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Macroscopic quantum superpositions in highly excited strongly interacting many-body systems

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We demonstrate a breakdown in the macroscopic (classical-like) dynamics of wave-packets in complex microscopic and mesoscopic collisions. This breakdown manifests itself in coherent superpositions of the rotating clockwise and anticlockwise wave-packets in the regime of strongly overlapping many-body resonances of the highly-excited intermediate complex. These superpositions involve $\sim 10^4$ many-body configurations so that their internal interactive complexity dramatically exceeds all of those previously discussed and experimentally realized. The interference fringes persist over a time-interval much longer than the energy relaxation-redistribution time due to the anomalously slow phase randomization (dephasing). Experimental verification of the effect is proposed.

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Understanding the boundary between microscopic (quantum) and macroscopic (classical) worlds has been a long-standing and key problem of modern science. In quantum mechanics a single object can be represented by different quasiclassical wave packets (WPs) simultaneously localized in two different locations. When these WPs spatially overlap this produces interference fringes, thereby demonstrating a coherent superposition of the two distinct locations of the same object. Such a superposition is generally referred to as ''Schrödinger cat state'' (SCS) following Schrödinger's discussion of quantum superposition of ''live cat'' and ''dead cat'' states [1], stressing a sharp contrast between the quantum world and our everyday macroscopic experience. The quantum-classical transition, driven by decoherence, occurs ever quicker with increasing size of the system [2–4]. SCSs have been realized, e.g., for the Rydberg atomic electron states [5], photons in a microwave cavity [3], and laser-cooled trapped ions [4]. A quantum superposition of distinct macroscopic states, involving $\sim 10^9$ Cooper pairs, was also reported [6]. Even though these examples represent quite different systems they have one common feature, namely, that SCSs [3–6] are quantum superpositions of a relatively small number ($\leqslant 10$) of *single-particle* (Fock) states reflecting either the *single-particle* nature of the system [4,5] or the *absence* of interaction between different degrees of freedom [3,6]. In particular, the data [6] can be understood in terms of a quantum superposition of just two *single* quasiparticle (Fock) states of the superconducting condensate, each of these states being occupied by a macroscopically large number of *noninteracting* Cooper pairs. Yet the *internal interactive complexity* and the *high excitation energy* are also characteristic properties of macroscopic objects with extremely small energy level spacings. This motivates a search for SCSs involving coherent superpositions of a very large number of highly excited strongly-mixed many-body configurations.

It was shown [7,8] that spontaneous off-diagonal spin correlations between strongly overlapping resonances of the deformed highly excited intermediate complex (IC), which can be formed in nuclear, molecular, and atomic cluster collisions, produce rotational WPs. These spatially localized WPs rotate in opposite, clockwise, and anticlockwise directions. The period of rotation is much shorter than the inverse average level spacing of the deformed IC, reflecting correlations between the very large number of complex spatially extended many-body configurations with different total spins. It should be noted that relatively stable WPs in highly excited many-body systems have been identified and associated with the existence of invariant manifolds in the classical phase space of the system [9].

In this paper, we demonstrate a breakdown of the simple WP classical-like dynamics [7] by revealing the interference between the two WPs. This is a manifestation of the coherent quantum, rather than statistical classical, nature of superposition of the two WPs representing the same system. The quantum superpositions discussed here involve $\sim 10^4$ highly excited individually ergodic many-body configurations, each of these being a superposition of $\sim 10^5$ single-particle states (e.g., Slater determinants). In this sense, and to the best of our knowledge, the *internal interactive complexity* of such SCSs *dramatically exceeds* all of those previously discussed and experimentally realized. Yet, in spite of the extreme complexity, it could be the smallest Schrödinger ''kitten,'' if created in heavy-ion collisions, in all SCS systems previously identified. Alternatively it might also be of nanometer size if created in atomic cluster collisions [10]. We propose an experimental test of the effect.

Consider a peripheral collision of heavy ions, or polyatomic molecules, or atomic clusters. Then suppose that the collision proceeds through formation of a deformed relatively long-living highly excited IC. This is a likely scenario provided that the double-folding potential between the collision partners has a pocket. The calculations indicate the existence of such pockets, e.g., for heavy-ion [11] and atomic cluster [12] systems. [It should be noted that the unambiguous experimental demonstration of quantum coherence for

hot fullerenes [13] strongly calls for a fully quantum-mechanical treatment of an atomic cluster (i.e., mesoscopic) collisions.] As the relative radial kinetic energy of the colliding partners transfers into intrinsic excitation they drop into this pocket, forming a highly excited deformed IC. We concentrate on this regime of the highly dense spectrum, where $D \ll \Gamma$, with D being an average level spacing, and $\hbar/\Gamma$ the average lifetime of the IC. In this domain of strongly overlapping resonances the energy spectrum is not resolved, and the dynamics of the IC is dominated by a very large number $\Gamma/D \gg 1$ of simultaneously excited many-body quasibound configurations. This regime is opposite in the extreme to that of Bose-Einstein condensation [14].

Let us consider the spinless collision partners in the entrance a and exit b channels. The time t and scattering angle θ dependent intensity of the decay of the IC is given by $P(t,\theta) \propto \lim_{\mathcal{I}\to\infty}(1/\mathcal{I})\int_{-\mathcal{I}/2}^{\mathcal{I}/2} d\varepsilon \exp(-i\varepsilon t/\hbar)\rho(\varepsilon,\theta)$ [15], where $\rho(\varepsilon,\theta) = \langle \delta f(E+\varepsilon)\delta f(E)^* \rangle$ is the amplitude energy autocorrelation function, and brackets $\langle \cdots \rangle$ denote the energy averaging. Here $\delta f(E) = \Sigma_J (2J+1)\exp(iJ\Phi)\delta S^J(E) P_J(\theta)$ is the oscillating around zero ($\langle \delta f(E) \rangle = 0$) collision amplitude, reflecting a time-delayed reaction mechanism [7]; $\delta S^J(E) = S^J(E) - \langle S^J(E) \rangle$ is the fluctuating ($\langle \delta S^J(E) \rangle = 0$) component of the S matrix $S^J(E)$ with total spin J, and $\langle S^J(E) \rangle$ is its energy averaged component. Φ is the deflection angle due to the J dependence of the potential phase shifts, and $P_J(\theta)$ are Legendre polynomials. We obtain

$$P(t,\theta) \propto H(t)\exp(-\Gamma t/\hbar) \sum_{JJ'} \sum_{\mu\nu} [W(J)W(J')]^{1/2} \overline{\tilde{c}_\mu^J \tilde{c}_\nu^{J'}} \times \exp[i\Phi(J-J') - i(E_\mu^J - E_\nu^{J'})t/\hbar] P_J(\theta) P_{J'}(\theta), \quad (1)$$

where $\tilde{c}_\mu^J = c_\mu^J / [\overline{(c_\mu^J)^2}]^{1/2}$, $c_\mu^J = \gamma_\mu^{Ja}\gamma_\mu^{Jb} - \overline{\gamma_\mu^{Ja}\gamma_\mu^{Jb}}$ ($c_\mu^J \equiv 0$), $\gamma_\mu^{Ja(b)}$ and $\gamma_\nu^{J'a(b)}$ are the partial width amplitudes, E_μ^J and $E_\nu^{J'}$ are the resonance energies, and $W(J) = \langle |\delta S^J(E)|^2 \rangle \propto \overline{(c_\mu^J)^2}$ is the average partial reaction probability. The overbars stand for the averaging over ensemble of $\gamma_\mu^{Ja(b)}$, $\gamma_\nu^{J'a(b)}$ which are considered as Gaussian random variables. The Heaviside step function $H(t)$ signifies that the IC cannot decay before it is formed at $t=0$. We take into account the spin off-diagonal correlation [7,16] $\overline{\tilde{c}_\mu^J \tilde{c}_\nu^{J'}} = (1/\pi) D\beta |J-J'| / \{[E_\mu^J - E_\nu^{J'} - \hbar\omega(J-J')]^2 + \beta^2 (J-J')^2\}$, while $\overline{\tilde{c}_\mu^J \tilde{c}_{\mu'}^J} = \delta_{\mu\mu'}$, the latter being a conventional assumption of random matrix theory [15,17,18]. In the correlator, $\beta \gg D$ is the spin dephasing width and $\omega \gg D/\hbar$ is the angular velocity of the coherent rotation of the highly excited IC. Changing from the (μ,ν) summation to the integration over $(E_\mu^J, E_\nu^{J'})$, which is an accurate approximation for $D \ll \beta$ and $D \ll \Gamma$, i.e., for $t \ll \hbar/D$, when the spectrum is not resolved, we obtain

$$P(t,\theta) \propto H(t)\exp(-\Gamma t/\hbar) \sum_{JJ'} [W(J)W(J')]^{1/2} \times \exp[i(\Phi - \omega t)(J-J') - \beta|J-J'|t/\hbar] \times P_J(\theta) P_{J'}(\theta). \quad (2)$$

Therefore although Eq. (1) involves (μ,ν) sums over $\sim \beta\mathcal{I}/D^2 \gg 1$ strongly overlapping ($\Gamma \gg D$) resonances, $P(t,\theta)$ reduces to a sum over (J,J').

We take the partial average reaction probability in the J-window form, $W(J) = \langle |\delta S^J(E)|^2 \rangle \propto \exp[-(J-I)^2/d^2]$, where I is the average spin and d is the J-window width. We use the asymptotic form of $P_J(\theta)$ for θ, $\pi - \theta \gg 1/I$ and employ the Poisson summation formula. For relatively slow dephasing, $d\beta/\Gamma \ll 1$, and, for $d \gg 1$, we obtain

$$P(t,\theta) \sim H(t)(1/\sin\theta)\exp(-\Gamma t/\hbar) \times \sum_{k=0}^{\infty} \left\{ \mathcal{P}_k^{(+)}(t,\theta)^2 + \mathcal{P}_{k+1}^{(-)}(t,\theta)^2 + 2\cos\left[(2I+1)\theta - \frac{\pi}{2}\right] \times \mathcal{P}_k^{(+)}(t,\theta)\mathcal{P}_{k+1}^{(-)}(t,\theta) \right\}, \quad (3)$$

where $\mathcal{P}_k^{(\pm)}(t,\theta) = \Delta^{-1}(t)\exp[-(\Phi \pm \theta + 2\pi k - \omega t)^2/2\Delta^2(t)]$ and $\Delta^2(t) = (d^{-2} + \beta^2 t^2/\hbar^2)$. We observe that $\mathcal{P}_k^{(+)}(t,\theta)$ and $\mathcal{P}_{k+1}^{(-)}(t,\theta)$ have the meaning of time dependent amplitudes for the decay into the angle θ, after k consecutive revolutions of the IC rotating in opposite directions. Due to the azimuthal symmetry of the problem the emission directions of the collision fragments, at $t=0$, are concentrated along a cone of angle $|\Phi|$. If we intersect this cone by the reaction plane at $t=0$ we obtain two spatially localized WPs, with an angular dispersion $\sim 1/d$, located symmetrically around the direction of the initial beam at $\theta = |\Phi|$ and $\theta = 2\pi - |\Phi|$. As time proceeds the WPs rotate with angular velocity ω, in opposite directions. This reflects an opening or closing (depending on the value of Φ) of the cone. The spin off-diagonal δS-matrix correlation is a necessary condition for formation of the WPs. The WPs meet and overlap in a vicinity of forward and backward angles producing interference fringes due to the interference term between the amplitudes $\mathcal{P}_k^{(+)}(t,\theta)$ and $\mathcal{P}_{k+1}^{(-)}(t,\theta)$ in Eq. (3).

As an example, we illustrate the dynamics and coherent superpositions of WPs for $^{12}C + ^{24}Mg$ elastic scattering [19]. For this system an analysis of the cross section energy autocorrelation function, obtained from the excitation function measured over the c.m. energy range 12.27–22.8 MeV at $\theta = \pi$, reveals [7] rotational WPs in spite of the strong overlap of resonance levels in the highly excited intermediate molecule. We calculate $P(t,\theta)$ with a set of parameters evaluated from the fit of $P(t,\theta=\pi)$ [7,8]: $\Phi = 0$, $d = 3$, $I = 14$, $\beta = 0.01$ MeV, $\hbar\omega = 1.35$ MeV, and $\Gamma = 0.3$ MeV. The quantity presented in Fig. 1 is $AP(t,\theta)/\langle \sigma(E,\theta) \rangle$, where $\langle \sigma(E,\theta) \rangle \propto \int_0^\infty dt P(t,\theta)$ is the energy average differential cross section for the time-delayed collision. The reason for this is that amplitudes of individual WPs are strongly and abruptly enhanced in the close vicinity of $\theta = 0, \pi$. This is because, in

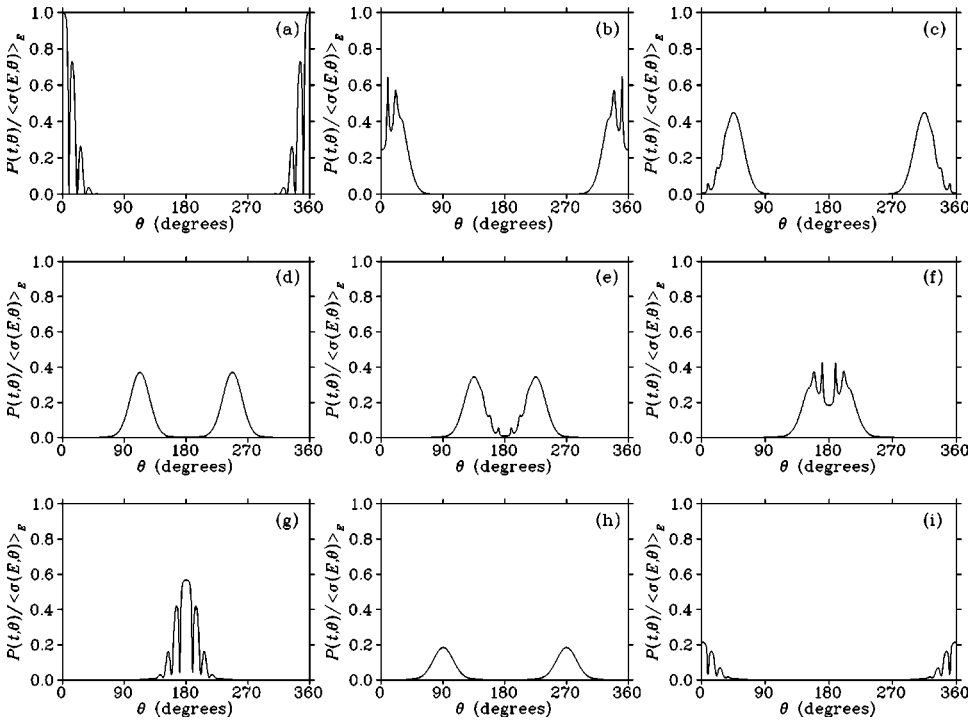

FIG. 1. Time and angle dependence of intensity of decay of the highly excited intermediate molecule created in a $^{12}C+^{24}Mg$ collision. Panel (a) corresponds to $t=0$; (b) $t=T/16$; (c) $t=T/8$; (d) $t=5T/16$; (e) $t=3T/8$; (f) $t=7T/16$; (g) $t=T/2$; (h) $t=3T/4$; (i) $t=T$. The period of one complete revolution of the molecule is $T=3.06\times10^{-21}$ sec (see the text).

contrast to an intermediate angular range $\pi/I\leqslant\theta\leqslant\pi-\pi/I$, for these forward and backward angles the reaction plane is ill defined, and the cone degenerates into a line so that all azimuthal angles ($0\leqslant\phi\leqslant2\pi$) contribute to the decay. However, this also results in a similar enhancement of $\langle\sigma(E,\theta)\rangle$ for $\theta\sim0,\pi$, so that the quantity $P(t,\theta)/\langle\sigma(E,\theta)\rangle$ permits us to probe interference between the two WPs, amplitudes of which depend smoothly on θ. The constant A is derived from the condition $AP(t=0,\theta=0)/\langle\sigma(E,\theta=0)\rangle=1$.

In Fig. 1, at the initial moment of time, $t=0$, the forward-oriented ($\theta\sim0,2\pi$) WPs completely overlap producing strong interference fringes. As the WPs move apart the amplitude of the fringes is reduced. Thus, in panel (d), of Fig. 1, the fringes are absent since the WPs are far apart and do not overlap. As the WPs begin to overlap around $\theta\sim\pi$, the interference fringes reappear and are most pronounced at $t=T/2$ ($T=3.06\times10^{-21}$ sec is the rotation period) when the backward-oriented WPs completely overlap [panel (g)]. Then the WPs pass each other and move apart again, first becoming isolated [Fig. 1, panel (h)], and finally, at $t=T$ [panel (i)], completely overlap around $\theta=0,2\pi$. As time proceeds the overall intensity of the decay decreases exponentially due to the $\exp(-\Gamma t/\hbar)$ factor in $P(t,\theta)$. Note that the interference fringes at $t=T$ are not as pronounced as at $t=0$ due to a relatively small, but *finite* dephasing width.

It has been shown [20] that rotational coherence effects in the molecular decay are manifestations of *the temporal evolution of the angular orientation of molecules*. Therefore, we interpret the interference fringes in $P(t,\theta)$ in Fig. 1 in terms of *a quantum mechanical superposition of distinct orientations of a highly excited molecule*.

Values of $P(t,\theta)$ in Fig. 1 are obtained for the high intrinsic excitation energy (~15 MeV) of the deformed intermediate molecule with average level spacing $D\sim10^{-5}$ MeV [7,8]. Consequently, the energy spectrum of the molecule is not resolved during its lifetime, $\hbar/\Gamma\ll\hbar/D$. The time for formation of ergodic many-body configurations, i.e., the time it takes the two-body interaction to redistribute the energy between the particles of the system, is $\tau_{erg}=\hbar/\Gamma_{spr}$, with Γ_{spr} being the spreading width [18]. For highly excited nuclear systems $\Gamma_{spr}\sim5-10$ MeV [18] and $\tau_{erg}\simeq10^{-22}$ sec. The fact that the SCSs in Fig. 1 persist for $t\gg\tau_{erg}$ clearly demonstrates their many-body nature, i.e., that they originate from the interference of a very large number ($\Gamma/D\sim10^4$) of strongly overlapping *many-body* states. Therefore, the internal interactive complexity of the SCS in Fig. 1 dramatically exceeds that for the SCSs realized previously [3–6], where macroscopically distinct quantum superpositions involve $\leqslant10$ *isolated* ($\Gamma<D$) *noninteracting* configurations. It should be noted that, contrary to the conventional theories of highly excited many-body systems [18], coherent superpositions of $\sim10^4$ strongly overlapping individually ergodic many-body states in Fig. 1 survive the process of the energy redistribution and persist over the time interval $t\gg\tau_{erg}$. Clearly, this essentially many-body aspect of the problem on the relationship between the energy redistribution and dephasing rates could not be addressed in the previous studies of the SCSs created in the single-particle non-interactive systems [3–6].

The SCS in Fig. 1 is a manifestation of the anomalously long spin dephasing time $\hbar/\beta\sim10^{-19}$ sec, as compared with $\tau_{erg}\simeq10^{-22}$ sec. In the limit of short dephasing time, $\beta\sim\Gamma_{spr}\gg\Gamma$, we would have $P(t,\theta)\propto\exp(-\Gamma t/\hbar)\langle\sigma(E,\theta)\rangle$, where $\langle\sigma(E,\theta)\rangle\propto\Sigma_J(2J+1)^2W(J)P_J(\theta)^2$ is the energy averaged cross section obtained in the limit of absence of spin off-diagonal correlations, $\beta\sim\Gamma_{spr}$. This corresponds to the limit of random matrix theory [15,17,18], which, for $\Gamma\ll\Gamma_{spr}$, yields $AP(t,\theta)/\langle\sigma(E,\theta)\rangle=\exp(-\Gamma t/\hbar)$, i.e., angle-

independent straight horizontal lines in Fig. 1 (not shown), completely washing out the WPs and their interference. Another source for the WP spreading and suppression of their interference is a possible J-dependence of ω. Then the WPs and SCSs are washed out for $d\dot{\omega}t \geqslant \pi$. For the J-independent moment of inertia $\mathcal{J}$ of the IC this condition reads $(d/I)\omega t \geqslant \pi$. For $\mathcal{J}/J = \text{const}$, $\dot{\omega} = 0$, and the interference fringes persist for any $d \geqslant 1$ provided $\beta t/\hbar < \pi$.

The bimolecular type of collision considered here cannot be experimentally studied by the methods of femtochemistry used to monitor unimolecular reactions [21]. However, it has been demonstrated [7,8] that measurements of collision cross sections with pure energy and angle resolutions allow one to extract $P(t,\theta)$, i.e., to obtain information concerning the underlying reaction dynamics equivalent to that obtained using real-time methods in femtochemistry. It follows that the SCS predicted here (Fig. 1) *can* be tested experimentally.

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