

# Momentum dependence of symmetry potential in asymmetric nuclear matter for transport model calculations

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For transport model simulations of collisions between two nuclei which have  $N/Z$  significantly different from unity one needs a one-body potential which is both isospin and momentum dependent. This work provides sets of such potentials.

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## I. INTRODUCTION

Momentum dependent mean-fields for transport model calculations of heavy ion collisions have been in usage for many years now [1–6]. So far the attention has been for a momentum dependent potential which does not distinguish between neutrons and protons. This is adequate for systems which have  $N \approx Z$  where  $N$  is the number of neutrons and  $Z$  the number of protons. One main focus of nuclear physics research today is to explore both structures of exotic nuclei in regions far-off the stability line and novel properties of neutron-rich nuclear matter. The latter can be investigated by using collisions induced by neutron-rich nuclei at intermediate to high energies. To interpret critically data from these collisions and to extract accurately properties of neutron-rich nuclear matter, advanced transport model calculations are necessary. In asymmetric nuclear matter, the one body potential seen by a proton is different from that seen by a neutron. This has been implemented in BUU (Boltzmann-Uehling-Uhlenbeck) calculations but with a simplification that the potentials are taken to be momentum independent [7]. The present work aims to correct this deficiency. That the momentum dependence will be different for neutrons and protons is of course well known and has been the subject of quite sophisticated many body calculations, see, e.g, Ref. [8] for a recent review. We do not aim to add anything fundamental in this regard. Our objective is to obtain a parametrized version which displays the main characteristics of momentum dependence in asymmetric matter and is still usable in practical BUU calculations. Major advances in this direction were already made: see articles by Bombaci [8] and Prakash *et al.* [9]. We add to this. We will not only extend the simplest momentum dependent potential [1] to include isospin but also extend the improved treatment [2–4] subsequently introduced to include isospin. Thus this is an extension of the work reported in Ref. [3,4].

## II. A MOMENTUM DEPENDENT POTENTIAL FROM A PHENOMENOLOGICAL INTERACTION

An effective momentum dependent potential can be deduced from phenomenological interactions. We take the Gogny interaction [10] to obtain an idea of the momentum dependence. There are many reasons for this choice. It has

been used in detailed fits for spectra in finite nuclei. It gives accepted values for binding energy, saturation density, compressibility and symmetry energy in nuclear matter. It has been verified already [2] that the interaction produces a reasonable parameterization for the real part of the optical potential in nuclear matter as a function of incident energy. The Skyrme interaction has a wrong asymptotic behavior as a function of energy. (This is amplified in Ref. [2].) Since we want to devise a momentum dependence which should hold for beam energy as high as 1 GeV/nucleon (this would allow investigation of symmetry energy at higher than normal nuclear density) we discard the Skyrme interaction.

For the purpose of this work we will define nuclear matter to be an infinite system but without the restriction  $N=Z$ . Using the Gogny interaction, we deduce  $U(\rho, \delta, p, \tau)$ , the one body potential a particle of momentum  $p$  and isospin  $\tau$  feels in cold nuclear matter with density  $\rho$  and asymmetry  $\delta \equiv (\rho_n - \rho_p)/(\rho_n + \rho_p)$ . One then generalizes to  $U$  in the case of heavy ion collisions. For BUU calculations  $U$  is the only quantity needed. But it is useful to also have an expression for  $V(\rho, \delta)$ , the potential energy density in cold nuclear matter with a given density  $\rho$  and asymmetry  $\delta$ . This allows one to deduce  $E/A$  as a function of  $\rho$  and  $\delta$  which is, of course, of importance. The expression for  $V(\rho, \delta)$  can also be generalized to the case of heavy ion collisions and can be used to check, for example, the accuracy of energy conservation in a BUU simulation.

Normally nuclear matter denotes an infinite nucleus with  $N=Z$ . Total potential energy in cold matter is deduced from

$$V_T = \frac{1}{2} \sum_{p_1 \sigma_1 \tau_1 p_2 \sigma_2 \tau_2} \langle \vec{p}_1, \sigma_1, \tau_1, \vec{p}_2, \sigma_2, \tau_2 | v(r) \times (|\vec{p}_1, \sigma_1, \tau_1, \vec{p}_2, \sigma_2, \tau_2\rangle - |\vec{p}_2, \sigma_2, \tau_2, \vec{p}_1, \sigma_1, \tau_1\rangle) \rangle, \quad (2.1)$$

where,

$$v(r) = \sum_{i=1,2} (W + B P_\sigma - H P_\tau - M P_\sigma P_\tau)_i e^{-r^2/\mu_i^2} + t_0 (1 + P_\sigma) \rho^\alpha \left( \frac{\vec{r}_1 + \vec{r}_2}{2} \right) \delta(\vec{r}_1 - \vec{r}_2). \quad (2.2)$$

There are two finite range Gaussians and a density dependent zero-range force. The values of the parameters are given in Ref. [10]. The one body potential  $U(\rho, \delta, p, \tau)$  is obtained from  $U(\rho, \delta, p, \tau) = \sum_{p', \sigma', \tau'} \langle \vec{p}, \sigma, \tau, \vec{p}', \sigma', \tau' | v(r) \times (|\vec{p}, \sigma, \tau, \vec{p}', \sigma', \tau'\rangle - |\vec{p}', \sigma', \tau', \vec{p}, \sigma, \tau\rangle) \rangle$  plus rearrangement term which for nuclear matter is  $(3/2)t_0\alpha\rho^{\alpha-1}(1/4)\rho^2(1-\delta^2)$ .

The momentum dependence in  $U$  comes entirely from the exchange term of the finite range part, i.e., from  $\langle \vec{p}, \vec{p}' | e^{-r^2/\mu^2} | \vec{p}', \vec{p} \rangle$ . Except for the momentum dependent part, very simple expressions for  $U$  and  $V_T/A$ , the potential energy per particle are obtained for the Gogny potential. Thus  $U(\rho, \delta, p, \tau) = X + Y + Z$  where  $X$  arises from the direct term of the finite range interaction,  $Y$  arises from the  $t_0$  term (density dependent two body term), and  $Z$  from the exchange term of the finite range interactions. For a given  $\tau$ , these are

$$X = \rho \left( \sum_{i=1,2} \pi^{3/2} \mu_i^3 (W + B/2)_i \right) - \rho \tau \left( \sum_{i=1,2} \pi^{3/2} \mu_i^3 (H + M/2)_i \right), \quad (2.3)$$

$$Y = \frac{3}{2} t_0 \rho^\alpha (\rho - \rho_\tau) + \frac{3}{2} t_0 \alpha \rho^{\alpha-1} \frac{1}{4} \rho^2 (1 - \delta^2), \quad (2.4)$$

and

$$Z = \sqrt{\pi} \left[ \sum_{i=1,2} Z_i(p, \tau) (-W - 2B + H + 2M)_i + \sum_i Z_i(p, \tau') (H + 2M)_i \right], \quad (2.5)$$

where,

$$Z_i(p, \tau) = \frac{1}{\mu_i k} \left[ e^{-\{\mu_i [k_F(\tau) - k]/2\}^2} - e^{-\{\mu_i [k_F(\tau) + k]/2\}^2} \right] + \frac{\sqrt{\pi}}{2} \left[ \operatorname{erf} \left( \frac{\mu_i}{2} [k_F(\tau) - k] \right) + \operatorname{erf} \left( \frac{\mu_i}{2} [k_F(\tau) + k] \right) \right]. \quad (2.6)$$

Here  $\tau' \neq \tau$ , the isospin of the particle whose one body potential is being sought.

Similarly,  $V_T/A$  = potential energy per particle, has contributions from the direct term of the finite range force, from the density dependent  $t_0$  term and the exchange term of the finite range force. Denoting the first two by  $X'$  and  $Y'$ , respectively, explicit expressions for these are

$$X' = \rho \left[ \sum_{i=1,2} \pi^{3/2} \mu_i^3 \left( \frac{W}{2} + \frac{B}{4} - \frac{H}{4} - \frac{M}{8} \right)_i \right] - \rho \delta^2 \left[ \sum_{i=1,2} \left( \frac{H}{4} + \frac{M}{8} \right)_i \right], \quad (2.7)$$

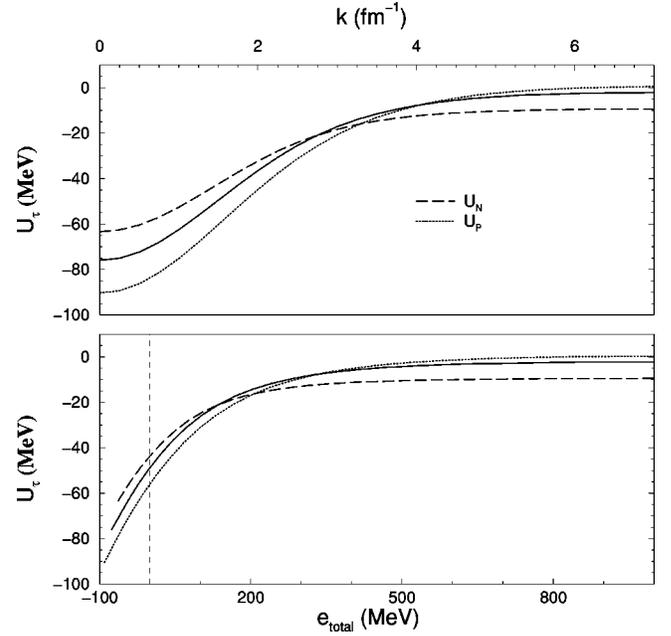
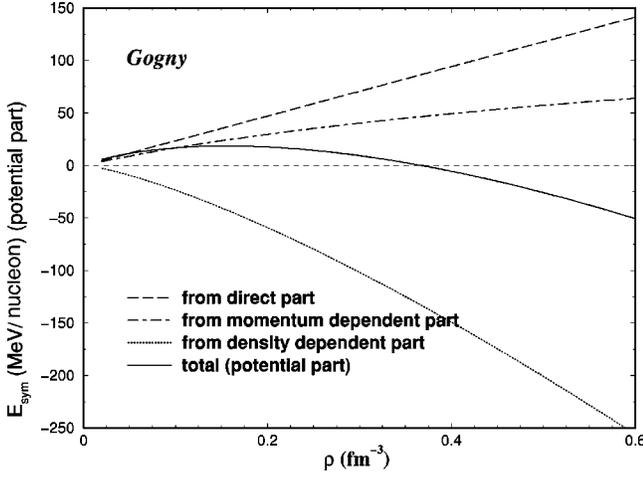


FIG. 1. Single particle potential  $U$  with respect to  $k$  and total single particle energy ( $e$ ) for neutron and proton for  $\delta=0.4$ . The solid line is the single particle potential for symmetric matter; here  $\rho=0.16 \text{ fm}^{-3}$ .

$$Y' = \frac{3}{8} t_0 \rho^{\alpha+1} (1 - \delta^2). \quad (2.8)$$

We do not write down the explicit expression for  $Z'$ . It is obtained from the expression of  $Z$  above after a further integration over  $p$ , sum over  $\sigma, \tau$  and dividing the answer by 2.

The one body potentials as a function of  $k$  for neutrons and protons in cold matter as predicted by the Gogny potential for  $\delta=0$  and  $\delta=0.4$  are shown in Fig. 1. They are quite similar to other calculations of  $U(\rho, \delta, p, \tau)$  available in the literature. We compare, in particular, to the  $U(\rho, \delta, p, \tau)$  given in Fig. 4 in Ref. [11]. That figure is for  $\rho=0.17$  and obtained from Brueckner-Hartree-Fock calculation. With Gogny interaction we find that the equation of state (EOS) of asymmetric nuclear matter can be written as  $(E/A)(\rho, \delta) \approx (E/A)(\rho, 0) + E_{\text{sym}}(\rho) \delta^2$ , in agreement with the empirical parabolic law found by all many body theories. Different calculations depending on the many body approaches and the interactions used, give very different behaviors for  $E_{\text{sym}}(\rho)$ , especially at high densities. In some, such as, the relativistic mean field model [12] and the Brueckner-Hartree-Fock [11], it is a continuously rising function of  $\rho$ . In others, such as, the variational many-body approach [13], it rises in the beginning and then begins to fall. Within the Hartree-Fock approach using all 86 Skyrme effective interactions widely used currently in nuclear structure studies, it was found that about 1/3(2/3) of them lead to symmetry energies in the first (second) category [14]. Gogny interaction with the default parameters has behavior in the second category. It is interesting to see what causes the fall and how the parameters of the interaction can be changed minimally to alter this behavior. In Fig. 2 we plot contributions to  $E_{\text{sym}}(\rho)$  from the Gogny interaction and we show separately the term coming


 FIG. 2.  $E_{\text{sym}}(\rho)$  as a function of  $\rho$ .

from (a) the direct term of the finite range part, (b) the exchange term of the finite range part, and (c) the density dependent zero range part. It is the latter that causes the bending over. For example, if we choose the density dependent term to be  $\delta$  independent,  $E_{\text{sym}}(\rho)$  will continue to rise.

Lastly, we write the potential energy density due to Gogny interaction in a form that is common practice in BUU literature. Thus

$$\begin{aligned}
 V(\rho, \delta) = & -52.41 \left( \frac{\rho^2}{\rho_0} \right) + 37.70 \left( \frac{\rho^2}{\rho_0} \right) \delta^2 + \frac{102.6}{\sigma+1} \frac{\rho^{\sigma+1}}{\rho_0^\sigma} (1 - \delta^2) \\
 & + \frac{1}{\rho_0} \sum_{\tau, \tau'} \int \int f_\tau(\vec{r}, \vec{p}) f_{\tau'}(\vec{r}, \vec{p}') \\
 & \times [(-41.51 + 46.02 \delta_{\tau, \tau'}) e^{-(\vec{k} - \vec{k}')^2 \mu_1^2/4} \\
 & + (-38.62 + 17.25 \delta_{\tau, \tau'}) e^{-(\vec{k} - \vec{k}')^2 \mu_2^2/4}] d^3 p d^3 p'. \quad (2.9)
 \end{aligned}$$

Here, as in previous work, all quoted numbers are in MeV,  $\rho_0 = 0.16 \text{ fm}^{-3}$ . Also  $\sigma = \alpha(\text{Gogny}) + 1 = 4/3$ .

Given that the momentum dependence generated by the Gogny potential comes from two Gaussians, we proceed to find a simpler version for momentum dependent potential for asymmetric matter. We will extend the parametrization of Refs. [3,4]. Subsequently we will find an even simpler version, the kind that was used in first applications of momentum dependent potentials for heavy ion collisions.

### III. A SIMPLE MOMENTUM DEPENDENT POTENTIAL FOR BUU CALCULATIONS

The simplest generalization of the potential energy density of Eq. (5.4) of Ref. [3] to asymmetric nuclear matter is

$$\begin{aligned}
 V(\rho, \delta) = & \frac{A_1}{2\rho_0} \rho^2 + \frac{A_2}{2\rho_0} \rho^2 \delta^2 + \frac{B}{\sigma+1} \frac{\rho^{\sigma+1}}{\rho_0^\sigma} (1 - x \delta^2) \\
 & + \frac{1}{\rho_0} \sum_{\tau, \tau'} C_{\tau, \tau'} \int \int d^3 p d^3 p' \frac{f_\tau(\vec{r}, \vec{p}) f_{\tau'}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}. \quad (3.1)
 \end{aligned}$$

The parameter  $x$  is introduced to cover the largely uncertain behavior of nuclear symmetry energy  $E_{\text{sym}}(\rho)$  as discussed in the previous section. For the choice of  $x=1$  (same as in Gogny) in the term containing  $B$ , the symmetry energy will bend over beyond a density  $\rho$ ; for the choice  $x=0$  the symmetry energy will continue to rise with density. In the above,  $C_{1/2, 1/2} = C_{-1/2, -1/2} = C_{\text{like}}$ , and  $C_{1/2, -1/2} = C_{-1/2, 1/2} = C_{\text{unlike}}$ . In terms of interactions between like and unlike particles, the above equation is equivalent to

$$\begin{aligned}
 V(\rho, \delta) = & \frac{A_u \rho_n \rho_p}{\rho_0} + \frac{A_l}{2\rho_0} (\rho_n^2 + \rho_p^2) + \frac{B}{\sigma+1} \frac{\rho^{\sigma+1}}{\rho_0^\sigma} (1 - x \delta^2) \\
 & + \frac{1}{\rho_0} \sum_{\tau, \tau'} C_{\tau, \tau'} \int \int d^3 p d^3 p' \frac{f_\tau(\vec{r}, \vec{p}) f_{\tau'}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}, \quad (3.2)
 \end{aligned}$$

where  $A_1 = (A_u + A_l)/2$  and  $A_2 = (A_l - A_u)/2$ . The one-body potential needed for BUU computations is given by

$$\begin{aligned}
 U(\rho, \delta, \vec{p}, \tau) = & A_u \frac{\rho_{\tau'}}{\rho_0} + A_l \frac{\rho_\tau}{\rho_0} + B \left( \frac{\rho}{\rho_0} \right)^\sigma (1 - x \delta^2) \\
 & - x \frac{B}{\sigma+1} \frac{\rho^{\sigma+1}}{\rho_0^\sigma} \frac{d\delta^2}{d\rho_\tau} \\
 & + \frac{2C_{\tau, \tau}}{\rho_0} \int d^3 p' \frac{f_\tau(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2} \\
 & + \frac{2C_{\tau, \tau'}}{\rho_0} \int d^3 p' \frac{f_{\tau'}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}. \quad (3.3)
 \end{aligned}$$

In the above  $\tau \neq \tau'$  and  $\partial \delta^2 / \partial \rho_n = 4 \delta \rho_p / \rho^2$  and  $\partial \delta^2 / \partial \rho_p = -4 \delta \rho_n / \rho^2$ .

The constants appearing in Eqs. (3.1) and (3.2) will be fixed by ensuring that properties of cold nuclear matter are reproduced. There  $f_\tau(\vec{r}, \vec{p}) = (2/h^3) \Theta[p_f(\tau) - p]$ . The integration in Eq. (3.1) is facilitated by noting that for a fixed  $\vec{p} \equiv (\vec{p}_1 - \vec{p}_2)/2$ , the center of mass momentum can be integrated out to give

$$\begin{aligned}
 & \int_0^{p_f(\tau)} \int_0^{p_f(\tau')} d^3 p_1 d^3 p_2 g(\vec{p}) \\
 & = \int_0^{q_f} \left[ \frac{16\pi}{3} [p_f^3(\tau) + p_f^3(\tau')] - 8\pi p [p_f^2(\tau) + p_f^2(\tau')] \right. \\
 & \quad \left. + \frac{16\pi}{3} p^3 - \frac{\pi}{p} [p_f^2(\tau) - p_f^2(\tau')]^2 \right] g(\vec{p}) d^3 p, \quad (3.4)
 \end{aligned}$$

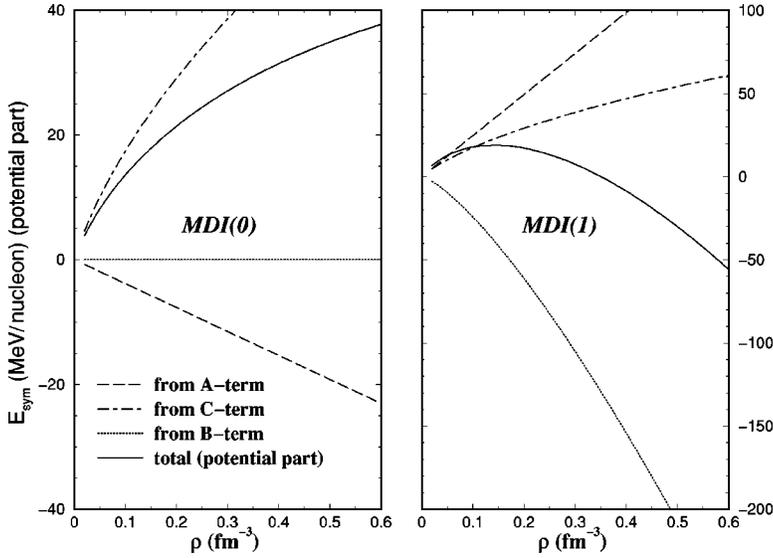


FIG. 3.  $E_{\text{sym}}(\rho)$  as a function of  $\rho$  for the two choices of  $x$ ; 0 (left panel) and 1 (right panel).

where  $q_f = [p_f(\tau) + p_f(\tau')]/2$ .

For completeness, for cold matter, we write down the values of the integrals appearing in Eqs. (3.1) and (3.2)

$$\begin{aligned}
 & \int \int d^3p d^3p' \frac{f_\tau(\vec{r}, \vec{p}) f_{\tau'}(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2} \\
 &= \left(\frac{2}{h^3}\right)^2 \frac{4}{3} \pi^2 \Lambda^2 \left[ \left\{ q_f - \frac{\Lambda}{2} \arctan\left(\frac{2q_f}{\Lambda}\right) \right\} \right. \\
 & \quad \times 4[p_f^3(\tau) + p_f^3(\tau')] - \left. \left\{ 3[p_f^2(\tau) + p_f^2(\tau')] + \frac{\Lambda^2}{2} \right\} q_f^2 + q_f^4 \right. \\
 & \quad \left. + \left\{ \frac{3\Lambda^2}{4} [p_f^2(\tau) + p_f^2(\tau')] + \frac{\Lambda^4}{8} - \frac{3}{8} [p_f^2(\tau) - p_f^2(\tau')]^2 \right\} \right. \\
 & \quad \left. \times \ln\left(1 + \frac{4q_f^2}{\Lambda^2}\right) \right]. \quad (3.5)
 \end{aligned}$$

Similarly, the value of the integral in Eq. (3.3) is

$$\begin{aligned}
 & \int d^3p' \frac{f_\tau(\vec{r}, \vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2} \\
 &= \frac{2}{h^3} \pi \Lambda^3 \left[ \frac{p_f^2(\tau) + \Lambda^2 - p^2}{2p\Lambda} \ln \frac{[p + p_f(\tau)]^2 + \Lambda^2}{[p - p_f(\tau)]^2 + \Lambda^2} \right. \\
 & \quad \left. + \frac{2p_f(\tau)}{\Lambda} - 2 \left\{ \arctan \frac{p + p_f(\tau)}{\Lambda} - \arctan \frac{p - p_f(\tau)}{\Lambda} \right\} \right]. \quad (3.6)
 \end{aligned}$$

We fix the force parameters by first optimally reproducing the variation of  $U(\rho_0, \delta, p)$  with  $p$  with that obtained by using a Gogny force (Brueckner-Hartree-Fock calculation gives similar results [11]). This fixes  $\Lambda$  of Eq. (3.3). Other parameters are then fixed by values of saturation density ( $0.16 \text{ fm}^{-3}$ ), binding energy (16 MeV) at saturation density,

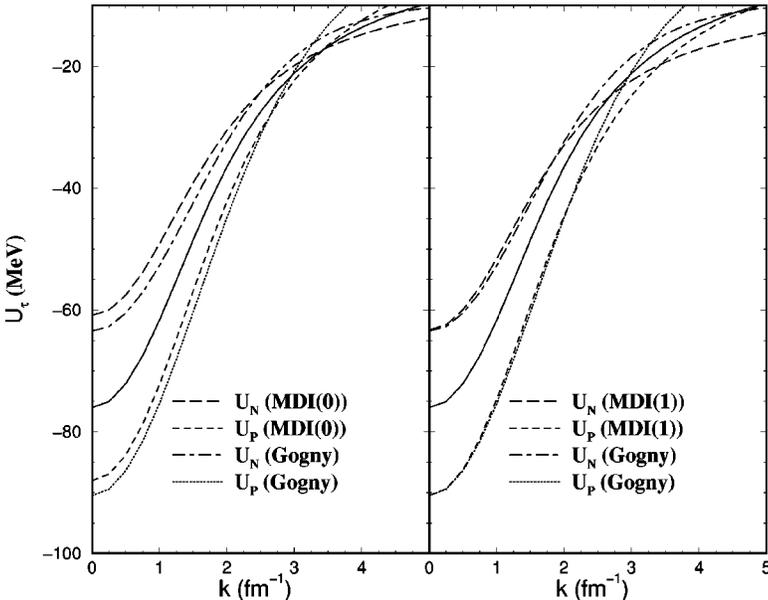


FIG. 4. One body potential in cold matter for  $\delta=0.4$ . As in Fig. 3, here also the two panels are for  $x=0$  and 1. Comparison with one-body potential as obtained with a Gogny interaction has also been shown. The solid line is the single particle potential for symmetric matter with MDI.

compressibility ( $\approx 210$  MeV) of  $N=Z$  nuclear matter at saturation density and symmetry energy at  $N=Z$  ( $\approx 30$  MeV). The values of the parameters for two choices (1)  $x=0$  [denoted by MDI(0)] and (2)  $x=1$  [denoted by MDI(1)] in Eq. (3.2) as in Gogny (this causes the value of the symmetry energy to bend over as a function of density) and are given in the table. Figure 3 shows the behavior of symmetry energy for the two choices. In Fig. 4 we show the one body potential in cold matter for  $\delta=0.4$ .

Before we close this section we like to add that BUU calculations with momentum dependent interactions are only slightly more complicated in the asymmetric case compared to the case where no distinction is made between neutrons and protons. In numerical solutions of Boltzmann equations, phase space densities are simulated by test particles, characterized by a position and a momentum. Now there will also be a tag on their charge but no major addition to the codes [6] are needed.

#### IV. REDUCTION TO THE GBD FORM

Although we do recommend the full formalism of the above section be implemented, it is possible to reduce the above to a GBD (Gale, Bertsch, and Das Gupta) form. A GBD potential, extended to asymmetric matter, already exists and is called BGBD [8]. There the extensions were made such that in the  $N=Z$  case one gets back exactly the original parameters [1]. The parameter  $x$  [Eq. (3.1)] was chosen to be  $1/15$ . In contrast, here we have chosen the momentum dependence of the Gogny potential as a reference curve and chosen  $x$  at 0 or 1. The values of the parameters of GBD(0) ( $x=0$ ), GBD(1) ( $x=1$ ), and BGBD ( $x=1/15$ ) are given in Table I.

We write the potential energy density coming from the momentum dependent part as

$$V_{\text{mom}}(\rho, \delta) = \frac{1}{\rho_0} \sum_{\tau, \tau'} C_{\tau, \tau'} \rho_{\tau} \int \frac{f_{\tau'}(\vec{r}, \vec{p}) d^3 p}{1 + (\vec{p} - \langle \vec{p} \rangle_{\tau})^2 / \Lambda^2}. \quad (4.1)$$

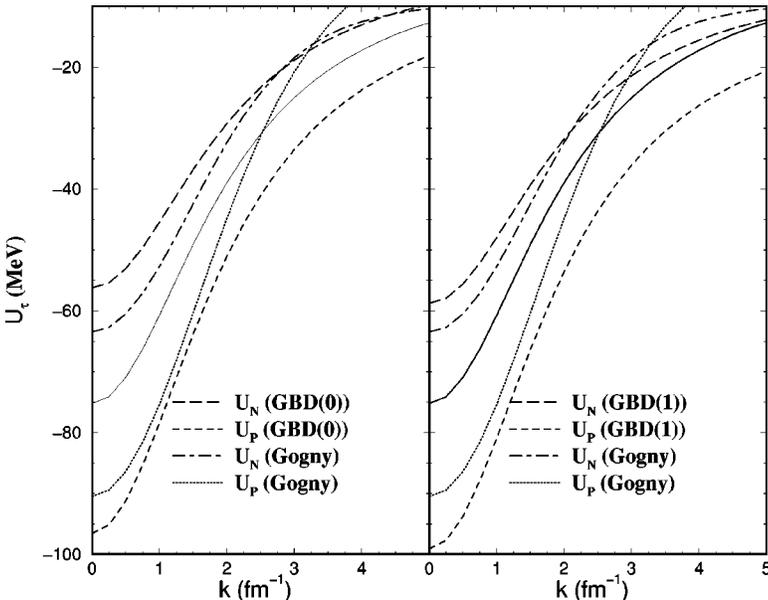


TABLE I. The values of the parameters of different interactions. Also gives the values  $K$  in the respective parametrizations. The saturation density for all parametrization is  $0.16 \text{ fm}^{-3}$ , except for BGBD(1/15) it is  $0.163 \text{ fm}^{-3}$ . The binding energy and the total symmetry energy including contribution from kinetic part are  $-16$  MeV/nucleon and  $31.623$  MeV/nucleon, respectively, in each case.

Parameters	MDI(0)	MDI(1)	GBD(0)	GBD(1)	BGBD(1/15)
$A_u$	-95.98	-187.27	-109.85	-299.69	-192.0
$A_l$	-120.57	-29.28	-191.30	-1.46	-96.0
$B$	106.35	106.35	205.66	205.66	203.3
$C_u$	-103.40	-103.40	-118.80	-118.80	-84.53
$C_l$	-11.70	-11.70	-26.26	-26.26	-65.472
$\sigma$	4/3	4/3	7/6	7/6	7/6
$\Lambda$	$1.0p_f^{(0)}$	$1.0p_f^{(0)}$	$1.5p_f^{(0)}$	$1.5p_f^{(0)}$	$1.5p_f^{(0)}$
$K$	210.68	210.68	214.68	214.68	215.0

The one-body potential generated by this piece of potential energy density for given  $\rho$ ,  $\delta$ ,  $\vec{p}$  and a given  $\tau$  is

$$U_{\text{mom}}(\rho, \delta, \vec{p}, \tau) = \frac{C_{\tau, \tau}}{\rho_0} \left[ \int \frac{f_{\tau}(\vec{r}, \vec{p}') d^3 p'}{1 + (\vec{p}' - \langle \vec{p} \rangle_{\tau})^2 / \Lambda^2} + \frac{\rho_{\tau}}{1 + (\vec{p} - \langle \vec{p} \rangle_{\tau})^2 / \Lambda^2} \right] + \frac{C_{\tau, \tau'}}{\rho_0} \left[ \int \frac{f_{\tau'}(\vec{r}, \vec{p}') d^3 p'}{1 + (\vec{p}' - \langle \vec{p} \rangle_{\tau'})^2 / \Lambda^2} + \frac{\rho_{\tau'}}{1 + (\vec{p} - \langle \vec{p} \rangle_{\tau'})^2 / \Lambda^2} \right]. \quad (4.2)$$

In the above equation,  $\tau' \neq \tau$ . As expected, the values of the constants in the force will have to be recalculated to reproduce the saturation properties. These are given in Table I. The GBD potential  $U(\rho, \delta, p, \tau)$  is plotted in Fig. 5. This does not track the Gogny potential as faithfully as the more

FIG. 5. One body potential in cold matter for  $\delta=0.4$ . As in Fig. 4, here also the two panels are for  $x=0$  and 1. Comparison with one-body potential as obtained with a Gogny interaction has also been shown. The solid line represents the single particle potential for symmetric matter with the GBD potential.

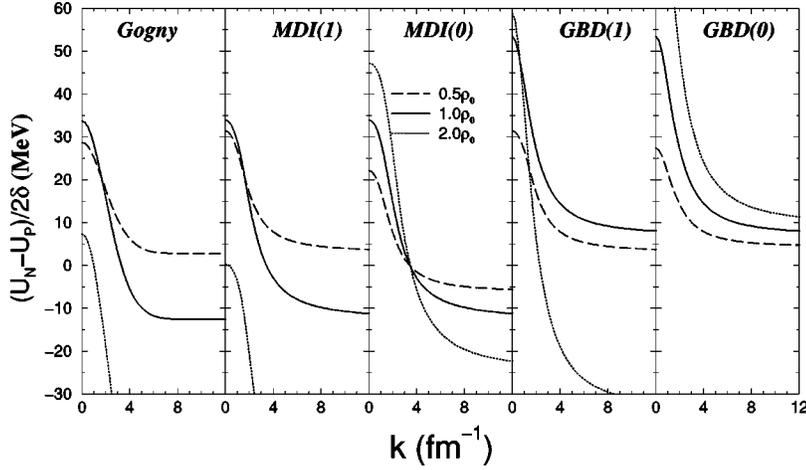


FIG. 6. Dependence of  $(U_N - U_p)/2\delta$  on momentum for different potentials at different densities. All the figures are for  $\delta=0.4$ .

sophisticated version of Sec. III does. The reader might wonder why the value of  $\Lambda$  is significantly bigger in GBD as opposed to in MDI(0) and MDI(1). We have tried to fit the variation of  $U$  with  $k$  as obtained in Gogny potential (or the Brueckner-Hartree-Fock calculation) by adjusting the value of  $\Lambda$ . For  $p$  close  $p_F$ , the contribution to  $U(p)$  in Sec. III comes mainly from  $p'$  near  $p_F$  whereas in GBD what counts is  $|\vec{p} - \langle \vec{p} \rangle_{\text{ave}}|$ : since,  $\langle \vec{p} \rangle_{\text{ave}}$  is zero, one requires a different value of  $\Lambda$  to mimic the variation with  $p$ . This point was not appreciated in Ref. [3].

### V. MOMENTUM DEPENDENCE OF THE SYMMETRY POTENTIAL

The single particle potentials derived in the previous sections can be used directly in transport model calculations. They combine the density, momentum and isospin dependences of both the isoscalar and symmetry potentials in a nontrivial way. In this section we evaluate the strength of the momentum dependence of the symmetry potential. To the leading order in  $\delta$ , the single nucleon potential can be cast to the form

$$U_{n/p}(\rho, \vec{p}, \delta) \approx U_0(\rho, \vec{p}) \pm U_{\text{sym}}(\rho, \vec{p}) \delta \quad (5.1)$$

in accordance with the Lane potential [17], where the  $\pm$  sign is for neutrons and protons, respectively. Thus the symmetry potential can be evaluated from  $U_{\text{sym}}(\rho, \vec{p}) = (U_n - U_p)/2\delta$ . Shown in Fig. 6 are the symmetry potentials as a function of  $k$  for the three densities. It is seen that the  $U_{\text{sym}}(\rho, \vec{p})$  is strongly momentum dependent for  $k \leq 5 \text{ fm}^{-1}$  in all models considered. Moreover, this dependence is particularly stronger at high densities. By construction, the results for Gogny and MDI(1) are very close. By comparing the results with  $x=0$  and  $x=1$ , it is seen that the momentum dependence is rather different mainly for  $\rho/\rho_0=2$ . This is because the symmetry energies with  $x=0$  and  $x=1$  are significantly different only in the region of  $\rho \geq \rho_0$  as shown in Fig. 3.

To our best knowledge, the symmetry potential has been assumed to be momentum independent in all previous studies. Our results above indicate that this is only a good ap-

proximation at high momenta. At momenta less than about 1 GeV/c, the momentum dependence of the symmetry potential is important.

### VI. SUMMARY

The goal of this work was to obtain parameters of a momentum dependent mean field potential which is applicable to highly isospin asymmetric nuclear matter but easy enough to use in a transport model calculation for heavy ion collisions at intermediate to high energies. We used the Gogny interaction as a guide. Published results of Brueckner-Hartree-Fock calculations were also used for choosing parameters. We have two versions. They both are quite flexible in the sense that parameters can be easily chosen according to the experimental data on binding energy, compressibility, etc. In each of these versions we have proposed two sets of parameters: one, in which the symmetry energy continues to rise as a function of density and another one where the symmetry energy first rises and then falls off with density. Our opinion is that introducing momentum dependence in symmetry potential will not make transport model simulations much harder or longer than they already are. Implementations of these potentials in BUU transport models are in progress.

Transport models with the momentum dependent symmetry potentials are more reliable tools for investigating the density dependence of nuclear symmetry energy and thus the EOS of neutron-rich matter. Knowledge on the symmetry energy  $E_{\text{sym}}(\rho)$  is essential for understanding not only the structure of radioactive nuclei but also many key issues in astrophysics. For instance, the  $E_{\text{sym}}(\rho)$  determines uniquely the proton fraction in neutron stars at  $\beta$  equilibrium [8]. A continuously rising symmetry energy leads to a growing proton fraction with increasing density, thus allowing for the fast cooling of protoneutron stars through the direct URCA process [15]. A falling symmetry energy at high densities forbids the direct URCA process to happen; moreover, it favors the formation of pure neutron domains in the cores of neutron stars [16]. Nuclear reactions induced by neutron-rich nuclei provide a great opportunity to pin down the density dependence of nuclear symmetry energy. Among the experimental

observables that have been found to be sensitive to the symmetry potential, the neutron/proton ratio of pre-equilibrium nucleon emissions, neutron-proton differential flow, and correlation functions, as well as the proton elliptic flow at high transverse momenta are expected to be most sensitive to the momentum dependence of the symmetry potential. These observables will be studied with the improved BUU transport models using the momentum dependent symmetry potentials. These results will be reported in a forthcoming publication.

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