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Lorentz covariant orbital-spin scheme for the effective N^*NM couplings

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For excited nucleon states N^* of arbitrary spin coupling to nucleon (N) and meson (M), we propose a Lorentz covariant orbital-spin (L-S) scheme for the effective N^*NM couplings. To be used for the partial wave analysis of various N^* production and decay processes, it combines merits of two conventional schemes, i.e., covariant effective Lagrangian approach and multipole analysis with amplitudes expanded according to angular momentum L. As examples, explicit formulas are given for $N^*\to N\pi$, $N^*\to N\omega$, and $\psi\to N^*\bar{N}$ processes which are under current experimental studies.

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I. INTRODUCTION

The study of the nucleon and its excited states N^* can provide us with critical insights into the nature of QCD in the confinement domain [1]. They are the simplest system in which the three colors of QCD neutralize into colorless objects and the essential non-abelian character of QCD is manifest. However, our present knowledge of N^* spectroscopy is still very poor, with information coming almost entirely from the old generation of πN experiments of more than 20 years ago [2] and with many fundamental issues not well understood [3]. Considering its importance for the understanding of the nonperturbative QCD, much effort has been devoted to the study of the N^* spectrum. A series of experiments on N^* physics with electromagnetic probes have been started at modern facilities such as TJNAF [4], ELSA [5], GRAAL [6], SPRING8 [7], and BEPC [8].

Abundant data have been accumulated for various N^* production and decay channels at these facilities in the last few years. Now an important task facing us is to a perform partial wave amplitude analysis (PWA) of these data to extract properties of N^* resonances, such as their spin parity, mass, width, decay branching ratios, and so on. For πN or γN to meson-nucleon final states, the most commonly used PWA formalism is the multipole analysis with amplitudes expanded according to angular momentum L of a meson-nucleon system [9–13]. This formalism is usually written in the meson-nucleon c.m. system, not in a covariant form, and hence is not very convenient to be used for multistep chain processes, such as $J/\psi\to N^*\bar{N}$ with N^* further decaying to meson-nucleon. For a multistep chain process, the covariant effective Lagrangian approach [14–17,8] is more convenient. In this approach, the effective N^*NM couplings are constructed by Rarita-Schwinger wave functions for particles of arbitrary spin [18], four-momenta of involved particles, Dirac γ matrices, etc., with constraint of general symmetries required by the strong interaction. A problem is this approach is that the amplitude is usually a mixture of various orbital angular momenta L. Hence the usual centrifugal barrier (Blatt-Weisskopf) factor [12,19], commonly used in multipole analysis and mesonic decays, cannot be used here since the barrier factor is L dependent. Instead vertex form factors with an exponential form or other forms are used in the effective Lagrangian approach. This makes a comparison to results from usual multipole approach very difficult.

In this paper we propose a covariant L-S scheme for the effective N^*NM couplings to be used for the partial wave analysis of N^* data. In this scheme, the amplitudes are expanded according to the orbital angular momentum L of two decay products, that are meanwhile Lorentz invariant. Hence it combines the merits of multipole analysis and the usual effective Lagrangian approach.

In nature, our formalism is equivalent to the standard approach of effective Lagrangians, but it has the advantage that terms with a definite angular momentum in the decay state are constructed on the Lagrangian level, which makes it easier to use L-dependent form factors and simplifies the interpretation of partial wave analyses. It should be used as an effective Lagrangian in the future.

II. GENERAL FORMALISM

In our construction of the covariant L-S scheme for the effective N^*NM couplings, we need to combine some knowledge from the covariant tensor formalism for meson decays [19] and covariant wave functions for hadrons of arbitrary spin [20]. For a given hadronic decay process $A\to BC$, in the L-S scheme on hadronic level, the initial state is described by its four-momentum P_μ and its spin state $\mathbf{S_A}$; the final state is described by the relative orbital angular momentum state of BC system $\mathbf{L}_{\rm BC}$ and their spin states $(\mathbf{S}_{\rm B},\mathbf{S}_{\rm C})$.

The spin states $(\mathbf{S_A},\mathbf{S_B},\mathbf{S_C})$ can be well represented by the relativistic Rarita-Schwinger spin wave functions for particles of arbitrary spin [18,19,21,17]. The spin-$\frac{1}{2}$ wave function is the standard Dirac spinor $u(p,s)$ or $v(p,s)$ and the spin-1 wave function is the standard spin-1 polarization four-vector $\varepsilon^\mu(p,s)$ for particle with momentum p and spin projection s:

$$\sum_{s=0,\pm1}\varepsilon_\mu(p,s)\varepsilon^*_\nu(p,s)=-g_{\mu\nu}+\frac{p_\mu p_\nu}{p^2}\equiv-\tilde{g}_{\mu\nu}(p). \quad (1)$$

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Spin wave functions for particles of higher spins are constructed from these two basic spin wave functions with C-G coefficients $(j_1,j_{1z};j_2,j_{2z}|j,j_z)$ as follows:

$$\varepsilon_{\mu_1\mu_2\cdots\mu_n}(p,n,s) = \sum_{s_{n-1},s_n}(n-1,s_{n-1};1,s_n|n,s)\varepsilon_{\mu_1\mu_2\cdots\mu_{n-1}} \times(p,n-1,s_{n-1})\varepsilon_{\mu_n}(p,s_n) \quad (2)$$

for a particle with integer spin $n\geqslant 2$, and

$$u_{\mu_1\mu_2\cdots\mu_n}(p,n+\tfrac{1}{2},s) = \sum_{s_n,s_{n+1}}(n,s_n;\tfrac{1}{2},s_{n+1}|n+\tfrac{1}{2},s)\varepsilon_{\mu_1\mu_2\cdots\mu_n} \times(p,n-1,s_n)u(p,s_{n+1}) \quad (3)$$

for a particle with half integer spin $n+\frac{1}{2}$ of $n\geqslant 1$.

The orbital angular momentum $\mathbf{L}_{BC}$ state can be represented by covariant tensor wave functions $\tilde{t}^{(L)}_{\mu_1\cdots\mu_L}$ as the same as for meson decay [19]. We define $r=p_B-p_C$, then

$$\tilde{t}^{(0)}=1, \quad (4)$$

$$\tilde{t}^{(1)}_{\mu}=\tilde{g}_{\mu\nu}(p_A)r^{\nu}\equiv\tilde{r}_{\mu}, \quad (5)$$

$$\tilde{t}^{(2)}_{\mu\nu}=\tilde{r}_{\mu}\tilde{r}_{\nu}-\tfrac{1}{3}(\tilde{r}\cdot\tilde{r})\tilde{g}_{\mu\nu}, \quad (6)$$

$$\tilde{t}^{(3)}_{\mu\nu\lambda}=\tilde{r}_{\mu}\tilde{r}_{\nu}\tilde{r}_{\lambda}-\tfrac{1}{5}(\tilde{r}\cdot\tilde{r})(\tilde{g}_{\mu\nu}\tilde{r}_{\lambda}+\tilde{g}_{\nu\lambda}\tilde{r}_{\mu}+\tilde{g}_{\lambda\mu}\tilde{r}_{\nu}), \quad (7)$$

$$\cdots.$$

In the L-S scheme, we need to use the conservation relation of total angular momentum:

$$\mathbf{S_A}=\mathbf{S_B}+\mathbf{S_C}+\mathbf{L}_{\rm BC} \quad \text{or} \quad -\mathbf{S_A}+\mathbf{S_B}+\mathbf{S_C}+\mathbf{L}_{\rm BC}=0. \quad (8)$$

Comparing with the pure meson case [19], here for N^*NM couplings we need to introduce the concept of relativistic total spin of two fermions.

For the case of A as a meson, B as N^* with spin $n+\frac{1}{2}$ and C as $\bar{N}$ with spin-$\frac{1}{2}$, the total spin of BC ($\mathbf{S_{BC}}$) can be either n or $n+1$. The two $\mathbf{S_{BC}}$ states can be represented as

$$\psi^{(n)}_{\mu_1\cdots\mu_n}=\bar{u}_{\mu_1\cdots\mu_n}(p_B,s_B)\gamma_5 v(p_C,s_C), \quad (9)$$

$$\Psi^{(n+1)}_{\mu_1\cdots\mu_{n+1}}=\bar{u}_{\mu_1\cdots\mu_n}(p_B,s_B) \times\left(\gamma_{\mu_{n+1}}-\frac{r_{\mu_{n+1}}}{m_A+m_B+m_C}\right)v(p_C,s_C) +(\mu_1\leftrightarrow\mu_{n+1})+\cdots+(\mu_n\leftrightarrow\mu_{n+1}) \quad (10)$$

for $\mathbf{S_{BC}}$ of n and $n+1$, respectively. As a special case of $n=0$, we have

$$\psi^{(0)}=\bar{u}(p_B,s_B)\gamma_5 v(p_C,s_C), \quad (11)$$

$$\Psi^{(1)}_{\mu}=\bar{u}(p_B,s_B)\left(\gamma_{\mu}-\frac{r_{\mu}}{m_A+m_B+m_C}\right)v(p_C,s_C). \quad (12)$$

Here r_{μ} term is necessary to cancel out the $\hat{\mathbf{p}}$-dependent component in the simple $\bar{u}\gamma_{\mu}v$ expression. In the A at-rest system, we have

$$\psi^{(0)}=C_{\psi}(-1)^{1/2-s_C}\delta_{s_B(-s_C)}, \quad (13)$$

$$\Psi^{(1)}_{i}=C_{\Psi}(-1)^{1/2-s_C}\chi^{\dagger}_{s_B}\boldsymbol{\sigma}_i\chi_{-s_C} \quad (14)$$

with two-component Pauli spinors $\chi^{\dagger}_{1/2}=(1,0)$ and $\chi^{\dagger}_{-1/2}=(0,1)$, and

$$C_{\psi}=\frac{(E_B+m_B)(E_C+m_C)+\mathbf{p_C^2}}{\sqrt{2m_B2m_C(E_B+m_B)(E_C+m_C)}}, \quad (15)$$

$$C_{\Psi}=\sqrt{\frac{(E_B+m_B)(E_C+m_C)}{2m_B2m_C}}\left(1+\frac{\mathbf{p_C^2}}{(E_B+m_B)(E_C+m_C)}\right). \quad (16)$$

In the nonrelativistic limit, both C_{ψ} and C_{Ψ} are equal to 1. Generally both of them have some smooth dependence on the magnitude of momentum. But both $\psi^{(0)}$ and $\Psi^{(1)}_{\mu}$ have no dependence on the direction of the momentum $\hat{\mathbf{p}}$, hence correspond to pure spin states with the total spin of 0 and 1, respectively.

For the case of A as N^* with spin $n+\frac{1}{2}$, B as N, and C as a meson, one needs to couple $-\mathbf{S}_A$ and $\mathbf{S}_B$ first to get $\mathbf{S}_{AB}\equiv-\mathbf{S}_A+\mathbf{S}_B$ states, which are

$$\phi^{(n)}_{\mu_1\cdots\mu_n}=\bar{u}(p_B,s_B)u_{\mu_1\cdots\mu_n}(p_A,s_A), \quad (17)$$

$$\Phi^{(n+1)}_{\mu_1\cdots\mu_{n+1}}=\bar{u}(p_B,s_B)\gamma_5\tilde{\gamma}_{\mu_{n+1}}u_{\mu_1\cdots\mu_n}(p_A,s_A) +(\mu_1\leftrightarrow\mu_{n+1})+\cdots+(\mu_n\leftrightarrow\mu_{n+1}) \quad (18)$$

for $\mathbf{S}_{AB}$ of n and $n+1$, respectively.

$$\phi^{(0)}=\bar{u}(p_B,s_B)u(p_A,s_A), \quad (19)$$

$$\Phi^{(1)}_{\mu}=\bar{u}(p_B,s_B)\gamma_5\tilde{\gamma}_{\mu}u(p_A,s_A), \quad (20)$$

with $\tilde{\gamma}_{\mu}=\tilde{g}_{\mu\nu}(p_A)\gamma^{\nu}$. In the A (N^*) at-rest system, we have

$$\phi^{(0)}=\sqrt{\frac{(E_A+m_A)(E_B+m_B)}{2m_A2m_B}}\delta_{s_As_B}, \quad (21)$$

$$\Phi^{(1)}_{i}=-\sqrt{\frac{(E_A+m_A)(E_B+m_B)}{2m_A2m_B}}\chi^{\dagger}_{s_B}\boldsymbol{\sigma}_i\chi_{s_A}. \quad (22)$$

Both have no dependence on the direction of the momentum $\hat{\mathbf{p}}$.

In effective Lagrangian approaches, the effective N^*NM couplings are constructed by p_A, r, $g^{\mu\nu}$, γ^{μ}, u or v, and may be mixture of various orbital angular momentum states. In our proposed covariant L-S scheme, the effective N^*NM couplings should be composed of p_A, $\tilde{t}^{(L)}$, $g^{\mu\nu}$, $\epsilon_{\alpha\beta\gamma\delta}$ (the full antisymmetric tensor), ψ (Ψ) or ϕ (Φ), corresponding to a pure orbital angular momentum L state. Then the procedure for constructing the effective N^*NM couplings is very similar to the case for pure mesons [19]. First the parity should be conserved, which means

$$\eta_A = \eta_B \eta_C (-1)^L, \tag{23}$$

where η_A, η_B, and η_C are the intrinsic parities of particles A, B, and C, respectively. From this relation, one knows whether L should be even or odd. Then from Eq. (8) one can figure out how many different L-S combinations, which determine the number of independent couplings. For a final state with orbital angular momentum of L, $\tilde{t}^{(L)}$ should appear once in the effective coupling without any other $\tilde{t}$ or r. This will guarantee a pure L final state. Then one can easily apply Blatt-Weisskopf centrifugal barrier factor for each effective coupling with an L final state if one wishes. We shall show the concrete procedure by examples in Sec. III.

III. EXAMPLES

We shall start with the simplest case for $N^*\to N\pi$ process, then for $N^*\to N\omega$ and $\psi\to N^*\bar{N}$ where ψ can be J/ψ or ψ' or any other heavy vector mesons.

A. $N^*\to N\pi$

For $N^*\to N\pi$, it is well known that only one possible L-S coupling for the $N\pi$ final state of each N^* decay. Since the nucleon has spin-parity $\frac{1}{2}^+$ and pion has spin-parity 0^-, $N^*(\frac{1}{2}^+)$ can only decay to $N\pi$ in P-wave with $S_{AB}=1$ to make $-\mathbf{S}_A+\mathbf{S}_B+\mathbf{S}_C+\mathbf{L}_{\mathbf{BC}}=\mathbf{S}_{\mathbf{AB}}+\mathbf{L}_{\mathbf{BC}}=0$, meanwhile satisfying the parity conservation relation [Eq. (23)]. Similarly we have $N^*(\frac{1}{2}^-)\to N\pi$ in the S wave, with $S_{AB}=0$; $N^*(\frac{3}{2}^+)\to N\pi$ in the P wave, with $S_{AB}=1$; $N^*(\frac{3}{2}^-)\to N\pi$ in the D wave; $N^*(\frac{5}{2}^+)\to N\pi$ in the F wave, with $S_{AB}=3$; $N^*(\frac{5}{2}^-)\to N\pi$ in the D wave, with $S_{AB}=2$; $N^*(\frac{7}{2}^+)\to N\pi$ in the F wave with $S_{AB}=3$; $N^*(\frac{7}{2}^-)\to N\pi$ in the G wave with $S_{AB}=4$; and so on. Then the effective $N^*N\pi$ couplings in the covariant L-S scheme are

$$N^*(\tfrac{1}{2}^+)\to N\pi: \quad \Phi^{(1)}_{\mu}\tilde{t}^{(1)\mu}, \tag{24}$$

$$N^*(\tfrac{1}{2}^-)\to N\pi: \quad \phi^{(0)}\tilde{t}^{(0)}, \tag{25}$$

$$N^*(\tfrac{3}{2}^+)\to N\pi: \quad \phi^{(1)}_{\mu}\tilde{t}^{(1)\mu}, \tag{26}$$

$$N^*(\tfrac{3}{2}^-)\to N\pi: \quad \Phi^{(2)}_{\mu\nu}\tilde{t}^{(2)\mu\nu}, \tag{27}$$

$$N^*(\tfrac{5}{2}^+)\to N\pi: \quad \Phi^{(3)}_{\mu\nu\lambda}\tilde{t}^{(3)\mu\nu\lambda}, \tag{28}$$

$$N^*(\tfrac{5}{2}^-)\to N\pi: \quad \phi^{(2)}_{\mu\nu}\tilde{t}^{(2)\mu\nu}, \tag{29}$$

$$N^*(\tfrac{7}{2}^+)\to N\pi: \quad \phi^{(3)}_{\mu\nu\lambda}\tilde{t}^{(3)\mu\nu\lambda}, \tag{30}$$

$$N^*(\tfrac{7}{2}^-)\to N\pi: \quad \Phi^{(4)}_{\mu\nu\lambda\delta}\tilde{t}^{(4)\mu\nu\lambda\delta}. \tag{31}$$

Here, for simplicity, we omit the vertex form factors. With properties of Rarita-Schwinger wave functions

$$\gamma^{\mu_i}u_{\dots\mu_i\dots}=0 \quad \text{and} \quad p^{\mu_i}u_{\dots\mu_i\dots}(p,s)=0, \tag{32}$$

one can easily obtain the relation between the covariant L-S couplings and the usual effective Lagrangian ones,

$$N^*(\tfrac{1}{2}^+)\to N\pi: \quad \Phi^{(1)}_{\mu}\tilde{t}^{(1)\mu}=\bar{u}_N\gamma_5\gamma_\mu u_* p^\mu_\pi\cdot C_\Phi, \tag{33}$$

$$N^*(\tfrac{1}{2}^-)\to N\pi: \quad \phi^{(0)}\tilde{t}^{(0)}=\bar{u}_N u_*\cdot 1, \tag{34}$$

$$N^*(\tfrac{3}{2}^+)\to N\pi: \quad \phi^{(1)}_{\mu}\tilde{t}^{(1)\mu}=\bar{u}_N u_{*\mu} p^\mu_\pi\cdot 2, \tag{35}$$

$$N^*(\tfrac{3}{2}^-)\to N\pi: \quad \Phi^{(2)}_{\mu\nu}\tilde{t}^{(2)\mu\nu}=\bar{u}_N\gamma_5\gamma_\mu u_{*\nu} p^\mu_\pi p^\nu_\pi\cdot 4C_\Phi, \tag{36}$$

$$N^*(\tfrac{5}{2}^+)\to N\pi: \quad \Phi^{(3)}_{\mu\nu\lambda}\tilde{t}^{(3)\mu\nu\lambda} = \bar{u}_N\gamma_5\gamma_\mu u_{*\nu\lambda} p^\mu_\pi p^\nu_\pi p^\lambda_\pi\cdot 12C_\Phi, \tag{37}$$

$$N^*(\tfrac{5}{2}^-)\to N\pi: \quad \phi^{(2)}_{\mu\nu}\tilde{t}^{(2)\mu\nu}=\bar{u}_N u_{*\mu\nu} p^\mu_\pi p^\nu_\pi\cdot 4, \tag{38}$$

$$N^*(\tfrac{7}{2}^+)\to N\pi: \quad \phi^{(3)}_{\mu\nu\lambda}\tilde{t}^{(3)\mu\nu\lambda}=\bar{u}_N u_{*\mu\nu\lambda} p^\mu_\pi p^\nu_\pi p^\lambda_\pi\cdot 8, \tag{39}$$

$$N^*(\tfrac{7}{2}^-)\to N\pi: \quad \Phi^{(4)}_{\mu\nu\lambda\delta}\tilde{t}^{(4)\mu\nu\lambda\delta} = \bar{u}_N\gamma_5\gamma_\mu u_{*\nu\lambda\delta} p^\mu_\pi p^\nu_\pi p^\lambda_\pi p^\delta_\pi\cdot 48C_\Phi, \tag{40}$$

with

$$C_\Phi=\left(1+\frac{m_N}{m_*}-\frac{m_\pi^2}{m_*^2+m_* m_N}\right), \tag{41}$$

u_N, and u_* are the Rarita-Schwinger wave functions of N and N^*, respectively; m_N, and m_* are the mass of N and N^*, respectively; and p_π is the 4-momentum of the pion. We see the two approaches are equivalent here up to some constants or a smooth m_*-dependent factor C_Φ. This is because, for any $N^*\to N\pi$ process, there is only one possible L-S coupling and hence only one independent coupling.

B. $N^*\to N\omega$

Unlike a pion with spin 0, here ω has a spin 1. For N^* with a spin $\frac{1}{2}$ there are two independent L-S couplings conserving parity [Eq. (23)] and total angular momentum [Eq. (8)]; for N^* with spin larger than $\frac{1}{2}$, there are three independent L-S couplings. Here we list them for N^* with spin up to 7/2.

$$(S_C, S_{AB}, L_{BC}): \quad \mathbf{S}_{AB}+\mathbf{S}_C+\mathbf{L}_{\mathbf{BC}}=0$$

$$N^*(\tfrac{1}{2}^+)\to N\omega(1,0,1):\quad \phi^{(0)}\varepsilon^*_\mu\tilde{t}^{(1)\mu}, \tag{42}$$

$$(1,1,1):\quad i\Phi^{(1)}_\mu\epsilon^{\mu\nu\lambda\sigma}\varepsilon^*_\nu\tilde{t}^{(1)}_\lambda\hat{p}_{*\sigma}, \tag{43}$$

$$N^*(\tfrac{1}{2}^-)\to N\omega(1,1,0):\quad \Phi^{(1)}_\mu\varepsilon^{*\mu}, \tag{44}$$

$$(1,1,2):\quad \Phi^{(1)}_\mu\varepsilon^*_\nu\tilde{t}^{(2)\mu\nu}, \tag{45}$$

$$N^*(\tfrac{3}{2}^+)\to N\omega(1,1,1):\quad i\phi^{(1)}_\mu\epsilon^{\mu\nu\lambda\sigma}\varepsilon^*_\nu\tilde{t}^{(1)}_\lambda\hat{p}_{*\sigma}, \tag{46}$$

$$(1,2,1):\quad \Phi^{(2)}_{\mu\nu}\varepsilon^{*\mu}\tilde{t}^{(1)\nu}, \tag{47}$$

$$(1,2,3):\quad \Phi^{(2)}_{\mu\nu}\varepsilon^*_\lambda\tilde{t}^{(3)\mu\nu\lambda}, \tag{48}$$

$$N^*(\tfrac{3}{2}^-)\to N\omega(1,1,0):\quad \phi^{(1)}_\mu\varepsilon^{*\mu}, \tag{49}$$

$$(1,1,2):\quad \phi^{(1)}_\mu\varepsilon^*_\nu\tilde{t}^{(2)\mu\nu}, \tag{50}$$

$$(1,2,2):\quad i\Phi^{(2)}_{\mu\alpha}\epsilon^{\mu\nu\lambda\sigma}\varepsilon^*_\nu\tilde{t}^{(2)\alpha}_\lambda\hat{p}_{*\sigma}, \tag{51}$$

$$N^*(\tfrac{5}{2}^+)\to N\omega(1,2,1):\quad \phi^{(2)}_{\mu\nu}\varepsilon^{*\mu}\tilde{t}^{(1)\nu}, \tag{52}$$

$$(1,2,3):\quad \phi^{(2)}_{\mu\nu}\varepsilon^*_\lambda\tilde{t}^{(3)\mu\nu\lambda}, \tag{53}$$

$$(1,3,3):\quad i\Phi^{(3)}_{\mu\alpha\beta}\epsilon^{\mu\nu\lambda\sigma}\varepsilon^*_\nu\tilde{t}^{(3)\alpha\beta}_\lambda\hat{p}_{*\sigma}, \tag{54}$$

$$N^*(\tfrac{5}{2}^-)\to N\omega(1,2,2):\quad i\phi^{(2)}_{\mu\alpha}\epsilon^{\mu\nu\lambda\sigma}\varepsilon^*_\nu\tilde{t}^{(2)\alpha}_\lambda\hat{p}_{*\sigma}, \tag{55}$$

$$(1,3,2):\quad \Phi^{(3)}_{\mu\nu\lambda}\varepsilon^{*\mu}\tilde{t}^{(2)\nu\lambda}, \tag{56}$$

$$(1,3,4):\quad \Phi^{(3)}_{\mu\nu\lambda}\varepsilon^*_\sigma\tilde{t}^{(4)\mu\nu\lambda\sigma}, \tag{57}$$

$$N^*(\tfrac{7}{2}^+)\to N\omega(1,3,3):\quad i\phi^{(3)}_{\mu\alpha\beta}\epsilon^{\mu\nu\lambda\sigma}\varepsilon^*_\nu\tilde{t}^{(3)\alpha\beta}_\lambda\hat{p}_{*\sigma}, \tag{58}$$

$$(1,4,3):\quad \Phi^{(4)}_{\mu\nu\lambda\sigma}\varepsilon^{*\mu}\tilde{t}^{(3)\nu\lambda\sigma}, \tag{59}$$

$$(1,4,5):\quad \Phi^{(4)}_{\mu\nu\lambda\sigma}\varepsilon^*_\delta\tilde{t}^{(5)\mu\nu\lambda\sigma\delta}, \tag{60}$$

$$N^*(\tfrac{7}{2}^-)\to N\omega(1,3,2):\quad \phi^{(3)}_{\mu\nu\lambda}\varepsilon^{*\mu}\tilde{t}^{(2)\nu\lambda}, \tag{61}$$

$$(1,3,4):\quad \phi^{(3)}_{\mu\nu\lambda}\varepsilon^*_\sigma\tilde{t}^{(4)\mu\nu\lambda\sigma}, \tag{62}$$

$$(1,4,4):\quad i\Phi^{(4)}_{\mu\alpha\beta\gamma}\epsilon^{\mu\nu\lambda\sigma}\varepsilon^*_\nu\tilde{t}^{(4)\alpha\beta\gamma}_\lambda\hat{p}_{*\sigma}. \tag{63}$$

where $\hat{p}_{*\sigma}=p_{*\sigma}/m_*$. In the N^* at-rest system, $\hat{p}_*=(1,0,0,0)$; $\epsilon^{\mu\nu\lambda\sigma}S_\mu L_\nu J_\lambda\hat{p}_{*\sigma}=(\mathbf{S}\times\mathbf{L})\cdot\mathbf{J}$ is the standard form for forming a total angular momentum $|\mathbf{J}|=1$ from two other angular momenta (S,L) of absolute value 1. In the covariant L-S tensor formalism, for S-L-J coupling, if $S+L+J$ is an odd number, then $\epsilon^{\mu\nu\lambda\sigma}\hat{p}_{A\sigma}$ is needed. These are the only possible independent couplings because the fact that $p_{*\sigma}t^{(n)\sigma\mu\cdots}=0$, $p_{*\sigma}\phi^{(n)\sigma\mu\cdots}=0$ and $p_{*\sigma}\Phi^{(n)\sigma\mu\cdots}=0$. The corresponding couplings from the simple effective Lagrangian approach are give in Ref. [17]. They have the same number of independent couplings as here and are linear combinations of couplings here. For example, for $N^*\frac{3}{2}^-\to N\omega$, the full amplitude in the covariant L-S scheme is

$$A=g_1\phi^{(1)}_\mu\varepsilon^\mu+g_2\phi^{(1)}_\mu\varepsilon_\nu\tilde{t}^{(2)\mu\nu}+g_3 i\Phi^{(2)}_{\mu\alpha}\epsilon^{\mu\nu\lambda\sigma}\varepsilon_\nu\tilde{t}^{(2)\alpha}_\lambda p_{*\sigma} \tag{64}$$

with vertex form factors g_1, g_2, and g_3, while in the simple effective Lagrangian approach [17] is

$$A=f_1\bar{u}_N u_{*\mu}\varepsilon^\mu+f_2\bar{u}_N\gamma_\nu u_{*\mu}p^\mu_N\varepsilon^\nu+f_3\bar{u}_N u_{*\mu}p^\mu_\omega\varepsilon_\nu p^\nu_N \tag{65}$$

with vertex form factors f_1, f_2, and f_3. These vertex form factors are smooth functions of m_* with practically constant m_N and m_ω; they have no dependence on angular variable. With some simple algebra and the identity [22]

$$\begin{aligned} i\epsilon_{\mu abc}=\gamma_5(&\gamma_\mu\gamma_a\gamma_b\gamma_c-\gamma_\mu\gamma_a g_{bc}+\gamma_\mu\gamma_b g_{ac}-\gamma_\mu\gamma_c g_{ab}\\ &-\gamma_a\gamma_b g_{\mu c}+\gamma_a\gamma_c g_{\mu b}-\gamma_b\gamma_c g_{\mu a}+g_{\mu a}g_{bc}\\ &-g_{\mu b}g_{ac}+g_{\mu c}g_{ab}), \end{aligned} \tag{66}$$

we have

$$\phi^{(1)}_\mu\varepsilon^\mu=\bar{u}_N u_{*\mu}\varepsilon^\mu, \tag{67}$$

$$\phi^{(1)}_\mu\varepsilon_\nu\tilde{t}^{(2)\mu\nu}=2\left(-1+\frac{m_N^2-m_\omega^2}{m_*^2}\right)\bar{u}_N u_{*\mu}p^\mu_\omega\varepsilon_\nu p^\nu_N+\frac{1}{3}\mathbf{r}^2\bar{u}_N u_{*\mu}\varepsilon^\mu, \tag{68}$$

$$\begin{aligned} i\Phi^{(2)}_{\mu\alpha}\epsilon^{\mu\nu\lambda\sigma}\varepsilon_\nu\tilde{t}^{(2)\alpha}_\lambda\hat{p}_{*\sigma} &=2\left(-3+\frac{m_N^2-m_\omega^2}{m_*^2}\right)\bar{u}_N u_{*\mu}p^\mu_\omega\varepsilon_\nu p^\nu_N+\mathbf{r}^2\bar{u}_N u_{*\mu}\varepsilon^\mu\\ &\quad+4\frac{(m_*+m_N)^2-m_\omega^2}{m_*}\bar{u}_N\gamma_\nu u_{*\mu}p^\mu_N\varepsilon^\nu, \end{aligned} \tag{69}$$

which give the relations between g_i and f_i vertex form factors:

$$f_1=g_1+\frac{1}{3}\mathbf{r}^2 g_2+\mathbf{r}^2 g_3, \tag{70}$$

$$f_2=4\frac{(m_*+m_N)^2-m_\omega^2}{m_*}g_3, \tag{71}$$

$$f_3=2\left(-1+\frac{m_N^2-m_\omega^2}{m_*^2}\right)g_2+2\left(-3+\frac{m_N^2-m_\omega^2}{m_*^2}\right)g_3. \tag{72}$$

g_i and f_i are related by some smooth m_* dependence factors. For an N^* with very broad width, this may cause some model dependence on the determination of their mass and width.

C. $\psi \to N^* \bar{N}$

Here we give an example of a vector meson decaying into the $N^*\bar{N}$ final state.

$$(S_A, S_{BC}, L_{BC}): \quad -\mathbf{S_A}+\mathbf{S_{BC}}+\mathbf{L_{BC}}=0$$

$$\psi \to N^*(\tfrac{1}{2}^+)\bar{N}(1,1,0): \quad \Psi^{(1)}_{\mu}\varepsilon^{\mu}, \tag{73}$$

$$(1,1,2): \quad \Psi^{(1)}_{\mu}\varepsilon_{\nu}\tilde{t}^{(2)\mu\nu}, \tag{74}$$

$$\psi \to N^*(\tfrac{1}{2}^-)\bar{N}(1,0,1): \quad \psi^{(0)}\varepsilon_{\mu}\tilde{t}^{(1)\mu}, \tag{75}$$

$$(1,1,1): \quad i\Psi^{(1)}_{\mu}\epsilon^{\mu\nu\lambda\sigma}\varepsilon_{\nu}\tilde{t}^{(1)}_{\lambda}\hat{p}_{(\psi)\sigma}, \tag{76}$$

$$\psi \to N^*(\tfrac{3}{2}^+)\bar{N}(1,1,0): \quad \psi^{(1)}_{\mu}\varepsilon^{\mu}, \tag{77}$$

$$(1,1,2): \quad \psi^{(1)}_{\mu}\varepsilon_{\nu}\tilde{t}^{(2)\mu\nu}, \tag{78}$$

$$(1,2,2): \quad i\Psi^{(2)}_{\mu\alpha}\epsilon^{\mu\nu\lambda\sigma}\varepsilon_{\nu}\tilde{t}^{(2)\alpha}_{\lambda}\hat{p}_{(\psi)\sigma}, \tag{79}$$

$$\psi \to N^*(\tfrac{3}{2}^-)\bar{N}(1,1,1): \quad i\psi^{(1)}_{\mu}\epsilon^{\mu\nu\lambda\sigma}\varepsilon_{\nu}\tilde{t}^{(1)}_{\lambda}\hat{p}_{(\psi)\sigma}, \tag{80}$$

$$(1,2,1): \quad \Psi^{(2)}_{\mu\nu}\varepsilon^{\mu}\tilde{t}^{(1)\nu}, \tag{81}$$

$$(1,2,3): \quad \Psi^{(2)}_{\mu\nu}\varepsilon_{\lambda}\tilde{t}^{(3)\mu\nu\lambda}, \tag{82}$$

$$\psi \to N^*(\tfrac{5}{2}^+)\bar{N}(1,2,2): \quad i\psi^{(2)}_{\mu\alpha}\epsilon^{\mu\nu\lambda\sigma}\varepsilon_{\nu}\tilde{t}^{(2)\alpha}_{\lambda}\hat{p}_{(\psi)\sigma}, \tag{83}$$

$$(1,3,2): \quad \Psi^{(3)}_{\mu\nu\lambda}\varepsilon^{\mu}\tilde{t}^{(2)\nu\lambda}, \tag{84}$$

$$(1,3,4): \quad \Psi^{(3)}_{\mu\nu\lambda}\varepsilon_{\sigma}\tilde{t}^{(4)\mu\nu\lambda\sigma}, \tag{85}$$

$$\psi \to N^*(\tfrac{5}{2}^-)\bar{N}(1,2,1): \quad \psi^{(2)}_{\mu\nu}\varepsilon^{\mu}\tilde{t}^{(1)\nu}, \tag{86}$$

$$(1,2,3): \quad \psi^{(2)}_{\mu\nu}\varepsilon_{\lambda}\tilde{t}^{(3)\mu\nu\lambda}, \tag{87}$$

$$(1,3,3): \quad i\Psi^{(3)}_{\mu\alpha\beta}\epsilon^{\mu\nu\lambda\sigma}\varepsilon_{\nu}\tilde{t}^{(3)\alpha\beta}_{\lambda}\hat{p}_{(\psi)\sigma}, \tag{88}$$

$$\psi \to N^*(\tfrac{7}{2}^+)\bar{N}(1,3,2): \quad \psi^{(3)}_{\mu\nu\lambda}\varepsilon^{\mu}\tilde{t}^{(2)\nu\lambda}, \tag{89}$$

$$(1,3,4): \quad \psi^{(3)}_{\mu\nu\lambda}\varepsilon_{\sigma}\tilde{t}^{(4)\mu\nu\lambda\sigma}, \tag{90}$$

$$(1,4,4): \quad i\Psi^{(4)}_{\mu\alpha\beta\gamma}\epsilon^{\mu\nu\lambda\sigma}\varepsilon_{\nu}\tilde{t}^{(4)\alpha\beta\gamma}_{\lambda}\hat{p}_{(\psi)\sigma}, \tag{91}$$

$$\psi \to N^*(\tfrac{7}{2}^-)\bar{N}(1,3,3): \quad i\psi^{(3)}_{\mu\alpha\beta}\epsilon^{\mu\nu\lambda\sigma}\varepsilon_{\nu}\tilde{t}^{(3)\alpha\beta}_{\lambda}\hat{p}_{(\psi)\sigma}, \tag{92}$$

$$(1,4,3): \quad \Psi^{(4)}_{\mu\nu\lambda\sigma}\varepsilon^{\mu}\tilde{t}^{(3)\nu\lambda\sigma}, \tag{93}$$

$$(1,4,5): \quad \Psi^{(4)}_{\mu\nu\lambda\sigma}\varepsilon_{\delta}\tilde{t}^{(5)\mu\nu\lambda\sigma\delta}. \tag{94}$$

Corresponding couplings in the effective Lagrangian approach are give in Ref. [17]. In the multipole approach, the amplitude for $\psi \to N^*\bar{N}$ generally takes the form

$$A=\sum_{L,m_L,S,m_S}(L,m_L;S,m_S|1,m_{\psi}) \times (S_B,m_B;S_C,m_C|S,m_S)Y_{Lm_L}(\hat{\mathbf{p}}_{\mathbf{N}})G_{LS}|\mathbf{p_N}|^L f_L(|\mathbf{p_N}|), \tag{95}$$

where G_{LS} is the coupling constant for the final state with orbital angular momentum L and total spin S, $\mathbf{p_N}$ is the momentum of $\bar{N}$ in the rest frame of ψ and $f_L(|\mathbf{p}_{bfN}|)$ is the vertex form factor. Taking $\psi \to N^*(\frac{1}{2}^+)\bar{N}$ as an example, the amplitude is

$$\begin{aligned}A=&(\tfrac{1}{2},m_B;\tfrac{1}{2}m_C|1,m_{\psi})Y_{00}(\hat{\mathbf{p}}_{\mathbf{N}})G_{01}f_0(|\mathbf{p_N}|)\\&+(2,m_L;1,m_S|1,m_{\psi})\\&\times(\tfrac{1}{2},m_B;\tfrac{1}{2}m_C|1,m_S)Y_{2m_L}(\hat{\mathbf{p}}_{\mathbf{N}})G_{21}|\mathbf{p_N}|^2 f_2(|\mathbf{p_N}|),\end{aligned} \tag{96}$$

with $m_S=m_B+m_C$ and $m_L=m_{\psi}-m_S$. With some simple algebra, the corresponding amplitude in the covariant L-S scheme can be reduced to the similar form:

$$\begin{aligned}A=&g_0\Psi^{(1)}_{\mu}\varepsilon^{\mu}f_0(|\mathbf{p_N}|)+g_2\Psi^{(1)}_{\mu}\varepsilon_{\nu}\tilde{t}^{(2)\mu\nu}f_2(|\mathbf{p_N}|)\\=&(\tfrac{1}{2},m_B;\tfrac{1}{2}m_C|1,m_{\psi})Y_{00}(\hat{\mathbf{p}}_{\mathbf{N}})g_0\sqrt{8\pi}C_{\Psi}f_0(|\mathbf{p_N}|)\\&+(2,m_L;1,m_S|1,m_{\psi})(\tfrac{1}{2},m_B;\tfrac{1}{2}m_C|1,m_S)\\&\times Y_{2m_L}(\hat{\mathbf{p}}_{\mathbf{N}})g_2\tfrac{8}{3}\sqrt{4\pi}C_{\Psi}|\mathbf{p_N}|^2 f_2(|\mathbf{p_N}|).\end{aligned} \tag{97}$$

Comparing Eqs. (96) and (97), we have

$$G_{01}=g_0\sqrt{8\pi}C_{\Psi}, \tag{98}$$

$$G_{21}=g_2\frac{8}{3}\sqrt{4\pi}C_{\Psi}. \tag{99}$$

In nonrelativistic limit, $C_{\Psi}=1$, and the covariant L-S scheme gives G_{01} and G_{21} as constants; but generally speaking, the covariant L-S scheme results in G_{01} and G_{21} smoothly dependent on $|\mathbf{p_N}|$.

As a concrete example, here we study the angular distribution and the relative ratio of D and S waves in the final states of $e^+e^- \to J/\psi \to p\bar{p}$. For this process of positron-electron collision, J/ψ spin projection is limited to be ± 1 along the beam direction. The differential decay rate of the J/ψ is related to the amplitude A as

$$\frac{d\Gamma}{d\Omega}=\frac{1}{32\pi^2}\overline{|A|^2}\frac{|\mathbf{p_N}|}{M^2_{\psi}}. \tag{100}$$

With A given by Eq. (97), we have

$$\overline{|A|^2}=\frac{m_\psi^2}{m_p^2}\left(\frac{1}{2}C_S^2+\frac{20}{9}C_D^2-\frac{2}{3}C_SC_D\cos\beta\right)(1+\alpha\cos^2\theta), \tag{101}$$

where

$$\alpha=\frac{2C_SC_D\cos\beta-\frac{4}{3}C_D^2}{\frac{1}{2}C_S^2+\frac{20}{9}C_D^2-\frac{2}{3}C_SC_D\cos\beta} \tag{102}$$

with $C_S\equiv|g_0|f_0(|\mathbf{p_N}|)$, $C_D\equiv|g_2|\mathbf{p_N}^2f_2(|\mathbf{p_N}|)$ and β the relative phase between C_S and C_D. If $C_D=0$, then $\alpha=0$ as expected for a pure S-wave decay; if $C_S=0$, then $\alpha=-\frac{3}{5}$ for the pure D-wave decay.

The relative ratio $R_{D/S}$ of D and S-wave decay rates is

$$R_{D/S}\equiv\frac{\Gamma_D}{\Gamma_S}=\frac{32C_D^2}{C_S^2}. \tag{103}$$

The experimental value of α for the $e^+e^-\to J/\psi\to p\bar{p}$ process is about 0.62 [23]. This gives the ratio $R_{D/S}$ to be in the range of 0.09∼1.9. The large uncertainty is due to the unknown relative phase β between S and D-wave amplitudes. For a full determination of the ratio $R_{D/S}$, the polarization information of final state particles is needed.

IV. DISCUSSION

Comparing with the simple effective Lagrangian approach, each coupling in the covariant L-S scheme corresponds to a single L final states while a coupling in the simple effective Lagrangian approach may be a mixture of two L final states. The number of independent couplings is the same in the two approaches, as it should be. In the simple effective Lagrangian approach, it is not necessary that the independent couplings be orthogonal to each other while in the covariant L-S scheme, they are orthogonal and make the partial wave analysis easier. The construction of the full amplitude in the covariant L-S scheme for a multistep process, e.g., $J/\psi\to N^*\bar{N}\to\omega N\bar{N}$, is similar to the simple effective Lagrangian approach [17]. The coupling constants for each couplings are fitted to the data in the procedure of partial wave analysis [8].

For a partial wave analysis, we only demand very basic requirements, i.e., Lorentz, CPT, and C and P invariances, for the amplitude and we make formalism more general. Various theories or models or assumptions can bring more constraints to the relations of various couplings, and hence reduce the number of independent couplings. For example, a chiral quark model calculation [24] results in a single coupling form for the $N^*(1675)(\frac{5}{2}^-)N\omega$ coupling, which corresponds to our (1,2,2) coupling of Eq. (55), while other quark model [25] gives different prediction. This can be checked in the future by partial wave analysis of processes involving $N^*(1675)(\frac{5}{2}^-)\to N\omega$. Some authors [16] assumed $N^*(\frac{3}{2}^\pm)N\omega$ couplings to have the same structure as $N^*(\frac{3}{2}^\pm)N\gamma$ couplings; hence only two independent couplings. In our general scheme, we have three independent couplings for $N^*(\frac{3}{2}^\pm)N\omega$ couplings; the gauge invariance requirement for the $N^*(\frac{3}{2}^\pm)N\gamma$ couplings reduces the number of independent couplings to two for the $N^*(\frac{3}{2}^\pm)N\gamma$ couplings.

In this paper we have given explicit formulas for $N^*\to N\pi$, $N^*\to N\omega$, and $\psi\to N^*\bar{N}$ as examples, since the relevant processes are understudy by experimental groups. For any baryon resonance decaying to a $\frac{1}{2}^+$ baryon plus a pseudoscalar meson through strong interaction, e.g., $N^*\to\Lambda K$, $N^*\to\Sigma K$, $\Lambda^*\to NK$, $\Lambda^*\to\Sigma\pi$, etc., the coupling has the same form as for $N^*\to N\pi$, the only difference is the coupling constants. For any baryon resonance strong decaying to a $\frac{1}{2}^+$ baryon plus a vector meson, the coupling has the same form as for $N^*\to N\omega$. For any vector meson strong decaying to a baryon resonance plus an anti $(\frac{1}{2}^+)$baryon, the coupling has the same form as for $\psi\to N^*\bar{N}$. Extension to other processes are straightforward by following the basic rules outlined in this work.

In our present L-S scheme for N^* decays, we have added the spin of the incoming nucleon resonance and the final nucleon. This is different with the usual L-S scheme where it is always the spin of the final state particles which are added to make the total spin S. The two schemes are simply related by recoupling various angular momenta involved. With recoupling technique in Ref. [26], we have the following relation between the two schemes:

$$[[\mathbf{S}_A\times\mathbf{S}_B]_{S_{AB}}\times\mathbf{S}_C]_{LM}=\sum_{S_{\mathbf{BC}}}\sqrt{(2S_{AB}+1)(2S_{BC}+1)}W(S_AS_BLS_C;S_{AB}S_{BC})\times[\mathbf{S}_A\times[\mathbf{S}_B\times\mathbf{S}_C]_{S_{BC}}]_{LM}, \tag{104}$$

where $W(S_AS_BLS_C;S_{AB}S_{\mathbf{BC}})$ is the usual Racah coefficients [26]. From this relation, after we obtain the coupling constants in our scheme, $g(S_{AB},L)$, we can easily obtain the corresponding coupling constants in the usual L-S scheme, $G(S_{BC},L)$, as

$$G(S_{BC},L)=\sum_{S_{AB}}g(S_{AB},L)\sqrt{(2S_{AB}+1)(2S_{BC}+1)}W\times(S_AS_BLS_C;S_{AB}S_{BC}). \tag{105}$$

Since the covariant L-S scheme combines the merits of two conventional schemes, i.e., covariant effective Lagrangian approach and the multipole analysis with amplitudes expanded according to the angular momentum L, we recommend it to be used in future partial wave analyses.

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