduced to make *f* dimensionless. Equation (2) holds in order $O(q^3)$ in the chiral expansion in small momenta *q* and the pion mass [25]. An additional c_2 term is not given in Eq. (1), since it does not contribute to the χ TPE potential to subleading order. However, it does contribute to the isoscalar πN scattering amplitude at the same order as c_1 and c_3 .

In subleading order the χ TPE potential gets contributions to the central, spin-spin, tensor, and spin-orbit potentials (cf. Table 1 in Ref. [7]). Important components are (i) A strong isospin-independent central attraction due to the c_3 term, (ii) an isospin-dependent tensor force opposite in sign to OPE due to the c_4 term, and (iii) an attractive isospin-independent spin-orbit force from nonstatic terms of the planar- and crossed-box diagrams. The values of the c_i 's are not fixed by chiral symmetry and must be determined from the experimental πN or NN scattering data.

The long-range χ TPE potential derived in the framework of the effective chiral Lagrangian is completely model independent. Any dynamical *model* [26,27] for the TPE *NN* interaction, containing, e.g., the ε (or " σ ") and ϱ mesons, the pomeron, and/or *N* and Δ isobars, has to reduce to this form for large *r*. These models should also predict values for the c_i 's consistent with the determinations from the πN and *NN* scattering data.

The breaking of charge independence due to the π^+ - π^0 mass difference in the OPE potential is taken into account exactly, as it was already in PWA93 [10]. In the χ TPE potential, we include the terms linear in the π^+ - π^0 mass difference.

ference, following Ref. [28]. One charge-independent pionnucleon coupling constant [20] is used in both the OPE and the χ TPE potentials. In the long-range interaction for r > bonly the chiral coupling constants c_i (i=1,3,4) remain then to be determined.

IV. RESULTS

In our previous study [7] of χ TPE it turned out that c_1 could not be determined accurately from the pp data base below 350 MeV. When we fitted c_1 , c_3 , and c_4 simultaneously, we found $c_1 = -4.4(3.4)/\text{GeV}$, where the error is statistical. A strong correlation was obtained between the values of c_1 and c_3 . Therefore, we used Eq. (2) to fix the value of c_1 . Assuming that the σ term has the low value $\sigma = 35(5)$ MeV [24,29], Eq. (2) gives $c_1 = -0.76(7)/\text{GeV}$, where the error is theoretical. We used the central value $c_1 = -0.76/\text{GeV}$ as input in the PWA, and determined in Ref. [7] the values $c_3 = -5.08(28)/\text{GeV}$ and $c_4 = 4.70(70)/\text{GeV}$, where the errors are statistical. One note that from the pp data below 350 MeV the value of c_3 could be extracted rather precisely, while c_4 was pinned down less accurately.

The value extracted for c_1 from the data below 500 MeV would also not be accurate enough to shed light on the value of the σ term, since the statistical error for c_1 obtained in Ref. [7] would have to be reduced at least by a factor of about 20. We therefore decided to use also here the value $c_1 = -0.76$ /GeV as input value, and to determine c_3 and c_4 from direct fits to the *pp* data and independently also from fits to the *np* data.

We analyzed 5109 pp data below 500 MeV using 33 BC parameters, and we reached χ^2_{min} = 5184.3. The optimal values for c_3 and c_4 and their (1 standard deviation) errors as determined from these pp data are

$$c_3 = [-4.78(11) + 80(f^2 - 0.0755)]/\text{GeV},$$

 $c_4 = [3.92(52) + 260(f^2 - 0.0755)]/\text{GeV},$ (3)

where also the dependence on the $NN\pi$ coupling constant f^2 is displayed. The correlation parameter is $\rho = -0.47$. These values for c_3 and c_4 are consistent with and more accurate than those found in Ref. [7] from the pp data below 350 MeV. The errors are statistically only. Systematic errors are difficult to assess and require further study.

For np scattering, we analyzed 4786 data below 500 MeV. In this case, we needed 40 BC parameters and reached $\chi^2_{min} = 4806.2$. The chiral coupling constants, their statistical errors, and their dependence on the $NN\pi$ coupling constant are in this np case,

$$c_3 = [-4.77(22) + 100(f^2 - 0.0755)]/\text{GeV},$$

 $c_4 = [3.97(24) + 40(f^2 - 0.0755)]/\text{GeV}.$ (4)

The correlation parameter is $\varrho = 0.22$. In Fig. 1, we show the results for c_3 and c_4 , for $f^2 = 0.0755$. Plotted are the positions of the χ^2 minima and the $\chi^2 = \chi^2_{min} + 1$ ellipses in the



FIG. 1. Ellipses of constant χ^2 in the (c_3, c_4) plane. Shown are the $\chi^2 = \chi^2_{min} + 1$ ellipses in the *pp* PWA and in the *np* PWA. The centers of the ellipses correspond to the minima in χ^2 .

 (c_3, c_4) plane, both for the *pp* and the *np* case. (These ellipses, of course, are determined with optimalization of all the BC parameters.)

The values of c_3 and c_4 determined from the pp and from the np data are in good agreement. The value for c_3 determined from the pp data is more than twice as accurate as the value from the np data, while for c_4 the situation is reversed: the value from the np data is twice as accurate as the value from the pp data.

We also determined the weighted averages with errors. We get

$$c_3 = [-4.78(10) + 84(f^2 - 0.0755)]/\text{GeV},$$

 $c_4 = [3.96(22) + 79(f^2 - 0.0755)]/\text{GeV}.$ (5)

These are our best values, following from all the pp and np scattering data below 500 MeV, which amounts to a total of almost 10,000 NN data. In Table I we list our results for c_3 and c_4 , for $f^2 = 0.0755$.

V. DISCUSSION AND SUMMARY

We have determined accurate values for the chiral coupling constants c_3 and c_4 from the pp and the np scattering data below 500 MeV. The values for the c_i 's (i=1,2,3,4) can also be determined from PWA's of the πN scattering data by fitting the amplitudes predicted by (heavy-baryon) chiral perturbation theory (χ PT) to the πN scattering amplitudes obtained from these PWA's. In the several such determinations (for a discussion of the status, see Ref. [30]) the value of c_3

TABLE I. Comparison of the chiral coupling constants c_i (i=1,3,4) (in units of 1/GeV) from different analyses. The πN results correspond to analyses to order $O(q^3)$ in χ PT. The input values of the σ term are in MeV. (For a discussion of the meaning of the errors, see the text.)

	Reference	σ	<i>c</i> ₁	<i>c</i> ₃	<i>c</i> ₄
πN	[31]	45(8)	-0.91(9)	-5.16(25)	3.63(10)
πN	[32]	40(8)	-0.81(12)	-4.70(1.16)	3.40(04)
рр	[7]	35(5)	-0.76(7)	-5.08(28)	4.70(70)
рр	This work	35(5)	-0.76(7)	-4.78(11)	3.92(52)
пр	This work	35(5)	-0.76(7)	-4.77(22)	3.97(24)
NN	This work	35(5)	-0.76(7)	-4.78(10)	3.96(22)

is found to lie in the range between -4.70 and -6.19, and the value of c_4 in the range between 3.25 and 4.12. The values that are found for c_3 and c_4 depend on the order in χ PT of the amplitudes, as well as on the specific πN PWA that is used. They also depend on what value is used for the σ term, because this value fixes the value of c_1 . In χ PT the isovector πN amplitudes can be predicted more accurately than the isoscalar amplitudes, because in leading order the latter are zero. Therefore, c_4 can probably be pinned down better than c_3 . The value of c_3 is, moreover, strongly correlated with the values of c_1 and c_2 . In Table I, we also listed the values obtained for the c_i 's in two πN analyses [to order $O(q^3)$ in χ PT] that assume, like us, an acceptably low value for the σ term. (From Ref. [31], we list only the c_i 's corresponding to one of the fits with $f^2 = 0.076$.)

A problem with these determinations of the c_i 's from the πN scattering data is that they are not determined directly from the data, but from fitting to the amplitudes of existing PWA's that have no reliable errors. For instance, in several analyses the amplitudes of the about 25-year-old Karlsruhe-Helsinki dispersion analysis were used. The amplitudes of that PWA have no associated errors and, what is worse, are in disagreement with the modern day πN data base. That analysis produced the high value $f^2 = 0.079$ for the pion-nucleon coupling constant [20]. The resulting errors on the c_i 's determined from the πN data, therefore, do not reflect the statistics of the data base, but are essentially rather arbitrary estimates.

Our results correspond to a long-range interaction that includes the leading and subleading χ TPE diagrams. Higherorder corrections for χ TPE and the leading three-pion exchange diagrams have been calculated by Kaiser [33], and can, in principle, be included as well. Work along these lines is continuing.

In summary, the long-range part of the χ TPE potential was included in energy-dependent PWA's of the pp and the np scattering data below 500 MeV. Good fits to the data were obtained. In the np PWA all the phase-shift parameters could be determined without input from the pp system. We conclude that OPE plus χ TPE provides a high-quality long-range strong two-nucleon interaction. Accurate values for the chiral coupling constants c_3 and c_4 of chiral perturbation theory were obtained from the pp and the np data separately. The values agree very well with each other, and they are also in good agreement with the range of values obtained from pion-nucleon scattering amplitudes. We consider this agreement to be experimental evidence that the χ TPE interaction, as predicted by chiral perturbation theory, is correct.

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