High spin states in ^{93}Sr

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Twenty-one γ transitions in ^{93}Sr were identified from $\gamma-\gamma-\gamma$ coincidences in the spontaneous fission of ^{252}Cf starting with the previously known three transitions in ^{93}Sr. The level structure of this nucleus is interpreted in part as arising from the weak coupling of the $1d_{5/2}$ neutron hole to the yrast states of the ^{94}Sr core. We have tried to give a quantitative description of the properties of ^{93}Sr by performing a shell-model study in which we assume that ^{88}Sr is a closed core. In this study we have also considered the neighboring isotopes ^{90,92,94}Sr. The calculated spectra have been compared with the experimental data.

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I. INTRODUCTION

The neutron rich Sr and Zr isotopes are a source of valuable information on the shell structure in the ^{100}Sn region. From this viewpoint the nuclei around ^{88}Sr, to which a doubly magic character is generally attributed, are of special interest. This has motivated recent studies [1–4] which have led to experimental data for these nuclei. There is now information for practically all the Sr isotopes up to $A=100$, with the exception of ^{93}Sr. The study of this missing nucleus is therefore important to complete our knowledge of this interesting group of nuclei. In the present work, 21 γ transitions in ^{93}Sr were identified from $\gamma-\gamma-\gamma$ coincidences in the spontaneous fission of ^{252}Cf starting with the previously known three transitions in ^{93}Sr.

As a first attempt to interpret the level scheme of ^{93}Sr, we have tried to identify the observed states as arising from the weak coupling of the $1d_{5/2}$ neutron hole to the states of the ^{94}Sr core. While some features of the experimental spectrum may be understood in this simple way, a detailed description can only be obtained in terms of the shell model. In principle, one may think of a large-scale calculation with ^{100}Sn as a closed core, where one has to consider 12 proton holes in the 50 closed shell and five neutrons beyond the 50 shell. A much simpler approach, which greatly reduces the dimensions of the model space, is based on the assumption that $Z=38$ is, to a large extent, a good magic number. This assumption has indeed been made in most of the existing calculations for the Sr and Zr isotopes.

Motivated by the data made available by the present experiment, we have found it interesting to perform a shell-model study of this nucleus as well as of the three adjacent isotopes, ^{90,92,94}Sr, assuming ^{88}Sr as a closed core and letting the 5, 2, 4, and 6 valence neutrons, respectively, occupy the five levels of the 50-82 shell. In our calculations we have employed a realistic effective interaction derived from the CD-Bonn nucleon-nucleon (NN) potential [5]. As a result, no adjustable parameter appears in our calculations.

The paper is organized as follows. In Sec. II we describe the experimental method and present the results of our measurements. In Sec. III we give an outline of our shell-model calculations and compare the experimental and calculated spectra. Section IV contains a summary of our conclusions.

II. EXPERIMENTAL METHODS AND RESULTS

In the present work, the measurements were carried out at the Lawrence Berkeley National Laboratory by using a spontaneously fissioning ^{252}Cf source inside Gammasphere. A ^{252}Cf source of strength ≈ 62 μCi was sandwiched between two Fe foils of thickness 10 mg/cm^2, and was mounted in a 7.62 cm diameter plastic ball to absorb β rays and conversion electrons. The source was placed at the center of the Gammasphere array which, for this experiment, consisted of 102 Compton suppressed Ge detectors. A total of 5.7×10^{11} triple and higher fold coincidence events were collected. The coincidence data were analyzed with the RADWARE software package [6]. The width of the coincidence time window was about 1 μsec.

Two partner fragments of ^{93}Sr in spontaneous fission of ^{252}Cf are ^{156}Nd$(3n)$ and ^{154}Nd$(5n)$. When we set double gates on two known transitions belonging to ^{154,156}Nd, the previously known 793.7, 986.1, and 1238.3 transitions in ^{93}Sr [7] are clearly seen in our spectra. Encouraged by the presence of these transitions we set out to identify the γ rays in ^{93}Sr and its level scheme. By double gating on these known transitions, we identified 21 transitions belonging to

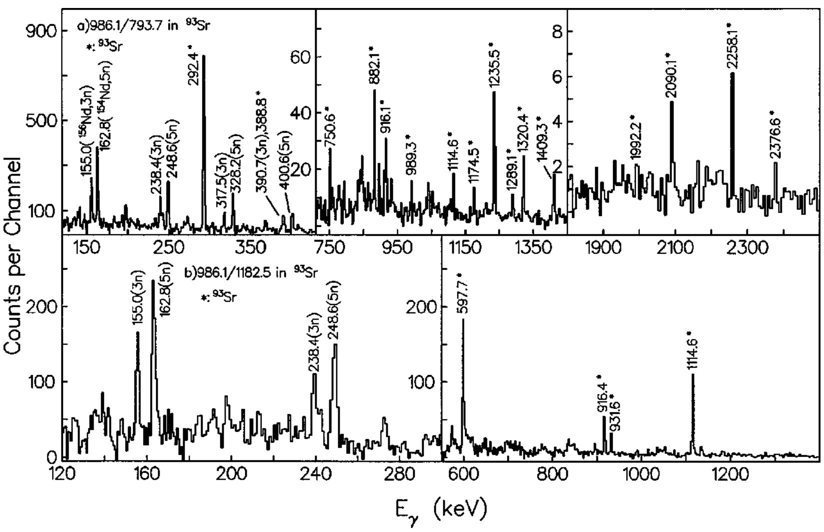

FIG. 1. Coincidence spectra with double gates set on 986.1- and 793.7-keV transitions (upper panel) and 986.1- and 1182.5-keV transitions (lower panel) in ^{93}Sr.

^{93}Sr as shown in Figs. 1(a) and 1(b). In Fig. 1(a), we clearly observe the transitions in the partner nuclei ^{154}Nd and ^{156}Nd. The corresponding number of neutrons evaporated is shown in parentheses in Fig. 1(a). The gating transitions are also shown in Figs. 1(a) and 1(b). The transitions belonging to ^{93}Sr are marked with an asterisk. By using the relative intensities and coincidence relations, we built the new level scheme of ^{93}Sr as shown in Fig. 2. The relative transition intensities are shown in Table I and Fig. 2. The intensity errors are about 5% for the strong transitions and about 30% for the weak transitions. The ordering of transitions in the bands is generally based on relative intensities, coincidence relationships, and the feeding and decaying intensity balances for levels.

The fact that the fission products are formed with an average of six or more units of angular momentum greatly simplifies the construction of bands and assignments of spins, because only yrast or near-yrast states are observed. We are further helped by the fact that the bands are interlaced with each other. Thus spin and parity assignments are quite constrained, since only $E2$, $M1$, and $E1$ multipolarities are expected to compete. Ideally, one would like to have internal conversion coefficients (ICCs) or directional gamma-gamma correlation measurements to confirm multipolarity assignments. Measurement of ICCs for prompt fission gamma radiation is not feasible due to complexity of the spectrum from so many isotopes. The Eurogam Collaboration has made a few angular correlation measurements [8,9]. In their case the fission fragments were stopped in a KCl salt pill, a diamagnetic medium in which the perturbing magnetic or electric fields at the stopped fission nuclei should be small. In all our Gammasphere experiments, we have stopped the fragments in metallic foils such as Fe and Ni, which could have large residual perturbing fields [10].

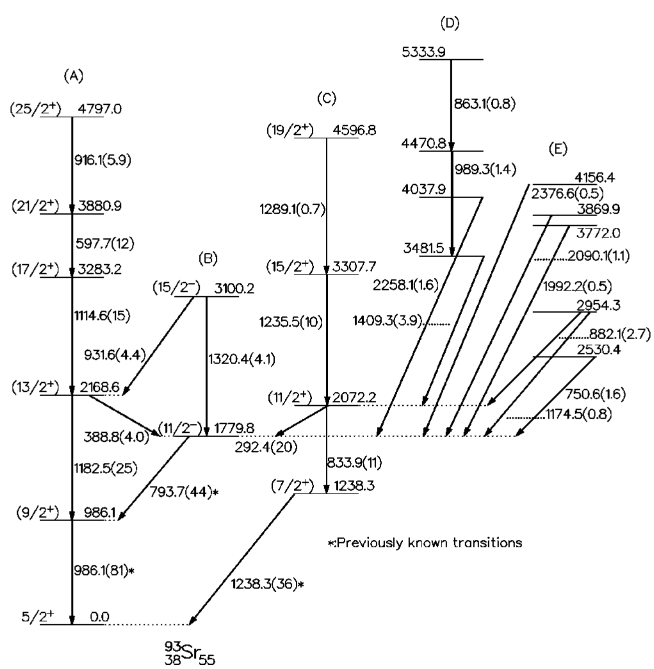

FIG. 2. Level scheme of ^{93}Sr. The asterisk (*) denotes the three previously known transitions.

In this situation, spin-parity assignments can only be tentative. Some guidance in making the assignments reported in Fig. 2 is provided by the following simple arguments. The level structure (bands A and B) of ^{93}Sr is in close correspondence with the yrast levels of the adjacent isotope ^{94}Sr [1]. This suggests that the level scheme of ^{93}Sr may arise from the weak coupling of the $1d_{5/2}$ neutron hole to the levels of the ^{94}Sr core. Then, spins and parities of bands A and B in ^{93}Sr are tentatively assigned assuming that their levels arise predominantly from stretched couplings (i.e., maximum spin) of the neutron $1d_{5/2}$ hole to states of the ^{94}Sr core, as Fig. 3 makes plausible. According to this interpretation, the $15/2^-$ state should originate from coupling to the first excited 5^- state in ^{94}Sr, which has not been observed to date. It should be mentioned, however, that this state has been identified in ^{90}Sr and ^{92}Sr at 3.15 and 2.77 MeV, respectively. Moreover, our shell-model calculations for ^{94}Sr predict a 5^- state at 2.75 MeV. As regards the levels of band C, they may be thought of as originating from the "stretched-minus-one" coupling of the $1d_{5/2}$ neutron hole to the ^{94}Sr core states.

TABLE I. Transition energies and intensities in ^{93}Sr. The intensity errors are about 5% for the strong transitions and about 30% for the weak transitions. All of spins and parities except $5/2^+$ are tentatively assigned in the present work.

E_γ (keV)	Relative I_γ	$I_i^\pi \rightarrow I_f^\pi$	E_γ (keV)	Relative I_γ	$I_i^\pi \rightarrow I_f^\pi$
292.4	20	$(11/2^+)\rightarrow(11/2^-)$	388.8	4.0	$(13/2^+)\rightarrow(11/2^-)$
597.7	12	$(21/2^+)\rightarrow(17/2^+)$	750.6	1.6	
793.7	44	$(11/2^-)\rightarrow(9/2^+)$	833.9	11	$(11/2^+)\rightarrow(7/2^+)$
863.1	0.8		882.1	2.7	
916.1	5.9	$(25/2^+)\rightarrow(21/2^+)$	931.6	4.4	$(15/2^-)\rightarrow(13/2^+)$
986.1	81	$(9/2^+)\rightarrow 7/2^+$	989.3	1.4	
1114.6	15	$(17/2^+)\rightarrow(13/2^+)$	1174.5	0.8	
1182.5	25	$(13/2^+)\rightarrow(9/2^+)$	1235.5	10	$(15/2^+)\rightarrow(11/2^+)$
1238.3	36	$(7/2^+)\rightarrow 5/2^+$	1289.1	0.7	$(19/2^+)\rightarrow(15/2^+)$
1320.4	4.1	$(15/2^-)\rightarrow(11/2^-)$	1409.3	3.9	
1992.2	0.5		2090.1	1.1	
2258.1	1.6		2376.6	0.5	

III. SHELL-MODEL CALCULATIONS

As already mentioned in Sec. I, our shell model calculations for the Sr isotopes with $N>50$ are performed assuming ^{88}Sr as a closed core, with the valence neutrons occupying the five single-particle (SP) levels $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $0g_{7/2}$, and $0h_{11/2}$ of the 50-82 shell. The assumption that ^{88}Sr is a closed core for the description of nuclei in the A ~100 mass region has been made in several shell-model calculations from the mid sixties to date [4,11–17]. However, most of these studies refer to nuclei with $Z>38$ and have been carried out by letting the valence protons occupy the $1p_{1/2}$ and $0g_{9/2}$ levels, while for the valence neutrons various model spaces have been used. To our knowledge, only in the studies of Refs. [4,14,17] concerning Zr isotopes have all the levels of the 50-82 shell been included. In [17] some results for ^{90,92,94,96,98}Sr were also reported. The Sr isotopes with $N=51$–54 have also been the subject of two recent shell-model studies [2,3], with the model space including the four single-proton orbitals of the 28-50 shell and the three neutron orbitals $1p_{1/2}$, $0g_{9/2}$, and $1d_{5/2}$ relative to a ^{66}Ni core. To cut down the calculations to a manageable size, at most four protons were allowed to occupy the $1p_{1/2}$ and $0g_{9/2}$ levels while neutron excitations across the $N=50$ shell were forbidden [2,3]. This study may be seen as complementary to ours. In fact, while it takes into account proton excitations from the $0f_{5/2}$ and $1p_{3/2}$ levels, which are neglected in our calculation, the valence neutrons were constrained to fill only the $1d_{5/2}$ level. In principle, an appropriate description of nuclei in the $A=100$ mass region would require the use of both proton and neutron full model spaces. However, such large-scale calculations are still impractical.

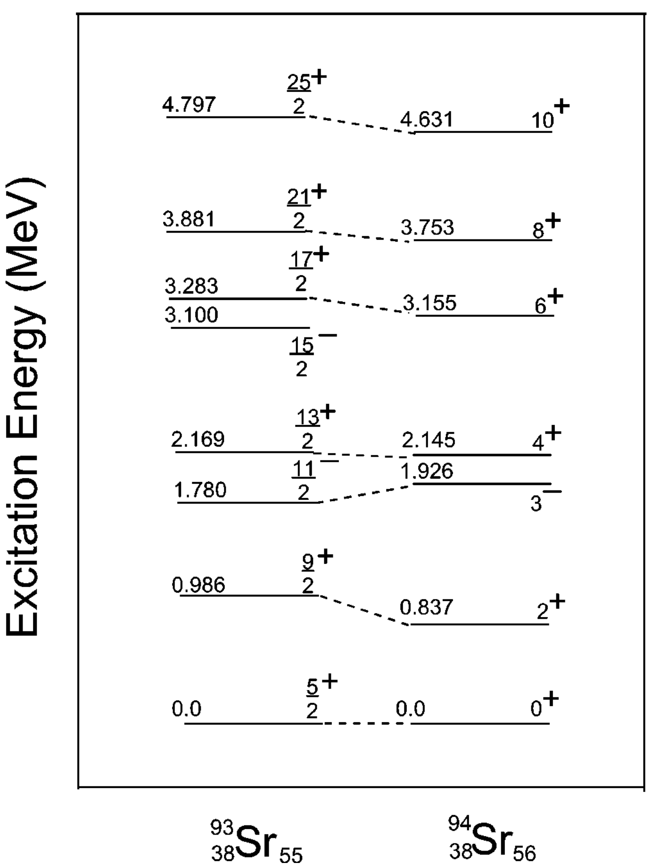

FIG. 3. Comparison between the experimental excited energies in ^{93}Sr and ^{94}Sr.

Let us now give a brief description of our calculations including the derivation of the neutron-neutron effective interaction and the choice of the SP energies. As mentioned in Sec. I, we have made use of a realistic neutron-neutron effective interaction derived from the CD-Bonn free NN potential [5]. This potential, as all modern NN potentials, contains a strong repulsive core which prevents its direct use in nuclear structure calculations. This difficulty is usually overcome by resorting to the well-known Brueckner G-matrix method. Here we have made use of an approach [18] which provides an advantageous alternative to the use of the above method. It consists in constructing a low-momentum NN potential, V_{low-k}, that preserves the physics of the original potential V_{NN} up to a certain cutoff momentum Λ. In particular, the scattering phase shifts and deuteron binding en-

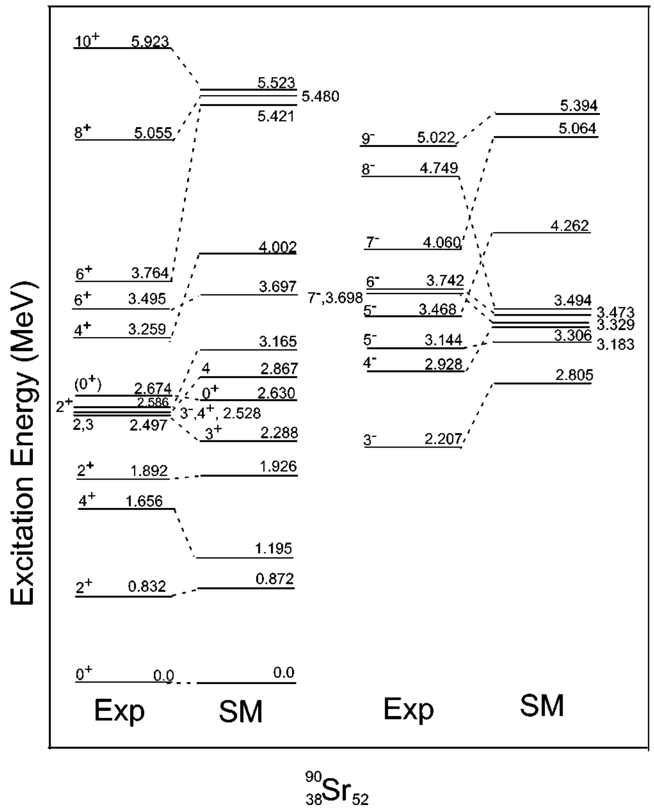

FIG. 4. Comparison of experimental levels (Exp) with shell model calculations (SM) in ^{90}Sr.

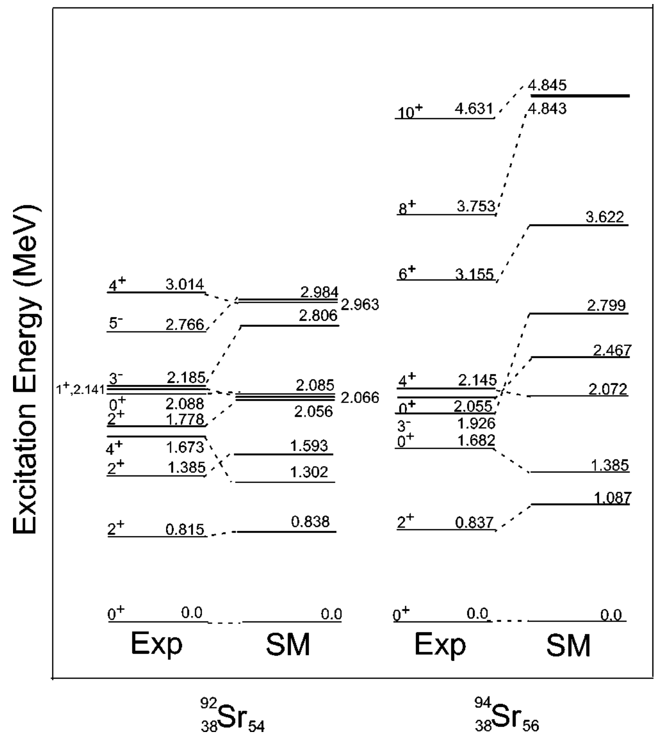

FIG. 5. Comparison of experimental levels (Exp) with shell model calculations (SM) in ^{92,94}Sr.

ergy calculated by V_{NN} are reproduced by V_{low-k}. The latter is a smooth potential that can be used directly as input for the calculation of shell-model effective interactions. A detailed description of our derivation of V_{low-k} can be found in Ref. [18], where a criterion for the choice of the cutoff parameter Λ is also given. We have used here the value $\Lambda = 2.1\ \mathrm{fm}^{-1}$. Once the V_{low-k} is obtained, the calculation of the matrix elements of the neutron-neutron effective interaction is carried out within the framework of a folded-diagram method, as described, for instance, in Ref. [19].

As regards the single-neutron energies, we have taken them from the experimental spectrum of ^{89}Sr as studied by means of (d,p) and $(\vec{d},p)$ reactions [20]. Given our assumption that ^{88}Sr is a closed core, this is certainly the most natural choice. The adopted values are (in MeV) $\epsilon_{d_{5/2}} = 0.0$, $\epsilon_{s_{1/2}} = 1.032$, $\epsilon_{d_{3/2}} = 2.008$, $\epsilon_{h_{11/2}} = 2.079$, and $\epsilon_{g_{7/2}} = 2.675$. These energies correspond to the observed lowest-lying levels in ^{89}Sr having a significant SP component, as indicated by the one-neutron spectroscopic factors [20]. It should be pointed out that our value of $\epsilon_{h_{11/2}}$ is quite different from that (3.5 MeV) of Refs. [14,17], while there are minor differences (at most 230 keV) for the other SP energies. All the calculations have been performed by using the OXBASH shell model code [21].

Before presenting the results for ^{93}Sr and the two adjacent isotopes, we would like to discuss our levels of ^{90}Sr which has only two valence neutrons to represent the best system to test the two body matrix elements and SP energies. In Fig. 4 we compare the experimental [2,22] and our calculated spectra of ^{90}Sr, the positive- and negative-parity levels being shown separately. We see that the structure of the positive-parity spectrum is reproduced by the theory, the only significant discrepancy being the position of the second 6^+ state which is predicted to lie at more than 1.5 MeV above the experimental one. As regards the quantitative agreement for the other levels, the discrepancies between theory and experiment are rather large for some states, ranging from a few to some hundreds keV. As for the negative-parity states, we see that the energies of the yrast levels are reproduced within at most 400 keV, with the exception of the 3^- and 8^- states. For these states and the non-yrast ones in Fig. 4 there are larger discrepancies, up to about 1.2 MeV. It should be pointed out that the results of Ref. [2], where proton excitations were explicitly taken into account, are not in substantially better agreement with experiment, at least as regards the positive-parity states. This may be taken as an indication of interplay between proton and neutron excitations and of the need, as mentioned above, of a more complete calculation for an accurate description of Sr isotopes.

The experimental spectra [1,3,22] of the two even nuclei ^{92}Sr and ^{94}Sr are shown in Fig. 5, together with the results of our calculations. Similar agreement as for ^{90}Sr is obtained for these isotopes. Differences larger than 500 keV are only found for the 3^- states in both nuclei and the 8^+ state in ^{94}Sr. It is worth noting that the almost constant $0^+ - 2^+$ spacings in ^{90,92,94}Sr are well reproduced by our calculations. In fact, we find that the 2^+ states in ^{90}Sr and ^{92}Sr are at

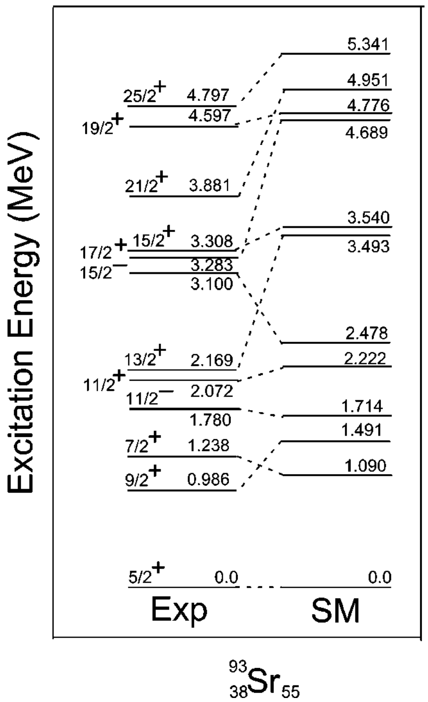

FIG. 6. Comparison of experimental levels (Exp) with shell model calculations (SM) in ^{93}Sr.

about the same position and the 2^+ energy increases only by 250 keV when going to ^{94}Sr. A different result was obtained in Ref. [17], where from ^{92}Sr to ^{94}Sr the 2^+ state moves upward by about 600 keV.

Let us now come to ^{93}Sr, which is the subject of the present experiment. From the above discussion it is evident that we are not in a position to attempt the identification of the observed levels on the basis of our calculations. Therefore, we limit ourselves to discuss the calculated spectrum as compared to the level scheme proposed in Fig. 2. This is made in Fig. 6. From this figure it appears that we can only support the identification of the levels of band C, for which the differences between the calculated and experimental values are at most 230 keV. The energies of the states of band A are not well reproduced, with the theoretical $13/2^+$, 17^+, and $21/2^+$ states pushed up 1.1–1.4 MeV. As regards the two levels of band B, we find that only the calculated energy of the $\frac{11}{2}^-$ state is in good agreement with experiment.

IV. SUMMARY

The main motivation of this work was to obtain experimental information on ^{93}Sr, for which few data were available as compared to the other neutron rich isotopes. Our study was carried out by using the spontaneous fission source of ^{252}Cf and Gammasphere. As a result, 21 γ transitions were identified starting with the three previously known transitions. The level scheme of ^{93}Sr has been interpreted in terms of weak coupling of the $1d_{5/2}$ neutron hole to the levels of the ^{94}Sr core.

Along with our experimental work, we have also performed realistic shell-model calculations for ^{93}Sr and for the three even isotopes ^{90,92,94}Sr. We emphasize that these calculations are free from adjustable parameters. It turns out that the calculated results provide only a partial interpretation of the experimental spectra of the above nuclei. In particular, for ^{93}Sr the calculated level energies are in quantitative agreement with the experimental ones only for band C. This is likely to be a consequence of our neglecting the proton degrees of freedom. However, as mentioned in Sec. I, only large-scale calculations may shed light on this point. On the experimental side, more detailed studies, including measurements of electromagnetic transition rates, are very much needed.

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