Scaling behavior at high p_T and the p/π ratio

Rudolph C. Hwa[1] and C. B. Yang[1,2]
[1]*Institute of Theoretical Science and Department of Physics, University of Oregon, Eugene, Oregon 97403-5203*
[2]*Institute of Particle Physics, Hua-Zhong Normal University, Wuhan 430079, People's Republic of China*
(Received 7 November 2002; published 10 March 2003)

We first show that the pions produced at high p_T in heavy-ion collisions over a wide range of high energies exhibit a scaling behavior when the distributions are plotted in terms of a scaling variable. We then use the recombination model to calculate the scaling quark distribution just before hadronization. From the quark distribution, it is then possible to calculate the proton distribution at high p_T, also in the framework of the recombination model. The resultant p/π ratio exceeds one in the intermediate-p_T region where data exist, but the scaling result for the proton distribution is not reliable unless p_T is high enough to be insensitive to the scale-breaking mass effects.

DOI: 10.1103/PhysRevC.67.034902 PACS number(s): 25.75.Dw, 24.85.+p

I. INTRODUCTION

There are three separate and independent aspects about the hadrons produced at large transverse momentum p_T in heavy-ion collisions at high energies that collectively contribute to a coherent picture to be addressed in this paper. One is the existence of a scaling behavior at large p_T that we have found by presenting the data in terms of a new variable. Another is the issue about the surprisingly large proton-to-pion ratio at moderate p_T (~ 2–3 GeV/c) discovered by PHENIX [1] in central Au-Au reactions at $\sqrt{s}=130$ and 200 GeV. The third issue concerns the hadronization process relevant for the formation of hadrons at large p_T and the applicability of the recombination model [2]. It is our goal to show that, in light of the scaling behavior of the π^0 produced, the recombination mechanism naturally gives rise to a p/π ratio that exceeds 1 in the $2<p_T<3$ GeV/c range.

Particle production in heavy-ion collisions at very high energies is usually described in terms of hydrodynamical flow [3], jet production at high p_T [4], thermal statistical model [5], or a combination of various hadronization mechanisms [6]. In none of the conventional approaches does one expect protons to be produced at nearly the same rate as the pions. If all hadrons with $p_T>2$ GeV/c are regarded as products of jet fragmentation, then the known fragmentation functions of quark or gluon jets would suppress the proton relative to the pion by the sheer weight of the proton mass. Such a discrepancy from the observed data led some to regard the situation as an anomaly and propose the gluonic baryon junction as a mechanism to enhance the proton production rate [7]. Their predictions remain to be verified experimentally.

The parton fragmentation functions have been used even at low p_T in string models where the production of particles in hadronic collisions is treated as the fragmentation of diquarks, as done in the dual parton model [8]. There has been a long-standing dichotomy on whether particle production in the fragmentation region can better be described by fragmentation [8,9] or by recombination [2,10]. It is possible that the two pictures might be unified in a more comprehensive treatment of hadronization in the future. Here we extend the recombination model to the central region at large p_T. It should be recognized that an essential part of the recombination model is the determination of the distribution functions of the quarks and antiquarks that are to recombine. In the case of large-p_T hadrons, the underlying physics is undoubtedly hard collisions of partons and the associated radiation of gluons. If the parton distributions can be calculated just before hadronization, then the final step of recombination can readily be completed. If those distributions cannot be determined in pQCD, then the step between the initiating large-p_T parton and the resultant hadrons can efficiently be described by a fragmentation function, determined phenomenologically from experiments. Thus, in that sense the two approaches, recombination and fragmentation, are not contradictory, but complementary.

We state from the outset that no attempt will be made here to perform a first-principles calculation of the parton distributions at large p_T before recombination. However, from the observed data on pion production in central Au-Au collisions at the Relativistic Heavy-Ion Collider (RHIC), it is possible to work backwards in the recombination model to determine the quark (and antiquark) distribution at large p_T. On the basis of the quark distributions inferred, it is then possible to calculate the proton distribution in the recombination model. The basic idea is that if there is a dense system of quarks and antiquarks produced in a heavy-ion collision, whatever the dynamics responsible for them may have been (gluons having been converted to $q\bar{q}$ pairs before hadronization), then the formation of pions and protons (and whatever else) is prescribed by the recombination model without any arbitrariness in normalization and momentum dependence.

One limitation of the recombination model, as it stands at present, is that it is formulated in a frame-independent way in terms of momentum fractions and is therefore inapplicable to a system where the particle momenta are low and the mass effects are large. The physics of recombination is still valid at low momentum, but the details of the wave functions of the constituent quarks become important; they have not been built into the recombination function that takes the simplest form in the infinite momentum frame. Thus our calculation of particles produced at midrapidity is not reliable when p_T

is of the order of the masses of the hadrons under consideration. For protons, we can trust the results only for $p_T > 3$ GeV/c. For pions, the lower limit of validity can be pushed much lower.

Since our approach makes crucial use of the experimental data on the pion spectrum as the input, it is essential to relate the spectra determined at different energies to an invariant distribution so that the scale-invariant recombination model can be applied. To discover the existence of an invariant distribution with no theoretical prejudices is a worthful problem in its own right. Fortunately, that turns out to be possible. The analysis for that part of the study will be presented below first to emphasize its independence from the theoretical modeling of hadronization. It should be mentioned that the scaling of transverse mass spectra has been investigated recently [11]. The emphasis there has been on the dependences on the particle species and centrality for $m_T < 3.8$ GeV, while our focus is on the dependence on energy ($17 < \sqrt{s} < 200$ GeV) for $p_T < 8$ GeV/c. Thus the two studies are complementary to each other.

II. A UNIVERSAL SCALING DISTRIBUTION

The preliminary data of the p_T distributions of π^0 produced at RHIC at $\sqrt{s}=130$ and 200 GeV were shown by the PHENIX Collaboration at Quark Matter 2002 [12] for central Au-Au collisions together with the WA98 data for Pb-Pb collisions at $\sqrt{s}=17$ GeV [13]. They show that the level of the tail at large p_T rises, as $\sqrt{s}$ is increased. We want to consider the possibility that the three sets of data points can be combined to form a universal curve.

The π^0 inclusive distributions at midrapidity are integrated over η for a range of $\Delta\eta=1$ so that the data points are given for the following quantity [12]:

$$f(p_T,s)=\frac{1}{2\pi p_T}\frac{dN}{dp_T}=\int_{\Delta\eta} d\eta(2\pi p_T N_{evt})^{-1}\frac{d^2N_{\pi^0}}{dp_T d\eta}. \tag{1}$$

In comparing the PHENIX data with those of WA98 one should recognize that in addition to the difference in the colliding nuclei there is a slight mismatch in centrality (top 10% for PHENIX and top 12.7% for WA98).

To unify the three datasets, it is natural to first consider a momentum fraction variable similar to x_F in longitudinal momentum. However, so much momenta are taken by the other particles outside the $\Delta\eta=1$ range, it is unwise to also use $\sqrt{s}/2$ as the scale to calculate the transverse momentum fraction. We assume that for every $\sqrt{s}$ there is a relevant scale K to describe the p_T behavior relative to that scale. Let us define

$$z=p_T/K \tag{2}$$

and transform $f(p_T,s)$ to a new function $\Phi(z,K)$, where

$$\Phi(z,K)=K^2 f(p_T,s)=\frac{1}{2\pi z}\frac{dN}{dz}. \tag{3}$$

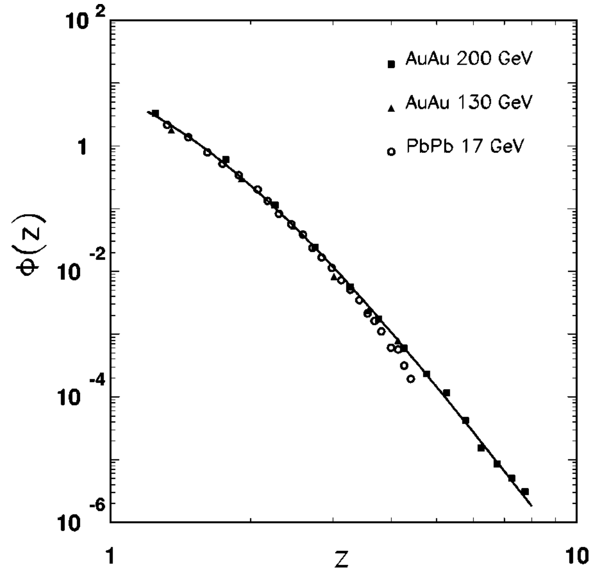

FIG. 1. Scaled transverse momentum distribution of produced π^0. Data are from Refs. [12,13]. The solid line is a fit of the data by Eq. (6).

We adjust K for each s and check whether all three datasets coalesce into one universal dependence on z, which we would simply label as $\Phi(z)$, if it is possible.

In Fig. 1 we show $\Phi(z)$, where the three symbols represent the three datasets for the three energies. Evidently, the universality exists and is striking. While this behavior needs to be confirmed by more data, and the theoretical implication remains to be explored, the existence of this scaling behavior is a significant phenomenological property of the p_T distributions that suggests some underlying simplicity. It is like the Koba-Nielson-Olesen scaling of the multiplicity distributions $P(n,s)$ in pp collisions, where for $\sqrt{s}<200$ GeV they can be expressed by one universal scaling function $\psi(z)$, with $z=n/\langle n\rangle$ [14,15].

The values of K that are used for the plot in Fig. 1 are in units of GeV: $K=1(200)$, 0.9 (130), and 0.717 (17), the quantities in the parentheses being the values of $\sqrt{s}$. The $\sqrt{s}$ dependence of K forms nearly a straight line, as shown in Fig. 2. Since the high and low energy data differ both in colliding nuclei and in centrality, one does not expect strict regularity in how K depends on $\sqrt{s}$. Nevertheless, an approximate linear dependence is a simple behavior expected on dimensional grounds. The straight line in Fig. 2 corresponds to the best fit

$$K(s)=0.69+1.55\times10^{-3}\sqrt{s}, \tag{4}$$

where $\sqrt{s}$ is in units of GeV. It should be recognized that the normalization of $K(s)$ is arbitrary; it is chosen to be 1 at $\sqrt{s}=200$ GeV for simplicity. If it is normalized to some other value at that point, the linear behavior in Fig. 2 is unchanged, only the scale of the vertical axis is shifted accordingly. The scaling property in Fig. 1 is also unchanged, the only modifications being the scales of the horizontal and vertical axes. Thus the absolute magnitude of the dimensionless variable z has no significance.

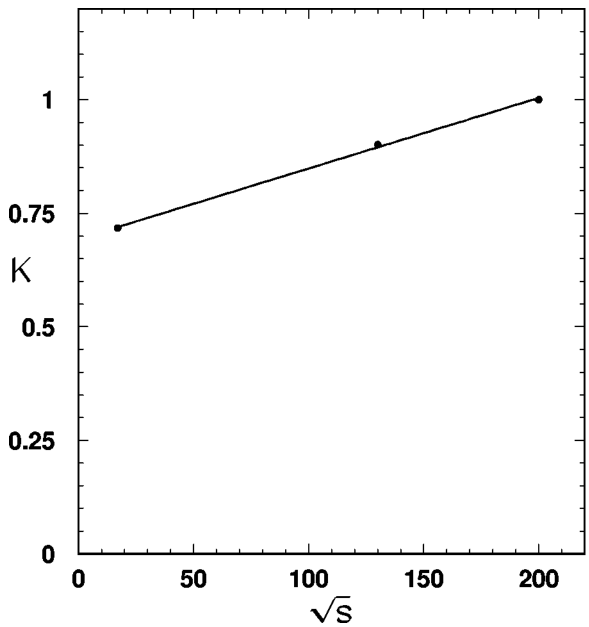

FIG. 2. The dependence of $K(s)$ on $\sqrt{s}$. The line is a linear fit.

If the z dependence of $\Phi(z)$ in Fig. 1 were strictly linear, so that it is a power-law dependence

$$\Phi(z) \propto z^{\alpha}, \tag{5}$$

then there would be no relevant scale in the problem. The fact that it is not a straight line implies that there is an intrinsic scale in the p_T problem, which is hardly surprising. What is significant is that while there is no strict scaling in z, there is no explicit dependence on s. That is, at any energy we have the same universal function $\Phi(z)$, which will be referred to as the scaling behavior in s. That function can be parametrized by

$$\Phi(z) = 1500(z^2+2)^{-4.9}, \tag{6}$$

which is represented by the smooth curve in Fig. 1. For large enough z, Eq. (6) does have the form of the power law given in Eq. (5) with $\alpha = 9.8$. It is a succinct statement of the universal properties at high p_T. The departure from Eq. (5) at small z reflects the physics at low p_T. Since there are no data on π^0 for $p_T < 1$ GeV/c, the extrapolation of $\Phi(z)$ to $z < 1$ is not reliable. However, there is a more accurate determination of $\Phi(z)$ that includes the low-z region when the charge π^+ data are considered; it is given in Ref. [16], and is not needed here. Note that there is no fixed scale in p_T that separates the high- and low-p_T physics. Equation (6) gives a smooth transition from one to the other in the variable z, thus implying different ranges of values of the transition p_T at different s.

While Eq. (6) gives a good parametrization of the scaling function $\Phi(z)$ throughout the whole range of z, one notices, however, that the WA98 data at 17 GeV show a slight departure from $\Phi(z)$ at the high-z end of that dataset. It should be recognized that those data points have $p_T > 3$ GeV/c, which represents a huge fraction of the available energy at $\sqrt{s} = 17$ GeV. In fact, one expects the violation of universality to be more severe at higher z at that $\sqrt{s}$, since energy conservation would suppress the inclusive cross section at higher p_T. What is amazing is that most of the WA98 data points are well described by $\Phi(z)$, even though the corresponding p_T values take up a much larger fraction of the available energy than the other data points from RHIC. It demonstrates the significance of the variable z in revealing the scaling property.

III. PION AND QUARK DISTRIBUTIONS IN THE RECOMBINATION MODEL

Having found a scaling distribution for the produced π^0 independent of s, we now consider the hadronization process in the recombination model in search of an origin of such a scaling behavior. In previous investigations the recombination model has been applied only to the fragmentation region where the longitudinal momenta are large and the transverse momenta are either held fixed at low p_T or integrated over [2,10,17]. We now consider the creation of pions in the central region of AA collisions and study the p_T dependence. Unlike the former case where the longitudinal momentum fractions of the partons are essentially known (from the structure functions), the p_T distributions of the partons in the latter case are essentially unknown. Indeed, it is the aim of this section to determine the parton p_T distributions from the π^0 distribution found in the preceding section.

Let us start by writing down the basic equation for recombination in the three-space

$$E\frac{d^3N_\pi}{d^3p} = \int \frac{d^3p_1}{E_1}\frac{d^3p_2}{E_2}\mathcal{F}(\vec{p}_1,\vec{p}_2)\,\mathcal{R}_\pi(\vec{p}_1,\vec{p}_2,\vec{p}), \tag{7}$$

where the left-hand side (LHS) is the inclusive distribution of the pion with energy momentum (E,p). $\mathcal{F}(\vec{p}_1,\vec{p}_2)$ is the probability of having a quark at p_1^μ and an antiquark at p_2^μ just before hadronization. $\mathcal{R}_\pi(\vec{p}_1,\vec{p}_2,\vec{p})$ is the invariant distribution, $Ed^3N_\pi^{q\bar{q}}/d^3p$, of producing a pion at p^μ given a q at p_1^μ and a $\bar{q}$ at p_2^μ. Note that $\mathcal{R}_\pi$ has the dimension (momentum)$^{-2}$, same as the LHS.

Writing the phase-space density in the form

$$\frac{d^3p}{E} = dy\, d\phi\, p_T\, dp_T, \tag{8}$$

we define the inclusive distribution in p_T, averaged over y and ϕ,

$$\frac{d^3N_\pi}{p_T\, dp_T} = \frac{1}{\Delta y}\int_{\Delta y} dy\, \frac{1}{2\pi}\int_0^{2\pi} d\phi\, E\frac{d^3N_\pi}{d^3p}, \tag{9}$$

where Δy is limited to one unit of rapidity in the central region. Our focus will be on the p_T distribution at high p_T. For the recombination distribution $\mathcal{R}_\pi(\vec{p}_1,\vec{p}_2,\vec{p})$ we need only consider the partons in the same transverse plane that contains $\vec{p}$, since at high p_T the partons with different y_i are not likely to recombine. Indeed, we assume not only y_1

$=y_2=y$, but also $\phi_1=\phi_2=\phi$ so that the partons and the pion are all colinear, and the kinematics can be reduced to that of a one-dimensional (1D) problem. As in the usual parton model, the parton momentum fractions in the hadron can vary between 0 and 1, but the deviation in the momentum components of the partons transverse to the hadron $\vec{p}$ must be severely limited because of the limited transverse size of the hadron. Thus we write

$$\mathcal{R}_\pi(\vec{p}_1,\vec{p}_2,\vec{p})=\mathcal{R}^0_\pi\delta(y_1-y_2)\,\delta(\phi_1-\phi_2)\times\delta\left(\frac{y_1+y_2}{2}-y\right)\delta^2(\vec{p}_{1_T}+\vec{p}_{2_T}-\vec{p}_T), \tag{10}$$

where $\mathcal{R}^0_\pi$ is dimensionless, since $\delta^2(\vec{p}_{1_T}+\vec{p}_{2_T}-\vec{p}_T)$ has the dimension of $\mathcal{R}_\pi(\vec{p}_1,\vec{p}_2,\vec{p})$. If this delta function is further written in the colinear form due to the $\delta(\phi_1-\phi_2)$ in Eq. (10)

$$\delta^2(\vec{p}_{1_T}+\vec{p}_{2_T}-\vec{p}_T)=\delta\left(\frac{\phi_1+\phi_2}{2}-\phi\right)\frac{1}{p_T}\delta(p_{1_T}+p_{2_T}-p_T), \tag{11}$$

then Eq. (7) can be reduced to the 1D form

$$\frac{dN_\pi}{p_T dp_T}=\int dp_{1_T}dp_{2_T}p_{1_T}p_{2_T}\mathcal{F}(p_{1_T},p_{2_T})\mathcal{R}^0_\pi p_T^{-2}\times\delta\left(\frac{p_{1_T}+p_{2_T}}{p_T}-1\right), \tag{12}$$

where $\mathcal{F}(p_{1_T},p_{2_T})$ is the $q\bar{q}$ distribution in p_{i_T} averaged over y and ϕ.

We can reexpress this equation in terms of the scaling variable $z=p_T/K$, introduced in Eq. (2), and obtain

$$\frac{dN_\pi}{zdz}=\int dz_1dz_2\,z_1\,z_2\,F(z_1,z_2)\,R_\pi(z_1,z_2,z), \tag{13}$$

where

$$F(z_1,z_2)=K^4\,\mathcal{F}(p_{1_T},p_{2_T}), \tag{14}$$

$$R_\pi(z_1,z_2,z)=\mathcal{R}^0_\pi z^{-2}\,\delta\left(\frac{z_1+z_2}{z}-1\right). \tag{15}$$

Since $\mathcal{F}(p_{1_T},p_{2_T})$ is the parton density in $p_{1_T}dp_{1_T}p_{2_T}dp_{2_T}$, $F(z_1,z_2)$ is the corresponding dimensionless density in $z_1dz_1z_2dz_2$. Equation (13) is now our basic formula for recombination in the scaled transverse-momentum variable. The total number of q and $\bar{q}$ is $\int dz_1dz_2\,z_1z_2F(z_1,z_2)$, which should be invariant under a change of scale

$$z=\lambda x, \tag{16}$$

so that

$$x=p_T/K', \quad K'=\lambda K. \tag{17}$$

The corresponding change on $F(z_1,z_2)$ is that it becomes

$$F'(x_1,x_2)=\lambda^4F(z_1,z_2). \tag{18}$$

Thus the normalization of $F(z_1,z_2)$ is scale dependent, as it should in view of Eq. (14).

So far, the recombination function $R_\pi(z_1,z_2,z)$ is not fully specified because $\mathcal{R}^0_\pi$ has not been. In Eq. (15) the factor z^{-2} is associated with the dimension of the pion density, and the delta function with momentum conservation. To introduce the pion wave function in terms of the constituent quarks, we rewrite Eq. (15) as

$$R_\pi(z_1,z_2,z)=R^0_\pi z^{-2}\,G_\pi(\xi_1,\xi_2), \tag{19}$$

where R^0_π is a normalization constant to be determined and $G_\pi(\xi_1,\xi_2)$ is the valon distribution of the pion [2,10]. Since the recombination of a q and $\bar{q}$ into a pion is the time-reversed process of displaying the pion structure, the dependence of $R_\pi(z_1,z_2,z)$ on the pion structure is expected. During hadronization the initiating q and $\bar{q}$ dress themselves and become the valons of the produced hadron without significant change in their momenta. The variable ξ_i in Eq. (19) denotes the momentum fraction of the ith valon, i.e.,

$$\xi_i=z_i/z, \tag{20}$$

which is denoted by y_i in the valon model [2,10], a notation that cannot be repeated here on account of the rapidity variables already used in Eq. (10). In general, the valon distribution of a hadron h has a part specifying the wave-function squared, $\tilde{G}_h$, and a part specifying momentum conservation

$$G_h(\xi_1,\ldots)=\tilde{G}_h(\xi_1,\ldots)\,\delta\left(\sum_i\xi_i-1\right), \tag{21}$$

where the functional form of $\tilde{G}_h$ is determined phenomenologically. Although, for proton, $\tilde{G}_p$ is found to be highly nontrivial [18], for pion, $\tilde{G}_\pi$ turns out to be very simple [10]

$$\tilde{G}_\pi(\xi_1,\xi_2)=1, \tag{22}$$

which is a reflection of the fact that the pion mass is much lower than the constituent quark masses, so tight binding results in a large uncertainty in the momentum fractions of the valons. Equation (22) implies that the valon momenta of the pion are uniformly distributed in the range $0<\xi_i<1$.

What remains in Eq. (19) for us to determine is R^0_π. At this point we need to be more specific about the quark and antiquark that recombine. If the colors of q and $\bar{q}$ are considered, then the probability of forming a color singlet pion is 1/9 in $3\times\bar{3}$. Similarly, for three quarks forming a proton the probability is 1/27 in $3\times3\times3$. In the parton distributions, $F_{q\bar{q}}$ for pion production involves two color triplets and F_{qqq} for proton production involves three color triplets so the color factors work out just right in that the factors of 9 for $q\bar{q}$ and 27 for qqq are canceled by the corresponding inverse factors in the recombination probabilities. In other words, for the p/π ratio to be considered later, we can ignore the factors associated with the color degrees of freedom and proceed with the determination of $F_{q\bar{q}}$ without specifying the quark colors and summing over them.

The situation with flavor is not the same. For a $u\bar{u}$ pair and a $d\bar{d}$ pair, they can form π^0 and η in the flavor octet. The branching ratio of η to $3\pi^0$ is 32.5% and to $\pi^+\pi^-\pi^0$ is 22.6%. Thus for every η produced there is, on average, $1.2\pi^0$. Due to the higher mass of η we make the approximation that the rate of indirect production of π^0 via η is roughly the same as the direct production from $u\bar{u}$ and $d\bar{d}$. If we now use $q\bar{q}$ to denote either $u\bar{u}$ or $d\bar{d}$, but not both $u\bar{u}$ and $d\bar{d}$, then each pair of $q\bar{q}$ leads to one π^0. Since, in a heavy-ion collision there are many quarks and antiquarks produced in the central region, it is reasonable to assume that the q distribution is independent of the $\bar{q}$ distribution so that we can write $F_{q\bar{q}}$ in the factorizable form

$$F_{q\bar{q}}(z_1,z_2)=F_q(z_1)F_{\bar{q}}(z_2), \tag{23}$$

where F_q stands for either u or d distributions, and similarly for $F_{\bar{q}}$, but, for π^0 production $\bar{q}$ should be the antiquark partner of q.

The fact that we consider η production above, but not the vector meson ρ, requires an explanation. We defer that discussion until the following section, after we have presented the formalism for the production of protons.

Returning now to the normalization of $R_\pi(z_1,z_2,z)$, we note that, using Eqs. (19), (21), and (22),

$$\int dz z R_\pi(z_1,z_2,z)=\int \frac{dz}{z} R_\pi^0 \delta\left(\frac{z_1+z_2}{z}-1\right)=R_\pi^0 \tag{24}$$

is the probability that a q at z_1 and a $\bar{q}$ at z_2 recombine to form a pion at any z. According to our counting above, the total probability for $q\bar{q}\rightarrow\pi^0$ integrated over all momenta is

$$\int_0^Z \frac{dz_1}{Z}\int_0^Z \frac{dz_2}{Z}\int dz\, z\, R_\pi(z_1,z_2,z)=1, \tag{25}$$

where Z is the maximum z_i, whatever it is. This normalization condition is scale invariant, and we find, using Eq. (24), that

$$R_\pi^0=1. \tag{26}$$

Substituting Eqs. (19)–(23) and (26) in Eq. (13), we obtain

$$\frac{dN_\pi}{zdz}=\int dz_1 dz_2 \frac{z_1 z_2}{z} F_q(z_1)F_{\bar{q}}(z_2)\,\delta(z_1+z_2-z). \tag{27}$$

This is obtained from Eq. (9) where y and ϕ are both explicitly averaged over. The LHS is to be identified with $\Phi(z)$. Note that the $1/2\pi$ factors in Eqs. (1) and (3), where $\Phi(z)$ is defined, are there to render $f(p_T,s)$ an average distribution in ϕ; that is the notation for the experimental distribution, defined in Ref. [12]. The distribution defined by us in Eq. (9) already includes the $1/2\pi$ factor, so our $dN_\pi/z\,dz$ is just the experimental $\Phi(z)$. As we have mentioned earlier, the normalization of z has no significance. By means of a scale change in Eq. (16) we can move from z to x, or vice versa, without changing the scale invariant form of Eq. (27). In Eq. (6) we found $\Phi(z)$ to have the form

$$\Phi(z)=A(z^2+c)^{-n}. \tag{28}$$

If we change z to x according to Eq. (16), then by keeping the total number of pions invariant, i.e.,

$$\int dz\, z\, \Phi(z,K)=\int dx\, x\, \Phi'(x,K'), \tag{29}$$

we have

$$\Phi'(x,\lambda K)=\lambda^2\Phi(\lambda x,K). \tag{30}$$

It thus follows that

$$\Phi'(x)=\lambda^{2(1-n)}A(x^2+c/\lambda^2)^{-n}. \tag{31}$$

Similarly, in the x variable the transformed quark distributions is

$$F'_q(x_1,K')=\lambda^2 F_q(z_1,K). \tag{32}$$

Without having to specify the arbitrary scale factor λ, let us work with the z variable and rewrite Eq. (27) as

$$\Phi(z)=\int_0^z dz_1 z_1\left(1-\frac{z_1}{z}\right)F_q(z_1)F_{\bar{q}}(z-z_1). \tag{33}$$

We must now consider how the q and $\bar{q}$ distributions differ. Unlike the structure functions of the nucleon, where q and $\bar{q}$ have widely different distributions, we are here dealing with the partons at high p_T in heavy-ion collisions just before recombination. The dynamics underlying their p_T dependences is complicated. Many subprocesses are involved, which include hard scattering, gluon radiation, jet quenching, gluon conversion to quark pairs, thermalization, hydrodynamical expansion, to name a few familiar ones. At very large p_T there are far more quark jets than antiquark jets, since the valence quarks have larger longitudinal momentum fractions than the sea quarks. By hard scattering, the quarks, therefore, can acquire larger p_T than the antiquarks. Thus in that way one would expect the p_T distribution of the quarks to be very different from that of the antiquarks. However, that view does not apply to our problem. Those are the q and $\bar{q}$ that initiate jets, along with jets initiated by gluons. The conventional approach is to follow the jet production by jet fragmentation, which can be modified by the dense matter that the initiating partons traverse. As discussed earlier, our approach is not to delve into the dynamical origins of the q and $\bar{q}$ distributions, but to consider the recombination of q and $\bar{q}$ just at the point of hadronization. Such q and $\bar{q}$ are not the partons that initiate jets, but are the parton remnants after the hard partons radiate gluons that subsequently convert to $q\bar{q}$ pairs. Those parton remnants have similar momentum distribution for q and $\bar{q}$, since gluon conversion creates q and $\bar{q}$ on equal basis; those partons are the ones that recombine to form hadrons. They are not to be confused with the jet-initiating hard partons that fragment into hadrons in the fragmentation model. In the recombination picture those hard partons that acquire large p_T immediately after hard scattering are not ready for recombination; they lose momenta and virtuality through gluon radiation until a large body of low-virtuality quarks and antiquarks are assembled for recombination—a view that is complementary to the fragmentation picture. Of course, there are more quarks than antiquarks, since the number of valence quarks of the participating nucleons cannot diminish. For that reason we allow $F_q(z)$ to differ in normalization from $F_{\bar{q}}(z)$. However, as a first approximation we assume that their z dependences are the same.

There is some indirect experimental evidence in support of our assumption. In Ref. [1] the $\bar{p}/p$ ratio for central collisions is reported to be essentially constant within errors; more precisely, it ranges between 0.6 and 0.8 for p_T in the range $0.5 < p_T < 3.8$ GeV/c. Since $\bar{p}$ is formed by the recombination of three $\bar{q}$, while p is formed from three q, a quick estimate of the $\bar{q}/q$ ratio is that it varies between $0.6^{1/3}$ and $0.8^{1/3}$, i.e., from 0.843 to 0.928. Such a narrow range of variation is sufficient for us to assume that $F_{\bar{q}}(z)$ has the same z dependence as $F_q(z)$. For their relative normalization we take the mean $\bar{p}/p$ ratio to be 0.7. Thus we set the $\bar{q}/q$ ratio to be

$$F_{\bar{q}}(z)/F_q(z) = F'_{\bar{q}}(x)/F'_q(x) = 0.7^{1/3}. \tag{34}$$

With this input, we are finally ready to infer the quark distribution from the pion distribution.

We parametrize $F_q(z)$ by

$$F_q(z) = a(z^2 + z + z_0)^{-m} \tag{35}$$

and adjust the three parameters a, z_0, and m to fit $\Phi(z)$ by using Eq. (33). We obtain an excellent fit with the values

$$a = 90, \quad z_0 = 1, \quad m = 4.65. \tag{36}$$

In Fig. 3 we show with a solid line the data represented by the formula in Eq. (6) and the dashed line showsthe result of the theoretical calculation using Eqs. (33)–(36). They coalesce nearly completely in the interval $1 < z < 8$. The quark distribution $F_q(z)$ is shown in Fig. 4. To appreciate the p_T range corresponding to z in Fig. 4, recall Eq. (2), $p_T = zK$, and Fig. 2 for K. Thus at $\sqrt{s} = 200$ GeV, p_T is z in GeV. Equations (35) and (36) represent the main result of this study. What is important is that we have found a scaling quark distribution that is independent of s from SPS to RHIC, and perhaps to LHC. It is a succinct summary of the effects of all the dynamical subprocesses in heavy-ion collisions. The nontrivial z dependence in Eq. (35) indicates that there are intrinsic scales in the low-p_T problem.

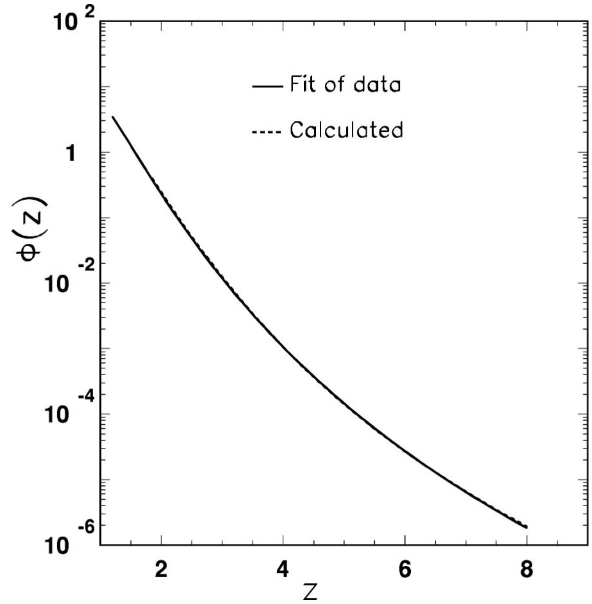

FIG. 3. The solid line is the fit of the data as shown in Fig. 1 (in a different scale) and the dashed line is the theoretical calculation of $\Phi(z)$ using the quark distribution in Fig. 4.

IV. THE p/π RATIO

The quark distribution obtained in the preceding section cannot be checked directly. Since it is the distribution at the end of its evolution, massive dileptons would not be sensitive to it due to their production at the early stages. Proton production provides the most appropriate test, since hadronization occurs near the end. We shall therefore calculate the proton distribution at high p_T and compare with the data on the p/π ratio. This is not a completely satisfactory venture,

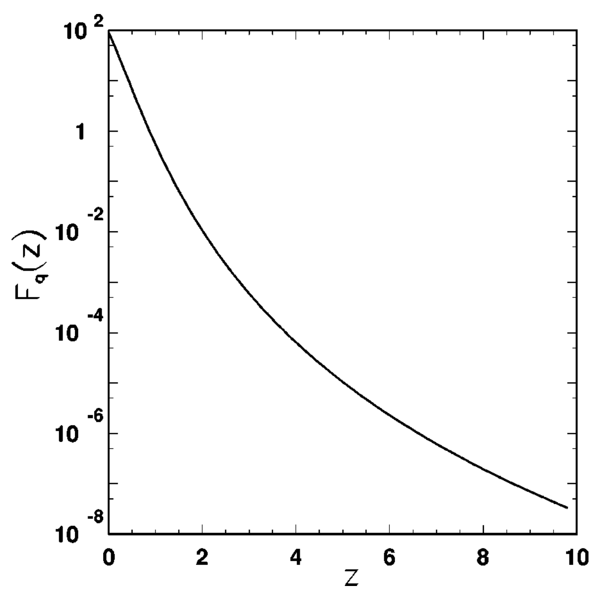

FIG. 4. Quark distribution in z.

since the proton mass is large, so only at very high p_T can our scale invariant calculation be valid without explicit consideration of the mass effect. Present data on the p/π ratio do not extend beyond $p_T \sim 3.8$ GeV/c [1]. Nevertheless, our calculation should provide some sense on the magnitude of the rate of proton production at the high-p_T end.

The inclusive distributions in the scaled p_T variable can be obtained in the recombination model by generalizing Eq. (13) to the recombination of three quarks

$$\frac{dN_p}{zdz} = \int dz_1 dz_2 dz_3 z_1 z_2 z_3 F(z_1,z_2,z_3) R_p(z_1,z_2,z_3,z), \tag{37}$$

where $F(z_1,z_2,z_3)$ is given the factorizable form

$$F(z_1,z_2,z_3) = F_u(z_1)F_u(z_2)F_d(z_3). \tag{38}$$

As in Eq. (19), we relate the recombination function R_p to the valon distribution G_p of the proton

$$R_p(z_1,z_2,z_3,z) = R_p^0 z^{-2} G_p(\xi_1,\xi_2,\xi_3), \tag{39}$$

where G_p has the general form given in Eq. (21), and R_p^0 remains to be determined. In Ref. [18] a detailed study of the proton structure functions has been carried out in deriving the valon distribution from the parton distributions that fit the deep inelastic scattering data. It is

$$G_p(\xi_1,\xi_2,\xi_3) = g(\xi_1\xi_2)^\alpha \xi_3^\beta \delta(\xi_1+\xi_2+\xi_3-1), \tag{40}$$

where

$$\alpha = 1.755, \quad \beta = 1.05, \tag{41}$$

$$g = [B(\alpha+1,\beta+1)B(\alpha+1,\alpha+\beta+2)]^{-1}. \tag{42}$$

Single-valon distributions $G_p(\xi_i)$ can be obtained from the three-valon distribution by integration and are peaked around $\xi = 1/3$, indicating that each of the three valons carries on average roughly 1/3 the momentum of the proton, their sum being strictly 1. Details of the valon model, described in Ref. [18], are not needed for the following. It is only necessary to recognize that the recombination of two u quarks with a d quark to form a proton has a probability proportional to the proton's valon distribution that accounts for the proton structure. The other point to bear in mind is that the valon distribution in the proton is obtained in the frame where the proton momentum is infinitely large so the finite masses of the proton and valons are unimportant. However, the validity of that result, when the proton momentum is only two or three times larger than its mass, is questionable. With that caveat we proceed with our scale invariant calculation and see what can emerge.

As discussed in the preceding section, there is no need to consider the color factors for either pion or proton formation since hadrons are color singlets, but the flavor octets for these hadrons do introduce some factors. The $|uud\rangle$ state appears in $10+8+8'$ of $3\times3\times3$; among them the first two contain Δ^+ and p. Thus the flavor parts of $|\langle\Delta^+|uud\rangle|^2$ and $|\langle p|uud\rangle|^2$ are 1/3 for each. Since Δ^+ decays to $p+\pi^0$ and $n+\pi^+$, $|\langle p|\Delta^+\rangle|^2$ gives another factor 1/2. The spin decomposition of $2\times2\times2$ is $4+2+2$, among which the Δ^+ component is 4/8 and p is 2/8. Putting the flavor and spin factors together, we have

$$|\langle p|uud\rangle|^2 + |\langle p|\Delta^+\rangle\langle\Delta^+|uud\rangle|^2 = \frac{1}{3}\times\frac{1}{4} + \frac{1}{3}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{6}. \tag{43}$$

We thus normalize R_p, as we have done in Eq. (25), by

$$\int_0^Z \prod_{i=1}^3 \frac{dz_i}{Z} \int dz\, z\, R_p(z_1,z_2,z_3,z) = \frac{1}{6}. \tag{44}$$

In view of Eqs. (21) and (39), we have

$$R_p^0 \int_0^Z \prod_{i=1}^3 \frac{dz_i}{Z} \tilde{G}_p\left(\frac{z_1}{z_t},\frac{z_2}{z_t},\frac{z_3}{z_t}\right) = \frac{1}{6}, \tag{45}$$

where $z_t = \Sigma_i z_i$. Using Eq. (40), the above integral can be transformed to

$$g\int_0^1 \prod_{i=1}^3 d\zeta_i \left(\frac{\zeta_1\zeta_2}{\zeta_t^2}\right)^\alpha \left(\frac{\zeta_3}{\zeta_t}\right)^\beta = 2.924 \tag{46}$$

with $\zeta_i = z_i/Z$ and $\zeta_t = \Sigma_i \zeta_i$. There is no explicit dependence on Z, and Eqs. (41) and (42) have been used to obtain the numerical value in Eq. (46). It thus follows that

$$R_p^0 = 0.057. \tag{47}$$

At this point we should address the question why we consider Δ^+ production above, but not ρ production in the preceding section. For the production of π^0, if we are to consider the contribution from $\rho^\pm$ (since ρ^0 does not decay strongly into π^0), we would be extending our scope to other flavored states besides $u\bar{u}$ and $d\bar{d}$. Then other vector mesons and higher resonances, such as K^*, that can decay into π^0 must also be included. Similarly, for p production the consideration of other states beside uud would involve many resonances that can decay into p. The system is not closed without more phenomenological input beside π^0. Thus for a closed system in which a prediction can be made, we limit ourselves to only the $u\bar{u}$ and $d\bar{d}$ in the meson states and uud in the baryon states; hence, only π^0, η, p, and Δ^+ are considered. To include $u\bar{d}$ and $d\bar{u}$, we must also include uuu and udd, and so on. We surmise that if more resonances are included in both the meson and baryon sectors, the p/π ratio to be determined below would change somewhat; however, the result is not likely to differ by a factor greater than 2.

With the recombination function R_p completely determined, and the quark distribution $F_q(z_i)$ given by Eqs. (35) and (36), we can now use Eq. (37) to calculate the proton distribution in z. The result is shown by the solid line in Fig. 5, where only the portion $z>2$ is exhibited. We have stated at the outset that the scale invariant form of dN_p/zdz cannot

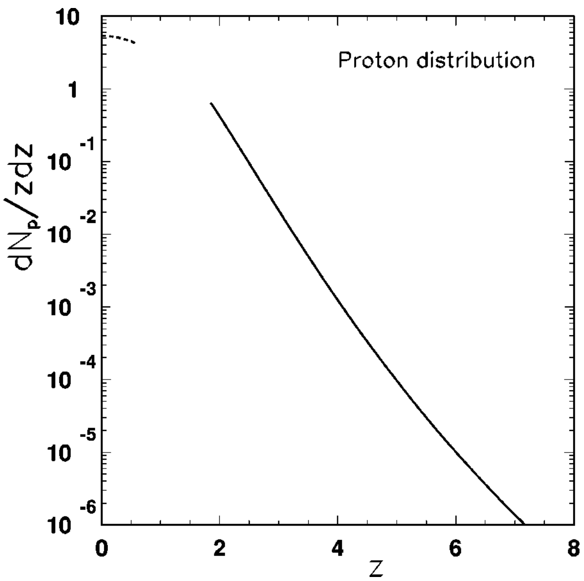

FIG. 5. Proton distribution in z. Solid line is the theoretical result; the dashed line is the fit of data at low-p_T [19].

be expected to be valid when the mass effect is important. The relevant value of z corresponding to the proton mass (let alone the Δ^+ mass) is

$$z_m = m_p/K, \tag{48}$$

which ranges from 1.3 at $\sqrt{s}=17$ GeV to 0.94 at 200 GeV. As expected, the scaling violating effects are energy dependent. Thus we should not regard the calculated result to be reliable for $z<3$. At very low p_T, the distributions of all hadrons can be given exponential fits in the transverse mass. The STAR data for most central collisions at $\sqrt{s}=130$ GeV [19] give for $\bar{p}$ production for $p_T<0.6$ GeV

$$\frac{1}{2\pi m_T}\frac{d^2N_{\bar{p}}}{dm_T dy} = 4\exp[-(m_T - m_p)/T_p], \tag{49}$$

where $m_T=(m_p^2+p_T^2)^{1/2}$ and $T_p=565$ MeV. To convert this distribution to that for p we assume that only the normalization at $p_T=0$ needs to be adjusted. The $\bar{p}/p$ ratio at low p_T is 0.6 [1]. Since $m_T dm_T = p_T dp_T$ and the distribution in p_T changes by a scale factor K^2 given in Eq. (2), where $K=0.9$ for $\sqrt{s}=130$ GeV, the factor 4 in Eq. (49) should therefore be changed to $4\times 0.81/0.6 = 5.4$. Expressing m_T in terms of z by use of Eq. (2) with $K=0.9$, we show the z dependence of the distribution for p in Fig. 5 by the short dashed line. The region $0.5<z<2$ is left blank because our scaling result cannot be reliably extended into that region. Nevertheless, it is gratifying to observe that the theoretical calculation without any free parameters produces a proton distribution at large z that is reasonable in normalization and shape and can smoothly be connected with the low-p_T distribution by interpolation.

With the proton distribution now at hand, we can calculate the p/π ratio. For the pion distribution we use $\Phi(z)$ given in

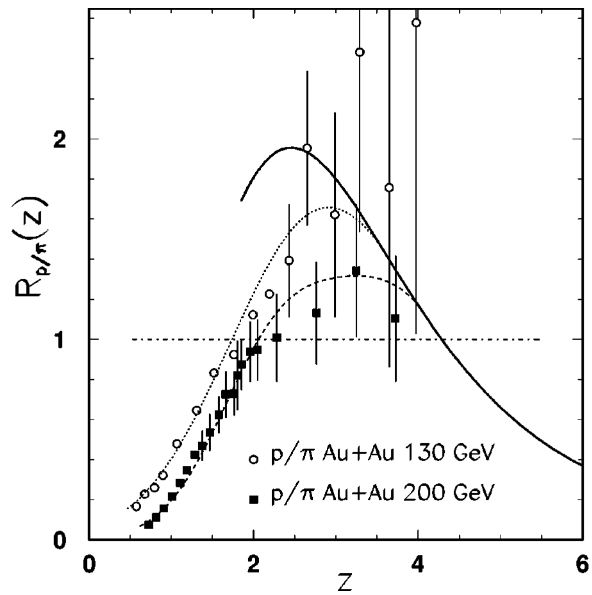

FIG. 6. Proton-to-pion ratio: solid line is the scaling distribution from calculation; data (preliminary) are from Ref. [1]. The dotted and dashed lines are eyeball fits of the data as extrapolations from the scaling result.

Eq. (6). For the proton we use the calculated result based on Eq. (37). Their ratio, defined by

$$R_{p/\pi}(z) = \frac{dN_p}{zdz}/\Phi(z) \tag{50}$$

is shown by the solid line in Fig. 6. The preliminary data on the p/π ratio were reported in Ref. [1], which we show also in Fig. 6 for both $\sqrt{s}=130$ and 200 GeV. Note that because it is a ratio, there is no change in the normalizations of $R_{p/\pi}$ for the two energies, but in transforming from p_T to z the factor K in Eq. (2) must be taken into account. Unlike the pion case, the effects of the proton mass are not negligible for $p_T \lesssim 3$ GeV/c, and one sees no scaling in s or z in Fig. 6. Our scale invariant calculation is unreliable for $z<3$ and shows a result that is obviously too high at $z\lesssim 2$. There seems to be a good chance that the theory and experiment can agree well for $z>4$. In Fig. 6 we show two curves that can connect our scaling result with the data. The dotted curve is an eyeball fit of the 130 GeV data with a connection at $z=3.5$, while the dashed curve fits the 200 GeV data with a connection at $z=4$. In the absence of a theoretical study that takes the mass-dependent effects into account in the intermediate p_T region, the only point we can make here is that it is not hard to produce a p/π ratio that exceeds 1 in the scale invariant calculation in the recombination model, but it does so in a region where both theory and experiment need refinement. Judging by what is self-evident in Fig. 6, we see no strong need for any exotic mechanism for proton production (as proposed in Ref. [7]) beyond the conventional subprocess where three quarks recombine to form a proton.

V. CONCLUSION

The discovery of a scale-invariant distribution $\Phi(z)$ for pion production at intermediate and high values of p_T in heavy-ion collisions, ranging over energies in excess of an order of magnitude of variation, is an important phenomenology observation that should be checked experimentally in great detail. Additional energy points should be added not only to strengthen the validity of the scaling behavior, but also to find the onset of scaling violation, if it exists.

The phenomenological properties of hadron production provide useful insights into the hadronization process and into the nature of the quark system just before they turn into hadrons. The usual approach to the study of heavy-ion collisions is from inside out, following the evolution of the dense matter, either in terms of hydrodynamical flow or of hard parton scattering and subsequent hadronization by fragmentation [20]. Our approach pursued here is from outside in, by starting from the observed scaling behavior of the pions produced and deriving the momentum distribution of the quarks that can give rise to such a behavior. This is accomplished by using the recombination model. There is no direct way to check the validity of the quark distribution thus obtained. However, we have used it to determine the proton distribution at high p_T, where the mass effects are unimportant. The data on proton production have not yet reached that regime where the predicted scaling distribution can be checked. In the region where data exist on the p/π ratio, we find that our calculated result, though not reliable, is in rough agreement with the imprecise data to the extent that the ratio exceeds 1, a feature that is notable.

While the recombination model needs further work to take the proton mass into account at intermediate p_T, its formulation in the invariant form has been developed here to treat the very high p_T region. We have made the assumption that the quark and antiquark distributions are the same, apart from normalization, just before recombination. This assumption is supported by the constancy of the $\overline{p}/p$ ratio in the PHENIX data in the central region. This experimental fact can also be used to lend credence to our general approach to hadronization that is treated as a recombination process, for which we have given arguments why the distributions of quarks and antiquarks should be similar before they recombine. In contrast, the fragmentation model would suggest a decreasing function of $\overline{p}/p$ in p_T because of the dominance of quark jets over antiquark jets at large p_T [4].

In this paper we have only considered the energy dependence of the p_T spectrum at fixed maximum centrality. It is natural to ask what the dependence is on centrality. We have investigated that problem by making a phenomenological analysis of the data on centrality dependence without using any hadronization model, and found a scaling behavior very similar to what is reported here. The scaling distribution found there [16] includes the very small p_T region in the fit, and is therefore more accurate. But the fits in the intermediate- and large-p_T regions are the same. The implication on the centrality dependence of the p/π ratio in the recombination model is still under study.

To have an invariant quark distribution independent of s, just before hadronization provides an unexpected picture of the quark system. It suggests that the evolution of the system proceeds toward a universal form whatever the collision energy. We expect that universal form to depend on rapidity. The origin of such a scaling distribution in z is not known at this point and can form the focus of a program of future theoretical investigations.

ACKNOWLEDGMENTS

We wish to thank D. d'Enterria for a helpful communication. This work was supported, in part, by the U.S. Department of Energy under Grant No. DE-FG03-96ER40972.

[1] T. Sakaguchi, PHENIX Collaboration, nucl-ex/0209030, talk given at Quark Matter 2002, Nantes, France, 2002.
[2] K.P. Das and R.C. Hwa, Phys. Lett. **68B**, 459 (1977); R.C. Hwa, Phys. Rev. D **22**, 1593 (1980).
[3] For a recent review see P. Houvinen, in *Proceedings of Quark Matter 2002, Nantes, France, 2002*, edited by H. H. Gutbrod and K. Werner (Elsevier, Amsterdam, 2003).
[4] X.-N. Wang, Phys. Rep. **280**, 287 (1997); Phys. Rev. C **58**, 2321 (1998); **61**, 064910 (2000).
[5] For an overview see A. Białas, in *Proceeding of Quark Matter 2002, Nantes, France, 2002* (Ref. [3]).
[6] For a summary see J.-P. Blaizot, *Proceedings of Quark Matter 2001* [Nucl. Phys. **A698**, 360c (2002)].
[7] I. Vitev and M. Gyulassy, Phys. Rev. C **65**, 041902(R) (2002).
[8] A. Capella, U. Sukhatme, C.-I. Tan, and J. Tran Thanh Van, Phys. Rep. **236**, 225 (1994).
[9] B. Andersson, G. Gustafson, and C. Peterson, Phys. Lett. **69B**, 221 (1977); **71B**, 337 (1977); B. Andersson, G. Gustafson, G. Ingelman, and T. Sjöstrand, Phys. Rep. **97**, 33 (1983).
[10] R.C. Hwa, and C.B. Yang, Phys. Rev. C **66**, 025205 (2002).
[11] L. McLerran and J. Schaffner-Bielich, Phys. Lett. B **514**, 29 (2001); J. Schaffner-Bielich, D. Kharzeev, L. McLerran, and R. Venugopalan, Nucl. Phys. **A705**, 494 (2002).
[12] D. d'Enterria, PHENIX Collaboration, hep-ex/0209051, talk given at Quark Matter 2002, Nantes, France, 2002.
[13] K. Reygers, WA98 and PHENIX Collaborations, nucl-ex/0202018.
[14] Z. Koba, H.B. Nielsen, and P. Olesen, Nucl. Phys. **B40**, 317 (1972).
[15] G.J. Alner *et al.*, UA5 Collaboration, Phys. Lett. **138B**, 304 (1984); T. Alexopoulous *et al.*, E735 Collaboration, Phys. Lett. B **435**, 453 (1998).
[16] R.C. Hwa and C.B. Yang, nucl-th/0301004.
[17] R.C. Hwa and C.B. Yang, Phys. Rev. C **65**, 034905 (2002).
[18] R.C. Hwa and C.B. Yang, Phys. Rev. C **66**, 025204 (2002).
[19] P.G. Jones, STAR Collaboration, *Proceedings of The Physics of the Quark-Gluon Plasma, Palaiseau, France, 2001* (Ecole Polytechnique, Palaiseau, 2001), p. 9.
[20] M. Gyulassy, I. Vitev, and X.-N. Wang, Phys. Rev. Lett. **86**, 2537 (2001).