Collision damping in the $\pi^3\text{He} \to d'N$ reaction near the threshold

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We present a simple quantum mechanical model exploiting the optical potential approach for the description of collision damping in the reaction $\pi^3\text{He} \to d'N$ near the threshold, which recently has been measured at TRIUMF. The influence of the open $d'N \to NNN$ channel is taken into account. It leads to a suppression factor of about 10 in the d' survival probability. Applications of the method to other reactions are outlined.

I. INTRODUCTION

In this paper we consider the inelastic final state interaction (FSI) between the neutron and the hypothetical d' resonance with the quantum numbers $T=0$, $J^P=0^-$ produced near threshold in the reaction $\pi^3\text{He} \to d'N$. This resonance has been suggested to explain the observed peculiar resonancelike behavior [6] of the pionic double charge exchange (DCX) on nuclei, $\pi^+ + A \to A' + \pi^-$, ranging from ^7Li to ^{93}Nb at $T_\pi \approx 50$ MeV (see Refs. [1–3], and references therein). Such a NN-decoupled dibaryon had been predicted by QCD-inspired models [4,5]. In a sequence of recent papers this behavior for the DCX reaction, on Ca isotopes $^{42,44,48}\text{Ca}$, was qualitatively reproduced by Nuseirat *et al.* [7] and, for the medium and heavy nuclei, in the generalized seniority model, by Wu and Gibbs [8]. Still there is no conventional model, which could reproduce these low-energy phenomena consistently for all nuclei. An independent test of the idea would be investigation of the $pp\pi^-$ invariant mass spectrum in the double pion production, $pp \to pp\pi^- \pi^+$, since the quantum numbers of the resonance forbid strong coupling to np. Recent measurements at CELSIUS [9] do not show evidence of the resonance at the predicted level in contrast to data from ITEP taken at a higher proton energy [10]. Thus the situation appears not yet fully settled. Another way of searching for the d' resonance is DCX on helium isotopes, where d' should manifest itself as a threshold phenomenon. At energies around 100 MeV the total cross section of the reaction $^4\text{He}(\pi^+, \pi^-)$ shows an excess over conventional calculations which could be ascribed to the contribution of the d' resonance [11]. At the same time, the contribution of this resonance for both, $^4\text{He}(\pi^+, \pi^-)$ [11] and $^3\text{He}(\pi^-, \pi^+)$ [12], occurred almost an order of magnitude smaller than expected. The reason for the suppression of the production cross sections near threshold can be due to the collision damping $d'N \to NNN$, and in the present paper we study this hypothesis using a simple quantum mechanical model to describe the propagation of an unstable resonance

in the nuclear medium. Although singularities near production threshold always attracted much attention and are described in many textbooks (see, e.g., Refs. [13,14], and references therein), we are not aware of similar approaches to the final state interaction suggested before.

The paper is organized as follows. In the second section we present a simple quantum mechanical model exploiting the optical potential approach to account for the effect of collision damping. In the third section we estimate the spreading width of the d' in the nuclear medium, namely, the virtual pion exchange contribution to the spreading is calculated in detail. In the next, fourth, section we give our numerical estimates of collision damping and of the d' survival probability in the nuclear medium, and compare the latter to the experimental data. The fifth section contains our conclusions and outlook.

II. A SIMPLE QUANTUM MECHANICAL MODEL

In this section we present a simple quantum mechanical model to describe the propagation of a resonance in the nuclear medium. As an example, we consider the behavior of the hypothetical resonance d' which can contribute to the pionic DCX on ^3He.

Due to the very short interaction time of the pion with the helium nucleus it is the sudden influence that is the appropriate mechanism for considering the $\pi^3\text{He} \to d'N$ reaction [15]. Thus the $d'N$ system is created at the initial moment, $t=0$, with the wave function of helium ψ_{He} and then separates with time as

$$\psi_{d'N}(\mathbf{r}, t) = \psi(\mathbf{r} - \mathbf{v}t, 0) = \psi_{\text{He}}(\mathbf{r} - \mathbf{v}t), \tag{1}$$

with the velocity v defined by the energy excess over the threshold ε,

$$v = \sqrt{\frac{2\varepsilon}{\mu}},$$

where $\mu \approx \frac{2}{3} m_N$ is the reduced mass of the $d'N$ system.

If the potential in the effective Schrödinger equation for the wave function (1) contains an imaginary part,

$$U(\mathbf{r}) = U_0(\mathbf{r}) + i U_1(\mathbf{r}), \quad U_1(\mathbf{r}) < 0, \tag{2}$$

then the probability to find the system in its initial state, $d'N$, decreases with time approaching a finite limit with the survival probability being simply

$$w_{surv} = \frac{|\psi_{d'N}(t \to \infty)|^2}{|\psi_{d'N}(t \to \infty)|^2_{U_1 = 0}}. \tag{3}$$

Let us, for simplicity, consider the steplike form of the potential $U_1(\mathbf{r})$,

$$U_1(\mathbf{r}) = \begin{cases} U_1 = \text{const}, & |\mathbf{r}| \leq R_1 \\ 0, & |\mathbf{r}| > R_1, \end{cases} \tag{4}$$

where the potential strength U_1 can be related to the elastic s-wave $d'N$ zero-angle amplitude using the Born approximation:

$$\text{Im} F(0) \approx -\frac{\mu}{2\pi} \int U_1(\mathbf{r}) d^3 r = -\frac{2}{3} \mu R_1^3 U_1, \tag{5}$$

whereas, according to the optical theorem,

$$\text{Im} F(0) = \frac{\mu v}{4\pi} \sigma_{tot} \approx \frac{\mu v}{4\pi} \sigma_{in}. \tag{6}$$

The problem of separation of the elastic and inelastic parts of the cross section in such kinds of reactions is a very subtle one. One of the relevant worries concerns the possible broadening of the ground state, so that a slight change in the resonance position could have been erroneously ascribed to the inelastic part. Bearing in mind that, in general, such an effect could lead to an overestimate of the damping in Eq. (6), we still do not expect it to change our estimates dramatically. Indeed, from *a posteriori* estimate [see Eq. (44) below] we find the elastic part of the cross section to be an order of magnitude smaller than the corresponding inelastic part, and thus it appears beyond the accuracy of the present paper.

From Eqs. (5) and (6) one finds

$$U_1 = -\frac{3}{8\pi} \frac{\sigma_{in} v}{R_1^3}. \tag{7}$$

Notice that we do not meet any constraints on the wavelength of the resonance since, in the Born approximation, the zero-angle amplitude (5) does not depend on the energy of d'.

As a direct consequence of the Schrödinger equation one has the probability conservation law at any moment in time in the form

$$\frac{d}{dt} \int_V \rho(\mathbf{r},t) d^3 r + \int_S \mathbf{j}(\mathbf{r},t) d\mathbf{s} = 2 \int_V U_1(\mathbf{r}) \rho(\mathbf{r},t) d^3 r, \tag{8}$$

where

$$\rho(\mathbf{r},t) = |\psi_{d'N}(\mathbf{r},t)|^2,$$

$$\mathbf{j}(\mathbf{r},t) = \frac{i}{2\mu} \psi_{d'N}(\mathbf{r},t) \nabla \psi^*_{d'N}(\mathbf{r},t) + \text{c.c.} \tag{9}$$

are the probability density and the probability flux through the corresponding surface S, respectively.

The term on the right hand side of Eq. (8) describes absorption of particles. In the absence of this term the density of particles would evolve in space and time, according to Eq. (1), as

$$\rho_0(\mathbf{r},t) = |\psi_{d'N}(\mathbf{r} - \mathbf{v}t,0)|^2 = |\psi_{\text{He}}(\mathbf{r} - \mathbf{v}t)|^2 = \rho_0(\mathbf{r} - \mathbf{v}t, 0), \tag{10}$$

if quantum spreading of the initial wave packet is neglected.

Equation (8) becomes especially simple if the size of the wave packet, hereafter called R_2, is small compared to the radius of the optical potential, R_1. Indeed, one can split the entire evolution time into two periods, $0 \leq t \leq T$ and $t > T$, where $T = R_1/v$ is the moment of time when d' and N leave the region of interaction. Then the solution to Eq. (8) reads

$$\rho(\mathbf{r},t) = \begin{cases} \rho_0(\mathbf{r},t) e^{-\Gamma t}, & t \leq T \\ \rho_0(\mathbf{r},t) e^{-\Gamma T}, & t > T, \end{cases} \tag{11}$$

with

$$\Gamma = 2|U_1|. \tag{12}$$

Hence the survival probability is

$$w_{surv} = \exp\left(\frac{2 U_1 R_1}{v}\right) = \exp\left(-\frac{3\sigma_{in}}{4\pi R_1^2}\right), \tag{13}$$

where we also used the relation (7).

The opposite limit, $R_2 \ll R_1$, leads to the expression similar to Eq. (13) with R_1 interchanged with R_2:

$$w_{surv} = \exp\left(-\frac{3\sigma_{in}}{4\pi R_2^2}\right). \tag{14}$$

To see this let us consider the following anzatz for the probability density:

$$\rho(\mathbf{r},t) = \rho_0(\mathbf{r},t) w(t), \tag{15}$$

so that the survival probability defined above is just $w_{surv} = w(\infty)$. After integration over an infinitely large volume V (bounded by an infinitely remote surface S) in Eq. (8) one finds the following equation for the function $w(t)$:

$$\frac{d}{dt} w(t) = -w(t) \int_V 2|U_1(\mathbf{r})| \rho_0(\mathbf{r} - \mathbf{v}t, 0) d^3 r, \quad w(0) = 1, \tag{16}$$

which can be easily solved with the result

$$w(t)=\exp\left[-\int_0^t dt'\int_V 2|U_1(\mathbf{r})|\rho_0(\mathbf{r}-\mathbf{v}t',0)d^3r\right].$$

$$(17)$$

We further simplify our estimates and consider the step-like form for both functions, $U_1(\mathbf{r})$ and $\rho_0(\mathbf{r},t)$:

$$U_1(\mathbf{r})=-\frac{\sigma_{in}v}{2V_1}\Theta(R_1-|\mathbf{r}|),$$

$$\rho_0(\mathbf{r},t)=\frac{1}{V_2}\Theta(R_2-|\mathbf{r}-\mathbf{v}t|),\quad V_{1,2}=\frac{4}{3}\pi R_{1,2}^3.\quad (18)$$

For the two limiting cases, $R_1\gg R_2$ and $R_1\ll R_2$, the integral $\int 2|U_1|\rho_0 d^3r$ entering Eq. (16) is

$$\frac{\sigma_{in}v}{V_1V_2}\int\Theta(R_1-|\mathbf{r}|)\Theta(R_2-|\mathbf{r}-\mathbf{v}t'|)d^3r$$

$$=\frac{\sigma_{in}v}{V_{big}}\Theta(R_{big}-vt'),\quad (19)$$

where

$$R_{big}=\max\{R_1,R_2\},\quad V_{big}=\max\{V_1,V_2\},\quad (20)$$

and we have neglected the edge phenomena.

Now the integral over t' in Eq. (17) can be done trivially, which gives for the survival probability:

$$w_{surv}=\exp\left(-\frac{\sigma_{in}R_{big}}{V_{big}}\right),\quad (21)$$

which coincides with Eqs. (13) and (14).

In the general case, $R_1\sim R_2$, the survival probability can be presented as

$$w_{surv}=\exp\left[-\frac{\sigma_{in}\bar{R}}{\bar{V}}F(R_1/R_2)\right],\quad \bar{R}=\sqrt{R_1^2+R_2^2},$$

$$\bar{V}=\frac{4}{3}\pi\bar{R}^3,\quad (22)$$

with F being a smooth function of R_1/R_2, $F(0)=F(\infty)=1$.

III. VIRTUAL PION EXCHANGE CONTRIBUTION TO THE SPREADING WIDTH OF THE d' RESONANCE IN THE NUCLEAR MEDIUM

In this section we estimate the spreading width of the d' resonance in the nuclear medium. Let us start from the virtual pion contribution described by the diagram in Fig. 1.

As shown in Ref. [16], at low energies there is only one Lorenz invariant structure describing the $d'NN\pi$ vertex and, therefore, the amplitude of the decay $d'\to NN\pi$ can be written as
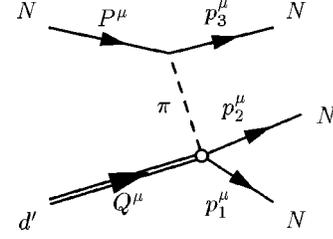
FIG. 1. The diagram for the virtual pion exchange contribution to the spreading width of the d' resonance in the nuclear medium.

$$\mathcal{M}_{d'\to NN\pi}=\frac{f}{2m_N}\bar{u}_1C\gamma_5(i\tau_2\vec{\tau})\bar{u}_2^T\vec{\pi},\quad (23)$$

where $u_{1,2}$ are bispinors and $C=\gamma_2\gamma_0$. The coupling constant f can be expressed through the $NN\pi$ decay width $\Gamma=\Gamma_{pp\pi^-}+\Gamma_{nn\pi^+}+\Gamma_{np\pi^0}$ as

$$f\approx\sqrt{\frac{128\pi^2\sqrt{2}M\Gamma}{3\,\eta_0(M-2m_N-m_\pi)^2}}\sqrt{\frac{m_N}{m_\pi}},\quad (24)$$

where m_N, m_π, and M are the nucleon, pion, and d' masses, respectively, η_0 being the enhancement factor due to the NN FSI in this decay, $\eta_0\approx4-5$. For $\Gamma\approx0.5$ MeV (as deduced from the data on the DCX reactions to discrete levels [1]) $f\approx14$.

The invariant matrix element of the process $d'N\to NNN$, when all initial and outgoing particles are on mass shell, can be written in the form

$$\mathcal{M}^{(0)}=\sum_{ijk}\varepsilon_{ijk}\mathcal{M}_{ijk}^{(0)}$$

$$=\frac{fg}{4m_N}\sum_{ijk}\bar{u}_iC\gamma_5(\tau_2\vec{\tau})\bar{u}_j^T\frac{\varepsilon_{ijk}}{(P-p_k)^2-m_\pi^2}\bar{u}_k\gamma_5\vec{\tau}u,$$

$$\sum_{i=1}^3 p_i^\mu=P^\mu+Q^\mu,\quad (25)$$

where g is the pseudoscalar $NN\pi$ coupling, $g^2/4\pi\approx14.3$, ε_{ijk} is the totally antisymmetrical tensor, and indices i,j, and $k=1,2$, and 3 numerate the outgoing nucleons. As a result, the amplitude $\mathcal{M}^{(0)}$ is totally antisymmetric under permutation of the final nucleons. See also Fig. 1 where the notations are explained.

The matrix element $\mathcal{M}^{(0)}$ contains three different contributions, hence, when squared, it produces three different diagonal terms and three cross terms. After integration over the three-particle phase space all diagonal terms equally contribute to the cross section. The same holds true for the cross terms. Hence, it is sufficient to consider only one diagonal term squared (e.g., $\{ijk\}=\{123\}$) and one of the cross terms (e.g., $\{ijk\}\times\{i'j'k'\}=\{123\}\times\{321\}$).

The diagonal term $\{123\}$ squared and averaged/summed over spins and isospins of the initial/final nucleons is

$$\langle|\mathcal{M}_{123}^{(0)}|^2\rangle=12\frac{(fg)^2}{m_N^2}(p_1p_2+m_N^2)\frac{(Pp_3-m_N^2)}{[(P-p_3)^2-m_\pi^2]^2}.$$

$$(26)$$

Integration over the phase space is essentially simplified if the initial particles are at rest, i.e., $P^\mu=(m_N,0,0,0)$ and $Q^\mu=(M,0,0,0)$, which means the limit $v\to0$. In this limit $\sigma_{in}=\sigma_{d'N\to NNN}\propto1/v$, so that the product $\sigma_{in}v$ remains constant. Thus, in this limit,

$$\langle|\mathcal{M}_{123}^{(0)}|^2\rangle$$

$$=24\frac{(fg)^2}{m_N}(E_3-m_N)\frac{(M+m_N)^2+m_N^2-2(M+m_N)E_3}{(m_\pi^2-2m_N^2+2m_NE_3)^2},$$

$$(27)$$

i.e., $\langle|\mathcal{M}_{123}^{(0)}|^2\rangle$ depends only on the energy of one of the nucleons, E_3, which ranges between m_N and

$$E_{max}=\frac{(M+m_N)^2-3m_N^2}{2(M+m_N)}.$$

$$(28)$$

Let us consider now the FSI between the nucleons in the relative s state. Since there are three identical nucleons in the exit channel, only two of them can be in the relative s wave with the third one being in the relative p wave. It is easy to see that the matrix element $\mathcal{M}_{ijk}^{(0)}$ (nucleon number k being in the $NN\pi$ vertex) is proportional to the three-momentum $|\mathbf{p}_k|$, therefore, this nucleon is in p wave in the rest frame of the initial d' and N and, hence, in p wave relative to the nucleons i and j in the $d'NN\pi$ vertex (see Fig. 1). At the same time nucleons from the $d'NN\pi$ vertex are in relative s wave [16]. It means that only the FSI between the nucleons in the $NN\pi$ vertex is important. It can be taken into account by multiplying $\mathcal{M}_{ijk}^{(0)}$ by $\psi_{\mathbf{q}}(0)$, where $\psi_{\mathbf{q}}(\mathbf{r})$ is the continuum NN wave function $[\mathbf{q}\approx(\mathbf{p}_i-\mathbf{p}_j)/2]$ containing the s-wave NN scattering amplitude, if the $d'NN\pi$ vertex is assumed to be pointlike:

$$\mathcal{M}_{ijk}\approx\mathcal{M}_{ijk}^{(0)}\left(1+\frac{R^{-1}}{-a_s^{-1}-iq_{ij}}\right),$$

$$(29)$$

where $R\approx0.8$ fm [16], a_s is the ^1S scattering length, and $\mathbf{q}_{ij}$ is the three-momentum of either nucleon, i or j, in their center-of-mass frame,

$$q_{ij}=\sqrt{\frac{1}{4}[(M+m_N)^2+m_N^2-2E_k(M+m_N)]-m_N^2}.$$

$$(30)$$

The Coulomb effects, which are important for very small invariant masses in the pp subsystem, can be neglected in the integrated cross section.

With the FSI taken into account, the diagonal term has to be replaced by

$$\langle|\mathcal{M}_{123}^{(0)}|^2\rangle\to\langle|\mathcal{M}_{123}|^2\rangle=\langle|\mathcal{M}_{123}^{(0)}|^2\rangle\left|1+\frac{R^{-1}}{-a_s^{-1}-iq_{ij}}\right|^2.$$

$$(31)$$

Finally, the summary contribution of all diagonal terms to the differential cross section is

$$d\sigma_{diag}=\frac{3}{(2\pi)^5}\frac{1}{3!}\frac{1}{4m_NMv}\langle|\mathcal{M}_{123}|^2\rangle$$

$$\times\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\frac{d^3p_3}{2E_3}\delta^4(Q+P-p_1-p_2-p_3),$$

$$(32)$$

where 3! accounts for the three identical particles in the final state.

As already mentioned, in the limit $v\to0$ the integration over the phase space is simplified since there is no angular dependence, and

$$\int\langle|\mathcal{M}_{123}|^2\rangle\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\frac{d^3p_3}{2E_3}\delta^4(Q+P-p_1-p_2-p_3)$$

$$=\pi^2\int\langle|\mathcal{M}_{123}|^2\rangle dE_2dE_3.$$

$$(33)$$

Now let us consider contributions of the cross terms (e.g., $\{123\}\times\{321\}$):

$$-\langle\mathcal{M}_{123}^{(0)}\mathcal{M}_{321}^{(0)*}+\mathcal{M}_{321}^{(0)}\mathcal{M}_{123}^{(0)*}\rangle$$

$$=-3\frac{(fg)^2}{m^2}\frac{(p_2p_3)(Pp_1-m_N^2)+(p_1p_2)(Pp_3-m_N^2)+m_N^2\left(P\sum_ip_i\right)-(p_1p_3)(Pp_2+m_N^2)-m_N^4}{[(P-p_3)^2-m_\pi^2][(P-p_1)^2-m_\pi^2]}.$$

$$(34)$$

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As $v \to 0$ this expression depends only on energies of the outgoing nucleons,

$$
\begin{aligned}
&-\langle \mathcal{M}_{123}^{(0)} M_{321}^{(0)*} + \mathcal{M}_{321}^{(0)} M_{123}^{(0)*} \rangle \\
&= -3\frac{(fg)^2}{m} \Bigg\{ (E_1 - m_N)[E_3(M + m_N) - m_N^2] \\
&\quad + (E_3 - m_N)[E_1(M + m_N) - m_N^2] \\
&\quad - M\Big[\frac{1}{2}(M^2 - 4m_N^2) - (M - E_1 - E_3)(M + m_N)\Big] \Bigg\} \\
&\quad \times \frac{1}{[(P - p_3)^2 - m_\pi^2][(P - p_1)^2 - m_\pi^2]},
\end{aligned} \tag{35}
$$

and the FSI can be taken into account in the way described above, so that each $\mathcal{M}_{ijk}^{(0)}$ has to be replaced by $\mathcal{M}_{ijk}$, as in Eq. (29).

The integration over the three-particle phase space is similar to the integration performed in Eq. (33), so that for the total inelastic cross section one finds

$$
\sigma_{in} = \frac{4.5}{v} \text{ mb}, \tag{36}
$$

where the contribution of the cross terms is positive and does not exceed 15% when $v \to 0$. The product $\sigma_{in}v$ is a smooth function of the energy over the threshold, ε, so that, e.g., for $\varepsilon = 20$ MeV (which corresponds to the kinetic energy of the nucleon of about 30 MeV in the d' rest frame) the contribution of the diagonal terms is only 10% larger than for $v \to 0$.

To proceed further we consider the d' propagation in the nuclear medium, which we assume to be infinite, for simplicity. Then, similarly to Eq. (12), one has for the spreading width,

$$
\Gamma_s = 2|\text{Im}U_A|. \tag{37}
$$

Here U_A is the summary effective potential created by nucleons in the nuclear matter,

$$
U_A = \sum_{i=1}^{N_1} U_i \approx \rho_A V_1 (U_0 + i U_1), \tag{38}
$$

where $\rho_A = \frac{1}{2}m_\pi^3$ is the density of the nuclear matter, $N_1 = \rho_A V_1$ being the number of nucleons in the interaction region of the volume V_1. Using Eqs. (7) and (37) one easily finds

$$
\Gamma_s = 2\rho_A V_1 |U_1| = \rho_A \sigma_{in} v, \tag{39}
$$

or, if the Pauli blocking factor, $\eta \approx 2.2$, is taken into account,[1] then

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[1] To estimate Pauli blocking in the reaction $d'N \to NNN$ we simply limit the phase space for the final nucleons by the constraint $p_N > P_F$, where P_F is the Fermi momentum.

$$
\Gamma_s = \rho_A(\sigma_{in}/\eta)v. \tag{40}
$$

With the help of the result (36) one can estimate the spreading width to be about 7 MeV. Notice, however, that the pion exchange (the diagram in Fig. 1) contributes only 30–40 % of the observed spreading width in nuclei, i.e., there must be an additional mechanism for the reaction $d'N \to NNN$, the total spreading width being

$$
\Gamma_s \approx 10\text{–}20 \text{ MeV}. \tag{41}
$$

Such a mechanism could be a σ exchange as discussed in Ref. [2].

Now we can check how well the approximation of the infinite nuclear medium works. To this end we have to ensure that the typical free-path length of the d' due to collision damping, $L_{d'} \sim v/\Gamma_s$, does not exceed the radius of the nucleus R_A ($\frac{4}{3}\pi R_A^3 \rho_A \equiv N_A$). For DCX to discrete levels this condition is fulfilled since for Γ_s given by Eq. (41) and $v \sim 1/10$, which corresponds to the energy $\varepsilon \approx 3$ MeV above the threshold, both lengths appear to be of the same order of magnitude, and $L_{d'} \lesssim R_A$.

Besides we can perform *a posteriori* check of the validity of the approximations made in Sec. II. Namely, we can justify the Born approximation used in Eq. (5):[2]

$$
\mu R_1^2 |U_{0,1}| \approx \frac{\eta}{4\pi} \frac{m_N \Gamma_s}{\rho_A R_1} \approx \frac{1}{3}, \tag{42}
$$

and the one used in Eq. (6) when the elastic part of the total cross section was neglected. Indeed, for the scattering off the steplike potential (2) one has

$$
\sigma_{el} = \frac{16}{9} \pi \mu^2 R_1^6 (U_0^2 + U_1^2), \quad \sigma_{in} = \frac{8}{3} \pi R_1^3 U_1 \frac{1}{v}, \tag{43}
$$

with the ratio

$$
\frac{\sigma_{el}}{\sigma_{in}} = \frac{2}{3} \mu^2 R_1^3 \frac{U_0^2 + U_1^2}{|U_1|} v \approx \frac{\eta}{9\pi} \frac{m_N^2 \Gamma_s}{\rho_A} v \approx \frac{1}{10}, \tag{44}
$$

where we put $|U_0| \sim |U_1|$ and substituted the velocity $v \sim 1/10$.

IV. NUMERICAL ESTIMATES

In this section we present the results of numerical calculations using Eqs. (22) and (40), and the estimate Eq. (41). Thus for the survival probability one, finally, has

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[2] Let us remind the reader that the Born approximation is valid for $|U_{0,1}| \ll 1/(\mu R_1^2)$.
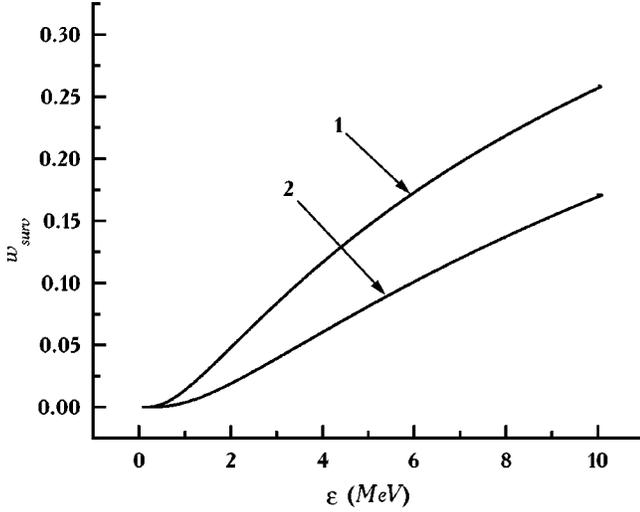
FIG. 2. Survival probability w_{surv} versus the excess of the pion kinetic energy over the threshold (in MeV) for $\Gamma_s = 20$ MeV, $\bar{R} = 1.6$ fm (curve 1), and $\bar{R} = 1.4$ fm (curve 2).

$$w_{surv} = \exp\left(-\frac{\eta\bar{R}\Gamma_s}{\rho_A \bar{V} v}\right), \tag{45}$$

where we put the function $F(R_1/R_2)$ equal to unity every-where for the sake of simplicity.

In Fig. 2 we depict the survival probability (45) as a function of the excess energy of the initial pion over the threshold for d' production using for the spreading width $\Gamma_s = 20$ MeV and for the radius $\bar{R}$ the values 1.4 fm and 1.6 fm, respectively. For the excess energy of 3 MeV above the threshold we find that only about 10% of all created resonances survive, so that the suppression factor is of order ten. Finally, in Fig. 3, we give the "bare" total cross section of the d' production in the reaction $\pi^3\text{He} \to d'N$ as well as the same cross section multiplied by the factor of the survival probability. We find our theoretical predictions to comply reasonably well with the experimental data given in Ref. [12] (dots with error bars in Fig. 3), where already the effects of collision damping had been discussed briefly.

V. CONCLUSIONS AND OUTLOOK

Using an optical potential approach we have shown that collision damping strongly decreases the survival probability of d' in presence of a nuclear medium. Even for ^3He it reduces the d' production cross section near threshold by an order of magnitude, thereby leading to a reasonable agreement with the data. Another way to approach the problem of collision damping of the d' resonance near its production threshold would be a full coupled-channel treatment. This requires the consideration of (at least) three channels: $\pi^3\text{He}$, $d'N$, and NNN (one could even neglect the vacuum width of the d' considering it as a stable particle). However, solving the coupled-channel problem requires a rather accurate knowledge of many reaction amplitudes (with $J^P = 1/2^-$) involved in this problem. In principle, this
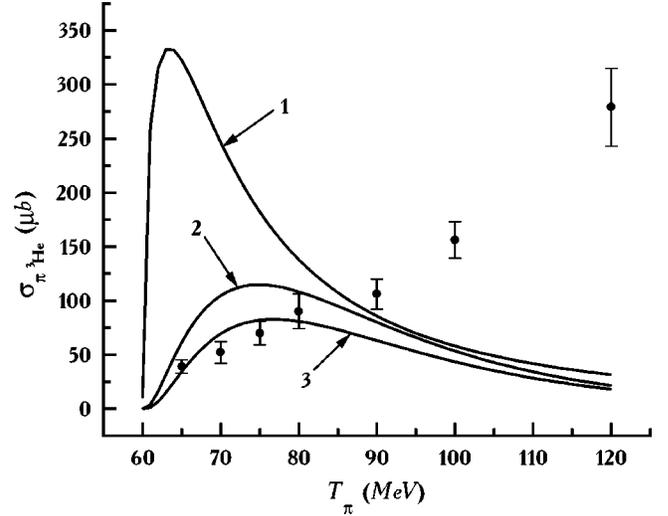
FIG. 3. The cross section for the d' production in the reaction $\pi^3\text{He} \to d'N$ (in μbarn) versus the kinetic energy of the pion T_π (in MeV). Curve 1 gives the "bare" result, without the damping effect taken into account (Ref. [17]); curves 2 and 3 represent the same cross section multiplied by the survival probability, Eq. (45), for $\Gamma_s = 20$ MeV and $\bar{R}$ taking the values 1.6 fm (curve 2) and 1.4 fm (curve 3). Experimental data (Ref. [12]) are shown by the dots with error bars.

would allow us to find the correction to the d' production amplitude by summing up all possible contributions to the amplitude $\pi^3\text{He} \to d'N$ including the route $\pi^3\text{He} \to NNN \to d'N$.

Let us note that such effects should equally renormalize the amplitude of the d' formation in heavier nuclei. However, it is just this amplitude (deduced from DCX transitions to discrete levels [1]) that was taken as an input to predict the d' production off ^3He [17]. The difference between these phenomena is, therefore, due to different details of the d' propagation following its initial production by the incoming pion. In this respect it appears noteworthy to compare with the situation of Λ and Σ production in the reaction $pp \to KNY$ near threshold, where the observed [18] surprisingly small Σ production cross section is interpreted as being due to $pp \to K^+p\Sigma^0 \to K^+p\Lambda$, i.e., a strong FSI between p and Σ^0, which transfers Σ^0 immediately into the energetically much more likely Λ—a situation very similar to that of $d'N \to NNN$ discussed above for the DCX on helium isotopes.

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