Neutron and proton densities and the symmetry energy

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(Received 30 May 2002; revised manuscript received 9 December 2002; published 17 March 2003)

The neutron/proton distributions in nuclei, in particular, the *n-p* difference, are considered in a ''macroscopic'' Thomas-Fermi approach. The density dependence $F(\rho)$ of the symmetry-energy density, where ρ is the total density, drives this difference in the absence of Coulomb and density-gradient contributions when we obtain an explicit solution for the difference in terms of F. If F is constant then the *n-p* difference and, in particular, the difference δR between the neutron and proton rms radii are zero. The Coulomb energy and gradient terms are treated variationally. The latter make only a small contribution to the *n-p* difference, and this is then effectively determined by F. The Coulomb energy reduces δR. Switching off the Coulomb contribution to the *n-p* difference then gives the maximum δR for a given F. Our numerical results are for ^{208}Pb. We consider a wide range of F; for these, both δR and the ratio χ of the surface to volume symmetry-energy coefficient depend, approximately, only on an integral involving F^{-1}. For $\delta R \lesssim 0.45$ fm this dependence is one valued and approximately linear for small δR, and this integral is then effectively determined by δR. There is a strong correlation between δR and χ, allowing an approximate determination of χ from δR. δR has a maximum of $\cong 0.65$ fm.

DOI: 10.1103/PhysRevC.67.034305 PACS number(s): 21.10.Ma, 21.30.−x

I. INTRODUCTION

We consider the neutron and proton densities ρ_N and ρ_P in a local density ''macroscopic'' Thomas-Fermi approach. In particular, we study the neutron-proton density difference and its consequence for the difference δR of rms radii and for the surface-symmetry energy. There has recently been much interest in δR because of the experimental possibility of directly measuring the neutron-distribution radius in ^{208}Pb via parity-violating electron scattering [1]. Hadronic probes cannot do this cleanly because of uncertainties from strong-interaction corrections: proton and pion (mostly LAMPF data) scattering determine the neutron radius, and thus the difference between the neutron and proton radii, with an uncertainty of at least 0.2 fm [2]. The relevance of δR for the neutron equation of state, and hence for neutron stars, has been realized and discussed by several authors [3–5], and it has been realized that the difference δR is principally determined by the density dependence of the symmetry-energy density. The critical role of this density dependence for δR was in fact shown long ago by one of the authors [6]. Recent studies are in the framework of mean-field models, both with nonrelativistic in particular, Skyrme-type approaches [7], and relativistic mean-field theory (RMFT) approaches. An excellent review is by Furnstahl [8], who gives an overview of mean-field functionals and extensive references especially for RMFT; see also Ref. [9] for a review of RMFT. A recently extended RMFT approach has also been used to calculate the neutron radius of ^{208}Pb and neutron-star properties [10]. A macroscopic approach closely related to ours can be found in Ref. [11], which emphasizes the importance of the density derivative of the symmetry-energy density.

Our Thomas-Fermi approach assumes local nuclear-matter conditions but includes density-gradient terms and of course the Coulomb interaction. We shall show that the gradient terms play only a small role for the relative density difference $\lambda = (\rho_N - \rho_P)/\rho$, whereas for the total density ρ such terms are essential. This gives us confidence that our approach has some validity in spite of some limitations. The smallness of the gradient terms is consistent with the ''naïve'' dimensional analysis of RMFT; a thorough discussion of these and related issues is given in Ref. [8]. In our approach the density distributions $\rho(r), \lambda(r)$ for a particular nucleus are determined as functions of the radial distance r from the center of the nucleus in terms of the parameters of the energy functional. δR is then determined from the densities. For the determination of $\lambda(r)$ the density dependence $F(\rho)$ [Eq. (9)] of the symmetry-energy density plays a central role, as was already shown in Ref. [6]. We note that $F(\rho)$ becomes a determined function of r for some given density distribution $\rho(r)$. To a very good approximation the problem separates into one involving the total density $\rho(r)$ and one for the relative difference $\lambda(r)$. In the absence of gradient and Coulomb terms we obtain an explicit solution $\lambda_m(r)$ in terms of $F(\rho)$ and $\rho(r)$. [If $F(\rho)$ is independent of ρ, and therefore also of r, then $\rho_N/\rho_P = N/Z$ and $\delta R = 0$.] The gradient and Coulomb terms are subsequently included variationally. We have limited ourselves to ^{208}Pb. We use a range of functions for $F(\rho)$, rather than some particular interactions. We show that, at least for these functions, the difference in radii δR, and also the surface-symmetry energy, are largely determined by the appropriate norm Φ of $F^{-1}(\rho)$. If $\delta R \lesssim 0.45$ fm then the dependence is one valued and approximately linear for small δR. The effect of the gradient terms as well as of the coupling between the problems for λ and ρ is small. A determination of δR will then approximately de-

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termine Φ, which is a purely nuclear-matter quantity. Comparisons with other results are made.

II. DENSITIES AND ENERGIES

Instead of ρ_N, ρ_P we use the variables ρ, λ, all considered to be functions of the radial distance r,

$$\rho=\rho_N+\rho_P, \quad \lambda=(\rho_N-\rho_P)/\rho, \tag{1a}$$

$$\rho_N=\rho(1+\lambda)/2, \quad \rho_P=\rho(1-\lambda)/2, \tag{1b}$$

subject to the constraints

$$\int \rho \ dv=A, \tag{2}$$

$$\int \rho\lambda \ dv=(N-Z)\equiv A\lambda_0, \tag{3}$$

where

$$\lambda_0=(N-Z)/A. \tag{4}$$

All integrals are spherically symmetric, with the integrands functions of r; integrations are over all space with the volume element $dv=4\pi r^2 \ dr$ and the range from 0 to ∞. If the neutron and proton distributions are the same, i.e.,

$$\text{if} \ \ \rho_N/\rho_P=N/Z,$$

then

$$\lambda\equiv\lambda_0=(N-Z)/A. \tag{5}$$

The mean-square radii are given by

$$\langle r^2\rangle=A^{-1}\int \rho r^2 \ dv, \quad \langle r_n^2\rangle=\tfrac{1}{2}N^{-1}\int \rho(1+\lambda)r^2 \ dv,$$

$$\langle r_p^2\rangle=\tfrac{1}{2}Z^{-1}\int \rho(1-\lambda)r^2 \ dv. \tag{6}$$

The difference between the neutron and proton rms radii is defined by

$$\delta R=\sqrt{\langle r_n^2\rangle}-\sqrt{\langle r_p^2\rangle}. \tag{7}$$

If $\rho_N/\rho_P=N/Z$ then $\langle r^2\rangle=\langle r_n^2\rangle=\langle r_p^2\rangle$ and $\delta R=0$.
The energy density we use is

$$\varepsilon=\varepsilon_V(\rho)+\varepsilon_\lambda+(\kappa_0/\rho_0)[(\nabla\rho_n)^2+(\nabla\rho_p)^2] +2(\kappa_1/\rho_0)\nabla\rho_n.\nabla\rho_p+\varepsilon_C, \tag{8}$$

$$\varepsilon_\lambda=u_\tau[(\rho_n-\rho_p)/\rho]^2\rho F(\rho), \tag{9}$$

$$\varepsilon_C=eV_C(r)\rho_p(r), \quad V_C(r)=(4\pi e/r)\int_0^r \rho_p(r')r'^2dr'. \tag{10}$$

$\varepsilon_V(\rho)/\rho$ is the energy per particle of symmetric nuclear matter of density ρ, saturating at normal nuclear-matter density $\rho_0\approx 0.165\ \text{fm}^{-3}$ with energy $u_V\cong -16$ MeV and with curvature determined by the incompressibility constant K. We shall not give an explicit form for $\varepsilon_V(\rho)$ since, as we shall discuss, it has only a small effect on the quantities involving λ.

ε_λ is the symmetry-energy density and $F(\rho)$ determines its density dependence. The magnitude $u_\tau\approx 35$ MeV is given by the semiempirical mass formula [12] and by related fits with RMFT [13]. a_4 is often used in the literature instead of u_τ. We require that $F(\rho_0)=1$ in order that the semiempirical value is obtained for $A\to\infty$. The gradient terms have the form consistent with finite-range interaction effects and are determined by the coefficients κ_0 and κ_1. The Weiszacker gradient terms arising from the kinetic energy are known to be small and their effect can in any case be absorbed by the dominant potential terms. The term $\propto\nabla\rho_n.\nabla\rho_p$ arises from the neutron-proton interaction. ε_C is the effective Coulomb energy density where we have neglected the small exchange contribution.

In terms of ρ and λ the total energy density is

$$\varepsilon(\rho,\lambda)=\varepsilon(\rho)+\varepsilon_\lambda(\rho,\lambda)+C_\lambda(\nabla\rho\lambda)^2+\varepsilon_C(\rho,\lambda), \tag{11}$$

$$\varepsilon(\rho)=\varepsilon_V(\rho)+C_0(\nabla\rho)^2, \tag{12}$$

$$\varepsilon_\lambda(\rho,\lambda)=u_\tau\lambda^2\rho F(\rho), \tag{13}$$

$$C_0=(\kappa_0+\kappa_1)/2\rho_0, \quad C_\lambda=(\kappa_0-\kappa_1)/2\rho_0. \tag{14}$$

$\varepsilon_C(\rho,\lambda)$ is obtained by using Eq. (1b) for ρ_P in Eq. (10). The total energy then becomes a functional of ρ and λ:

$$E[\rho,\lambda]=E[\rho]+E_\lambda[\rho,\lambda]. \tag{15}$$

$E[\rho]$ comprises both the symmetric volume and surface energies, resulting in only an A dependence, and also the usual direct Coulomb energy $E_{C0}[\rho]\propto(\text{Ze/A})^2$ obtained for $\lambda\equiv\lambda_0$. Thus

$$E[\rho]=\int \varepsilon(\rho) \ dv+E_{C0}[\rho], \tag{16}$$

$$E_{C0}[\rho]=\int \varepsilon_C(\rho,\lambda_0)dv =(4\pi Ze/A)^2\int_0^\infty(1/r)\rho r^2dr\int_0^r\rho r'^2dr'. \tag{17}$$

The term

$$E_\lambda[\rho,\lambda]=E_{V\lambda}[\rho,\lambda]+E_{C\lambda}[\rho,\lambda]+E_{G\lambda}[\rho,\lambda] \tag{18}$$

contains all the dependence on $\lambda-\lambda_0$, i.e., on the difference in neutron and proton distributions, but also includes the standard volume-symmetry energy $u_\tau\lambda_0^2A=u_\tau(N-Z)^2/A$ obtained for $\lambda\equiv\lambda_0$. The total Coulomb energy is

$$E_C[\rho,\lambda]=\int \varepsilon_C(\rho,\lambda)dv$$

$$=(4\pi e/2)^2\int_0^\infty (1/r)\rho(r)\{1-\lambda(r)\}r^2dr$$

$$\times\int_0^r \rho(r')\{1-\lambda(r')\}r'^2dr'. \quad (19)$$

The λ-dependent part is

$$E_{C\lambda}[\rho,\lambda]=E_C[\rho,\lambda]-E_{C0}[\rho]. \quad (20)$$

$E_{C\lambda}$ is positive (negative) if $\lambda-\lambda_0$ is positive (negative), corresponding to a less (more) extended proton than neutron distribution.

$$E_{V\lambda}[\rho,\lambda]=\int \varepsilon_\lambda dv, \quad (21)$$

for

$$\lambda\equiv\lambda_0: E_{V\lambda}=u_\tau\lambda_0^2 A, \quad (22)$$

as already mentioned. The λ-dependent gradient term gradient has the simple form

$$E_{G\lambda}[\rho,\lambda]=C_\lambda\int(\nabla\rho\lambda)^2dv. \quad (23)$$

III. VARIATIONAL CALCULATION

We consider the variation of $E[\rho,\lambda]$ with respect to ρ and λ, subject to the conditions of Eqs. (2)–(4). The term $E[\rho]$, depending only on ρ, gives the (volume and surface) energy dependent only on A and also the usual Coulomb energy. For a given nucleus, variation of $E[\rho]$ with respect to ρ determines this energy as well as $\rho(r)$ in terms of C_0 and of the parameters of $\varepsilon_V(\rho)$. In particular, a fit to the data gives $C_0\cong 30$ MeV [14]. Minimization of $E_\lambda[\rho,\lambda]$ with respect to λ, for a fixed $\rho(r)$ appropriate to the nucleus considered, then determines $\lambda(r)$ and E_λ in terms of $F(\rho)$ and of C_λ. Variation of the total energy $E[\rho,\lambda]$ with respect to ρ will lead to a coupling of the problems for ρ and λ and will result in some additional decrease of energy. Because of the stiffness of the equation of state, i.e., of $\varepsilon_V(\rho)$, this effect is quite small, $\propto\lambda_0^4$/K. The smallness of this contribution is supported by simultaneous minimization with respect to ρ and λ for a particular case. This conclusion is also reached in Ref. [11], where the input involves similar physics. Further discussion is in Sec. VI.

We thus consider the energy $E_\lambda[\rho,\lambda]$ for a given $\rho(r)$ satisfying Eq. (2). For numerical results $\rho(r)$ is chosen to be consistent with the data for the nucleus considered. First we consider only the term due to ε_λ:

$$E_{V\lambda}[\rho,\lambda]=\int\varepsilon_\lambda dv\equiv u_\tau\int\lambda^2\rho F(\rho)dv. \quad (24)$$

Minimizing this with respect to $\lambda(r)$, subject to the constraint of Eq. (3), we obtain an explicit, exact, and simple solution $\lambda_m(r)$, E_λ^m in terms of the given $\rho(r)$:

$$\lambda_m(r)=\lambda_0 F^{-1}[\rho(r)]\Phi^{-1},\quad E_\lambda^m=u_\tau\lambda_0^2 A\Phi^{-1}, \quad (25)$$

$$\Phi=A^{-1}\int\rho F^{-1}(\rho)dv, \quad (26)$$

with

$$\int\rho\lambda_m dv=A\lambda_0. \quad (27)$$

With $\lambda=\lambda_m+\delta\lambda$, we obtain for the energy

$$E_{V\lambda}(\lambda_m+\delta\lambda)=E_\lambda^m+\int(\delta\lambda)^2\rho F(\rho)dv, \quad (28)$$

where

$$\int\rho\ \ \delta\lambda\ \ dv=0, \quad (29)$$

which follows from $\int\rho\lambda\ dv=\int\rho\lambda_m\ dv=A\lambda_0$. Since there is no term linear in $\delta\lambda$ in Eq. (28) this demonstrates explicitly that λ_m gives the minimum of $E_{V\lambda}$, provided that $F(\rho)\geqslant 0$. This condition is satisfied for reasonable $F(\rho)$, since $F(\rho=0)\geqslant 0$, $F(\rho_0)=1$, and since $0\leqslant\rho\leqslant\rho_0$ includes all ρ in the surface. To obtain the surface-symmetry energy appropriate to E_λ^m we subtract out the volume part $u_\tau\lambda_0^2 A$:

$$E_{S\lambda}^m=E_\lambda^m-u_\tau\lambda_0^2 A=u_\tau\lambda_0^2 A(\Phi^{-1}-1). \quad (30)$$

We draw some important conclusions from these results: For

$$F(\rho)\equiv 1:\Phi=1,$$

$$\lambda_m\equiv\lambda_0,\quad \delta R=0,\quad E_{S\lambda}^m=0. \quad (31)$$

Thus if there is no density dependence of the symmetry-energy density—and if also the Coulomb and gradient terms are neglected—then the neutron and proton distributions are the same, i.e., $\rho_n/\rho_p\equiv N/Z$, and $\delta R=0$; also there is then no surface-symmetry energy. It is thus the variation of $F(\rho)$, equivalent to a differential neutron-proton pressure, which drives the separation of the two densities; thus $\Phi-1>0$ corresponds to different neutron and proton distributions. In fact if $\Phi-1\ll 1$ then $E_{S\lambda}^m\propto\Phi^{-1}-1\propto\int\rho(\rho-1)F'(\rho)dv$ and thus $E_{S\lambda}^m\propto F'(0)$.

We note that $\lambda_m(r)$ is independent of u_τ and that any dependence on u_τ can occur only through the gradient and Coulomb terms and, furthermore, predominantly through the latter. The gradient terms, as we will show, make only a small contribution.

We also note that if $F(\rho)\leqslant 1$ for $\rho\leqslant\rho_0$, and since $A^{-1}\int\rho dv=1$, then $\Phi=A^{-1}\int\rho F^{-1}(\rho)dv\geqslant 1$ and we have $E_{S\lambda}^m\leqslant 0$. This implies the same (negative) relative sign to the volume-symmetry energy as for the ordinary surface energy

relative to the volume energy. This condition, although very reasonable, and satisfied in all our calculations, is not required.

We now return to the full problem for λ including the gradient and Coulomb terms $E_{G\lambda}[\rho,\lambda]+E_{C\lambda}[\rho,\lambda]$. We put $\lambda=\lambda_m+\delta\lambda$, subject to the constraint of Eq. (29), and use a trial function for $\delta\lambda\equiv\lambda_{\rm var}$. The total (surface-)symmetry energy is now

$$E_{S\lambda}[\rho,\lambda]=E_\lambda-u_\tau\lambda_0^2A$$
$$=E_{S\lambda}^m+u_\tau\int(\delta\lambda)^2\rho F(\rho)dv+E_{G\lambda}+E_{C\lambda}. \tag{32}$$

Clearly, if the Coulomb and gradient terms are neglected then the minimum $E_{S\lambda}^m$ of $E_{S\lambda}$ (equivalent to the minimum of E_λ) is obtained for $\delta\lambda\equiv0$. The constraint of Eq. (29) requires that we must choose $\delta\lambda\equiv\lambda_{\rm var}$ such that the ρ-weighted average of $\delta\lambda$ is zero. The functional form of $\lambda_{\rm var}$ must reflect this by having both negative and positive parts. We use the following form:

$$\delta\lambda\equiv\lambda_{\rm var}=\Lambda(r^m-\Re^m)G(r),$$
$$G(r)=(1+ar+br^2)\{1+\exp[(r-R)/w]\}^{-1}, \tag{33}$$

with

$$\Re^m=\int r^m\rho G(r)dv/\int\rho G(r)dv \tag{34}$$

to satisfy Eq. (29). $\delta\lambda$ then depends on six variational parameters: Λ, m, a, b, R, and w; in particular Λ determines the overall magnitude of $\delta\lambda$.

The Coulomb energy $E_{C\lambda}$ gives a decrease in $\delta\lambda$, and $E_{C\lambda}$ alone gives a more extended proton than neutron distribution. We note that even for $\delta\lambda\equiv0$ both $E_{C\lambda}[\rho,\lambda_m]$ and $E_{G\lambda}[\rho,\lambda_m]$ are nonzero unless $\lambda_m\equiv\lambda_0$.

For the dependence on C_λ there is a linear contribution $E_{G\lambda}[\rho,\lambda_m]=C_\lambda\int[\nabla(\rho\lambda_m)]^2dv$, which is independent of $\delta\lambda$. There is a further contribution $\propto C_\lambda$ to $\delta\lambda$, which occurs through a bilinear contribution $\lambda_m\ \delta\lambda$, in $E_{C\lambda}[\rho,\lambda]$. Higher order terms in C_λ will arise both from $E_{G\lambda}$ and $E_{C\lambda}$ and also from the term $u_\tau\int(\delta\lambda)^2\rho F(\rho)dv$. Our numerical results, discussed below, show that the dependence on C_λ is fairly small and approximately linear for a large range of values of C_λ.

The surface-symmetry coefficient $u_{S\tau}$, defined in the semiempirical mass formula by the term $u_{S\tau}\lambda_0^2A^{2/3}$, is given in terms of $E_{S\lambda}$ by

$$u_{S\tau}/u_\tau=(E_{S\lambda}/u_\tau\lambda_0^2)A^{-2/3}. \tag{35}$$

$u_{S\tau}$ is the analog of the surface-energy coefficient u_S of the surface energy $u_SA^{2/3}$; empirically $u_S/u_V\approx-1.1$. Simple models of the surface show that (neglecting Coulomb and gradient terms) $u_{S\tau}/u_\tau=-O(t/r_0)=-O(1)$, where $t\cong2$ fm is the surface thickness and $r_0\cong1.1$ fm is the usual radius constant.

IV. THE DENSITY DEPENDENCE $F(\rho)$

It is convenient to use the nondimensional variable $x=\rho/\rho_0$ instead of ρ. Then $F'(x)=\rho_0F'(\rho)=O(1)$ and $F(\rho_0)=1\rightarrow F(x=1)=1$. If $\varepsilon_\lambda(\rho,\lambda)$ for small ρ arises entirely from the Fermi kinetic energy then $F(\rho)\cong\alpha x^{2/3}$ with $\alpha\cong0.2$. Since $\lambda_m\propto F^{-1}(\rho)$ the results are sensitive to small ρ, and thus to large r, and we use a cutoff $x_{\rm cut}$. This is desirable anyway, since the local density approximation will not be valid for large r where quantum penetrability effects (wave function tails) will become important. The dependence on $x_{\rm cut}$ is discussed further below. The condition $0\leqslant F(x)\leqslant1$ is satisfied for most of the $F(x)$ we use; in particular, there are no negative values.

We have mostly used two functional forms which depend explicitly on the first derivative F' since we expect that this will largely determine the variation of $F(x)$; we have already discussed this for $\Phi-1\ll1$. More generally, because of the constraints on $F(x)$, this turns out to be only partially true. The functional form we have mostly used is

$$F(x)=F(x_C)+(x-x_C)F'(x_C)+\gamma(x-x_C)^3$$

with

$$F''(x_C)=0. \tag{36}$$

The latter condition together with $F(1)=1$ then determines $F(x_C)$ and γ in terms of the two parameters x_C and $F'(x_C)$. We have considered mostly the values $x_C=0.5$ and $x_C=1$. We also require

$$F(x)=\alpha x^{2/3}$$

if

$$F(x)\leqslant\alpha x^{2/3}. \tag{37}$$

This determines the small ρ behavior and implies $F(x=0)=0$. $F(x)$ then depends on four parameters: x_C, $F'(x_C)$, α, and $x_{\rm cut}$. We have also considered the linear functional form:

$$F(x)=1+(x-1)F'(1). \tag{38}$$

This depends on the derivative at ρ_0 and, together with Eq. (37), depends on the three parameters $F'(1)$, α, and $x_{\rm cut}$ for $F'(1)\geqslant1$; however, for $F'(1)\leqslant1$ the condition $F(x=0)=0$ is not satisfied, there is no dependence on α, and $x\leqslant x_{\rm cut}$ effectively replaces $F(x=0)=0$. We note that $F(x)=x$ if $F'(x_C)=1$ ($\gamma=0$) in Eq. (36); this is equivalent to Eq. (38) with $F'(1)=1$. However, for the same derivative at ρ_0, i.e., the same $F'(1)$ ($\neq1$), Eqs. (36) and (38) give different $F(x)$. In particular, Eq. (36), especially for smaller $F'(1)<0.5$, gives substantially smaller $F(x)$ than Eq. (38) because of the (negative) cubic term in Eq. (36) and because then $F(0)\geqslant0$ for Eq. (38). Some $F(x)$ are shown in Fig. 4 below and are discussed in Sec. V. Instead of $F'(1)$, other authors, e.g., see Ref. [8], use $p_0=u_\tau\rho_0F'(1)$ for the derivative at ρ_0 which then depends on u_τ.

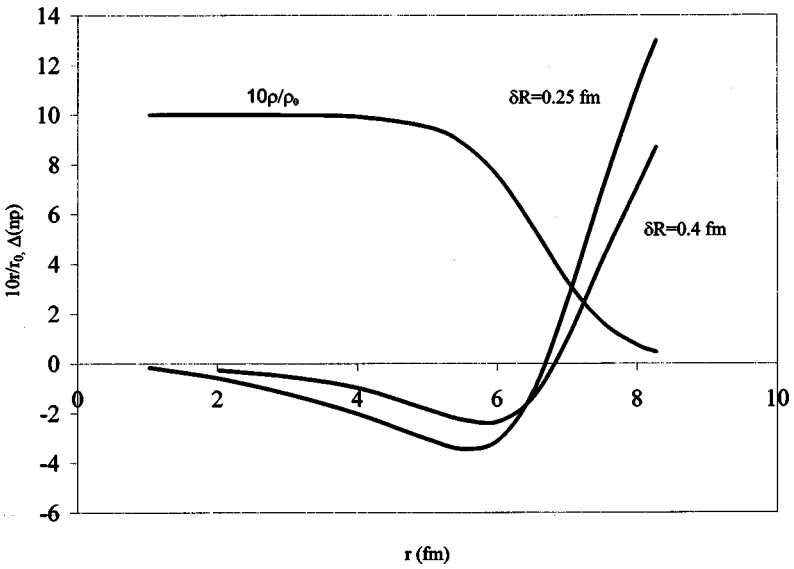

FIG. 1. Change $\Delta(n\text{-}p)$ between the number of neutrons and protons in a shell of radius r and thickness 1 fm due to the difference $\lambda-\lambda_0$. Results are for two $F(x)$ corresponding to $\Phi=1.85$, $\delta R=0.40$ fm, and to $\Phi=1.40$, $\delta R=0.25$ fm. Also shown is $10\rho/\rho_0$.

V. RESULTS

All our numerical results are for ^{208}Pb, for which $\lambda_0=0.2115$. We use a standard empirical fit [15] to the electron scattering data:

$$\rho=a[1+\exp(r-c)/w]^{-1};\quad c=6.62\ \text{fm},$$

$$w=0.546\ \text{fm},\quad a=0.1604, \tag{39}$$

with a determined from Eq. (2). We neglect the small difference between ρ and ρ_P; this is unimportant for our purposes since our focus is on the determination of λ for a given ρ. Some further related discussion is given in Ref. [8]. We have checked numerically that without gradient and Coulomb terms we indeed obtain $\lambda=\lambda_m\equiv\lambda_0$ for $F\equiv1$, whereas for $F\neq1$ we obtain $\delta\lambda=0$, i.e., $\lambda=\lambda_m$, in agreement with λ_m giving the minimum. For $F\equiv1$ but with the Coulomb and gradient terms and with $C_\lambda=30$ MeV, we obtain $\delta R=-0.058$ ($u_\tau=35$) and -0.082 fm ($u_\tau=25$ MeV). The negative values reflect the dominance of the Coulomb interaction, which gives a more extended proton than neutron distribution.

Figure 1 shows the difference $\Delta(n-p)$, resulting from $\lambda-\lambda_0$, between the number of neutrons and protons in a shell of radius r and of nominal thickness 1 fm. Results are shown for two $F(x)$ corresponding to rather different values of δR. The average (over r) is $\langle\Delta(n-p)\rangle=0$. The contribution from λ_0 is omitted since it corresponds to the same neutron and proton distributions: thus $\Delta(n-p)\equiv0$ for $\lambda\equiv\lambda_0$. It is noteworthy that the neutrons are pushed out quite far beyond the protons, even for the smaller $\delta R=0.25$ fm, justifying the usage ''neutron skin.'' Wave function tails would enhance this effect.

The dependence on the coefficient C_λ of the gradient term is shown in Table I for $u_\tau=35$ MeV and for two $F(x)$ with different values of Φ. The dependence of both δR and of the surface-symmetry energy (per nucleon) $E_{S\lambda}/A$ on C_λ is linear to a rather good approximation except for quite negative values of C_λ, for which the positivity of the energy is not assured; the gradient term $E_{G\lambda}$ on its own is less linear with C_λ. In the absence of Coulomb forces the dependence on C_λ is smaller and more strictly linear. Importantly, for a given $F(x)$, the differences in δR for a wide range of C_λ are quite small, $\leqslant10\%$, and comparable to or less than other uncertainties discussed below. This small dependence on C_λ—and thus on the gradient term—has the important implication that δR depends predominantly only on a nuclear-matter quantity involving $F(x)$. The small dependence is consistent with the ''naïve'' dimensional analysis of effective RMFT [8].

For the cutoff we consider the values $0.01\leqslant x_{\text{cut}}\leqslant0.1$, the corresponding number of nucleons outside x_{cut} being 1.1

TABLE I. Dependence on C_λ (MeV) for $u_\tau=35$ MeV, $x_{\text{cut}}=0.05$. $\Delta(E_{S\lambda}/A)$ and $\Delta(\delta R)$ denote the changes of $E_{S\lambda}/A$ and δR, respectively, with respect to their values for $C_\lambda=0$. Results are for $F'=1$, $x_C=0.5$, $\alpha=0.2$, and $\Phi=1.847$. δR are in fm; E in MeV.

C_λ	$E_{G\lambda}$	$E_{S\lambda}/A$	δR	$\Delta(E_{S\lambda}/A)$	$\Delta(\delta R)$
0	0	−0.5661	0.392	0	0
15	0.844	−0.5620	0.397	0.0041	0.005
−15	−0.883	−0.5703	0.386	−0.0042	−0.006
30	1.658	−0.5579	0.401	0.0082	0.009
−30	−2.174	−0.5746	0.380	−0.0085	−0.012
60	3.213	−0.5500	0.408	0.016	0.016
−60	−2.208	−0.5837	0.367	−0.0176	−0.025

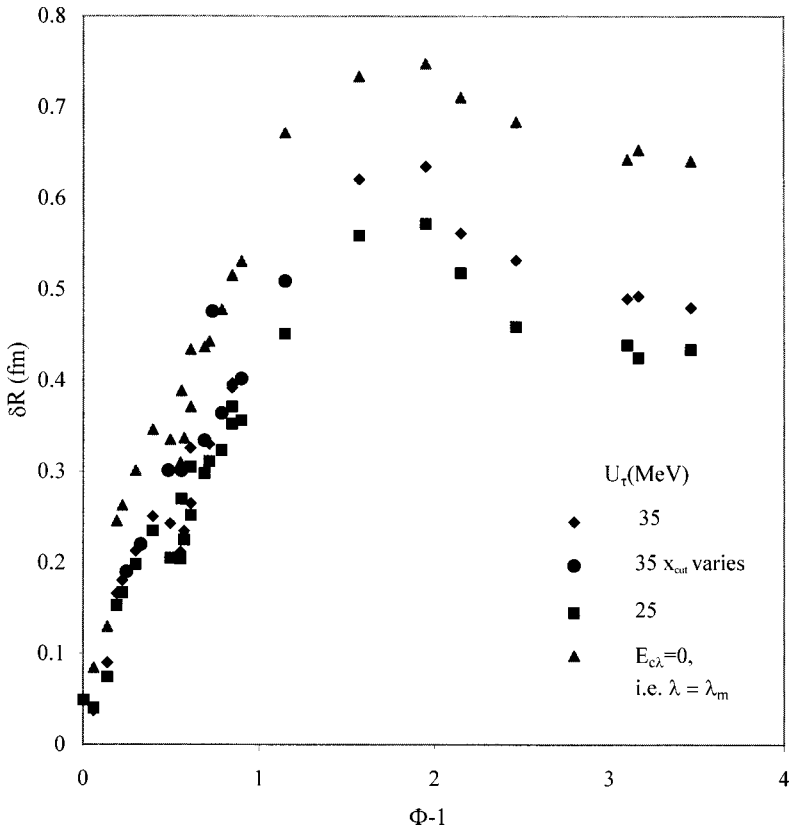

FIG. 2. Difference δR between the neutron and proton rms radii vs $\Phi-1$ for the $F(x)$ of Eqs. (36) and (37) for a large number of parameters and for the indicated values of u_τ. The full circles correspond to different $x_{\rm cut}$ all with $u_\tau=35$ MeV. The values for $E_{C\lambda}=0$ are also those for $\lambda\equiv\lambda_m$ as well as those for $u_\tau=\infty$ but with the Coulomb contribution. See the text for further discussion.

$\leqslant A_{\rm out}\leqslant 9.4$. Results for the two forms of $F(x)$ for this range of $x_{\rm cut}$ are shown in Figs. 2 and 3 as full circles. If we plot our results vs $\Phi-1$, where Φ is the integral given by Eq. (26), then the results for δR and $u_{S\tau}/u_\tau$ vs $\Phi-1$ for different $x_{\rm cut}$ are very well consistent with the general pattern discussed below. Thus Figs. 2 and 3 show that δR and $u_{S\tau}/u_\tau$, for a large number of parameters for the $F(x)$ of Eqs. (36) and (38), are approximately determined by just $\Phi-1$; the dependence on $x_{\rm cut}$ is just a special case of this general dependence. Most of our results are then for $x_{\rm cut}=0.05$ with $A_{\rm out}\cong 4$, which seems reasonable.

The following discussion of the choice of parameters of $F(x)$ is illustrated by Fig. 4. The curves a–e are for progressively increasing Φ (and δR), corresponding to increasingly overall differences from $F(x)\equiv 1$, for which $\Phi=1$. For F' we consider $0\leqslant F'\leqslant 3.5$ for both Eqs. (36) and (38); this covers all plausible values. For the parameter x_C we consider effectively the whole range from 0 to 1 for Eq. (36); for Eq.

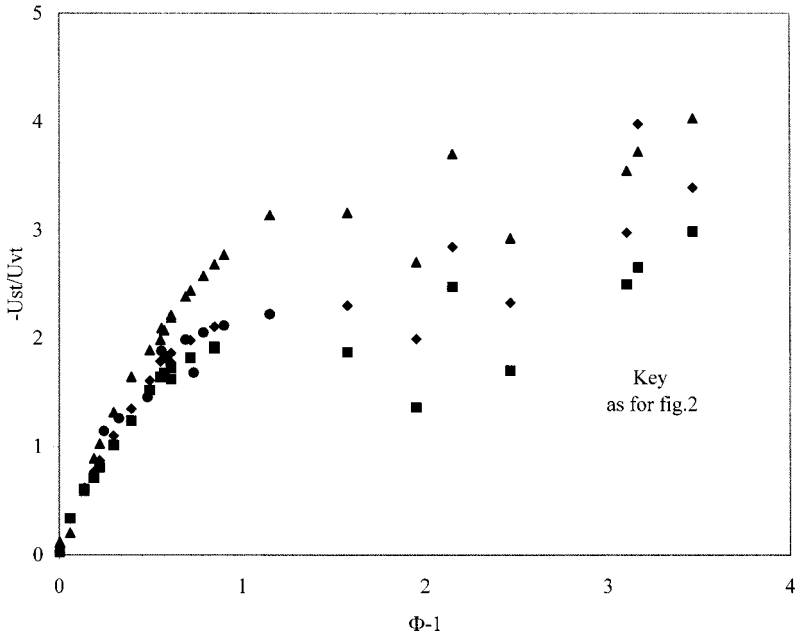

FIG. 3. The ratio $-u_{S\tau}/u_\tau$ of the surface-symmetry-energy coefficient to the volume coefficient vs $\Phi-1$. See the caption for Fig. 2.

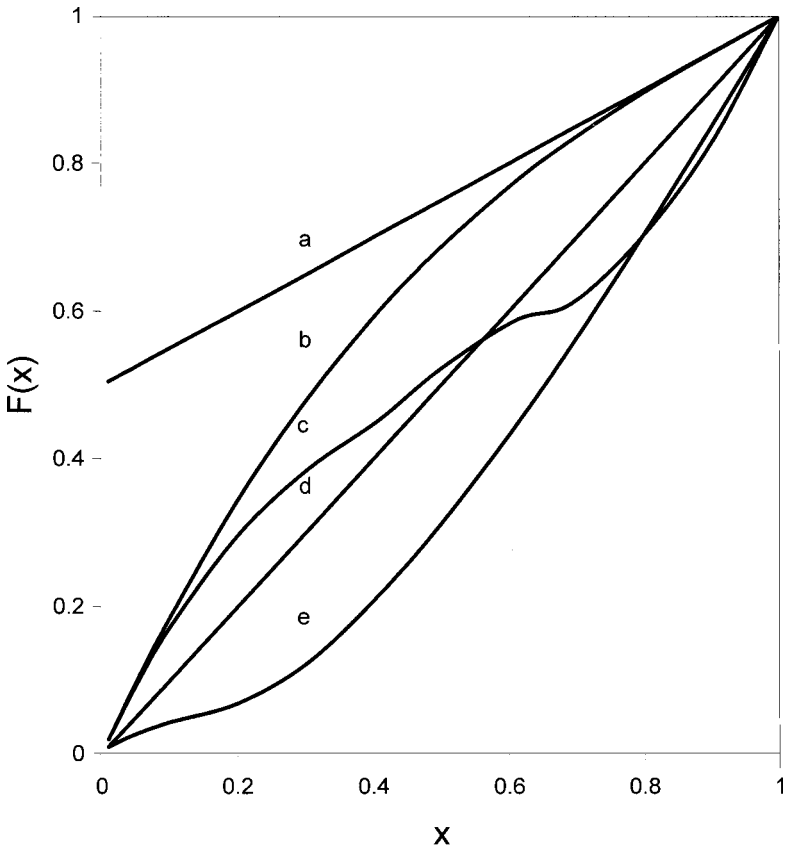

FIG. 4. Density dependence $F(x)$ vs x. Curve a is for Eq. (38) with $F'=0.5$, $\Phi=1.137$, $\delta R=0.08$ fm. Curves b–e are for Eq. (36); for b: $F'=0.5$, $x_C=1$, $\Phi=1.395$, $\delta R=0.25$ fm; for c: $F'=0.5$, $x_C=0.5$, $\Phi=1.611$, $\delta R=0.27$ fm, for d: $F'=1$, all x_C, $\Phi=1.847$, $\delta R=0.40$ fm; for e: $F'=1.5$, $x_C=1$, $\Phi=3.152$, $\delta R=0.56$ fm. Values are for $u_\tau=35$ MeV.

(38), $x_C\equiv 1$. The end point conditions $F(x=1)=1$, $F(x=0)=0$ [$\geqslant 0$ for Eq. (38)] play an important role. For $F'=1$ (curve d in Fig. 4) $F(x)=x$ (except for small x) and is independent of x_C. The case $x_C=1$ ($\rho=\rho_0$) has already been touched upon, and is considered further in Sec. VI where the dependence on p_0 (equivalently, on F') is shown. This shows no universality, especially for small F' when the nonlinear terms in Eq. (36) are particularly important and when $F(0)\geqslant 0$ for Eq. (38). This is in striking contrast to the dependence on Φ. For $x_C=0.5$, which corresponds to fixing F' at $\frac{1}{2}o_0$, the function $F(x)$ is forced (for small $F'\lesssim 0.5$) to be quite linear in the half-density region and thus overall to be very different from 1; Φ is then also significantly different from 1 (Fig. 4, curve c). On the other hand, also for small F' but with x_C close to 1, both $F(x)$ and Φ are much closer to 1 with correspondingly small δR (Fig. 4, curves a and b). In Fig. 2 the latter values are those in the lower left hand corner with both small $\Phi-1$ and small δR, whereas the former are those with $\Phi-1\approx 0.5-0.7$ and $\delta R\approx 0.2-0.25$ fm; these points are mostly those that hang down to the right in this region. For large $F'\gtrsim 1.5$ (Fig. 4, curve e), and correspondingly large Φ and δR, the difference between Eqs. (36) and (38) is relatively small. For α, which mostly determines the low density dependence, we have mostly used $\alpha=0.2$. However, we have considered values from 0.01 to 0.5. For $\Phi-1\lesssim 1$, especially smaller $\Phi-1$, there is little dependence on α; for $\Phi-1>1$ the results are again consistent with the general dependence on Φ.

As already noted, λ and $u_{S\tau}/u_\tau$ depend on u_τ only through the Coulomb term $E_{C\lambda}$ (neglecting the small dependence on C_λ). For $E_{C\lambda}=0$ we have $\lambda\equiv\lambda_m$, which is independent of u_τ. This is then equivalent to $u_\tau=\infty$ if $E_{C\lambda}$ is included. Thus the difference $\lambda-\lambda_m$ is the result of competition between u_τ and the Coulomb energy $E_{C\lambda}$. Reducing u_τ is equivalent to increasing the relative contribution of $E_{C\lambda}$, and will reduce δR and $u_{S\tau}/u_\tau$ from their maximum values attained for $E_{C\lambda}=0$ appropriate to λ_m. This is because the Coulomb repulsion favors a more extended proton distribution. The dependence on u_τ is depicted in Fig. 5 for $\Phi=1.494$, $C_\lambda=30$ MeV [$F'(1)=1,\alpha=0.2,x_{\rm cut}=0.05$]. The values for $E_{C\lambda}=0$ are nicely consistent with those for $u_\tau=\infty$. The results shown in Fig. 2 for δR and for $u_{S\tau}/u_\tau$ in Fig. 3 also demonstrate this dependence. These results, for a large number of $F(x)$, are for $u_\tau=25$ and 35 MeV, and $E_{C\lambda}=0$ (i.e., $u_\tau=\infty$). The dependence of δR on u_τ for a given $\Phi-1$ is consistent with a simple scaling:

$$\delta R(u_\tau=\infty)-\delta R(u_\tau)=[\,\delta R(u_\tau=\infty)-\delta R(u'_\tau)](u'_\tau)/u_\tau), \tag{40}$$

which allows estimates for empirically reasonable values of u_τ from those for $E_{C\lambda}=0$ and either $u'_\tau=25$ or 35 MeV.

We emphasize that for a particular $F(\rho)$ we vary only u_τ. Our results for the dependence of δR on u_τ show a similar trend as the results of RMFT when just the ρ-meson coupling, which determines u_τ, is varied [8]. Both sets of results, as expected, are consistent with small values of δR when extrapolated to $u_\tau=0$. Our results show a somewhat stronger dependence on u_τ than the RMFT results; this could

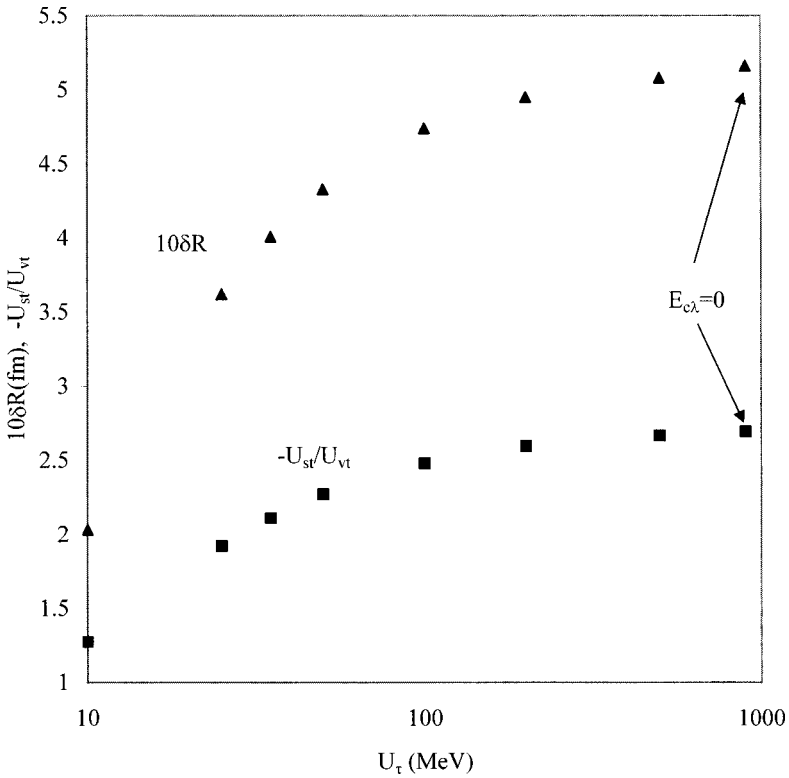

FIG. 5. Dependence of δR and $-u_{S\tau}/u_\tau$ on u_τ for $\Phi=1.494$. See the caption for Fig. 2.

be because for the latter the derivative is also changing. The results for a wide range of RMFT, which are shown in Fig. 7 of Ref. [8], show a much larger slope ($\cong$0.018 fm/MeV), which extrapolates to unreasonably large negative values of $\delta R\cong -0.43$ fm for $u_\tau=0$. This is presumably because other parameters in addition to u_τ are varied.

As discussed, the dependence of both δR and $u_{S\tau}/u_\tau$ on $\Phi-1$ is roughly universal for a given u_τ. Thus if $\delta R\lesssim 0.4-0.5$ fm, a given (experimental) value of δR will determine Φ. However if δR is larger than about 0.45 fm, then δR is not single valued, and Φ cannot then be uniquely determined from δR. In fact, evidence indicates that δR is very likely to be smaller than 0.4 fm. Our results show that there is an upper limit to δR, absolutely, of $\cong$0.75 fm attained for $E_{C\lambda}=0$, and of $\cong$0.65 fm for more realistic $u_\tau \cong 25-35$ MeV.

Comparison of Figs. 2 and 3 shows that $-u_{S\tau}/u_\tau$ vs $\Phi-1$ follows a similar pattern to that for δR, in particular, for $\Phi-1\leqslant 1$. The associated correlation between $-u_{S\tau}/u_\tau$ and δR is shown in Fig. 6. This shows that for $\delta R\lesssim 0.4$ fm the value of $-u_{S\tau}/u_\tau$ is reasonably well determined by δR, and will thus provide important input for the semiempirical mass formula, and should result in a better determination of u_τ. If indeed $\delta R\lesssim 0.4$ fm then it follows that $-u_{S\tau}/u_\tau\lesssim 2$.

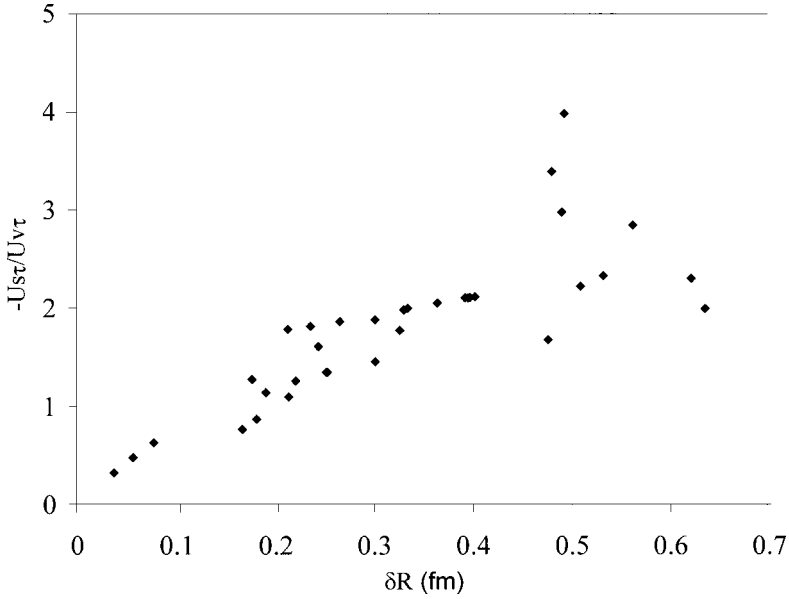

FIG. 6. Relation between $-u_{S\tau}/u_\tau$ and δR for the results shown in Figs. 2 and 3.

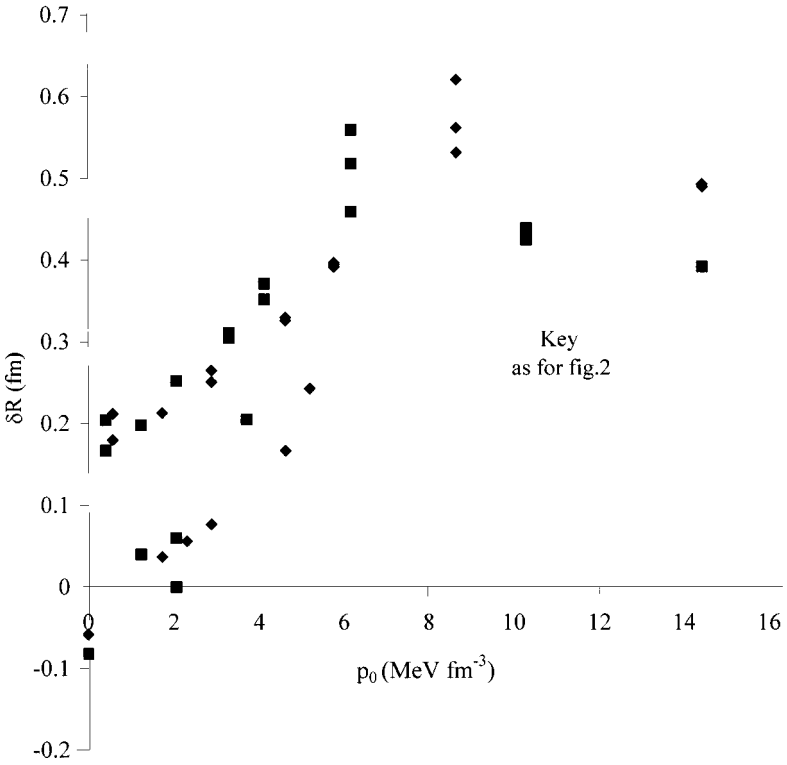

FIG. 7. Difference δR between the neutron and proton rms radii vs the derivative p_0. See the text for details.

VI. DISCUSSION

We have demonstrated the central role that $F(\rho)$ plays for the determination of $\lambda=(\rho_N-\rho_P)/\rho$, and therefore of δR and $u_{S\tau}/u_\tau$; in particular, the exact solution λ_m obtained in the absence of Coulomb and gradient terms exhibits this explicitly. We expect this dependence to persist also in other approaches. For the large set of $F(\rho)$ that we have considered, this dependence occurs, approximately, only through the integral Φ of Eq. (26). It seems plausible that this dependence is quite general, but this needs confirmation. In our approach Φ is not uniquely determined by $F(\rho)$, in particular, not by its derivative. This was discussed in Sec. V and is also demonstrated by the following discussion.

In a comparison with other calculations we show in Fig. 7 our results for δR vs $p_0=u_\tau\rho_0F'(1)$, instead of vs $F'(1)$, where p_0 is the derivative (at ρ_0) used by other authors [8]. For $p_0\lesssim 6$ MeV fm^{-3} there are two distinct branches for $F(x)$, corresponding to Eq. (36) for the upper branch and Eq. (38) for the lower. In Fig. 2 on a plot vs $\Phi-1$, these branches are barely distinguished. The upper branch in Fig. 7 is more linear with p_0 with a slope $\cong 0.04$ as compared to $\cong 0.043$ MeV fm^{-3} for a wide variety of mean-field models (Fig. 11 in Ref. [8]), which have a somewhat smaller δR. The lower branch has a slope $\cong 0.033$ MeV fm^{-3} and considerably smaller δR. As previously discussed, $F(x)$ for Eq. (36) has a nonlinear term, which, especially for smaller $F'(1)<0.5$, reduces $F(x)$ and increases Φ, as compared to using Eq. (38) for which $F(0)\geqslant 0$ [for $F'(1)\leqslant 0$], and which should correspond more closely to the results in Ref. [8]. For $F'(1)=1$ the results for both Eqs. (36) and (38) are identical and for $F'(1)>1$ the corresponding values, in particular, of Φ, are quite similar. Since Eq. (38) for small F' does not satisfy $F(0)=0$ this may indicate that the upper branch may be more realistic. In any case in plots of $F(x)$ vs Φ the two branches are hardly distinguishable.

Our approach shows the critical importance of the interplay between u_τ and the Coulomb energy. For $E_{C\lambda}=0$, and also neglecting the small gradient contribution, the solution $\lambda_m(r)$, which is exact, is given in terms of $F(\rho)$ by Eq. (25) and is independent of u_τ. If the Coulomb energy $E_{C\lambda}$, due to the difference of neutron and proton distributions, is included, then λ will decrease (from λ_m) and so will δR. Thus in the limit $u_\tau=\infty$ the difference δR attains its maximum, corresponding to $E_{C\lambda}=0$, $\lambda\equiv\lambda_m$.

The coupling between ρ and λ will give some additional decrease of energy. This is small because of the stiffness of the equation of state. Limited calculations we have made, for the simultaneous minimization with respect to ρ and λ, support this. Thus with only $\varepsilon_V(\rho)$ and a rather schematic equation of state, with an incompressibility constant $K\cong 200$ MeV, we obtain for the total energy per particle $E/A=-7.962$ MeV. Including the symmetry-energy contribution, but without coupling between the problems for ρ and λ, and for $u_\tau=25$ MeV and $\Phi\cong 2.1$, gives $E/A=-8.382$ MeV. This is changed by $\leqslant 0.005$ MeV by the coupling, and $\delta R=0.27$ fm is changed by $\leqslant 0.01$ fm. Our conclusion is consistent with that of Ref. [11]. The small effect on δR is also consistent with results for δR vs K, extracted from numerous calculations in Ref. [8]; these seem consistent with a random scatter and show no significant dependence on K. For the A dependence of δR for constant Z or N, i.e., for isotope and isotone shifts, which intrinsically depend on the ρ-λ coupling or its equivalent in other approaches, realistic calculations need to be more precise. In

fact the dependence of the isotope shift on K through the Coulomb energy was already proposed by Wilets, Hill, and Ford [16], and through the (normal) nuclear-matter density dependence of the symmetry energy by Bodmer [16].

We briefly mention the implications for neutron stars. The pressure of neutron matter ($\lambda=1$) with energy density $\varepsilon(\rho)=\varepsilon_V(\rho)+u_\tau\rho F(\rho)$ [Eq. (11) without gradient and Coulomb terms] is $P(\rho)=P_0(\rho)+u_\tau\rho^2F'(\rho)$, where $P_0(\rho)$ is the pressure of symmetric nuclear matter. The additional pressure is thus proportional to $F'(\rho)$, making a direct connection between the neutron-matter equation of state and δR. This connection and the implications for neutron stars have been studied in Refs. [3–5], [10]. A larger δR implies a larger Φ and hence a larger $F'(\rho)$, which in turn results in a harder equation of state and therefore a larger neutron-star radius. The accuracy of the connection to δR assumes that higher powers of λ^2 in $\varepsilon_\lambda(\rho,\lambda)$ are small. This needs investigation, not only for $\lambda=1$, but also for λ relevant to nuclei well away from the stability line.

The magnitude of the symmetry-energy term u_τ plays a central role in all the above considerations. A redetermination of u_τ that includes the surface-symmetry term would involve a refit of the mass formula, which could make use of the correlation between $u_{S\tau}/u_\tau$ and δR where the experimental value of δR would be an input.

Our Fermi-Thomas version of the local density approximation for the neutron-proton difference should have some validity because of the small effect of the gradient terms. The use of fairly general density functionals avoids specifying some particular set of interactions. More detailed connections need to be made, but we do not pursue these here. Our approach, even with small gradient terms, is of course limited by the absence of shell and pairing effects, as well as wave function tails. These limitations can become especially severe for nuclei far from the stability line. Other approaches such as the RMFT approach will not suffer from some or most of these limitations. Nevertheless, our approach clearly demonstrates the general features of neutron-proton distributions, which are less evident in other approaches.

ACKNOWLEDGMENTS

We would like to thank C. H. Horowitz, S. W. Schramm, and especially R. J. Furnstahl for helpful discussions and comments.

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