Level densities for $20 \leqslant A \leqslant 110$

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A recent study of nuclear level densities for $20 \leqslant A \leqslant 70$ found evidence that the level densities for nuclei off the stability line were lower than those for nearby nuclei on the stability line. This analysis has been extended to cover the mass range $20 \leqslant A \leqslant 110$ with results that support the original conclusions. As part of the study, the variations with energy and mass number of the parity ratio and spin cutoff parameter are examined.

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I. INTRODUCTION

Most of the research in the area of nuclear level densities has been based on the work of Bethe [1]. Using the assumption that the nucleons were noninteracting, he was able to show that

$$\rho(u) = \frac{\sqrt{\pi}\,\exp(2\sqrt{au})}{12\,a^{1/4}u^{5/4}}. \tag{1}$$

In this expression, $\rho(u)$ is the density of states at an excitation energy u and a is the level density parameter that is proportional to the density of single particle states at the Fermi level. This density, in turn, is expected to be proportional to A, the nucleon number. From this expression, the density of levels can be inferred. A state is one of the $2J+1$ J_z projections of a level of spin J. If the state distribution is Gaussian in J_z, then

$$\rho_L(u) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\rho(u), \tag{2}$$

where σ is the spin cutoff parameter,

$$\sigma = \langle J_z^2 \rangle^{1/2} = \sqrt{\tfrac{1}{3}\langle J(J+1)\rangle} \tag{3}$$

and is usually energy dependent and $\rho_L(u)$ is the level density at energy u. The most important refinement to this model is the replacement of u by $u-\delta$, where δ is a pairing (or pairing plus shell) shift.

Numerous analyses of level density parameters have used the form

$$a = \alpha A, \tag{4}$$

where α is a fitting constant. In a recent paper [2], two alternative forms have been investigated:

$$a = \alpha A/\exp[\beta(N-Z)^2], \tag{5}$$

$$a = \alpha A/\exp[\gamma(Z-Z_0)^2]. \tag{6}$$

The presence of the $(N-Z)$ factor in Eq. (5) causes level densities for a series of nuclei of given A to be maximum for $N=Z=A/2$ and to decrease as the neutron or proton component becomes dominant. If $N=Z$, then the Z component of isospin T_z is 0 $(=(N-Z)/2)$. Thus, levels of isospin $T=0,1,2,\ldots$ are allowed. As $|N-Z|$ increases, the minimum T becomes T_z or $|N-Z|/2$, excluding levels of low isospin. A reduction of the total number of levels with increasing $|N-Z|$ would be expected for a given A.

The Z_0 in Eq. (6) is the value of Z for the particular value of A that is beta stable. For nuclei with $|Z-Z_0|$ large, the drip line will be approached. Arguments that imply a reduction in level density for nuclei that are very proton or neutron rich have been discussed in Ref. [2]. For low A, $Z_0 \approx A/2$ and Eqs. (5) and (6) are equivalent. Beyond $A=40$, the two equations give different predictions for a.

The analysis of Ref. [2] found that both Eqs. (5) and (6) provided a better fit to the data for $20 \leqslant A \leqslant 70$ than Eq. (4). There was a clear preference for Eq. (6) based on the χ^2, although it was pointed out that a better test would require more information about nuclei with $|Z-Z_0| \geqslant 2$.

The purpose of the present paper is to examine the database over a wider range in A than was done in Ref. [2]. That paper only discussed level density. Since both the spin cutoff parameter and the parity ratio are of relevance in making Hauser-Feshbach calculations, the database was also used to derive these quantities. A study of the systematic behavior of these quantities was also completed.

II. ANALYSIS

A large fraction of the level density information currently available comes from neutron resonance analysis. At low energies, the neutron can only interact with the nucleus in an S state. This means that for a zero spin target, all compound states observed will be $J=\frac{1}{2}$. Furthermore, if the parity of the target was +, all compound states will be $\frac{1}{2}^+$. More generally, if the target spin is J_s, the compound states will be $J_s \pm \frac{1}{2}$, with the same parity as the target.

To convert the observed level density to the total level density, both the spin cutoff factor and the parity ratio

$$\pi(u) = \frac{\rho_+(u)}{\rho_+(u)+\rho_-(u)} \tag{7}$$

TABLE I. Forms for the dependence of a on various parameters.

	Form	Parameter values	χ^2 relative to Eq. (4)
(A)	αA	$\alpha=0.1016$	1
(B)	$\alpha A+\beta A^{2/3}$	$\alpha=0.0481$	0.933
		$\beta=0.2037$	
(C)	$\alpha A/\exp[\beta(N-Z)^2]$	$\alpha=0.1062$	0.916
		$\beta=0.00051$	
(D)	$\alpha A/\exp[\gamma(Z-Z_0)^2]$	$\alpha=0.1068$	0.891
		$\gamma=0.0389$	
		$Z_0=0.5042A/(1+0.0073A^{2/3})$	
(E)	$\alpha A/\exp[\beta(N-Z)^2+\gamma(Z-Z_0)^2]$	$\alpha=0.1073$	0.881
		$\beta=0.00022$	
		$\gamma=0.0289$	
		$Z_0=0.5042A/(1+0.0073A^{2/3})$	
(F)	$\alpha A/\exp[(\beta(N-Z)^2+1)\gamma(Z-Z_0)^2]$	$\alpha=0.1076$	0.881
		$\beta=0.00084$	
		$\gamma=0.0527$	
		$Z_0=0.5042A/(1+0.0073A^{2/3})$	

are needed. Moreover, the nuclei reached in such studies typically have $|Z-Z_0|\leqslant 1$.

For this reason, the analysis of Ref. [2] was based on level counting at low energy. This technique can only be used for nuclei whose level schemes are believed to be complete up to a given excitation energy. By plotting the observed density as a function of energy, it is often possible to see where levels are missed. Since the level density increases exponentially, a very rapid increase in the number of lost levels occurs at a particular energy.

Two other tests have been applied. The spin cutoff factor is expected [3] to depend on A as $A^{7/12}$ and u as $u^{1/4}$. Although microscopic effects based on a particular orbit being filled cause some deviation from these predictions, truly substantial discrepancies signal missing levels.

The parity ratio also has very characteristic behavior. Ericson [3] has shown that, in general, a large basis has nearly equal numbers of positive and negative parity states. Thus, at large energies $\pi(u)=0.5$. At low energies $\pi(u)$ is 1 for even-even nuclei. For odd A, $\pi(u)$ will be 1 for nuclei in which the unpaired nucleon is in an even parity orbit, and 0 for nuclei in which the odd nucleon is in an orbit of odd parity. In a few cases, $\pi(u)$ can be near 0.5 for odd A at low u if a positive and negative parity orbit are nearly degenerate at the Fermi level. If experiments have missed levels of a particular parity at a few MeV, the parity ratio will not approach 0.5 as the energy increases.

Levels were taken from the ENSDF data file [4]. The number of states in each 0.5-MeV bin of excitation energy was tabulated. The same level information file was used to construct a table of values for the spin cutoff parameter and the parity ratio. The former quantity is simply

$$\sigma(U)=\left(\frac{\sum_J J(J+1)\rho(U,J)}{\sum_J 3\rho(U,J)}\right)^{1/2} \qquad (8)$$

where ρ (U,J) is the density of levels of spin J at energy U. Similarly, the parity ratio was constructed by calculating the ratio of the number of positive parity levels to the total number of levels in that bin.

The analysis of Ref. [2] focused on the range $20\leqslant A \leqslant 70$, where it was found that 130 nuclei had a constructed level density that appeared complete up to 2.5 MeV. This meant that the inferred level density did not show a flattening in slope or a decrease in the two top energy bins and that the spin cutoff and parity ratio showed reasonable behavior in this range.

Data from nuclei with $70\leqslant A\leqslant 110$ were treated in a similar way, yielding 111 additional nuclei that met the same criteria as were applied to the previous dataset. An effort was made to extend the range in A from 110 to 140. To check for an isotope effect, 16 of the tin isotopes were analyzed. Two additional nuclei at $A=140$ were added to ensure correct behavior of the fits at large A. This extended the number of nuclei to 257. Since nearly all of these nuclei are between atomic mass 20 and 110, this is the claimed region of validity, even though 3% of the nuclei were from the range $110 \leqslant A\leqslant 140$.

In addition to fits of the state density with the forms shown in Eqs. (4)–(6), fits with the forms

TABLE II. Mass formula parameters.

$M(Z,N)(\text{MeV})=Zm_p+Nm_n-a_vA+a_sA^{2/3}$ $+(a_{co}-a_{ci}/A^{1/3})Z^2/A^{1/3}+a_a(N-Z)^2/A$
$a_v=14.769$ MeV
$a_s=15.780$ MeV
$a_{co}=0.6909$ MeV
$a_{ci}=0.4469$ MeV
$a_a=19.22$ MeV

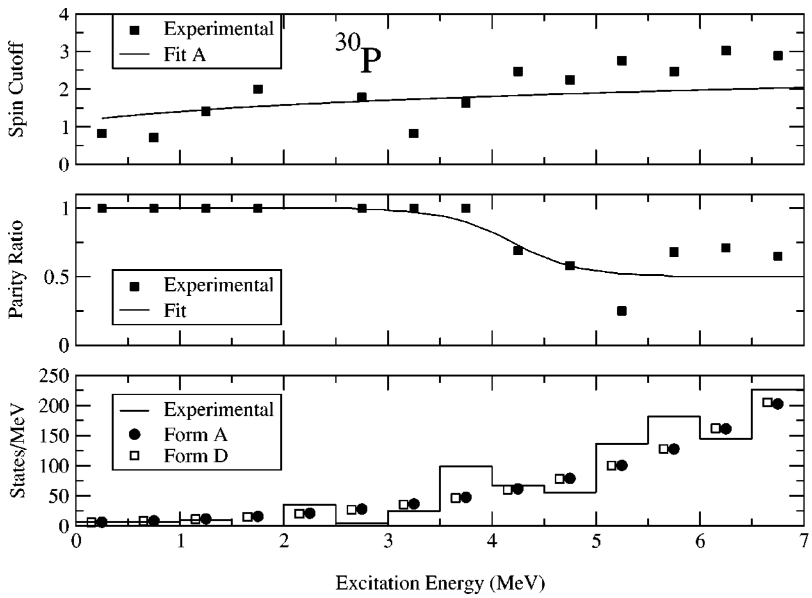

FIG. 1. Fits to the level density, the parity ratio and the spin cutoff parameter of ^{30}P. The level density forms fitted are forms *A* and *D*. One bin of the level density beyond the region fit is shown. The fitted points shown are offset slightly for clarity. Form *A* points are in the center of the bin, while the form *D* points are evaluated at the same energy but are offset for clarity.

$$a = \alpha A + \beta A^{2/3}, \tag{9}$$

$$a = \alpha A / \exp\{[\beta(N-Z)^2 + 1][\gamma(Z-Z_0)^2]\}, \tag{10}$$

$$a = \alpha A / \exp[\beta(N-Z)^2 + \gamma(Z-Z_0)^2] \tag{11}$$

were also attempted. The first form [Eq. (9)] has been used previously and is expected to be appropriate on theoretical grounds; Mughabghab and Dunford [5] have presented a summary of these results. Equations (10) and (11) are two forms that allow a dependence of *a* on $(N-Z)$ and $(Z-Z_0)$ as well as *A*.

The results of these fits are shown in Table I. In each case, the χ^2 minimization included as one step the choice of δ (energy shift) for the value of *a* that produced minimum χ^2.

Each of the alternative forms achieves a lower χ^2 than the form $a = \alpha A$. The best two-parameter form is $\alpha A/\exp[\gamma(Z-Z_0)^2]$; this agrees with the results of Ref. [2]. Slightly poorer fits were obtained with forms that depended on $(N-Z)$ but not $(Z-Z_0)$ and the form that included $A^{2/3}$.

Both of these forms involving three parameters gave slightly better χ^2 values than the form with only $(Z-Z_0)$, but the differences were so small that it was felt the additional complexity was not required.

In Table II, the parameters of a semiempirical mass formula fit to the ground state masses are presented. These parameters, as well as those in the expression for Z_o [$Z_o = 0.5042A/(1+0.0073A^{2/3})$] were obtained by a best fit to the nuclei included in the present study. The predicted masses from this formula are compared with the actual masses to calculate a predicted δ; this value was compared with the best-fit value for δ in the level density fits. Typical fits to the level density are shown in Figs. 1–5. As was stressed in Ref. [2], the test for the presence of a term with $(Z-Z_0)$ would be more definitive if more data off the sta-

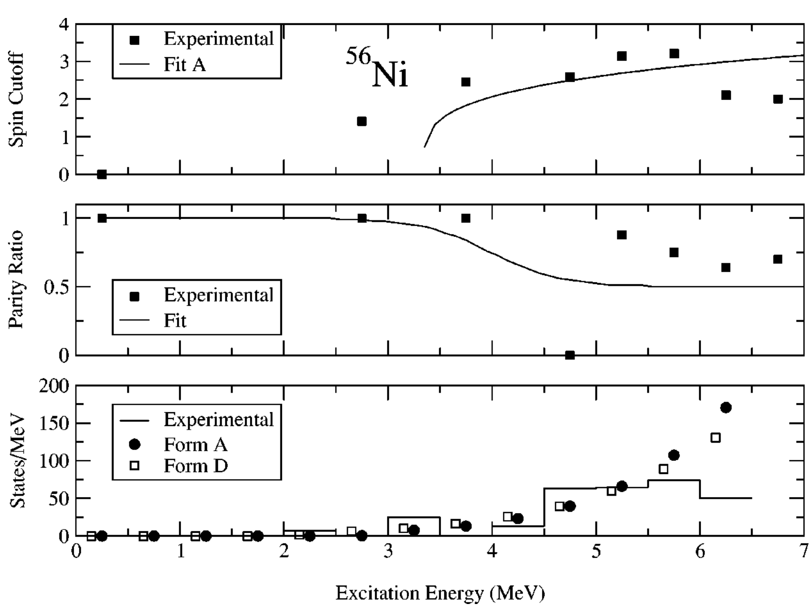

FIG. 2. Same as Fig. 1 for ^{56}Ni.

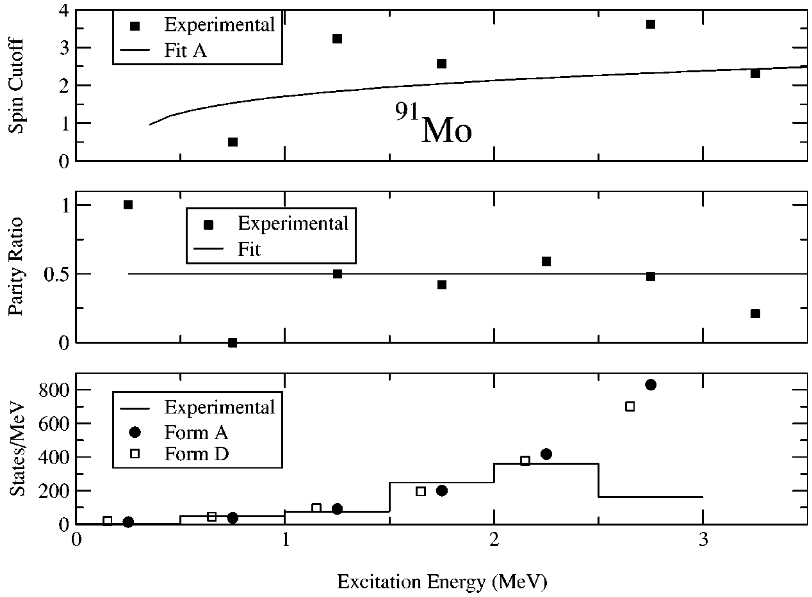

FIG. 3. Same as Fig. 1 for ^{91}Mo.

bility line were available. Of the nuclei shown, ^{30}P was chosen because the level scheme is thought to be complete to a fairly high energy. Because Z is approximately Z_0 for this nucleus, there is very little difference between the fits with forms A and D. The other nuclei shown are ones for which $|Z-Z_0|\approx 2$, so larger differences between forms A and D are seen. As was found in Refs. [2,6], a systematic behavior of the best-fit δ parameters is observed. This parameter is sometimes assumed to be a pairing energy shift. Since shell effects are also known in level densities, the assumption that shell effects are not in δ forces them to be incorporated in a. This result is unfortunate [6], since it causes them to grow as energy increases.

The present results for δ are consistent with the procedure described in Refs. [2,6]. Comparing the actual mass of a given nucleus with that predicted using a semiempirical mass formula without a shell or pairing correction gives an empirical shell plus pairing correction; this was found to correlate well with the best-fit δ values.

In Fig. 6 the difference ΔE between the empirical δ (that found from the level density fit) and 0.525 times the difference between the experimental mass and the predicted liquid drop mass (Table II) is plotted. This shift could be added to the difference plotted to make use of the level density form (D) in predicting level densities. The appropriate parameters for calculating the shift are given in Table III. As part of the analysis procedure, the data for parity ratios were fit with the form

$$\pi(u)=\frac{1}{2}\left(1\pm\frac{1}{1+\exp\left[c(u-\delta_p)\right]}\right), \tag{12}$$

where $+$ was used for nuclei for which $\pi(u)$ approached 1 at low u and $-$ was used for those which approached 0.

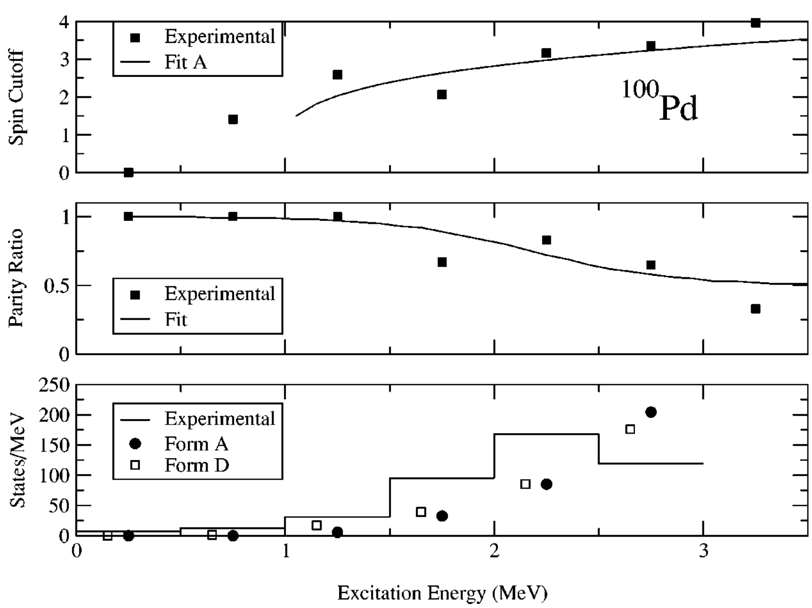

FIG. 4. Same as Fig. 1 for ^{100}Pd.

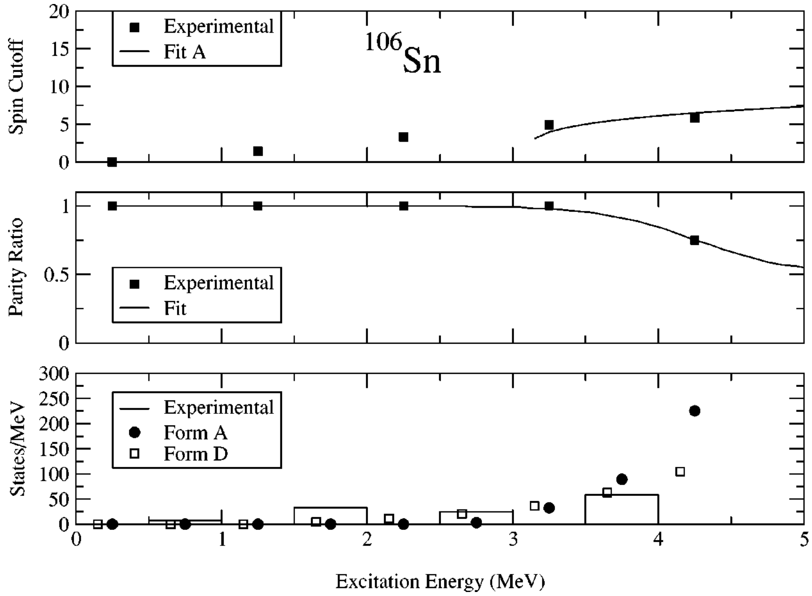

FIG. 5. Same as Fig. 1 for ^{106}Sn.

Nuclei for which the parity ratio approaches zero as U goes to zero are odd A nuclei with $21\leq Z\leq 40$ or $21\leq N\leq 40$. It was found that good fits to the parity ratio could be obtained by fixing c at 3 MeV^{-1} and allowing the shift δ_p to vary. Figure 7 shows this variation with A for even-even, even-odd, odd-even, and odd-odd nuclei. The behavior shows plausible systematics, with a smooth decreasing variation with A for even-even nuclei. For the other cases, very low values of δ_p can result if a positive orbital and a negative parity orbital are nearly degenerate at the Fermi level. It can be seen that for certain ranges in A this appears to occur.

In Table IV the parameters needed for predicting the parity ratios are given. There is a consistent trend for the parity ratio to approach 0.5 fastest for odd-odd nuclei, somewhat slower for even-odd and odd-even nuclei and slower still for even-even nuclei.

Previous studies of the parity ratio have been based on theoretical calculations [7–11] or on theory and experiment [6]. The experimental results obtained here are reasonably consistent with predictions although the previous results were not parametrized in precisely this way.

Note the tendency of δ_p to decrease as the value of A increases. The implication of these results is that at 7 MeV (typical nucleon binding energy near the stability line) the parity ratio is close to asymptotic for $A>50$. For nucleosynthesis calculations, many of the nuclei of interest will have binding energies less than 7 MeV. If the binding energy is 4 MeV, for example, the parity ratio is not asymptotic until the A value exceeds 75. It has been pointed out [11] that this will probably lead to situations near the drip line for $A<70$, where either the s or p wave strength function will be zero for bombarding energies of interest in nucleosynthesis calculations.

Fits were also constructed to the spin cutoff factor as a function of A. Two forms of the spin cutoff factor were examined in Ref. [6]:

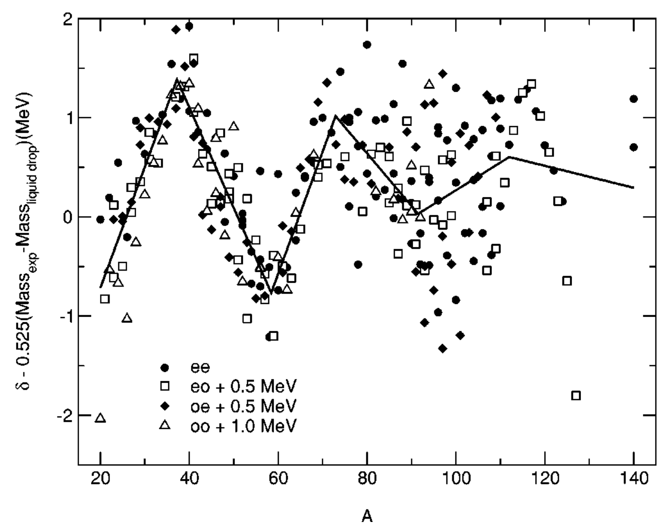

FIG. 6. Comparison of the best fit δ with 0.525 ($\mathrm{mass}_{\mathrm{exp}}$ $-\mathrm{mass}_{\mathrm{liquiddrop}}$) as a function of A. The line represents a straight line connecting the points indicated in Table III.

TABLE III. Delta segment points.

A	ΔE + Offset (MeV)
20.00	-0.7216
37.24	1.395
58.45	-0.7683
72.93	1.0334
91.50	0.0298
112.00	0.6039
140.00	0.2941
	Offset (MeV)
Even-even	0.0
Odd-even	0.5
Even-odd	0.5
Odd-odd	1.0

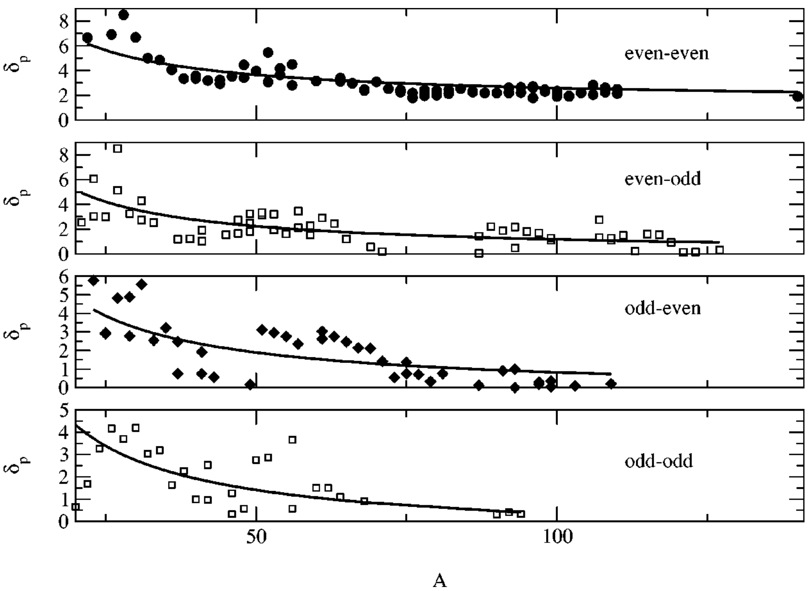

FIG. 7. Comparison of δ_p as a function of A for even-even, even-odd, odd-even, and odd-odd nuclei.

$$\sigma^2 = 0.0145 A^{5/3} \sqrt{\frac{u-\Delta}{a}}, \tag{13}$$

$$\sigma^2 = 0.1461 \sqrt{a(u-\Delta)} A^{2/3}. \tag{14}$$

The first expression (form A) is based on a rigid body model of the nucleus, while the second (form B) results from a statistical mechanical calculation using $\langle J_z^2 \rangle$ values averaged over the appropriate single particle states. The data were fit equally well with the two forms, and the appropriate constants obtained from the least squares search are close to those expected theoretically.

The constant in Eq. (13) is obtained under the assumption that the nucleus can be approximated by a hard sphere of radius $1.25A^{1/3}$ fm. The best fit of form (13) to the sigma data yields a value 0.88 times as large as this at the low A end and 0.71 times as large at mass 100. Figure 8 shows the plot of the σ values as a function of A. This indicates that the spin cutoff parameter at energies of a few MeV is slightly less than the rigid body value, with a small tendency to be closer to this value at mass 20–30 than at mass 100.

TABLE IV. Parity ratio delta parametrization.

Parametrization $\delta_p = a_0 + a_1/A^{a_2}$
Even-even
$a_0 = 1.34$
$a_1 = 75.22$
$a_2 = 0.89$
Even-odd
$a_0 = -0.08$
$a_1 = 75.22$ [a]
$a_2 = 0.89$ [a]
Odd-even
$a_0 = -0.42$
$a_1 = 75.22$ [a]
$a_2 = 0.89$ [a]
Odd-odd
$a_0 = -0.90$
$a_1 = 75.22$ [a]
$a_2 = 0.89$ [a]

[a]Variable held constant in fit.

III. SUMMARY

An analysis of level densities inferred from low energy excited levels has provided support for a previous analysis of a more limited database. In each case, the analysis indicates that level densities decrease as nuclei move away from the valley of stability. The present analysis suggests slightly different parameter values for this dependence than were proposed in Ref. [2]

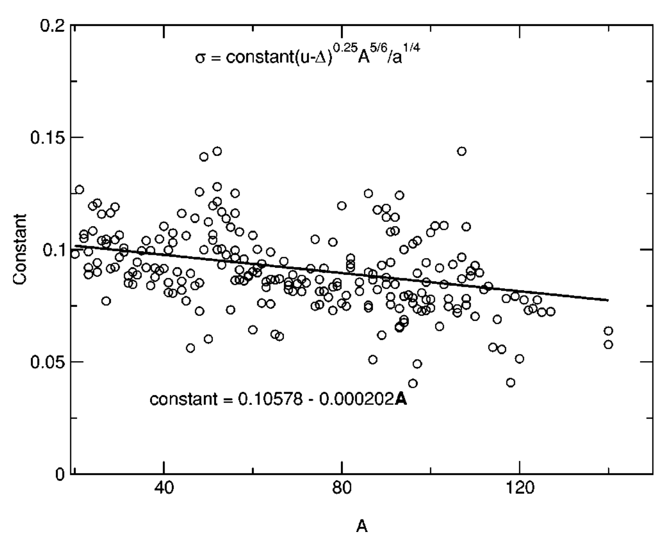

FIG. 8. Spin cutoff parameters as functions of A.

As was clear in the analysis described in Ref. [2], the differences between level density forms A and D are accentuated at high energy and for $|Z-Z_0| \geqslant 2$. Experiments are planned that will populate nuclei with larger values of $|Z-Z_0|$ through evaporation processes.

Examination of the systematic behavior of the spin cutoff factor and parity ratio has yielded information on the variation of these parameters with A. Both these quantities are necessary ingredients in Hauser-Feshbach calculations; both are also needed in evaluating level density systematics from neutron resonance counting.

ACKNOWLEDGMENTS

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