# Effects of the form factor on inclusive $(e,e')$ reactions in the quasielastic region

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We discuss the effect of nuclear medium on the inclusive $(e,e')$ reactions for $^{40}$Ca and $^{208}$Pb targets by introducing the form factors of the nucleons inside nuclei in the quasielastic region. These form factors include the change of nucleon properties in nuclear medium. Longitudinal and transverse structure functions for given momenta and energy transfers are extracted from the cross section calculated by using the form factors with the Rosenbluth separation method. A relativistic Hartree single particle model for the bound state and continuum nucleon wave functions is used and the effects of the electron Coulomb distortion are included in the calculation. We compare the results with the experimental data measured at Bates and Saclay.

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## I. INTRODUCTION

Medium energy electron scattering has been one of the useful tools to study the nucleon properties inside nuclei, especially in the quasielastic region. There have been many experiments [1–10] studying the quasielastic electron scattering from medium and heavy nuclei and a number of theoretical attempts [11–18] to fit the measured cross section and to separate longitudinal and transverse structure functions. The Fermi gas model in the impulse approximation provides a good description of the inclusive $(e,e')$ cross sections, but fails in describing the structure functions. For instance, there appeared to be a large suppression (about 40%) of the longitudinal structure function dubbed the missing strength in the sum rule of the longitudinal structure function. There was also disagreement between the extracted transverse structure functions and the predictions of the Fermi gas model, but this was expected since exchange currents, pion production, and other processes induced by transverse photons were not included in the Fermi gas model. Many attempts have been carried out to explain this missing strength of the longitudinal structure function by improving the nuclear bound state, the modification of nucleon form factors in nuclear medium, the final state interaction, and the relativistic dynamics effect. In particular, Boucher and Van Orden [12] studied the $(e,e')$ reaction by the random-phase approximation (RPA) with inclusion of many-body correlations and final state interactions. But the missing strength was not sufficiently explained.

The comparison of experimental $(e,e')$ data to theory comprises two ingredients. The first is the inclusion of electron Coulomb distortion effects and the second is the model used to calculate the nuclear transition current. In the early 1990's, the Coulomb distortion for the reactions $(e,e')$ and $(e,e'p)$ in the quasielastic region was treated exactly by the Ohio University group [8,11,19–21] using partial wave expansion of the electron wave functions. Such partial wave treatments are referred to the distorted wave Born approximation (DWBA), since the static Coulomb distortion was included exactly by numerically solving the radial Dirac equation containing the Coulomb potential for a finite nuclear charge distribution to obtain the distorted electron wave functions. While this calculation permits the comparison of different nuclear models against measured cross sections and provides an invaluable check on various approximate techniques of including Coulomb distortion effects, it is numerically challenging and computation time increases rapidly with higher incident electron energies. Furthermore, the calculated cross section cannot be separated into a longitudinal contribution and a transverse contribution.

In order to avoid these difficulties associated with the DWBA analyses at higher electron energies and to look for a way to still define structure functions, Kim and Wright [16,22,23] developed an approximate treatment of the Coulomb distortion based on the works of Knoll [24] and of Lenz and Rosenfelder [25]. The essence of the approximation is to calculate the four potential arising from the electron four current in the presence of the static Coulomb field of the nucleus. The Coulomb distortion is included in the four potential by the electron phase shifts for the elastic scattering and by letting the magnitude of the electron momentum include the effect of the static Coulomb potential. This last step leads to an $r$-dependent momentum. The approximate method turned out to allow the separation of the cross section into a "longitudinal" term and a "transverse" term. More recently, Kim and Wright [17] developed the treatment of the Coulomb distortion in the $(e,e')$ reaction by using a simple *ad hoc* procedure with the $\sigma$-$\omega$ model [26]. For medium and heavy nuclei, a good treatment of Coulomb distortion effects is necessary in order to extract the "longitudinal" and "transverse" structure functions.

Before continuing with a discussion of explaining the "longitudinal" suppression, it should be noted that not all investigators find such a suppression. For example, the Ohio group analyzed the Bates data [8] for $^{40}$Ca$(e,e')$ under the relativistic $\sigma$-$\omega$ model potential for the bound and continuum nucleons along with the relativistic current operator coupled with a good description of Coulomb distortion. This calculation was in good agreement with the Bates data with no evidence of longitudinal suppression [19]. This same model, coupled with our approximate treatment of Coulomb distortion, was compared to the Saclay quasielastic data on $^{208}$Pb

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taken with both electrons and positrons. Our calculation of Coulomb distortion by electrons and positrons was not consistent with the data quoted by Saclay, so it was not possible to extract a "longitudinal" structure function [17]. Furthermore, we investigated the approximation used by the Saclay group for electron Coulomb corrections and found it to be not a good approximation [16,17].

Alberico *et al.* [27], and Cheon and Jeong [28,29] argued that the postulated suppression could not be understood by the RPA theory alone and suggested the modification of the nucleon form factors in nuclear medium. Cheon and Jeong considered the pion's equation of motion in nuclear medium to obtain a density dependent modification of the form factors. By applying the pion's equation to the cloudy bag model (CBM) of the nucleon, electric and magnetic form factors of the nucleon in the nucleus were obtained. In particular, the size of the nucleon charge distribution inside nuclei, expressed as the root-mean-square radius, was found to be increased due to such nuclear medium effect, i.e., swollen nucleon. For $^{40}$Ca and $^{56}$Fe they found that the modified nucleon form factors corresponding to an electromagnetic radius increased by about 20% with respect to the free nucleon. In our previous paper [30], we applied the modified proton form factors to the exclusive $(e,e'p)$ reaction for $^{40}$Ca and $^{208}$Pb. However, the effect of the modified proton form factors was too small to be discerned from the error bars on the available experimental data. In a related consideration, Kelly [31] found that the density dependence of the nucleon form factors from the quark-meson coupling (QMC) model reduces the polarization ratio at high four momentum transfer, $Q^2=0.8\ (\mathrm{GeV}/c)^2$, in $(\vec{e},e'\vec{p})$ reactions. Saito, Tsushima, and Thomas [32] found that the longitudinal response function and the Coulomb sum are reduced by about 20% because of the modified form factors from nuclear matter by comparing with the Hartree contribution under the QMC model.

In the present work, we investigate the nuclear medium effect (swollen nucleon) for the $(e,e')$ reaction. We study the effect by replacing the form factors of free nucleons by those of the bound nucleons in the nucleus. For the treatment of the electron Coulomb distortion we just adopt the method in Ref. [17]. To avoid the violation of current conservation and gauge invariance we use the same potentials for the bound state and continuum state of nucleons obtained from the $\sigma$-$\omega$ model. In Sec. II, the treatment of the nuclear medium effect through the pion's equation of motion is briefly discussed. In Sec. III, we apply the nuclear medium effect to the inclusive $(e,e')$ reaction and in Sec. IV, our calculations are compared with the experimental data measured at Bates [8] for $^{40}$Ca and at Saclay [9,10] for $^{208}$Pb. Finally the conclusions are given in Sec. V.

## II. MEDIUM EFFECT

It is well known that coupling constants such as the pion-nucleon coupling constant and the pion decay constant, as well as masses of the pion and the nucleon, are modified in nuclear medium. In this section we recapitulate how to obtain the effective coupling constants necessary to calculate form factors of the nucleon in nuclear medium. The equation of motion for the pion in nuclear medium can be expressed by

$$(-\nabla^2+m_\pi^2)\phi_\pi(\mathbf{r})=\frac{i}{\sqrt{2}}\frac{g_{\pi NN}}{m_N}\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\,\delta(r)-m_\pi V(\mathbf{r})\phi_\pi(\mathbf{r}), \tag{1}$$

where the second term on the right-hand side represents an optical potential for the pion. We use the nonlocal Kisslinger type potential, $-m_\pi V(\mathbf{r})\phi_\pi(\mathbf{r})=\boldsymbol{\nabla}\cdot\alpha(\mathbf{r})\nabla\phi_\pi(\mathbf{r})$ [33]. The $\alpha(\mathbf{r})$, which is a dimensionless quantity, accounts for a dependence on a nuclear density. For a free pion, which is the case without the second term on the right-hand side in Eq. (1), the solution in momentum space is given by

$$\phi_\pi(\mathbf{q})=\frac{g_{\pi NN}}{\sqrt{2}m_N}\frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}. \tag{2}$$

If $\alpha(\mathbf{r})$ is assumed to be constant, which means a uniform nuclear density, the solution can be found as

$$\phi_\pi(\mathbf{q})=\frac{g_{\pi NN}}{\sqrt{2}m_N}\frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{(1+\alpha)\mathbf{q}^2+m_\pi^2}. \tag{3}$$

But the short-range repulsive character of the nuclear force in free space leads to nonuniform nuclear short-range correlations inside a nucleus. This effect can be taken into account by introducing the Gaussian type correlation function $f(|\mathbf{r}|)=\exp(-\beta\mathbf{r}^2)$ into $\alpha(\mathbf{r})=\alpha[1-f(|\mathbf{r}|)]$, where $\beta$ is a free parameter constrained by the short range of the nuclear force [34]. Then Eq. (1) can be rephrased as follows

$$(-\nabla^2+\tilde{m}_\pi^2)\tilde{\phi}_\pi(\mathbf{r})=\frac{\tilde{g}_{\pi NN}}{\sqrt{2}m_N}\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\,\delta(\mathbf{r})-\tilde{\alpha}\boldsymbol{\nabla}f(|\mathbf{r}|)\cdot\boldsymbol{\nabla}\tilde{\phi}_\pi(\mathbf{r}), \tag{4}$$

where $\tilde{m}_\pi=m_\pi/\sqrt{1+\alpha}$, $\tilde{\alpha}=\alpha/(1+\alpha)$, and $\tilde{g}_{\pi NN}=g_{\pi NN}/(1+\alpha)$. The solution for this nonhomogeneous equation can be obtained by the iteration method in the following way:

$$\tilde{\phi}_\pi=\phi_\pi+\frac{1}{\tilde{m}_\pi^2-\nabla^2}\tilde{V}\phi_\pi+\frac{1}{\tilde{m}_\pi^2-\nabla^2}\tilde{V}\frac{1}{\tilde{m}_\pi^2-\nabla^2}\tilde{V}\phi_\pi+\cdots, \tag{5}$$

where $\tilde{V}=-\tilde{\alpha}\boldsymbol{\nabla}f(|\mathbf{r}|)\cdot\boldsymbol{\nabla}$. The first term is the solution for the case of uniform nuclear density. It just corresponds to the case without the second term on the right-hand side in Eq. (4), while the second and third terms represent the rescattering of the pion from the optical potential in nuclear medium. Fourier transform of the second term is given by

$$\Sigma_1(\mathbf{q}) = -\frac{\tilde{\alpha}}{(2\pi)^3}\int d\mathbf{r}\exp[i\mathbf{q}\cdot\mathbf{r}]\frac{1}{\tilde{m}_\pi^2-\nabla^2}\boldsymbol{\nabla}\cdot[f(|\mathbf{r}|)\boldsymbol{\nabla}\phi_\pi(\mathbf{r})]$$
$$=\frac{\tilde{\alpha}}{(2\pi)^3}\int d\mathbf{p}d\mathbf{k}d\mathbf{r}\exp[i(\mathbf{q}-\mathbf{p}-\mathbf{k})\cdot\mathbf{r}] \times\frac{\mathbf{k}\cdot(\mathbf{p}+\mathbf{k})}{\tilde{m}_\pi^2+(\mathbf{p}+\mathbf{k})^2}f(|\mathbf{p}|)\phi_\pi(\mathbf{k})$$
$$=\tilde{\alpha}\int d\mathbf{k}\frac{\mathbf{k}\cdot\mathbf{q}}{\tilde{m}_\pi^2+\mathbf{q}^2}f(|\mathbf{q}-\mathbf{k}|)\phi_\pi(\mathbf{k}), \tag{6}$$

where $f(|\mathbf{p}|)$ and $\phi_\pi(\mathbf{k})$ are the Fourier transforms of $f(|\mathbf{r}|)$ and $\phi_\pi(\mathbf{r})$, respectively. Under the soft pion limit $(\mathbf{q}\to 0)$, the second term in Eq. (5) is finally expressed as

$$\Sigma_1(\mathbf{q})=\frac{\tilde{g}_{\pi NN}}{\sqrt{2}m_N}\frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{\mathbf{q}^2+\tilde{m}_\pi^2}\frac{\tilde{\alpha}}{3}\zeta(\beta), \tag{7}$$

where

$$\zeta(\beta)=\frac{1}{2\sqrt{\pi}}\beta^{-3/2}\int_0^\infty\frac{k^4}{k^2+\tilde{m}_\pi^2}\exp\left(\frac{-k^2}{4\beta}\right)dk. \tag{8}$$

Likewise the third term is obtained as

$$\Sigma_2(\mathbf{q})\simeq\frac{\tilde{g}_{\pi NN}}{\sqrt{2}m_N}\frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{\mathbf{q}^2+\tilde{m}_\pi^2}\frac{\tilde{\alpha}^2}{9}\zeta(\beta). \tag{9}$$

Finally, Eq. (5) becomes

$$\tilde{\phi}(\mathbf{q})=\frac{\tilde{g}_{\pi NN}}{\sqrt{2}m_N}\frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{(\mathbf{q}^2+\tilde{m}_\pi^2)}\left[1+\frac{\tilde{\alpha}}{3}\zeta(\beta)+\frac{\tilde{\alpha}^2}{9}\zeta(\beta)+\cdots\right]$$
$$=\frac{\tilde{g}_{\pi NN}}{\sqrt{2}m_N}\frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{(\mathbf{q}^2+\tilde{m}_\pi^2)}\left[1+\frac{\tilde{\alpha}/3}{1-\tilde{\alpha}/3}\zeta(\beta)\right]. \tag{10}$$

Therefore the solution of the pion's equation can be summarized just like the free space solution of Eq. (2),

$$\tilde{\phi}(\mathbf{q})=\frac{\tilde{g}^*_{\pi NN}}{\sqrt{2}\tilde{m}_N}\frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{(\mathbf{q}^2+\tilde{m}_\pi^2)}, \tag{11}$$

where $\tilde{g}^*_{\pi NN}=g_{\pi NN}(1+\alpha)^{-3/2}[1+\tilde{\alpha}/3/(1-\tilde{\alpha}/3)\zeta(\beta)]$, $\tilde{m}_\pi=m_\pi(1+\alpha)^{-1/2}$, and $\tilde{m}_N=m_N(1+\alpha)^{-1/2}$. Since $\zeta(\beta)\simeq 1$, we approximate $\tilde{g}^*_{\pi NN}$ as

$$\tilde{g}^*_{\pi NN}=g_{\pi NN}(1+\alpha)^{-3/2}\left[1+\frac{\alpha/(1+\alpha)}{3-\alpha/(1+\alpha)}\right]. \tag{12}$$

Similarly we can do the same calculation for $\tilde{f}^*_\pi$. The $\tilde{g}^*_\pi$ and $\tilde{f}^*_\pi$ obtained this way are the input data to calculate the electric and magnetic form factors, $G_E$ and $G_M$, of the nucleon inside the nucleus. Detailed calculations of the form factors by the CBM were discussed in Ref. [29] and are skipped in the present paper. The free parameter $\alpha$ above was determined to satisfy the experimental data of the $(e,e')$ scattering in Ref. [1]. For example, $\alpha=0.20,0.26,0.23$ were used for $^{12}C$, $^{40}Ca$, $^{56}Fe$, respectively [28]. In principle, the values can be calculated from the microscopic model based on the QCD. In this work we adopt the values of the form factors and investigate the effect of nuclear medium by applying to the realistic model on the $(e,e')$ reaction.

## III. APPLICATION TO THE INCLUSIVE PROCESS

In the plane wave Born approximation (PWBA) in which the leptons are described as Dirac plane waves, the cross section for the inclusive quasielastic $(e,e')$ reaction can be written as

$$\frac{d^2\sigma}{d\Omega d\omega}=\sigma_M\left\{\frac{q_\mu^4}{q^4}S_L(q,\omega)+\left[\tan^2\frac{\theta}{2}-\frac{q_\mu^2}{2q^2}\right]S_T(q,\omega)\right\}, \tag{13}$$

where $q_\mu^2=\omega^2-\mathbf{q}^2$ is the four momentum transfer and $\sigma_M$ is the Mott cross section given by $\sigma_M=(\alpha/2E)^2\cos^2\theta/2/\sin^4\theta/2$. $S_L$ and $S_T$ are the longitudinal and transverse structure functions that depend only on the three momentum transfer $q$ and the energy transfer $\omega$. It is possible to extract the two structure functions by keeping the momentum and energy transfers fixed while varying the electron energy $E$ and the scattering angle $\theta$. The longitudinal and transverse structure functions in Eq. (13) are bilinear products of the Fourier transform of the components of the nuclear transition current density integrated over the outgoing nucleon angles. Explicitly, the structure functions for a given bound state with angular momentum $j_b$ are given by

$$S_L(q,\omega)=\sum_{\mu_b s_p}\frac{\rho_p}{2(2j_b+1)}\int|N_0|^2d\Omega_p, \tag{14}$$

$$S_T(q,\omega)=\sum_{\mu_b s_p}\frac{\rho_p}{2(2j_b+1)}\int(|N_x|^2+|N_y|^2)d\Omega_p, \tag{15}$$

with the outgoing nucleon density of states $\rho_p=pE_p/(2\pi)^2$. The $\hat{\mathbf{z}}$ axis is taken to be along the momentum transfer $\mathbf{q}$. $\mu_b$ and $s_p$ are $z$ components of the angular momentum of the bound and continuum states. The Fourier transform of the nuclear current $J^\mu(\mathbf{r})$ is simply given by

$$N^\mu=\int J^\mu(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}d^3r, \tag{16}$$

where $J^\mu(\mathbf{r})$ denotes the nucleon transition current. The continuity equation has been used to eliminate the $z$ component $(N_z)$ via the equation $N_z=(\omega/q)N_0$ with the Coulomb gauge.

The *ad hoc* expressions for the longitudinal and transverse structure functions with the inclusion of the electron Coulomb distortion (see Ref. [17] for details) are given by

$$N_0^{\rm ad\,hoc}=\int\left(\frac{q'_\mu(r)}{q_\mu}\right)^2\left(\frac{q}{q'(r)}\right)^2 e^{i\langle\delta(\kappa_i^2)+\delta(\kappa_f^2)\rangle}e^{i\mathbf{q}'(r)\cdot\mathbf{r}}J_0(\mathbf{r})d^3r, \tag{17}$$

$$\mathbf{N}_T^{\rm ad\,hoc}=\left(\frac{p'_i(0)}{p_i}\right)\int e^{i\mathbf{q}'(r)\cdot\mathbf{r}}\mathbf{J}_T(\mathbf{r})d^3r, \tag{18}$$

where $\langle\delta(\kappa_{i,f}^2)\rangle$ denotes an average over the angles of the vector $\mathbf{r}$. That is, $\langle\kappa_{i,f}^2\rangle=\langle(\mathbf{r}\times\mathbf{p}_{i,f})^2\rangle=r^2p_{i,f}^2(3-\cos^2\theta_{p_{i,f}})/4$. The $r$-dependent momentum transfer is given by $\mathbf{q}'(r)=\mathbf{p}'_i(r)-\mathbf{p}'_f(r)$. The local effective momentum $\mathbf{p}'(r)$ is given in terms of the Coulomb potential of the target nucleus by

$$\mathbf{p}'(\mathbf{r})=\left(p-\frac{1}{r}\int_0^r V(r)dr\right)\hat{\mathbf{p}}. \tag{19}$$

The nucleon transition current in the relativistic single particle model is given by

$$J_\mu(\mathbf{r})=e\bar{\psi}_p(\mathbf{r})\hat{J}_\mu\psi_b(\mathbf{r}), \tag{20}$$

where $\hat{J}_\mu$ is a free nucleon current operator. $\psi_p$ and $\psi_b$ are the wave functions of the knocked out nucleon and the bound state, respectively. For a free nucleon, the operator consists of two ingredients, the Dirac contribution and the contribution of an anomalous magnetic moment $\mu_T$,

$$\hat{J}^\mu=F_1\gamma^\mu+F_2\frac{i\mu_T}{2M}\sigma^{\mu\nu}q_\nu, \tag{21}$$

where $\mu_T$ is the nucleon anomalous magnetic moment (for a proton $\mu_T=1.79$ and for a neutron $\mu_T=-1.91$). The Dirac form factor $F_1$ and the Pauli form factor $F_2$ which are functions of the four momentum transfer squared $q_\mu^2$ are related to the electric and magnetic form factors $G_E$ and $G_M$ given by

$$G_E=F_1+\frac{\mu_T q_\mu^2}{4M^2}F_2,$$

$$G_M=F_1+\mu_T F_2. \tag{22}$$

We choose the following standard dipole form [35]:

$$G_E=\frac{G_M}{(\mu_T+1)}=\frac{1}{\left(1-\frac{q_\mu^2}{0.71}\right)^2}, \tag{23}$$

where $q_\mu^2$ is in units of $(\mathrm{GeV}/c)^2$.

## IV. COMPARISON WITH THE EXPERIMENTAL DATA FOR $(e,e')$ REACTION

An alternative form of the cross section in terms of the total structure function in the PWBA is given by

$$\frac{d^2\sigma}{d\Omega d\omega}=\left(\frac{\sigma_M}{\epsilon(\theta)}\right)\left(\frac{q_\mu^4}{q^4}\right)S_{total}(q,\omega,\theta), \tag{24}$$

where $\epsilon(\theta)$ denotes the virtual photon polarization given by

$$\epsilon(\theta)=\left(1-\frac{2q^2}{q_\mu^2}\tan^2\frac{\theta}{2}\right)^{-1}. \tag{25}$$

The total structure function is written as

$$S_{total}(q,\omega,\theta)=\epsilon(\theta)S_L(q,\omega)-\frac{1}{2}\left(\frac{q^2}{q_\mu^2}\right)S_T(q,\omega). \tag{26}$$

Equation (26) describes $S_{total}$ as a straight line in the independent variable $\epsilon(\theta)$ with slope $S_L(q,\omega)$ and intercept proportional to $S_T(q,\omega)$ by keeping the momentum transfer $q$ and the energy transfer $\omega$ fixed. In the quasielastic scattering, the longitudinal structure function roughly carries the charge distribution of the nucleons and the transverse structure function carries the current and magnetization distributions of the nucleons.

In our analyses of the quasielastic scattering, we use the relativistic bound and continuum single particle wave functions for the nucleons. For the inclusive $(e,e')$ reaction we use continuum solutions for the outgoing, but unobserved, nucleons which are in the same Hartree potential as for the bound states [26]. This choice guarantees current conservation, gauge invariance, and orthogonality of initial and final states. In this calculation we do not change the Dirac form factor $F_1$ while the Pauli form factor $F_2$ is changed via the anomalous magnetic moments as in Refs. [28,29] due to the nuclear medium effect. Note that we take into account the electron (positron) Coulomb distortion in all the calculations.

In Fig. 1 we compare the cross sections as a function of the energy transfer $\omega$ for two cases; incident electron energy $E=290$ MeV, scattering angle $\theta=140°$ and $E=375$ MeV, $\theta=90°$. The solid lines are calculated with $\mu_p=1.79$ for the proton and $\mu_n=-1.91$ for the neutron, which are the experimental values for the free nucleons. The dotted and dashed lines are the results with $\mu_p=1.71$, 2.01 and $\mu_n=-1.92$, $-2.32$, respectively. The experimental data are from Bates [8]. The differences between the solid lines and the dotted lines are less than 5% for the two angles around the peak positions. Although the dashed lines show very large differences from the other two curves, the lines are still more or less within the error bars. For the backward angle all the curves are within the error bars, but for the other case the dashed lines slightly overestimate the data. By changing the anomalous magnetic moments the effect of the modified form factors appears larger on the transverse structure function than on the longitudinal structure function because the longitudinal contribution is very small for the backward angle. Hence the modified form factors due to the magnetic anomalous moment in nuclear medium play a more impor-

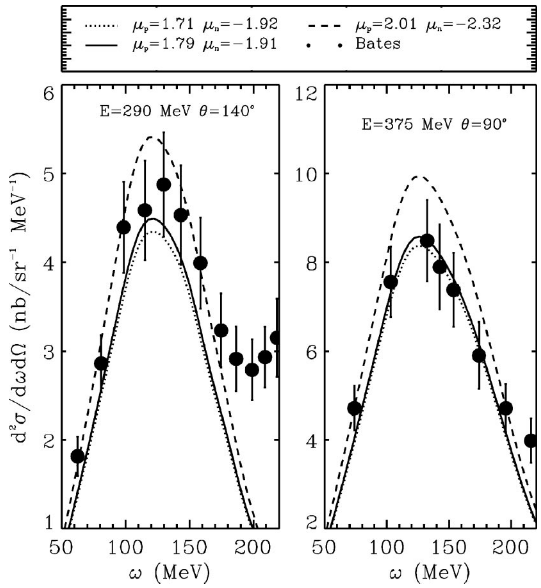


FIG. 1. The cross sections for [40]Ca at two different electron energies and scattering angles; incident electron energy $E = 290$ MeV, scattering angle $\theta = 140°$ and $E = 375$ MeV, $\theta = 90°$. The solid lines are calculated with $\mu_p = 1.79$ for the proton and $\mu_n = -1.91$ for the neutron. The dotted and dashed lines are the results with $\mu_p = 1.71$, 2.01 and $\mu_n = -1.92$, $-2.32$, respectively. The experimental data are from Bates [8].

tant role on the transverse structure function than on the longitudinal structure function. This may not be enough to explain the suppression for the longitudinal structure function. Note that changing the anomalous magnetic moment does not affect the shape but changes the magnitude of the calculated cross sections.

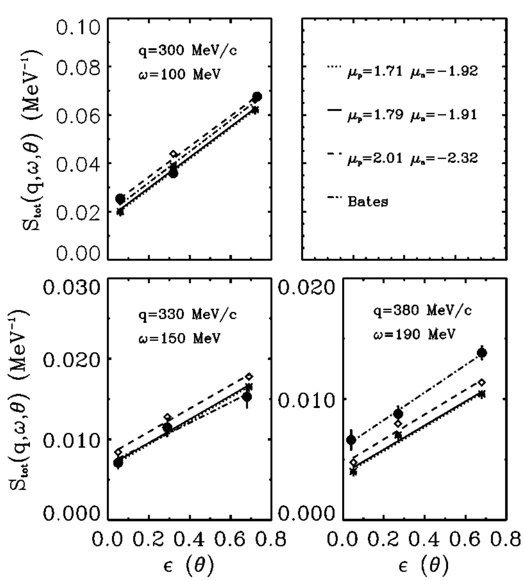


FIG. 2. Rosenbluth separation plots of the cross sections for [40]Ca at given $q$ and $\omega$. + (dotted line) is for $\mu_p = 1.71$ and $\mu_n = -1.92$, × (solid line) is for $\mu_p = 1.79$ and $\mu_n = -1.91$, ◇ (dashed line) is for $\mu_p = 2.01$ and $\mu_n = -2.32$, and ● (dashed-dotted line) represents the experimental data extracted from Ref. [8].

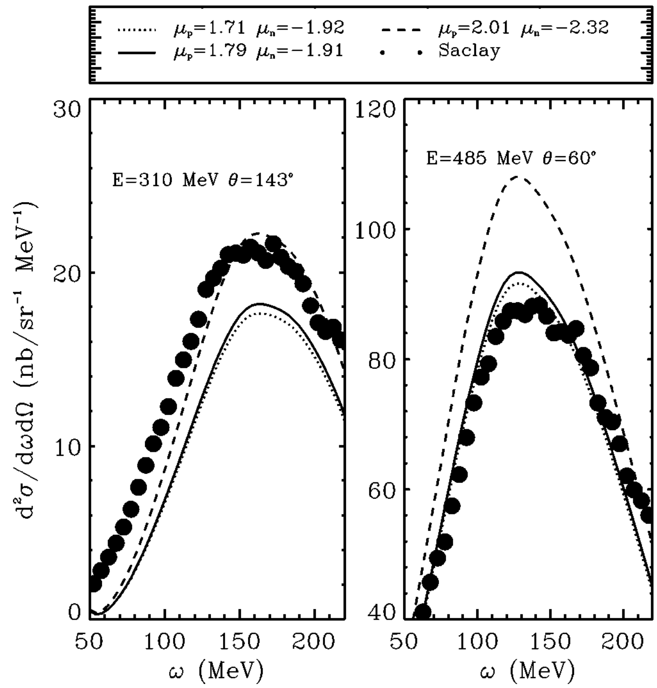


FIG. 3. The cross sections for [208]Pb at two different electron energies and scattering angles; incident energy $E = 310$ MeV, scattering angle $\theta = 143°$ and $E = 485$ MeV, $\theta = 60°$. The solid lines are calculated with $\mu_p = 1.79$ for the proton and $\mu_n = -1.91$ for the neutron. The dotted and dashed lines are the results with $\mu_p = 1.71$, 2.01 and $\mu_n = -1.92$, $-2.32$, respectively. The experimental data are from Saclay [9].

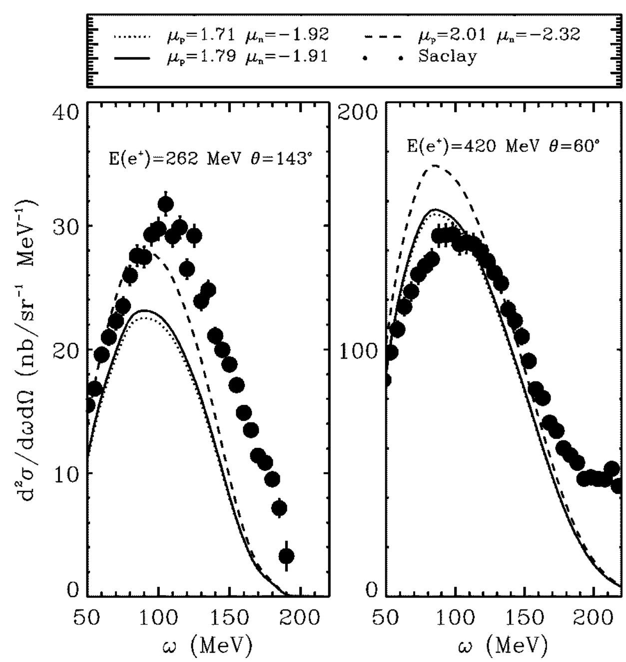


FIG. 4. The cross sections for [208]Pb at two different positron energies and scattering angles; incident energy $E = 262$ MeV, scattering angle $\theta = 143°$ and $E = 420$ MeV, $\theta = 60°$. The solid lines are calculated with $\mu_p = 1.79$ for the proton and $\mu_n = -1.91$ for the neutron. The dotted and dashed lines are the results with $\mu_p = 1.71$, 2.01 and $\mu_n = -1.92$, $-2.32$, respectively. The experimental data are from Saclay [10].

In Fig. 2 we compare the structure functions extracted from $^{40}$Ca for three different momentum and energy transfers using the Rosenbluth separation. We obtain three points of the $S_{total}$ for the scattering angle $\theta$ at given $q$ and $\omega$. We draw the best fit at each points as a straight line. + (dotted line) is for $\mu_p = 1.71$ and $\mu_n = -1.92$, $\times$ (solid line) is for $\mu_p = 1.79$ and $\mu_n = -1.91$, $\diamond$ (dashed line) is for $\mu_p = 2.01$ and $\mu_n = -2.32$, and ● (dashed-dotted line) represents the experimental data extracted from Ref. [8]. In each case we find a good fit to a straight line while the fit for data is slightly off for $q = 300$ MeV and $\omega = 100$ MeV case. The slopes agree with each other but the intercepts for $q = 380$ MeV and $\omega = 190$ MeV do not agree with the data. As shown in Fig. 1, changing the anomalous magnetic moments affects the transverse structure function. From these calculations we do not find the suppression of the longitudinal structure function in $^{40}$Ca.

We also consider $^{208}$Pb as a next target nucleus. Figures 3 and 4 show a comparison of the cross sections by changing the anomalous magnetic moments with the incident electron and positron, respectively. As shown in Fig. 1, the shape of the cross sections does not change but the magnitude changes, and the differences between the solid and the dotted lines are also less than 5%. The effect of the form factor for $^{208}$Pb contributes similarly as for $^{40}$Ca, and the calculated results of the positron are the same as the electron. It shows that the effect is not affected by the incident lepton type or the target nuclei but depends on the momentum transfer. Although in our previous papers [16,17] the theoretical cross sections are slightly above the Saclay data for the forward angle and are below the data for the large backward angle, using the different anomalous magnetic moments (dashed lines) produces better agreement with the experimental data for the backward angle but worse for the forward angle. We still do not find the suppression of the longitudinal structure function by including the nuclear medium effect.

## V. CONCLUSIONS

The purpose of this work is to investigate the effect of the modified form factors inside a nucleus in the $(e,e')$ reaction. The modified form factors through the pion's equation of motion are described with effective coupling constants and masses of mesons and nucleons in the framework of the CBM. The effect of the anomalous magnetic moment obtained from the modified form factors contributes more to the transverse structure function than to the longitudinal structure function. From this result, we do not find a good explanation for the suppression of the longitudinal structure function even in the presence of the electron Coulomb distortion. Hence, implementing the various values of the anomalous magnetic moments obtained from the CBM does not sufficiently explain the suppression. Unless significantly improved theoretical descriptions for the nuclear model reduce other uncertainties such as the off-shell effect, the two-body current, and so on; it is not possible to furnish the effect of the form factor in nuclear medium.

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