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α decay and proton-neutron correlations

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We study the influence of proton-neutron (p-n) correlations on α-decay width. It is shown from the analysis of α Q values that the p-n correlations increase the penetration of the α particle through the Coulomb barrier in the treatment following Gamow's formalism, and enlarges the total α-decay width significantly. In particular, the isoscalar p-n interactions play an essential role in enlarging the α-decay width. The so-called "α condensate" in $Z \geqslant 84$ isotopes are related to the strong p-n correlations.

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The α decay has long been known as a typical decay phenomenon in nuclear physics [1]. Various microscopic approaches to estimating the formation amplitude of the α cluster have been proposed [2–5]. The calculations [6–8] showed that $J=0$ proton-proton (p-p) and neutron-neutron (n-n) pairing correlations cause substantial α-cluster formation on nuclear surface. This suggests that the BCS approach with a pairing force offers a promising tool to describe the α decay. Proton-neutron (p-n) correlations are also significantly important for the α-decay process in a nucleus [9,10]. The effect of the p-n correlations on the α-formation amplitude was studied by a generalization of the BCS approach including the p-n interactions [11], though it was shown that the enhancement of the formation amplitude due to the p-n interactions is small. The authors of Ref. [12] pointed out that continuum part of nuclear spectra plays an important role in the formation of α cluster. On the other hand, a shell model approach including α-cluster-model terms [13] gave a good agreement with the experimental decay width of the α particle from the nucleus ^{212}Po. It is also interesting to investigate the effect of deformation on the α-decay width. According to Ref. [12], the contribution of deformation improves theoretical values for deformed nuclei such as ^{244}Pu .

The p-n interactions are expected to become strong in $N \approx Z$ nuclei because valence protons and neutrons in the same orbits have large overlaps of wave functions [14]. In fact, this can be seen in the peculiar behavior of the binding energy at $N=Z$. The double differences of binding energies are good indicators to evaluate the p-n interactions [15,16]. We have recently studied [17] various aspects of the p-n interactions in terms of the double differences of binding energies, using the extended $P+QQ$ force model [18]. The concrete evaluation confirmed that the p-n correlations become very strong in the $N \approx Z$ nuclei. It was shown in Ref. [19] that the isoscalar ($T=0$) p-n pairing force persists over a wide range of $N>Z$ nuclei. One of the double differences of binding energies was also discussed as a measure of α *particle* superfluidity in nuclei [20,21]. (We abbreviate the α-like correlated four nucleons in a nucleus to "α-*particle*" in italic letters. The α *particle* is not a free α particle but a correlated unit in a nucleus.) We expect that the p-n correlations must play an important role in the barrier penetration of the α decay.

Experimental evidence of the α clustering appears in the systematics of α Q values (Q_α) [22], i.e., a large Q_α value coincides with a large α-decay width in the vicinity of the shell closures $Z=50$, $Z=82$, and $Z=126$ [23]. The Q_α value is essentially important for penetration [24–27]. It is known that if experimental Q_α values are used for the α decay between ground states, Gamow's treatment [1] describes qualitatively well the penetration of the α particle through the Coulomb barrier, even though the α *particle* is assumed to be "a particle" in the nucleus. The penetration probability is expected to be sensitive to the p-n component of the Q_α value. How much is the p-n correlation energy included in the Q_α value? What is the role of the p-n correlations in the barrier penetration? In this paper, we study these things and the effect of the p-n correlations on the α decay.

The total α-decay width is given by the well-known formula [28]

$$\Gamma = 2P_L \frac{\hbar^2}{2M_\alpha r_c} g_L^2(r_c), \tag{1}$$

where L and M_α denote, respectively, the angular momentum and the reduced mass of α particle, and r_c is the channel radius. The α-decay width depends on two factors, the penetration factor P_L and the α-formation amplitude $g_L(r_c)$. The α penetration is known as a typical phenomenon of "quantum tunneling" in quantum mechanics.

Since the α-decay width depends sensitively upon the Q_α value, we first discuss the Q_α value, which is written in terms of the binding energy $B(Z,N)$ as follows:

$$Q_\alpha(Z,N) = B(Z-2,N-2) - B(Z,N) + B_\alpha, \tag{2}$$

where B_α is the binding energy of ^{4}He. Experimental mass data show that the Q_α values are positive for β-stable nuclides with mass number greater than about 150. The Q_α value remarkably increases for nuclei above the closed shells, $N=50$, 82, and 126. This is attributed to dramatic increase of separation energy at the closed shells with large shell gap. The Q_α value becomes largest (about 10 MeV) above $N=128$, and the α particle can penetrate a high Coulomb potential barrier (which is 25 MeV for ^{212}Po, though it prohibits the emission of the α particle classically).

It has been shown in Refs. [15–17] that the double difference of binding energies defined by

$$\delta V^{(2)}(Z,N) = -\frac{1}{4}[B(Z,N) - B(Z,N-2) - B(Z-2,N) + B(Z-2,N-2)] \quad (3)$$

is a good measure for probing the p-n correlations. We have recently studied global features of the p-n correlations in $A = 40$–165 nuclei by calculating the values of $\delta V^{(2)}$ with the extended $P+QQ$ force model accompanied by an isoscalar ($T=0$) p-n force [19]. The analysis has revealed that the $T=0$ p-n pairing interaction makes an essential contribution to the double difference of binding energies, $\delta V^{(2)}$. The graph of $\delta V^{(2)}$ as a function of A exhibits a smooth curve of $40/A$ on average, and deviations from the average curve $40/A$ are small, in the region of the mass number $80 < A < 160$ [16,17,19]. Observed $\delta V^{(2)}$ can be reproduced with the use of semiempirical mass formula based on the liquid-drop model, and the symmetry energy term is the main origin of $\delta V^{(2)}$ [29]. However, the parameters of the liquid-drop model are adjusted to experimental binding energies and the liquid-drop model does not give sufficient information about correlations in many-nucleon systems [30]. Our analysis [17,19] indicates that the symmetry energy in the liquid-drop model is attributed dominantly to the J-independent $T=0$ p-n pairing force when considering in the context of correlations. [In SO(5) symmetry model, the contributions of the J-independent $T=0$ p-n pairing and the $J=0$ isovector ($T=1$) pairing forces to $\delta V^{(2)}$ are estimated to be 73% and 27%, respectively.]

In Fig. 1, we show the values of $\delta V^{(2)}$ observed in isotopes with proton number $Z=84$–100. This figure displays dramatic deviations from the average curve $40/A$, in contrast with that for $80 < A < 160$ shown in Ref. [19]. The large deviations, however, seem to be different from those in $N=Z$ nuclei with $N<30$ discussed in Refs. [16,17], because nuclei with $N>128$, having a large number of excess neutrons are in a very different situation from the $N=Z$ nuclei. The large deviations from the average curve $40/A$ for $N>128$ cannot be explained by only the symmetry energy or the J-independent $T=0$ p-n pairing force which smoothly varies with nucleon number and is almost insensitive to the shell effects. Nuclei with large $\delta V^{(2)}$ in Fig. 1 are simply those with short half-lives (i.e., large α-decay widths), above the double-closed-shell nucleus ^{208}Pb. The plots of $\delta V^{(2)}$ for the Po and Rn nuclei extracted from Fig. 1 are shown in Figs. 2(a) and 2(b). We can see dramatic changes of $\delta V^{(2)}$ at $N=128$ both in the Po and Rn nuclei.

It is important to note that $\delta V^{(2)}$ is largest for ^{212}Po with one α *particle* in Fig. 2(a) and ^{216}Rn with two α *particle*s in Fig. 2(b) (and so on), outside the double-closed-shell core ^{208}Pb. The peaks are intimately related to the even-odd staggering of proton or neutron pairs from the "α-condensate" point of view discussed by Gambhir *et al.* [20]. They defined the following quantities for the correlations between pairs:

$$V_{pair}^{even}(A) = \frac{1}{2}[B(Z-2,N) + B(Z,N-2)] - B(Z,N), \quad (4)$$

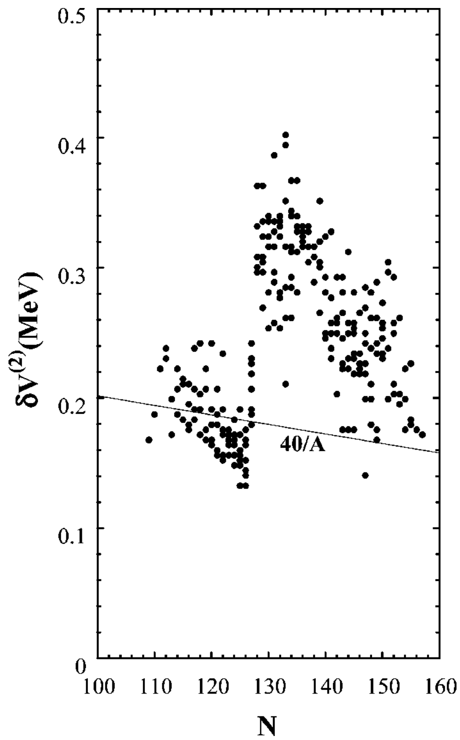

FIG. 1. The experimental double difference of binding energies, $\delta V^{(2)}$ for nuclei with proton number $Z=84$–100 and neutron number $N=110$–157 as a function of neutron number N along neutron chain.

$$V_{pair}^{odd}(A-2) = B(Z-2,N-2) - \frac{1}{2}[B(Z-2,N) + B(Z,N-2)]. \quad (5)$$

In Fig. 3, $V_{pair}^{even}(A)$ for even pair number and $V_{pair}^{odd}(A-2)$ for odd pair number are plotted along the α-line nuclei that can be regarded as many α *particles* outside the core ^{132}Sn or ^{208}Pb. The magnitude of the staggering corresponds to the double difference of binding energies, i.e., $\delta V^{(2)}(Z,N) = [V_{pair}^{even}(A) - V_{pair}^{odd}(A-2)]/4$. We can see that the magnitudes (about 1.33 MeV) for the isotopes with $N>126$ are almost twice as large as those (about 0.78 MeV) for the lighter isotopes with $N<126$. Thus the peaks of $\delta V^{(2)}$ observed in Fig. 3 are related to the superfulid condensate of α *particles* proposed by Gambhir *et al.* The strong α-like $2p$-$2n$ correlations, which enlarge $\delta V^{(2)}$, are important for $A>208$ nuclei with large α-decay widths.

As mentioned earlier, the barrier penetration in the α decay is very sensitive to the Q_α value. It is interesting to examine how much the p-n correlation energy is included in the Q_α value. What roles do the p-n interactions play in the α decay? When we use the one-proton (neutron) separation energy $S_p(S_n)$,

$$S_p(Z,N) = B(Z,N) - B(Z-1,N), \quad (6)$$

$$S_n(Z,N) = B(Z,N) - B(Z,N-1), \quad (7)$$

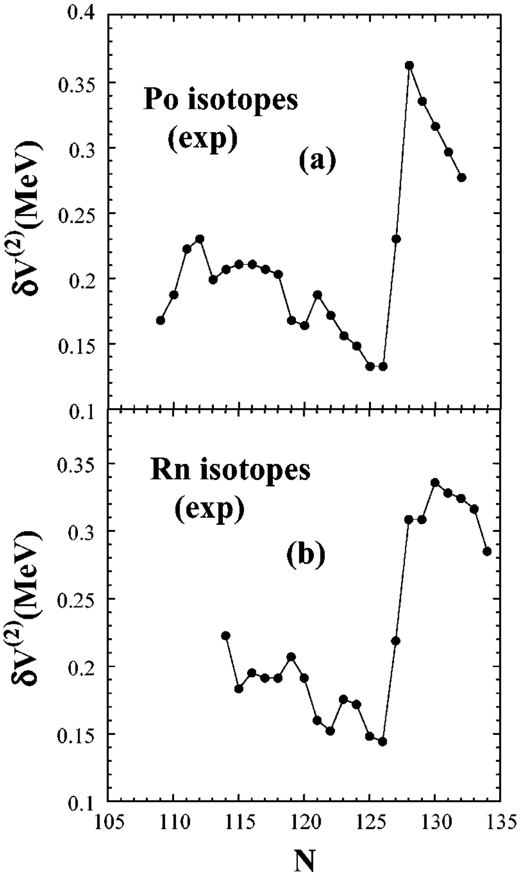

FIG. 2. The experimental double difference of binding energies, δV^2, for (a) the Po nuclei and (b) the Rn nuclei as a function of neutron number N.

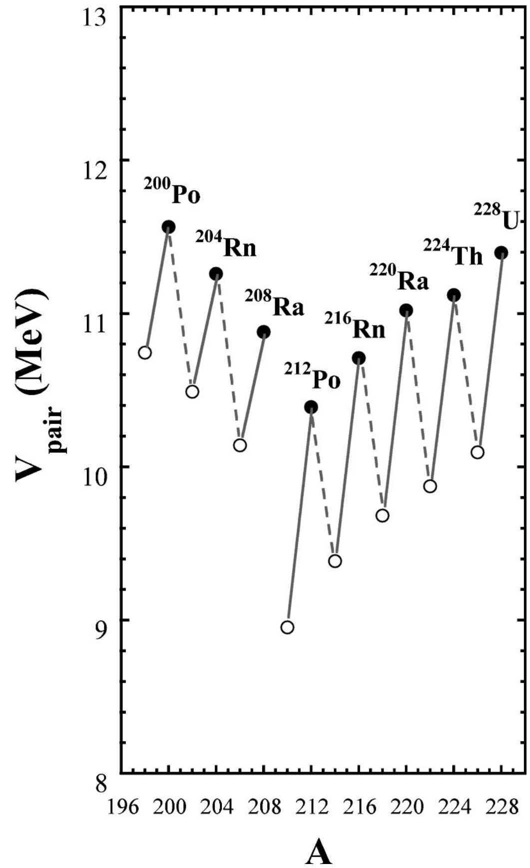

FIG. 3. Even-odd staggering along the α line as a function of mass number. The solid circles denote $V_{pair}^{even}(A)$ for even pair number and the open circles $V_{pair}^{odd}(A-2)$ for odd pair number.

and the three-point odd-even mass difference for proton and neutron,

$$\Delta_p(Z,N) = \frac{(-1)^Z}{2}[B(Z+1,N) - 2B(Z,N) + B(Z-1,N)], \tag{8}$$

$$\Delta_n(Z,N) = \frac{(-1)^N}{2}[B(Z,N+1) - 2B(Z,N) + B(Z,N-1)], \tag{9}$$

the Q_α value is expressed as

$$Q_\alpha(Z,N) = Q_{pn} + Q_{pair} + Q_S + B_\alpha, \tag{10}$$

$$Q_{pn} = 4\,\delta V^{(2)}(Z,N), \tag{11}$$

$$Q_{pair} = 2[(-1)^N \Delta_n(Z,N-1) + (-1)^Z \Delta_p(Z-1,N)], \tag{12}$$

$$Q_S = 2[S_n(Z,N) + S_p(Z,N)]. \tag{13}$$

Since $\delta V^{(2)}$ represents the p-n correlations [17,19], the p-n component Q_{pn} corresponds to the p-n correlation energy of each α *particle*. Here, note that the number of p-n bonds in an α *particle* is four as illustrated in Fig. 4. The p-p and n-n pairing component Q_{pair} is given by the proton and neutron odd-even mass differences.

The upward discontinuity of Q_α value at $N=128$ is highest for ^{210}Pd, ^{211}Bi, and ^{212}Po, and decreases monotonously both in lighter and heavier elements when $|Z-82|$ increases. Similar behavior is observed in systematics of separation energy, namely, this increase comes mainly from the neutron separation energy S_n. The magic character of Q_α seems to be strongest at $Z=82$, $N=128$, though it occurs near other doubly magic or submagic nuclei. The special increase of the Q_α value at $N=128$ is attributed, in the first place, to the

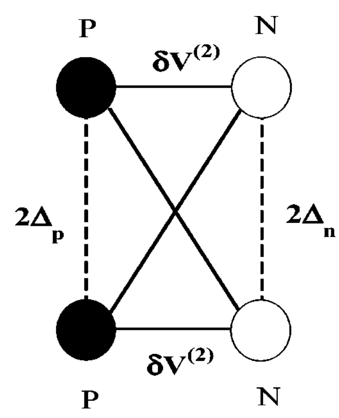

FIG. 4. Schematic illustration of the p-p (n-n) correlations and the p-n correlations in a correlated unit, α *particle*.

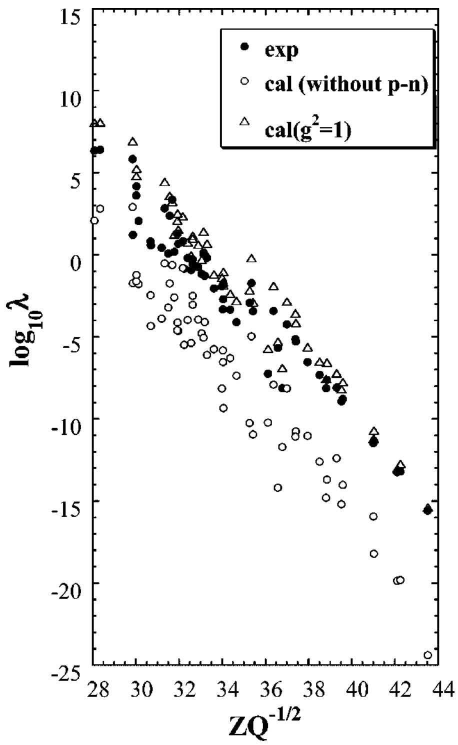

FIG. 5. The $\log_{10}\lambda_{exp}$ in nuclei with $Z=84$–100, $N=110$–154 as a function of $ZQ^{-1/2}$. The solid circles represent the experimental values of $\log_{10}\lambda_{exp}$. The open circles denote the values neglecting the p-n correlation energies $4\delta V^{(2)}$.

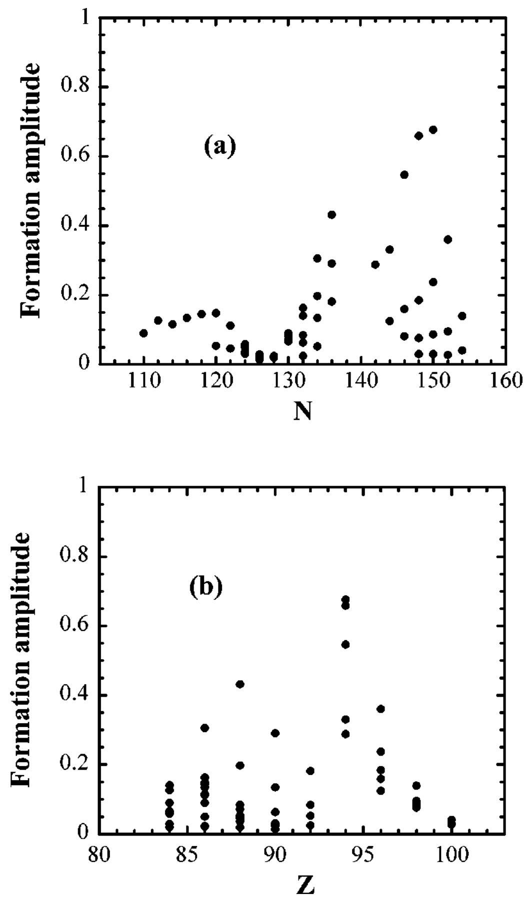

FIG. 6. The formation amplitude g^2 for nuclei with proton number $Z=84$–100 and neutron number $N=110$–157. (a) and (b) show g^2 as a function of the neutron number N and as a function of the proton number Z, respectively.

single-particle energy gaps in the magic nuclei. However, a similar systematics is also observed in the double difference of binding energies $[\delta V^{(2)}(Z,N)]$ in Figs. 1 and 2. Since Q_{pn} is proportional to $\delta V^{(2)}(Z,N)$, the p-n component $Q_{p\text{-}n}$ must contribute to the α decay. In fact, if we remove $Q_{p\text{-}n}$ from the experimental Q_α value and assume $g_L(r_c)=1.0$, the common logarithm of the decay constant ($\log_{10}\lambda$) in the Wentzel-Kramers-Brillouin (WKB) approximation is largely reduced as shown in Fig. 5. Figure 5 shows the significant influence of $Q_{p\text{-}n}$ on the α decay. Thus the p-n correlation energy $Q_{p\text{-}n}$ increases the α-decay width, though it is smaller than the separation energy and the odd-even mass difference. The previous analysis using the extended $P+QQ$ force model tells us that $\delta V^{(2)}$ mainly corresponds to the J-independent $T=0$ p-n interactions [17,19]. Therefore, Fig. 5 testifies that the α-decay transition is enhanced by the p-n correlations through the Q_α value. The $T=0$ p-n interaction is crucial to the α-decay phenomenon.

Although we have approximated $g_L(r_c)=1.0$ in the above consideration, the α-decay width depends on the α-formation amplitude $g_L(r_c)$ as well as on the Q_α value. The α-formation amplitude is very important for the α decay from the viewpoint of nuclear structure. Almost all the studies of the α decay have concentrated on this problem. We can get a rough estimation of the α-decay width using $g_L(r_c)=1.0$ in the largest limit. This assumption means a situation that an α particle is moving in the potential between the daughter nucleus and the α particle. The values of $\log_{10}\lambda$ calculated using the experimental Q_α values and $g_L(r_c)=1.0$ are also plotted in Fig. 5 where the channel radius r_c is taken to be beyond the touching point of the daughter nucleus and α particle, that is, $r_c=1.2A^{1/3}+3.0$ fm. The values agree quite well with the experimental ones [31] in nuclei with $Z=84$–100 and $N=110$–154. In particular, the agreement is good for nuclei with large values of $ZQ^{-1/2}$. It is notable that the effect of the α-formation amplitude on the α-decay width is smaller than that of the p-n correlations mentioned above. The α decay is fairly well understood in terms of tunneling in quantum mechanics when we use the experimental Q_α values. There are, however, still differences between the calculation and experiment. These discrepancies should be improved by appropriate evaluation of the α-formation amplitude. The correlated unit, α *particle*, in the nucleus can be regarded as the α particle only with some probability, and the realistic formation amplitude is not 1.0.

There are several approaches to the calculation of the formation amplitude, the shell model, the BCS method [12], the hybrid (shell model–α-cluster) model [13], etc. The effect of continuum states on the α decay is known to be very large [12]. One therefore needs very large shell model basis to obtain the experimental values of α-formation amplitude. The hybrid model by Varga *et al.* [13], which treats a large shell model basis up to the continuum states through the wave function of the spatially localized α cluster, explains well the experimental decay width. We can estimate the experimental α-formation amplitude from the ratio

$$g^2_{exp}(r_c) = \frac{\lambda_{exp}}{\lambda_{cal}[g^2(r_c)=1]}, \quad (14)$$

where λ_{exp} is the experimental α-decay constant and $\lambda_{cal}[g^2(r_c)=1]$ denotes the α-decay constant calculated using $g^2(r_c)=1$ in the WKB approximation. Figures 6(a) and 6(b) show the experimental α-formation amplitude g^2_{exp} as functions of N and Z, respectively. A remarkable feature is that g^2_{exp} is quite small when N or Z is a magic number, and becomes larger in the middle of the major shell. A typical example is $g^2_{exp}=0.020$ in ^{212}Po which is known as a spherical nucleus. This value is very close to that obtained with the hybrid model [13] and the BCS approach [12]. On the other hand, nuclei in midshell exhibit typical rotational spectra, and are considered to be deformed nuclei. The enhancement of g^2_{exp} may be closely related to deformation. In fact, g^2_{cal} is considerably improved by introducing the deformation [12], while a spherical BCS method cannot explain the experiment. The effect of the deformation on the α-formation amplitude seems to be remarkably large. We end our discussion by commenting that the dynamical correlations due to the p-n interactions in addition to the static contribution to the Q_α value are probably driving correlations of the α *particle* [21].

In conclusion, we have shown that the nuclear correlations reveal themselves in the α decay through the Q_α value that affects the α penetration factor. We estimated the effects of the p-n correlations on the α-decay transition from the experimental double difference of binding energies, $\delta V^{(2)}$. The p-n correlations related to the Q_α value increase the rate of the α-decay transition, and play an important role particularly in the penetration process. However, nuclei with $N > 128$ have large deviations from the average curve $40/A$ of $\delta V^{(2)}$, which cannot be explained by the symmetry energy or the J-independent $T=0$ p-n pairing force. This suggests that there would be other interactions or correlations to describe the specific feature of $\delta V^{(2)}$ in this region. This problem should be studied further. The ‘‘α-condensate’’ point of view suggests that the strong p-n correlations in $A > 208$ nuclei cause the α-like $2p$-$2n$ correlations. The α-like correlations are important for the penetration as well as the formation of the α *particle*.

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