Scattering lengths for p-^{3}He elastic scattering from an effective-range phase shift analysis

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We have extended an earlier phase-shift analysis of p-^{3}He scattering data by including additional low-energy measurements in the dataset. The p-^{3}He zero-energy scattering lengths obtained from this analysis depend sensitively on a group of cross section and analyzing power measurements at proton energies below 1 MeV. When this group of measurements is included in the dataset, two possible phase-shift solutions are found. The first solution yields a singlet scattering length of $a_s = 15.1$ fm and a triplet scattering length of $a_t = 7.9$ fm, while the second solution yields values of $a_s = 7.2$ fm and $a_t = 10.4$ fm. Without this group of measurements, the best-fit scattering lengths are $a_s = 10.3$ fm and $a_t = 8.2$ fm. We believe that the scattering lengths are currently not well determined experimentally and that additional low-energy p-^{3}He data are needed in order to clarify the situation.

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In the last few years, considerable progress has been made in performing accurate quantum-mechanical calculations of observables in four-nucleon ($4N$) systems. As a result, $4N$ systems have become increasingly important as testing grounds for models of the nuclear force. Among the observables of fundamental interest in $4N$ systems are the $N+3N$ zero-energy scattering lengths. Precise experimental values of these scattering lengths are useful not only for testing theoretical models and methods, but also as input for calculations of quantities such as cross sections for weak proton capture on ^{3}He [1,2].

Carbonell has recently summarized some theoretical and experimental results for $A=4$ scattering [3]. As Carbonell points out in Ref. [3], theoretical values calculated by different groups for the singlet (a_s) and triplet (a_t) p-^{3}He scattering lengths are more widely scattered than in the n-^{3}H case. In the top half of Table I, we list the results of three representative calculations of p-^{3}He scattering lengths. The Pisa group has used the Kohn variational principle and the correlated hyperspherical harmonics technique (CHH) with the AV18 and Urbana IX potentials to obtain values for the scattering lengths for n-^{3}H and p-^{3}He zero-energy scattering [4]. Filikhin and Yakovlev have calculated scattering lengths for p-^{3}He by using the cluster-reduction method (CRM) to solve the differential Yakubovsky equations in the s-wave approximation with the Malfliet-Tjon (MT) I–III NN potential, and then parametrizing the resulting phase shifts with an eighth-order polynomial expansion in scattering energy [5]. Carlson *et al.* have determined the p-^{3}He triplet scattering length by means of a variational Monte Carlo method using the AV14 and Urbana VII potentials [2]. We note that the calculated n-^{3}H scattering lengths have been found to scale with the ^{3}H binding energy [4]. Therefore, for a meaningful comparison with experiment, the method and potentials used to calculate scattering lengths should also produce a $3N$ binding energy that is consistent with the experimental value. This is the case for all the theoretical results shown in Table I. Still, there is a difference of up to 30% between scattering lengths obtained using different methods and potentials.

Experimentally, Tegnér and Bargholtz [1] have obtained a value for a_t of 10.2 ± 1.4 fm by fitting an effective-range expansion to the $^3S_1$ phase shifts of Berg [6] over the range from 0 to 1 MeV, and of Tombrello [7] over the range from 1 to 11.3 MeV. More recently, Alley and Knutson [8,9] carried out a modified effective-range phase-shift analysis of a large set of p-^{3}He data from 0 to 12 MeV. Based on this analysis, values of $a_s = 10.8 \pm 2.6$ fm and $a_t = 8.1 \pm 0.5$ fm were obtained [3,4]. These two sets of experimental values are given in the first two rows of the bottom half of Table I. The errors are statistical only. Even without systematic errors taken into account, it is clear that the precision of the experimental values is not high.

In order to clarify the experimental situation, we have reexamined the analysis of Ref. [8]. In particular, we have included additional low-energy measurements in the dataset. We have also carried out fits with different sets of parameters and slightly different datasets in order to investigate possible systematic errors in the scattering lengths.

The original dataset used in Ref. [8] consisted of measurements of the differential cross section, proton and ^{3}He analyzing powers, and spin correlation coefficients for p-^{3}He elastic scattering between $E_p = 1.01$ and 12.79 MeV. A total of 1085 data points were included (a detailed list can be found in Ref. [8]). Since the publication of Ref. [8], additional proton analyzing power measurements at $E_p = 1.60$ and 2.25 MeV have been reported [10]. Although we find that these new measurements do not affect the phase-shift fits significantly, we have added them to the dataset for completeness. Additionally, there exists a group of measurements of cross sections and proton analyzing powers at very low energies (from 0.1 to 1.0 MeV) [6], which was not included in the original dataset of Ref. [8]. Adding the Ref. [6] data to the dataset reduces the lowest energy represented in the

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TABLE I. Theoretical and experimental values for singlet (a_s) and triplet (a_t) scattering lengths for p-^{3}He elastic scattering.

		Theory	
a_s (fm)	a_t (fm)	method (force model)	Ref.
11.5	9.13	CHH (AV18+UrIX)	[4]
8.2	7.7	CRM (MTI−III)	[5]
	10.1±0.5	VMC (AV14+UrVII)	[2]
		Experiment	
a_s (fm)	a_t (fm)	dataset	Ref.
	10.2±1.4	Refs. [6]+ [7]	[1]
10.8±2.6	8.1±0.5	Ref. [8]	[3,4,8]
15.1±0.8	7.9±0.2	Refs. [8]+ [10]+ [6], solution 1	This work
7.2±0.8	10.4±0.4	Refs. [8]+ [10]+ [6], solution 2	This work

dataset from 1.01 to 0.1 MeV. As we will discuss below, we find that these low-energy data from Ref. [6] have a significant effect on the phase-shift fits and the scattering lengths.

The formalism we use in performing the phase-shift fits is that of Ref. [8]. We give a brief description here; the reader is referred to Refs. [8,9] for more details. In order to perform a global fit of the measurements over the range of energies from 0 to 12 MeV, we use a modified effective-range expansion to parametrize the phase shifts δ^j_{ls}, and a power series expansion in k^2 for the mixing parameters $\epsilon(j^\pi)$. Three terms in each expansion were used in the fit. We ignore the effects of inelastic scattering, since the first inelastic channel ($p+{}^3\mathrm{He}\rightarrow d+2p$) does not open up until $E_p=7.3$ MeV, and all possible breakup reactions below 12 MeV have very low cross sections. Consequently, all the phase shifts are taken to be real. The S matrix is parametrized according to the Blatt-Beidenharn convention [11]. The phase-shift analysis code uses MINUIT [12] to perform the least-squares minimization.

The dataset is divided into groups, each group consisting of measurements that are thought to have a common normalization. The group normalizations are not, strictly speaking, variable parameters in the fit, but are calculated so as to minimize χ^2 each time after the phase-shift parameters are adjusted by the fitting program. There were a few groups of data for which an uncertainty in the normalization was not given in the original reference. In these cases, the normalization uncertainty was estimated based on typical uncertainties for similar types of data. Fits with somewhat different estimates for these unknown normalization uncertainties did not produce significantly different results.

The analysis of Ref. [8] indicated that nine phase shifts ($^1S_0$, $^3S_1$, $^1P_1$, $^3P_0$, $^3P_1$, $^3P_2$, $^1D_2$, ^3D_j, ^3F_j) and three mixing parameters [$\epsilon(1+)$, $\epsilon(1-)$, and $\epsilon(2-)$] were necessary for obtaining a good fit to the data and that including additional parameters did not improve the fit very much, typically changing the χ^2 per degree of freedom (χ^2_ν) by only 1–2%. Applying these 36 fitting parameters (three expansion coefficients for each of 12 phase shifts and mixing parameters) to the original Ref. [8] dataset produces the results mentioned above, that is, $a_s=10.8\pm2.6$ and $a_t=8.1\pm0.5$. This fit, which we will call the Ref. [8] solution, has a χ^2 per degree of freedom of $\chi^2_\nu=0.919$. Adding the 1.60 and 2.25 MeV proton analyzing power data of Ref. [10] to the database decreases χ^2_ν only slightly, to 0.917, and does not change the scattering lengths significantly, giving $a_s=10.3\pm2.7$ fm and $a_t=8.2\pm0.6$ fm.

The addition of the very low energy Ref. [6] data to the dataset, however, changes the phase-shift fits qualitatively as well as quantitatively. An examination of the χ^2 surface obtained when the Ref. [6] data are included indicates that there are two possible solutions. Solution 1 (with $\chi^2_\nu=0.958$ for 1233 degrees of freedom) yields scattering lengths $a_s=15.1\pm0.8$ fm and $a_t=7.9\pm0.2$ fm; solution 2 (with $\chi^2_\nu=0.973$) yields scattering lengths $a_s=7.2\pm0.8$ fm and $a_t=10.4\pm0.4$ fm. The errors quoted for these scattering lengths are purely statistical and are obtained by determining the change in the corresponding effective-range parameter that increases the total χ^2 by 1 (with all other parameters allowed to vary freely). Figure 1 shows how the χ^2 surface depends on the zeroth-order effective-range expansion coefficient for the $^1S_0$ phase shift, $b_0=-1/a_s$. The two minima at $b_0=-0.066$ and $b_0=-0.139$ correspond to solutions 1 and 2. In contrast, fitting with the Ref. [6] data omitted pro-

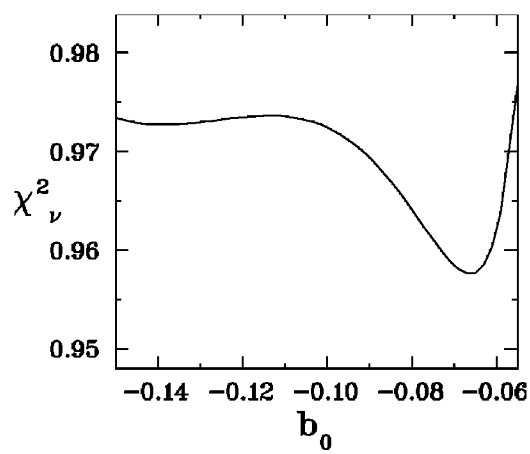

FIG. 1. Dependence of the reduced χ^2 on the zeroth-order effective-range expansion coefficient for the $^1S_0$ phase shift, $b_0=-1/a_s$, for fits to the full dataset. At each value of b_0, the other parameters have been varied to minimize χ^2.

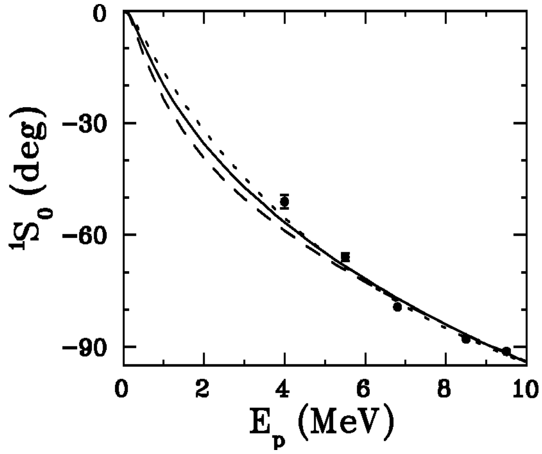

FIG. 2. Energy dependence of the best-fit $^1S_0$ phase shift for three different phase-shift solutions. The solid line corresponds to the Ref. [8] solution; the dashed (dotted) line corresponds to solution 1 (solution 2) for the full dataset including the Refs. [6,10] data. The points are single-energy phase-shift solutions from Ref. [14].

duces a dependence of χ^2 on this parameter that is parabolic, with a single minimum at $b_0 = -0.093$. We note that fits with the Refs. [6,10] data included consistently indicate that the analyzing powers of Ref. [6] are too large by 4–5%, whereas the other proton analyzing powers in the dataset with energies below 3 MeV, those of Refs. [10] and [13], are too small by about the same amount.

The main differences among these three solutions (the solution for the Ref. [8] dataset, and solutions 1 and 2 for the full dataset) are in the $^1S_0$ and $^3S_1$ phase shifts (particularly below $E_p \approx 4$ MeV) and the $\epsilon(1+)$ mixing parameter. There is also a small but statistically significant difference in the $^3P_0$ phase-shift parameters. Figure 2 shows the energy dependence of the best-fit $^1S_0$ phase-shift parameter for the three solutions. All three solutions give very similar phase shifts at $E_p \approx 5$ MeV and above; this is most likely due to the large number of spin correlation measurements between 4 and 10 MeV, and especially at 5.54 MeV, in the dataset.

Fits done with either the cross section or the proton analyzing power data of Ref. [6] omitted indicate that it is the cross section data that are responsible for the two minima in the χ^2 surface. If these analyzing power data are included in the dataset but the cross section data are omitted, a single solution is found, with scattering lengths of about $a_s = 16.5$ fm and $a_t = 7.6$ fm. On the other hand, if the cross section data are included but the analyzing power data are omitted, two solutions are again found. These two solutions produce a_s and a_t values that are not significantly different from the values produced when all data are included in the dataset.

With the data from Refs. [10] and [6] included in the dataset, we reexamined the effects of using a different set of fitting parameters. Omitting $\epsilon(2-)$ increases χ^2_ν by about 3%, and omitting either $\epsilon(1+)$ or $\epsilon(1-)$ produces an even worse fit, as does omitting $^1D_2$, ^3D_j, or ^3F_j. We therefore retained all of these parameters. We find that including the $\epsilon(2+)$ or $^1F_3$ parameters produces only a negligible change

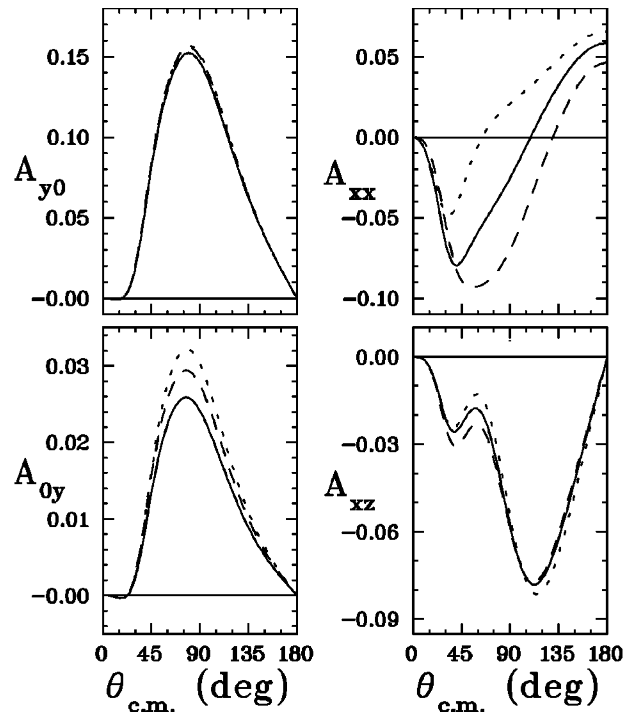

FIG. 3. Predictions of the three phase-shift solutions for several observables at a proton lab energy of 2 MeV. The solid line corresponds to the Ref. [8] solution; the dashed (dotted) line corresponds to solution 1 (solution 2) for the full dataset including the Refs. [6,10] data.

in the quality of the fit ($\approx 1\%$ change in χ^2_ν). When ^3D_j splitting is included, χ^2_ν decreases by 1–2%, and when ^3F_j splitting is included, χ^2_ν decreases by 5–6%. There is very little change in the scattering length values when any of these additional parameters are included. Because including additional parameters does not produce a large change in χ^2_ν or in the scattering lengths, the results we report here are for fits with the original Ref. [8] set of 36 parameters.

Our solutions may be compared with those of Yoshino *et al.* [14], who carried out single-energy p-^{3}He phase-shift fits at proton energies of 4.0, 5.5, 6.8, 9.5, and 19.48 MeV using the Matsuda-Watari [15] (MW) parametrization of the S matrix. At 4.0 MeV they also performed a fit using the Blatt-Beidenharn (BB) parametrization. Their set of fitting parameters included ^3D and ^3F splitting, the $^1F_3$ phase shift, and (at some energies) $\epsilon(2+)$ and $\epsilon(3-)$ mixing parameters. The Yoshino *et al.* phase shifts are generally consistent with our solutions over the 4–9.5 MeV energy range, and do not clearly favor any one of our three possible solutions. As an example, the best-fit values for $^1S_0$ from Ref. [14] are shown in Fig. 2 along with our three energy-dependent solutions. (The BB and MW representations are the same for this parameter.)

We note that the statistical errors in a_s and a_t obtained from our individual fits do not give a complete representation of the uncertainty in the values of the scattering lengths. Additional systematic error may be due to the choice of fitting parameters, the choice of energy range, the particular effective-range parametrization used, or systematic error in one group of data. We have not, however, attempted to estimate systematic errors in the scattering lengths from these sources, since these errors would most likely be smaller than the uncertainty that arises from the existence of several possible phase-shift solutions.

It is natural to ask what types of new measurements would be most effective at reducing the ambiguity in the phase-shift analysis. To investigate this question, we have carried out calculations of a variety of p-^{3}He observables for several proton energies between 1 and 4 MeV, using the phase-shift parameters of each of the three solutions. For the differential cross sections, the differences in the predictions of the three different solutions are small (about 1%) except at energies below about 3 MeV. For example, at 2 MeV, the differences are as large as 2–4% at some angles. For the proton analyzing powers (A_{y0}), the differences are rather small (0.004 or less) at energies between 1 and 4 MeV, with the largest differences at energies around 2 MeV near the peak of the angular distribution (see Fig. 3). These small differences are not very surprising, because the dataset already contains cross section and proton analyzing power data at these low energies, with uncertainties in the data of typically a few percent. On the other hand, there are few ^{3}He analyzing power points and no spin correlation coefficient data for energies below 4 MeV in the dataset, and so we might expect that the three solutions make rather different predictions for these observables at low energies. This is exactly what we find. Figure 3 shows that at $E_p=2$ MeV the maximum difference in the ^{3}He analyzing power A_{0y} among the three solutions is 0.006 (about 20%) at the peak of the angular distribution. This difference is about the same (in an absolute sense) at 4 MeV and at 1 MeV. For the spin correlation coefficients, the differences can be even larger. Predictions for A_{xx} and A_{xz} at 2 MeV are shown in Fig. 3. The differences in A_{xx} at this energy are particularly large; A_{yy} and A_{zz} also show large differences.

We conclude that the phase-shift parameters, particularly those needed to determine the scattering lengths, are sensitive to the very low energy cross section and proton analyzing power data of Ref. [6]. Unfortunately, when these data are included, the phase shift fit does not produce a unique solution, and for this reason we believe that the p-^{3}He scattering lengths are currently not well determined experimentally. Precise measurements of cross sections or analyzing powers at low energies, or measurements of spin correlation coefficients below 4 MeV, are needed in order to obtain reliable phase-shift fits and a more precise determination of the scattering lengths a_s and a_t.

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