# Λ-Λ interaction and $^{6}_{\Lambda\Lambda}$He

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A one-boson exchange (OBE) potential model, based on the Nijmegen model D potential, for the $^1S_0$, $S=-2$ interaction is analyzed with emphasis on the role of coupling between the $\Lambda\Lambda$, $N\Xi$, and $\Sigma\Sigma$ channels. Singlet scalar exchange, an approximation to two-pion exchange, is significant in all channels; surprisingly, the one-pion exchange component is almost negligible. The size of the channel coupling as a function of the overall strength of the OBE model potential is examined. Implications of the analysis for the binding energy of $^{6}_{\Lambda\Lambda}$He are considered; the new experimental datum may suggest a consistency between the extracted $\Lambda\Lambda$ matrix element and the relation implied by SU(3) among OBE baryon-baryon interactions.

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A recent $^{6}_{\Lambda\Lambda}$He binding energy measurement [1] yielding a $\Lambda\Lambda$ separation energy of

$$\Delta B_{\Lambda\Lambda}=B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He})-2B_{\Lambda}(^{5}_{\Lambda}\mathrm{He}) =1.01\pm 0.20^{+0.18}_{-0.11}\ \mathrm{MeV} \quad (1)$$

suggests that the effective $\Lambda\Lambda$ interaction is considerably weaker than that inferred from the earlier measurement (≈4.7 MeV) reported by Prowse [2]. We examine the implication of this new measurement within the framework of one-boson exchange (OBE) models that employ SU(3) symmetry to determine the baryon-baryon strangeness $S=-2$ interaction.

If one assumes flavor SU(3) is a good symmetry, then one can express the matrix elements of an OBE potential in terms of the irreducible representations of $8\otimes 8$ as

$$\langle nn|V|nn\rangle = V_{27},$$

$$\langle \Lambda N|V|\Lambda N\rangle = \frac{36}{40}\ V_{27}+\frac{4}{40}\ V_{8_s},$$

$$\langle \Lambda\Lambda|V|\Lambda\Lambda\rangle = \frac{27}{40}\ V_{27}+\frac{8}{40}\ V_{8_s}+\frac{5}{40}\ V_1. \quad (2)$$

Considering that $V_{8_s}$ and $V_1$ are repulsive while $V_{27}$ is attractive [3], we may conclude that

$$|\langle V_{nn}\rangle| > |\langle V_{\Lambda N}\rangle| > |\langle V_{\Lambda\Lambda}\rangle|. \quad (3)$$

From the three earlier measurements of $\Lambda\Lambda$ hypernuclei binding energies ($^{6}_{\Lambda\Lambda}$He [2], $^{10}_{\Lambda\Lambda}$Be [4,5], and $^{13}_{\Lambda\Lambda}$B [6–8]) which implied that the $\Lambda\Lambda$ matrix element $|\langle\Lambda\Lambda|V|\Lambda\Lambda\rangle|$ was ≈4–5 MeV, it was suggested that the breaking of SU(3) symmetry and the coupling between the $\Lambda\Lambda$, $N\Xi$, and $\Sigma\Sigma$ channels in the $^1S_0$ partial wave could bridge the gap between experiment ($|\langle V_{nn}\rangle| > |\langle V_{\Lambda\Lambda}\rangle| > |\langle V_{\Lambda N}\rangle|$) and the SU(3) expectations expressed in Eq. (3).

To examine this issue and the implications of the new experimental result, we consider the Nijmegen OBE potential model D [9]. If we require all coupling constants be determined by the SU(3) rotation of those parameters as fixed in the nucleon-nucleon (*NN*) and hyperon-nucleon (*YN*) sectors, then the only free parameters are those of the short range component of the interaction. These we vary within the constraint that the long range part of the potential be predominantly OBE in origin. This allows us to examine the $\Lambda\Lambda$ matrix element as a function of the strength of the $\Lambda\Lambda$ interaction and the importance of the coupling of the $\Lambda\Lambda$ channel to the $N\Xi$ and $\Sigma\Sigma$ channels.

To perform an SU(3) rotation on an OBE potential defined in the $S=0,-1$ sectors, one writes the Lagrangian in terms of the baryon octet coupled with the mesons which are either a singlet or a member of an octet. If the interaction is taken to be of the Yukawa type, then the interaction Lagrangian takes the form [10]

$$L_{int}=-\{g_s[B^\dagger B]_s M_s + g_{8_1}[B^\dagger B]_{8_1}M_8 + g_{8_2}[B^\dagger B]_{8_2}M_8\}, \quad (4)$$

where *B* and *M* are the baryon and meson field operators. In writing this Lagrangian, which is a scalar, the initial and final baryons are coupled to either a flavor singlet or an octet. Because there exist two irreducible octet representations, one needs a different coupling constant for each of the representations. That is, one has one coupling constant for each singlet meson $g_s$ and two coupling constants $g_{\{8_1\}}$ and $g_{\{8_2\}}$ for each meson octet. These coupling constants can then be determined by fitting the *NN* and *YN* experimental data.

The Nijmegen model D potential [9] postulates for the exchanged mesons the pseudoscalar octet $\{\pi,\eta,\eta',K\}$, the vector octet $\{\rho,\phi,\omega,K^*\}$, and a scalar meson $\{\varepsilon\}$. The masses of the mesons and baryons are taken from experiment, while the coupling constants are adjusted to fit the data in the $S=0,-1$ sectors; a hard core models the short range

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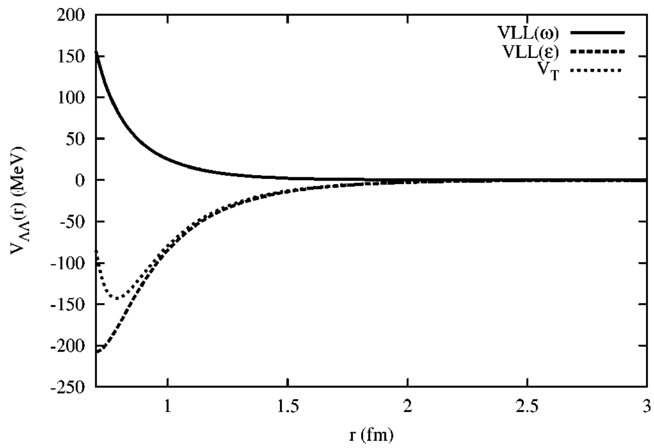


FIG. 1. The $\Lambda\Lambda$ potential in the ${}^1S_0$ channel. The solid and dashed lines indicate the contributions of the $\omega$ and $\varepsilon$ exchange, while the dotted line is the total potential. $C$ was adjusted so that the $\Lambda\Lambda$ scattering length is $a_{\Lambda\Lambda}=-1.91$ fm.

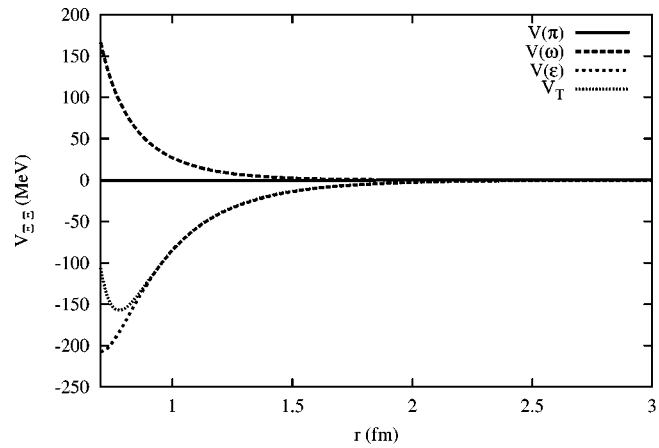


FIG. 2. The ${}^1S_0$ $N\Xi$-$N\Xi$ potential. The contributions of the $\pi$, $\omega$, and $\varepsilon$ exchange are represented by solid, dashed, and dotted lines, respectively. The total potential is represented by a dense dotted line. $C$ was adjusted to give $a_{\Lambda\Lambda}=-1.91$ fm.

interaction. This, in principle, determines the long range part of the potential which should be described in terms of meson-baryon degrees of freedom. These same coupling constants can be used to construct an OBE potential for $S \leqslant -2$. Flavor SU(3) is explicitly broken as a result of using physical masses for the baryons and mesons and the difference in the short range properties of the potential as we proceed from the $S=0$ to the $S=-1$ and $S=-2$ channels.

Such a procedure was followed by Carr *et al.* [11]. They considered only the $S$-wave interaction and ignored the tensor component. Their potential for the exchange of the $i$th meson was of the form

$$V_i(r)=V_c^{(i)}(r)+\vec{\sigma}_1\cdot\vec{\sigma}_2 V_\sigma^{(i)}(r), \tag{5}$$

where the radial potential $V_\alpha^{(i)}, \alpha=c,\sigma,$ for a meson of mass $m_i$ was assumed to be

$$V_\alpha^{(i)}(r)=V_0^{(i)}\left[\frac{e^{-m_i r}}{m_i r}-C\ \left(\frac{M}{m_i}\right)\ \frac{e^{-Mr}}{Mr}\right], \quad \alpha=c,\sigma. \tag{6}$$

To guarantee a one-parameter short range repulsion, the mass $M=2500$ MeV was used in all partial waves. Then the remaining parameter $C$ determined the strength of the short range interaction. This new parameter $C$ was constrained to ensure that the potential for $r\geqslant 1.0$ fm is unchanged and that the short range interaction is always repulsive. In Fig. 1 we illustrate the $\Lambda\Lambda$ potential in the ${}^1S_0$ channel. Included in the figure are the contributions from the $\varepsilon$ (dashed line) and $\omega$ (solid line) exchange as well as the full potential, which includes the sum of contributions from all allowed meson exchanges. In this case the parameter $C$ was adjusted so that the potential gives a $\Lambda\Lambda$ scattering length $a_{\Lambda\Lambda}=-1.91$ fm. We note that the dominant contribution to the potential is from $\varepsilon$ exchange, which is not part of any meson octet and was introduced to give medium range attraction and to emulate two-pion exchange.

We now turn to the $N\Xi$-$N\Xi$ potential where $\pi$ exchange is allowed. In Fig. 2 we present the most important contributions to the potential as well as the contribution from $\pi$ exchange. Surprisingly, $\pi$ exchange is negligible, as again the dominant contribution is from $\varepsilon$ exchange. One can make the same observation for the $\Sigma\Sigma$-$\Sigma\Sigma$ potential where $\pi$ exchange is an order of magnitude smaller than $\varepsilon$ exchange. This is a reflection of the fact that in the ${}^1S_0$ channel the strength of the $\pi$ exchange includes a factor of $m_\pi/m$, where $m$ is a hadron mass. Thus, we conclude that the diagonal elements of the potential contain little contribution from $\pi$ exchange and are dominated by $\varepsilon$ exchange. If one examines the coupling between the three channels $\Lambda\Lambda$, $N\Xi$, and $\Sigma\Sigma$, one observes that $\pi$ exchange contributes to the transition between the $\Lambda\Lambda$ and $\Sigma\Sigma$ channels. However, in this case the other isovector exchange, the $\rho$, is dominant.

Continuing the analysis, we consider the importance of the coupling between the three channels in our OBE model-D-based approach. We should point out that if the coupling is important, then the extraction of the $\Lambda\Lambda$ interaction from light $S=-2$ hypernuclei will require that we include this coupling in the analysis of the data. To illustrate this point let us consider the effective matrix element for the $\Lambda\Lambda$ interaction in second order in perturbation theory, i.e.,

$$V_{\Lambda\Lambda}^{\text{eff}}\approx\langle\Lambda\Lambda V|\Lambda\Lambda\rangle-\left|\frac{\langle\Lambda\Lambda|V|N\Xi\rangle|^2}{\Delta E}\right|, \tag{7}$$

where $\Delta E\approx 25$ MeV. In free space, as a result of the small difference between the $\Lambda\Lambda$ and $N\Xi$ threshold, this coupling is more important than that between the $NN$ and $N\Delta$ in the $S=0$ channel. On the other hand, in the nuclear medium, the transition from $\Lambda\Lambda$ to $N\Xi$ is Pauli blocked. As a result the additional attraction from the second order term is suppressed in nuclei. This implies that the effective $\Lambda\Lambda$ matrix element should be less attractive in the nuclear medium than in free space. This is true provided the coupling is, in general, large in free space. Therefore, we consider the effective role of the coupling as the size of the $\Lambda\Lambda$ scattering length $a_{\Lambda\Lambda}$ is changed.

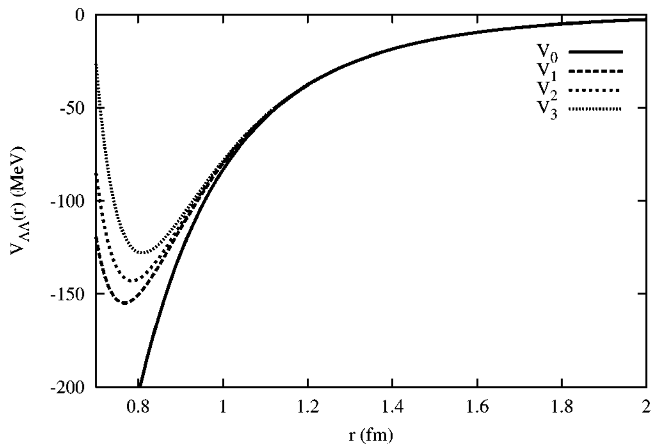


FIG. 3. The $\Lambda\Lambda$ potential in the ${}^1S_0$ channel. The solid line labeled $V_0$ is the OBE with no cutoff. The curves $V_1$, $V_2$, and $V_3$ correspond to the potential with no channel coupling, with coupling to the $N\Xi$ channel only, and with the full coupling to the $N\Xi$ and $\Sigma\Sigma$ channels. The parameter $C$ was adjusted to obtain a scattering length $a_{\Lambda\Lambda}=-1.91$ fm.

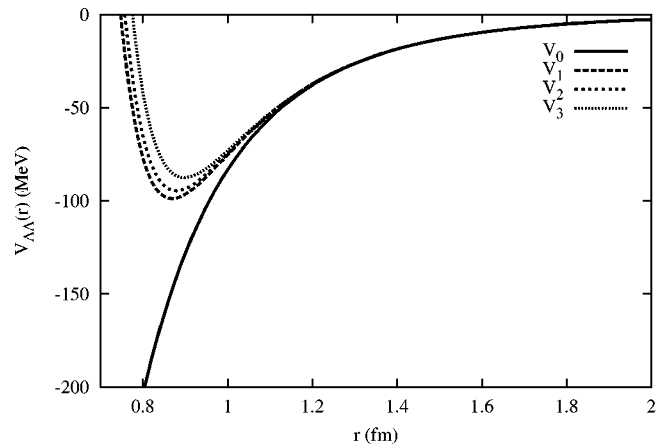


FIG. 4. The ${}^1S_0$ $\Lambda\Lambda$ potential for the case when the potential, including coupling to all the channels gives a scattering length of $a_{\Lambda\Lambda}=-0.5$ fm. The curves have the same labeling as in Fig. 3.

In Fig. 3 we present the $\Lambda\Lambda$ potential with no coupling ($V_1$), with coupling to the $N\Xi$ channel ($V_2$), and with the full coupling to both $N\Xi$ and $\Sigma\Sigma$ channels ($V_3$). The short range parameter $C$ was adjusted so that in each case the potential has a scattering length $a_{\Lambda\Lambda}=-1.91$ fm. This potential gives a ${}^{6}_{\Lambda\Lambda}$He [11] binding energy of some 10 MeV in the case of $V_1$ and about 9.7 MeV in the case of $V_2$, which are somewhat smaller than the experimental result (10.9 MeV) of Prowse [2]. From the figure we observe that as one includes first the $N\Xi$ and then the $\Sigma\Sigma$ channel, the $\Lambda\Lambda$ potential becomes shallower. This suggests that the coupling will reduce the binding energy of $\Lambda\Lambda$ hypernuclei, in agreement with the result observed by Carr *et al.* [11]. Surprisingly, the coupling to the $\Sigma\Sigma$ channel is quite important, even though the threshold for the $\Sigma\Sigma$ channel is some 160 MeV above the $\Lambda\Lambda$ threshold. One would, therefore, anticipate that a free space $\Lambda\Lambda$ interaction somewhat stronger than the one considered with $a_{\Lambda\Lambda}=-1.91$ fm would be required to reproduce the Prowse datum.

In contrast, the new measurement of the $\Lambda\Lambda$ binding energy in ${}^{6}_{\Lambda\Lambda}$He [1] suggests that the $\Lambda\Lambda$ potential is, in fact, much weaker than implied by the earlier measurement. We therefore have considered a potential that gives a scattering length $a_{\Lambda\Lambda}=-0.5$ fm. This is consistent with the results for the later Nijmegen soft core potential [12]. In Fig. 4 we present the $\Lambda\Lambda$ potential with no coupling ($V_1$), with coupling to the $N\Xi$ channel ($V_2$), and with coupling to both the $N\Xi$ and $\Sigma\Sigma$ channels ($V_3$) for $a_{\Lambda\Lambda}=-0.5$ fm: There are two distinct differences between the results for $a_{\Lambda\Lambda}=-0.5$ fm and those for $a_{\Lambda\Lambda}=-1.91$ fm: (i) In general the smaller scattering length gives a potential that is 30% shallower. (ii) Of more significance is the fact that the importance of the coupling is reduced. (However, even in this case, the coupling to the $\Sigma\Sigma$ channel is more important than just including the coupling to the $N\Xi$ channel.) This suggests that as we reduce the strength of the $\Lambda\Lambda$ interaction in our OBE model-D-based potential, the role of the coupling is reduced. Perhaps more important is the distinct possibility that we may need to include the coupling to the $\Sigma\Sigma$ channel even though the $\Sigma\Sigma$ threshold is some 160 MeV above that of $\Lambda\Lambda$ channel.

To give some quantitative measure to the variation in the $\Lambda\Lambda$ matrix element with changes in the scattering length, we recall the results of Ref. [11] in Table I for ${}^{6}_{\Lambda\Lambda}$He. Here we tabulate the $\Lambda\Lambda$ scattering length $a_{\Lambda\Lambda}$ and the binding energy of ${}^{6}_{\Lambda\Lambda}$He with and without coupling between the $\Lambda\Lambda$ and $N\Xi$ channels. Also included are the $\Lambda\Lambda$ matrix elements $\{\Delta B=[\mathrm{BE}(\Lambda\Lambda-N\Xi)-6.14]\approx-|\langle\Lambda\Lambda|V|\Lambda\Lambda\rangle|\}$ in this hypernucleus. These results confirm our expectation that the coupling between the channels becomes weaker in our OBE potential (based upon the Nijmegen model D) as the scattering length $a_{\Lambda\Lambda}$ becomes smaller and negative, i.e., as the $\Lambda\Lambda$ interaction becomes weaker.

This change in the binding energy, with and without the coupling to the $N\Xi$ channel, as one varies the $\Lambda\Lambda$ scattering length, is illustrated in Fig. 5. Here we plot the binding energy as a function of $a_{\Lambda\Lambda}^{-1}$. In particular, the (+) and (×) are the results of Ref. [11] with and without coupling between the $\Lambda\Lambda$ and $N\Xi$ channels. Also included are the recent results of Filikhin and Gal (FG) [13] which are calculated with only the $\Lambda\Lambda$ channel (i.e., no channel coupling is included). From the results of Ref. [11] we can clearly see that the role of coupling for a small, negative scattering length would be negligible, while the results of FG [13] suggest that the new experimental result [1] for the binding energy of ${}^{6}_{\Lambda\Lambda}$He of

TABLE I. Variation in the $\Lambda\Lambda$ interaction with changes in the strength of the $\Lambda\Lambda$ potential as measured by $a_{\Lambda\Lambda}$.

| $a_{\Lambda\Lambda}$ (fm) | BE($\Lambda\Lambda\alpha-N\Xi\alpha$) (MeV) | BE($\Lambda\Lambda\alpha$) (Mev) | $\Delta B$ (MeV) |
|---|---|---|---|
| −1.91 | 9.738 | 10.007 | 3.60 |
| −21.1 | 12.268 | 14.138 | 6.13 |
| 7.82 | 15.912 | 17.842 | 9.77 |
| 3.37 | 19.836 | 23.342 | 13.70 |

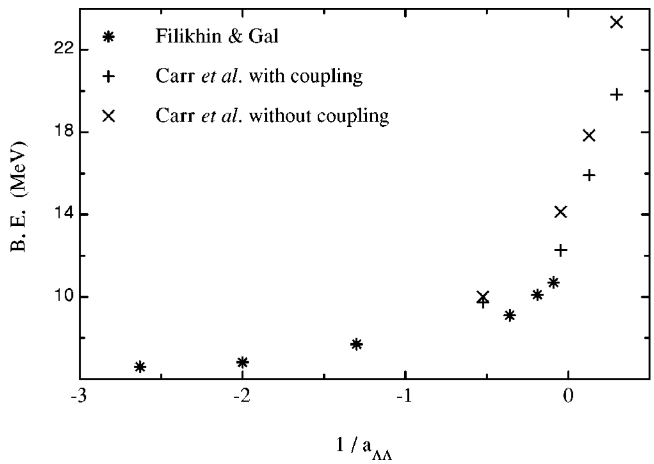


FIG. 5. Plot of the binding energy (B.E.) of ${}^{6}_{\Lambda\Lambda}$He as a function of $a_{\Lambda\Lambda}^{-1}$. Here (+) and (×) are the results of Carr *et al.* with and without coupling to the $N\Xi$ channel. Also included are the results of Filikhin and Gal (*).

$7.25 \pm 0.2^{+0.18}_{-0.11}$ MeV will imply a $\Lambda\Lambda$ scattering length of $\approx -0.5$ fm.

In conclusion, we have demonstrated that within the framework of an OBE model and flavor SU(3) corresponding to the Nijmegen model D one can generate a one-parameter set of potentials that preserve the OBE tail. The short range repulsion can then be adjusted to give the $\Lambda\Lambda$ scattering length. The primary concern with this procedure is the fact that the potential is dominated by the exchange of the scalar $\varepsilon$ meson. This meson was introduced in the strangeness $S=0$ sector to give medium range attraction and to model two-pion exchange. Its dominance in the $S=-2$ channel suggests that one should go back and include explicit two-pion exchange within a framework that will still allow one to perform a flavor SU(3) rotation of the potential to generate the $\Lambda\Lambda$ interaction. From the analysis of the importance of coupling between the ($\Lambda\Lambda$, $N\Xi$, and $\Sigma\Sigma$) channels in the strangeness $S=-2$, ${}^{1}S_{0}$ partial wave, we found that for a small, negative $\Lambda\Lambda$ scattering length the coupling between the channels is relatively weak. If we now combine this observation with the recent measurement of the binding energy of ${}^{6}_{\Lambda\Lambda}$He, one may conclude that a confirmation of this measurement could constrain the $\Lambda\Lambda$ scattering length to $a_{\Lambda\Lambda} \approx -0.5$ fm with good accuracy. Such a feeble interaction would not require inclusion of the coupling to the $N\Xi$ and $\Sigma\Sigma$ channels, which is a complication in the calculation of energies of light hypernuclei, if the OBE model used here is a valid representation of the physics. Finally, if the new measurement of the $\Lambda\Lambda$ matrix element [1] is correct, then it would confirm the validity of the SU(3) prediction for the relative strengths of the interactions in the $S=0$, $-1$, and $-2$ sectors as stated in Eq. (3).

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