Δ excitation and its influences on neutron stars in relativistic mean field theory

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The relativistic mean field models with the different parameter sets are extended to investigate the properties of Δ-excited nuclear matter. The calculated results show that the critical densities for the Δ-excited nuclear matter predicted in a chiral hadronic model are larger than those obtained in the nonlinear Walecka model with the parameter sets Tm1 and NL1. The inclusion of the Δ leads to the decrease in the maximum mass of neutron stars in a chiral hadronic model, while the opposite behavior is shown in the nonlinear Walecka model.

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The investigation of hadronic matter shows many new and interesting results. Different composition of hadronic matter may lead to different phase structures under extreme conditions. These new states of dense matter are expected to be found in relativistic heavy ion collisions or the conjecture of astronomical observations. In the framework of the nonlinear Walecka model it is predicted that there is a phase transition from nucleonic matter to Δ-excited nuclear matter, and the occurrence of this transition depends on the coupling constants [1,2]. Meanwhile, the calculated results in the derivative scalar coupling model illustrate that the properties of Δ-excited nuclear matter are insensitive to the choices of the model parameter [3]. In fact, whether stable Δ-excited nuclear matter exists or not is still controversial since little is known about the coupling constants of the Δ with the scalar and vector mesons. Recently, by means of the finite-density QCD sum rules and the numerical calculations the range of values for the coupling constants concerned with the Δ has been confined within the triangle of the coupling constants [4], the corresponding coupling constants of the Δ with the scalar and vector mesons restrained in this way are useful when the properties of Δ-excited nuclear matter are studied.

Recently, a chiral hadronic model has been proposed by Furnstahl, Serot, and Tang (also referred to as the FST model) [5], the features of this model incorporate the nonlinear chiral symmetry of the strong interaction, broken scale invariance, and the phenomenology of vector dominance. With the model parameter sets calibrated at the equilibrium properties of nuclear matter, the FST model has been successfully applied to study nuclear matter and finite nuclei [6,7]. In this Brief Report, the FST model and the nonlinear Walecka model are extended to investigate the properties of Δ-excited nuclear matter, and then the influences of the Δ excitation on the maximum mass of neutron stars are discussed.

With the inclusion of the Δ the FST model is adopted to show our formulas with the Lagrangian density as [5]

$$\mathcal{L}=\overline{\psi}_N\left[i\gamma_\mu\mathcal{D}^\mu+g_A\gamma^\mu\gamma_5 a_\mu-M_N^0+g_s\phi-\frac{1}{2}g_\rho\gamma_\mu\vec{\tau}\cdot\vec{b}^\mu\right]\psi_N +\overline{\psi}_\Delta[i\gamma_\mu\mathcal{D}^\mu+g_A\gamma^\mu\gamma_5 a_\mu-M_\Delta^0+g_s^\Delta\phi]\psi_\Delta +\frac{1}{2}\left[1+\eta\frac{\phi}{S_0}+\cdots\right]\left[\frac{1}{2}f_\pi^2\mathrm{tr}(\partial_\mu U\partial^\mu U^+)+m_v^2V_\mu V^\mu\right] -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{4!}\xi(g_v^2V_\mu V^\mu)^2+\frac{1}{2}\partial_\mu\phi\partial^\mu\phi -H_q\left(\frac{S^2}{S_0^2}\right)^{2/d}\left(\frac{1}{2d}\ln\frac{S^2}{S_0^2}-\frac{1}{4}\right)-\frac{1}{4}\vec{G}_{\mu\nu}\cdot\vec{G}^{\mu\nu} +\frac{1}{2}m_\rho^2\vec{b}_\mu\cdot\vec{b}^\mu+\cdots, \tag{1}$$

where ψ_N ($N=n,p$) and ψ_Δ stand for the nucleon and Δ fields, the additional coupling constants of the Δ with the scalar and vector mesons are introduced with the superscript of the Δ, the other notations in the above formula are the same as those in Ref. [5].

In the mean field approximation (MFA), the meson fields should be replaced by the constants, i.e., $\phi\to\phi_0\equiv\langle\phi\rangle$, $V_\mu\to\langle V_\mu\rangle\equiv\delta_{\mu,0}V_0$, $\vec{b}_{\mu,3}\to\langle\vec{b}_{\mu,3}\rangle\equiv\delta_{\mu,0}b_0$, and the other quantities related to the pion meson field vanish. The Lagrangian density for the FST model in the MFA can be rewritten as

$$\mathcal{L}_{MFA}=\overline{\psi}_N\left[i\gamma_\mu\partial^\mu-(M_N^0-g_s\phi_0)-g_v\gamma^0V_0-\frac{1}{2}g_\rho\tau_3\gamma^0b_0\right]\psi_N+\overline{\psi}_\Delta[i\gamma_\mu\partial^\mu-(M_\Delta^0-g_s^\Delta\phi_0) -g_v^\Delta\gamma^0V_0]\psi_\Delta+\frac{1}{2}m_v^2V_0^2\left(1+\eta\frac{\phi_0}{S_0}\right)+\frac{1}{4!}\xi(g_vV_0)^4 +\frac{1}{2}m_\rho^2b_0^2-H_q\left(1-\frac{\phi_0}{S_0}\right)^{4/d}\left[\frac{1}{d}\ln\left(1-\frac{\phi_0}{S_0}\right)-\frac{1}{4}\right]. \tag{2}$$

With Eq. (2) the equations of motion can be derived as

$$\left[i\gamma_\mu\partial^\mu-(M_N^0-g_s\phi_0)-g_v\gamma^0V_0-\frac{1}{2}g_\rho\tau_3\gamma^0b_0\right]\psi_N=0, \tag{3}$$

$$[i\gamma_\mu\partial^\mu-(M_\Delta^0-r_sg_s\phi_0)-r_vg_v\gamma^0V_0]\psi_\Delta=0, \tag{4}$$

for the nucleon and Δ fields, and

$$\frac{\eta}{2S_0}m_v^2V_0^2+m_s^2S_0\left(1-\frac{\phi_0}{S_0}\right)^{[4/d-1]}\ln\left(1-\frac{\phi_0}{S_0}\right)$$
$$=-g_s[\rho_n^s+\rho_p^s+r_s\rho_\Delta^s],$$
$$\frac{1}{6}\xi g_v^4V_0^3+\left(1+\eta\frac{\phi_0}{S_0}\right)m_v^2V_0=g_v[\rho_n+\rho_p+r_v\rho_\Delta],$$
$$m_\rho^2b_0=\frac{1}{2}g_\rho\langle\overline{\psi_N}\tau_3\psi_N\rangle\equiv\frac{1}{2}g_\rho\rho_3, \tag{5}$$

for the meson fields, where $r_s=g_s^\Delta/g_s$ and $r_v=g_v^\Delta/g_v$ are the ratios of the scalar and vector couplings for the Δ to those for the nucleon. The scalar and vector densities are denoted by

$$\rho_i^s\equiv\langle\overline{\psi_i}\psi_i\rangle,\quad \rho_i\equiv\langle\overline{\psi_i}\gamma^0\psi_i\rangle,\quad i=(n,p,\Delta). \tag{6}$$

The energy density for Δ-excited nuclear matter is derived as

$$\epsilon=\sum_{i=N,\Delta}\frac{\gamma(i)}{(2\pi)^3}\int_0^{k_{F_i}}(k^2+M^{*2}_i)^{1/2}d^3k+g_vV_0(\rho_n+\rho_p$$
$$+r_v\rho_\Delta)+\frac{g_\rho^2\rho_3^2}{8m_\rho^2}-\frac{1}{2}\left(1+\eta\frac{\phi_0}{S_0}\right)m_v^2V_0^2-\frac{1}{4!}\xi(g_vV_0)^4$$
$$+H_q\left\{\left(1-\frac{\phi_0}{S_0}\right)^{4/d}\left[\frac{1}{d}\ln\left(1-\frac{\phi_0}{S_0}\right)-\frac{1}{4}\right]+\frac{1}{4}\right\}, \tag{7}$$

where the degeneracy factors are taken to be $\gamma(N)=2$ ($N=n,p$) and $\gamma(\Delta)=16$, $k_{F_i}(i=n,p,\Delta)$ is for the Fermi momentum with the particle species i. The effective nucleon and Δ masses are given by

$$M_N^*=M_N^0-g_s\phi_0,\quad M_\Delta^*=M_\Delta^0-r_sg_s\phi_0, \tag{8}$$

with the bare nucleon and Δ masses M_N^0 and M_Δ^0, the baryon density is defined as

$$\rho=\rho_n+\rho_p+\rho_\Delta\equiv\frac{1}{3\pi^2}(k_{F_n}^3+k_{F_p}^3+8k_{F_\Delta}^3). \tag{9}$$

The chemical stability is described by [4]

$$E_F(N)=E_F(\Delta), \tag{10}$$

where $E_F(N)=g_vV_0\mp g_\rho^2\rho_3/4m_\rho^2+\sqrt{k_{F_N}^2+M^{*2}_N}$, "$-$" for $N=n$ and "$+$" for $N=p$, $E_F(\Delta)=r_vg_vV_0+\sqrt{k_{F_\Delta}^2+M^{*2}_\Delta}$.

Based on the empirical knowledge that there is no real Δ excitation at the saturation density of normal nuclear matter, Eq. (10) should have no real solution for k_{F_Δ}, so that the constrained condition for the ratios of the scalar and vector couplings r_s and r_v can be derived from Eq. (10) for symmetric nuclear matter as

$$r_s\leqslant ar_v+b,$$
$$a=\frac{g_vV_0}{g_s\phi_0}\bigg|_{\rho=\rho_0},\quad b=\frac{M_\Delta^0-g_vV_0-\sqrt{k_{F_N}^2+M_N^{*2}}}{g_s\phi_0}\bigg|_{\rho=\rho_0}. \tag{11}$$

Our numerical results show that the different relativistic nuclear models, e.g., the nonlinear Walecka model with the parameter sets Tm1 [8], NL1 [9], Boguta2 [10], and the linear Walecka model used in Ref. [11] as well as a chiral hadronic model with the parameter sets $T1$, $T2$, and $T3$ [5], give the approximate same values for the coefficients a and b, the obtained mean values of a and b are $a\approx0.81$ and $b\approx0.85$, thus, one has

$$r_s\leqslant0.81r_v+0.85, \tag{12}$$

for the relation between r_s and r_v. However, this relation imposes a weaker restriction on the coupling constants for the Δ [4]. In terms of the constraints that there are no real Δ present at the saturation density of nuclear matter, the appearance of the real Δ at higher density leads to a metastable state which is not the ground state of nuclear matter, and by means of the numerical calculations, a more restrictive constraint instead of Eq. (12) is phenomenologically summarized as [4]

$$r_s\leqslant1.01r_v+0.38. \tag{13}$$

The Δ vector and scalar self-energies have been studied in the framework of the finite-density QCD sum rules [12], the conclusion is that the Δ vector self-energy is weaker than that of the nucleon, while its scalar self-energy is stronger. Therefore, the other two constraints for r_s and r_v can be expressed as

$$r_s\geqslant1,\quad r_v\leqslant1. \tag{14}$$

Combining Eq. (14) with Eq. (13), the triangle relation for the Δ coupling constants has been constructed in Ref. [4]. In principal, one can choose any possible coordinates within this triangle as input coupling constants for the Δ. Without loss of generality, one can take the three sets of the coordinates at the three vertices of this triangle as input coupling constants for the Δ, so that the obtained results can contain any other possible results using the coupling constants for the Δ within this triangle. Moreover, the three sets of the coupling constants corresponding to the three vertices of this triangle for the Δ give the similar conclusions, it is enough for us to adopt one of the three parameter sets at the three vertices of this triangle for the Δ to show our calculated results, thus, the coordinates at a vertex of this triangle, i.e., $r_s=1.39$, $r_v=1.0$, are chosen as the input coupling constants for the Δ.

Since the results with the parameter set $T2$ for the FST model lie between those with the parameter sets $T1$ and $T3$, the parameter sets $T1$ and $T3$ for the FST model listed in Table 1 of Ref. [5] are used, in addition, there are many parameter sets for the nonlinear Walecka model, as an example, the quite successful parameter sets Tm1 in Table 2 of Ref. [8] and NL1 in Table 3 of Ref. [9] are also used in the present calculation.

Figure 1 shows the effective baryon masses for nucleon and Δ against the ratio of baryon density. It is seen that both the effective nucleon mass (a) and the effective Δ mass (b) decrease as the ratio of baryon density increases. Comparing figures (a) with (b), it is shown that the curves of the effective Δ mass are lower than those of the effective nucleon mass due to $r_s>1$.

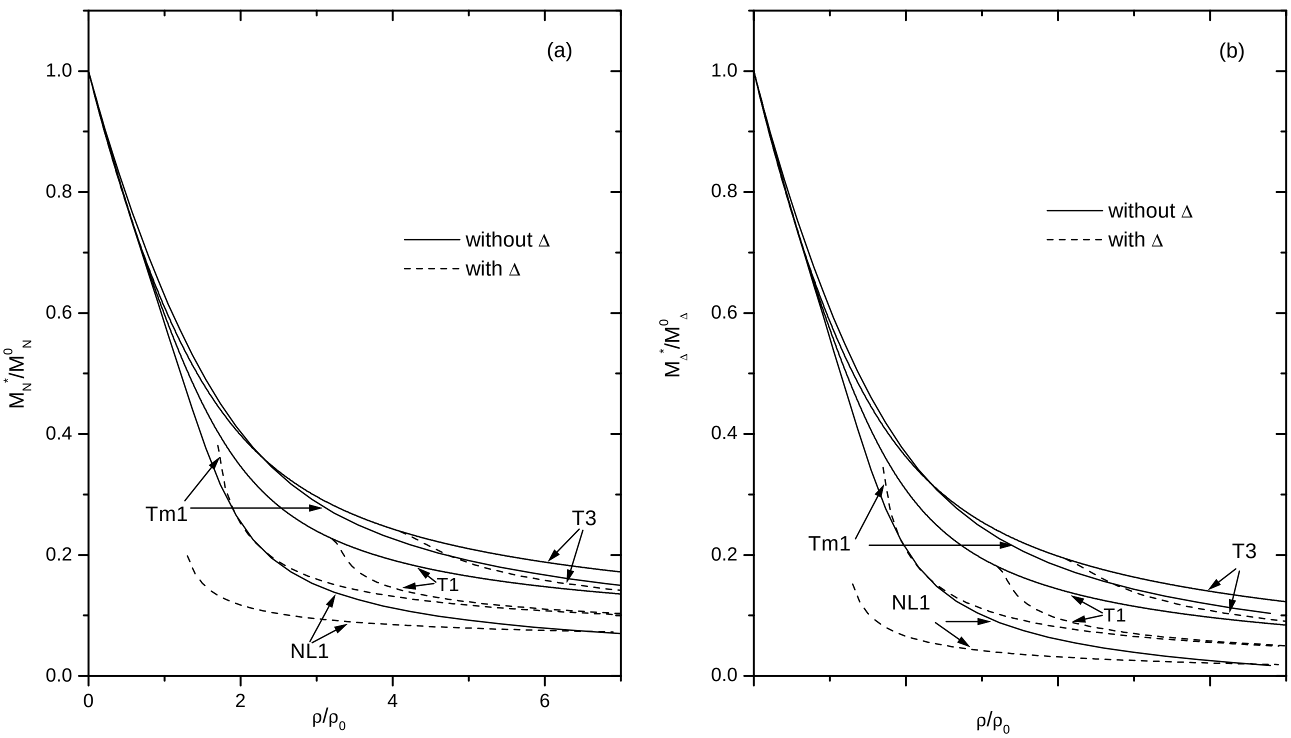

FIG. 1. Ratios of the effective baryon masses against the ratio of the baryon density with $r_s = 1.39$ and $r_v = 1.0$.

The binding energy per baryon is given in Fig. 2. It is observed that the critical density $(4.15\rho_0)$ for the presence of the Δ in the FST model with the parameter set $T3$ is larger than that $(3.22\rho_0)$ calculated with the parameter set $T1$, and both critical densities in the FST model are larger than those obtained in nonlinear Walecka model with the parameter set Tm1$(1.72\rho_0)$ and NL1$(1.25\rho_0)$. The result with the parameter set $T1$ makes the second minimum of the binding energy appear in the FST model, and no second minimum is formed with the parameter set $T3$. It is seen that the value of the second minimum is positive, and the depth of the second minimum in the FST model is shallower, this result means that the FST model leads to the existence of the metastable state of Δ-excited nuclear matter. In contrast, the results given by Tm1 and NL1 indicate that the ground state of nuclear matter is energetically favorable at higher densities $(\sim 3\rho_0)$ for Tm1 and $(\sim 2\rho_0)$ for NL1, and the real ground state of nuclear matter would be a Δ-excited matter at a density of about $(\sim 3\rho_0)$ for Tm1 and $(\sim 2\rho_0)$ for NL1. Recalling one of the conditions that the second minimum should lie above the saturation energy of normal nuclear matter when the triangle relation has been constructed [4], one can conclude that the triangle relation is valid for a chiral hadronic model, and invalid for the nonlinear Walecka model with the parameter sets Tm1 and NL1.

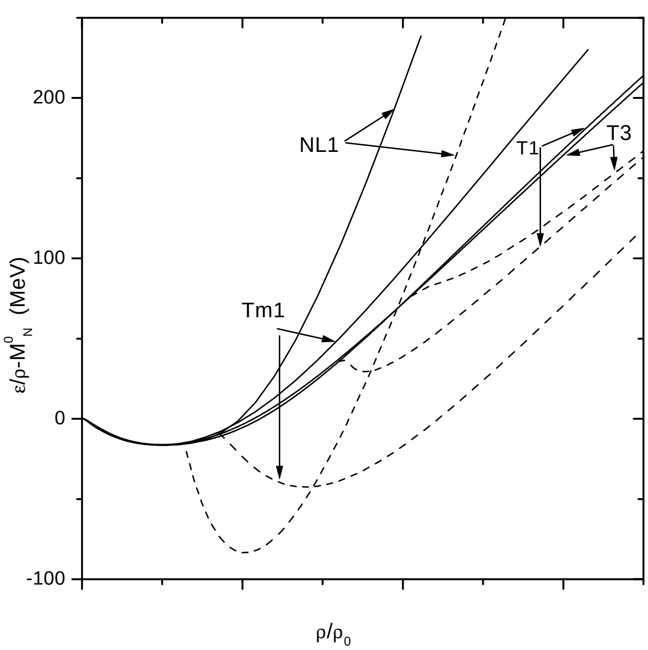

FIG. 2. Binding energy per baryon against the ratio of the baryon density with $r_s = 1.39$ and $r_v = 1.0$.

Comparison among the different models with the different coupling constants of the Δ is presented in Fig. 3. It is seen that with the fixed value of $r_v = 1.0$ the increasing of r_s, e.g., from $r_s = 1.31$ to 1.39, the second minimum of each curve shifts downwards, and its depth becomes deeper, whereas with the fixed value of $r_s = 1.39$, the decreasing of r_v, e.g., $r_s = 1.0$ to 0.8, the second minimum of each curve also moves downwards, and its depth becomes deeper. Therefore, the general trend is that the increasing of r_s or decreasing of r_v are in favor of the existence of Δ-excited matter. In comparison with the results given by Tm1 and NL1, the results obtained by the FST model is not sensitive to the variation of the Δ coupling constants, and can guarantee the triangle relation to be valid. Moreover, though the critical densities for the appearance of the Δ are varied with the variation of r_s and r_v, it is clear that the critical densities for the appearance

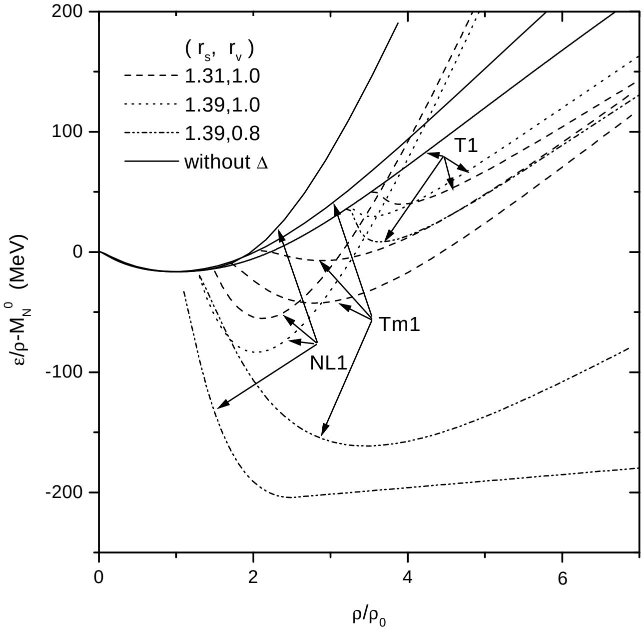

FIG. 3. Binding energy per baryon against the ratio of the baryon density with the different values of r_s and r_v.

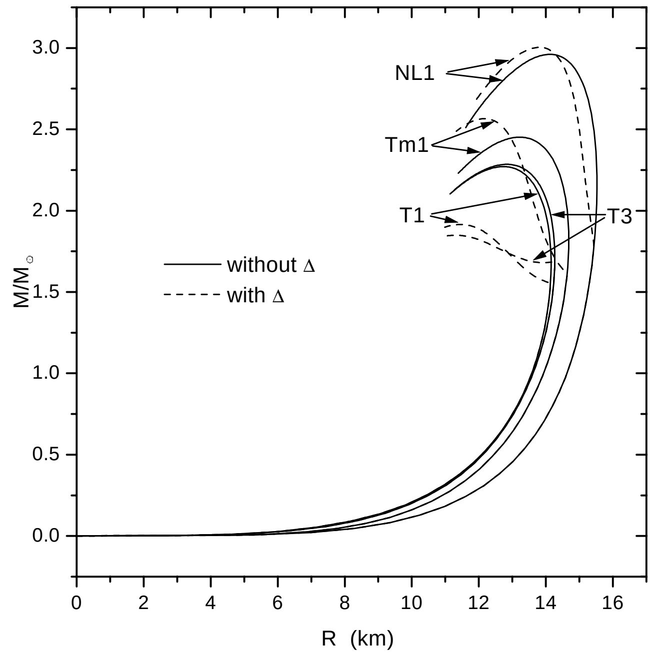

FIG. 4. Ratios of masses for neutron stars against the radius with $r_s = 1.39$ and $r_v = 1.0$.

of the Δ in the FST model are still larger than those given by the nonlinear Walecka model with the parameter sets Tm1 and NL1.

Using the equation of state for neutron matter with or without the inclusion of the Δ, and after solving the Tolman-Oppenheimer-Volkoff equation [13], one can examine the influence of the Δ excitation on neutron stars with $r_s = 1.39$ and $r_v = 1.0$. The mass and radius relation for neutron stars is displayed in Fig. 4. It is clear that the appearance of the Δ leads to the decrease in the maximum mass of neutron stars, i.e., $M/M_\odot \approx 2.27$, 2.28 without the Δ to $M/M_\odot \approx 1.91$, 1.85 with the Δ in the FST model with the parameter sets $T1$ and $T3$, respectively, while the inclusion of the Δ in the nonlinear Walecka model with the parameter sets Tm1 and NL1 is different from that of the FST model, the appearance of the Δ increases the maximum mass of neutron stars from $M/M_\odot \approx 2.45$(Tm1), 2.91 (NL1) without the Δ to $M/M_\odot \approx 2.56$(Tm1), 3.0 (NL1) with the Δ. These results indicate though both the FST model and the nonlinear Walecka model can reproduce the basic properties of nuclear matter, the prediction for the maximum mass of neutron stars with and without the Δ is different.

In conclusion, the different relativistic nuclear models in the MFA are applied to study the properties of Δ-excited nuclear matter, in comparison with the results obtained in the nonlinear Walecka model with the parameter sets Tm1 and NL1, the critical densities for the appearance of the Δ are larger, and the triangle relation for the Δ couplings is valid in the FST model and fail in the nonlinear Walecka model with the parameter sets Tm1 and NL1. The influences of the Δ leads to the decrease in the maximum mass of neutron stars in the FST model, while the opposite trend is shown in the nonlinear Walecka model.

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[1] B.M. Waldhauser, J. Theis, J.A. Maruhn, H. Stöcker, and W. Greiner, Phys. Rev. C **36**, 1019 (1987).
[2] Z.X. Li, G.J. Mao, Y.Z. Zhuo, and W. Greiner, Phys. Rev. C **56**, 1570 (1997).
[3] S.K. Choudhury and R. Rakshit, Phys. Rev. C **48**, 598 (1993).
[4] D.S. Kosov, C. Fuchs, B.V. Martemyanov, and A. Faessler, Phys. Lett. B **421**, 37 (1998).
[5] R.J. Furnstahl, H.B. Tang, and B.D. Serot, Phys. Rev. C **52**, 1368 (1995).
[6] R.J. Furnstahl, B.D. Serot, and H.B. Tang, Nucl. Phys. **A615**, 441 (1997).
[7] R.J. Furnstahl, H.B. Tang, and B.D. Serot, Nucl. Phys. **A598**, 539 (1996).
[8] Y. Sugahara and H. Toki, Nucl. Phys. **A579**, 557 (1994).
[9] P.G. Reinhard, M. Rufa, J. Maruhn, W. Greiner, and J. Friedrich, Z. Phys. A **323**, 13 (1986).
[10] T. Nakai and S. Takagi, Prog. Theor. Phys. **71**, 1005 (1984), and references therein.
[11] K. Wehrberger, C. Bedau, and F. Beck, Nucl. Phys. **A504**, 797 (1989).
[12] X. Jin, Phys. Rev. C **51**, 2260 (1995).
[13] G. Baym, Nucl. Phys. **A590**, 233c (1995), and references therein.