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Transition probabilities and isospin structure in the $N=Z$ nucleus ^{46}V

O. Möller, K. Jessen, A. Dewald, A. F. Lisetskiy, P. von Brentano, A. Fitzler, J. Jolie, A. Linnemann, B. Saha, and K. O. Zell

Institut für Kernphysik, Universtät zu Köln, Zülpicher Straße 77, D-50937 Köln, Germany

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Picosecond lifetimes in ^{46}V and ^{46}Ti were determined using the recoil distance Doppler-shift technique with a plunger device coupled to a setup of five HP Ge detectors enhanced by one EUROBALL CLUSTER detector. The experiment was carried out using the ^{32}S(^{16}O,pn) reaction at 38 MeV at the Cologne FN TANDEM facility. The differential decay curve method in coincidence mode was employed to derive lifetimes for five excited states in each nucleus. The resulting $E2$ transition probabilities are compared with existing shell model calculations and a comparison within the $T=1$ isospin triplet is given. Absolute $E1$ strengths of the 2^- decay in ^{46}V are discussed.

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The investigation of $N=Z$ nuclei is an exciting topic in nuclear structure physics, to which a lot of experimental and theoretical work has been devoted recently. Self-conjugate nuclei are symmetric with respect to the isospin degree of freedom and allow a sensitive testing of the isospin symmetry. This symmetry leads to selection rules, e.g., $E1$ transitions between low lying states with $T=0$ character are strictly forbidden. The only way to enhance $E1$ strengths between such states is to assume an admixture of $T=1$ components of the wave function, caused by the Coulomb interaction or by isospin violating parts of the strong interaction.

The determination of isospin mixing matrix elements via lifetime measurements is of special interest in odd-odd $N=Z$ nuclei with valence particles in the $f_{7/2}$ shell. In ^{46}V, much interesting data have been accumulated recently [1–6]. A very peculiar finding was the isospin forbidden $2^-\rightarrow 1^+$ transition with a relative $E1$ strength six times stronger than the strength of a competing allowed one with $\Delta T=1$ [1]. In order to achieve a better comparison between allowed and forbidden $E1$ transitions, it is important to determine (more and) absolute transition probabilities in this nucleus. The present work reports on the decay properties of the $2_1^-,T=0$ state and explains the observed relative $E1$ strengths.

Aside from this, a comparison with ^{46}Ti, the isobaric analog partner of ^{46}V, is presented. In the $T=1$ triplet the reduced $E2$ matrix elements have a linear dependence upon T_z, which follows from general arguments based on the ideas of isospin symmetry [7], provided that isospin is a good quantum number. Precise $B(E2;2_1^+\rightarrow 0^+)$ values provide a stringent test to this theoretical relation. The new data are in better agreement with the theoretical description than a previous value.

In ^{46}V recent model calculations in the full pf shell without any truncation for the positive parity states describe the experimental level order and branching ratios well [1]. Nevertheless, reliable data on transition probabilities are necessary for an overall comparison of experimental data with shell model calculations. Despite its relevance such information is still scarce.

We performed a recoil distance Doppler-shift (RDDS) experiment with the Köln coincidence plunger device [8] at the FN TANDEM facility at the University of Cologne. Excited states of ^{46}V were populated using the ^{32}S(^{16}O,pn) reaction at a beam energy of 38 MeV. In addition, states of ^{46}Ti were populated via the $2n$ exit channel of the compound reaction. The target was a 1 mg/cm^2 foil of ZnS backed onto a 2 mg/cm^2 tantalum foil. A gold foil of 8 mg/cm^2 stopped the recoiling nuclei, which had a velocity of $v/c=1.7\%$. The setup consisted of one EUROBALL CLUSTER detector [9,10] at 0° relative to the beam axis and five large volume HP Ge detectors at an angle of 143°. Thus the detectors were grouped into three angular rings: the inner segment of the EUROBALL CLUSTER detector ($\theta_1=0°$), its six outer segments ($\theta_2=34°$), and the five detectors in backward direction ($\theta_3=143°$). All of the detectors were positioned very close (≈11 cm) to the target, increasing the total γ detection efficiency of this setup to about 2.4% at 1.3 MeV.

Coincidence data were collected for 17 different target-to-stopper distances, between 1 and 7200 μm. Altogether 10^{10} $\gamma\gamma$ coincidence events were collected and sorted into 136 $4k\times 4k$ $\gamma\gamma$ matrices. Figure 1 shows the $5_1^+\rightarrow 3_1^+$ decay of ^{46}V in gated coincidence spectra, illustrating the quality of the data. The spectra shown are gated on ring 2 alone. Additional spectra were obtained by analyzing all the other rings.

In order to determine lifetimes, the differential decay curve (DDC) method [11] was used in coincidence mode, avoiding the disturbing effects of sidefeeding. From the spectra gated on the shifted components of feeding γ transitions we obtained the peak intensities of γ transitions depopulating the level of interest at different target-to-stopper distances x. In our notation the abbreviations s and u stand for ‘‘shifted’’ and ‘‘unshifted’’ components, respectively, of a given γ transition denoted by a capital index. The DDC method gives the lifetime $\tau(x)$ of a state as

$$\tau(x)=\frac{I_{su}^{BA}(x)-\alpha I_{su}^{CA}(x)}{v\,\frac{d}{dx}I_{ss}^{BA}(x)}, \tag{1}$$

where v denotes the recoil velocity and the factor α is the ratio

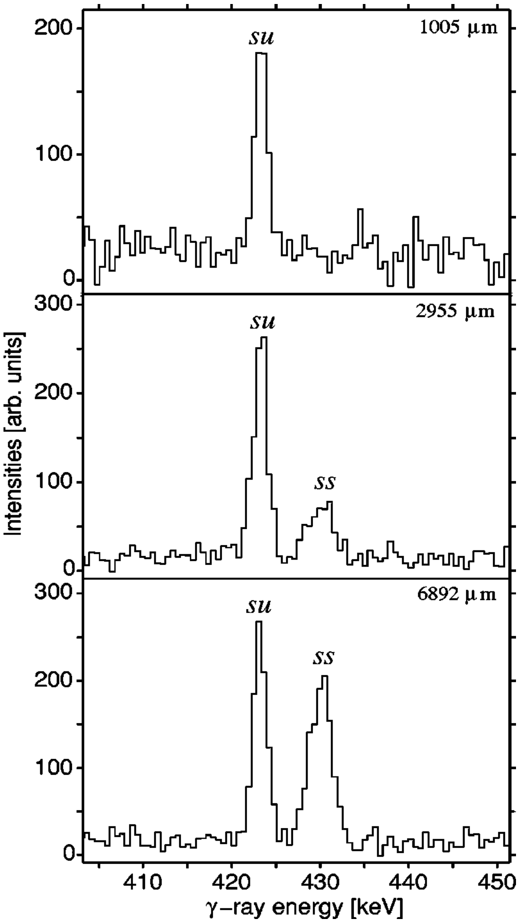

FIG. 1. Spectra at three different target-to-stopper distances detected from the polar angle $\theta_3 = 143°$. The $5_1^+ \rightarrow 3_1^+$ transition in ^{46}V ($K = 3^+$ band) gated by the shifted component of the $7_1^+ \rightarrow 5_1^+$ transition at $\theta_2 = 34°$.

$$\alpha = \frac{I_{su}^{CA}(x) + I_{ss}^{CA}(x)}{I_{su}^{CB}(x) + I_{ss}^{CB}(x)}.$$

The quantities $I_{su}^{BA}(x)$ and $I_{ss}^{BA}(x)$ denote the measured intensities of the depopulating γ transition A in coincidence with (the shifted component of) a populating γ transition B. The intensities $I_{su}^{CA}(x)$, $I_{ss}^{CA}(x)$, and so on are defined analogously. The derivative, $(d/dx)I_{ss}^{BA}(x)$, was determined by fitting piecewise continuously differentiable second order polynomials to the intensity values. The analysis involved is illustrated in Fig. 2; a detailed description of this method and the derivation of Eq. (1) can be found in Refs. [11,12].

An additional experiment, performed with two HP Ge detectors and hence lower gamma-ray efficiency in Cologne, used the same reaction at a beam energy of 34 MeV and a pulsed O^{16} beam. Its pulse width was determined to be 2 ns. The recorded γ-t events were sorted into a γ-energy–time matrix, which allowed the analysis of background subtracted time spectra, from which the lifetime of the 5_2^+ state in ^{46}V was determined (see Ref. [13] for details).

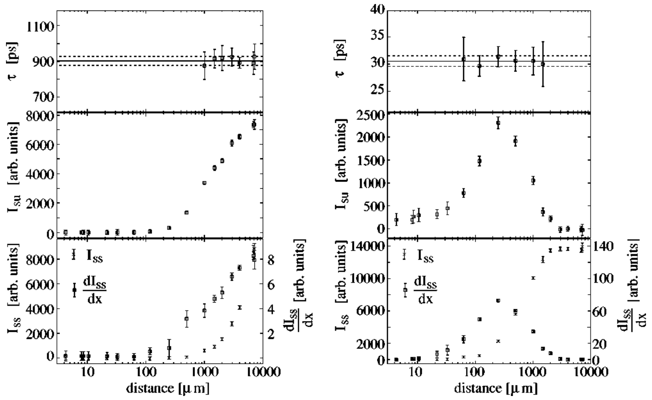

FIG. 2. DDCM analysis of the $5_1^+ \rightarrow 3_1^+$ decay in ^{46}V gated by the direct feeder from the 7_1^+ state (left) and the $3^- \rightarrow 4_1^+$ decay in ^{46}Ti gated by the direct feeder from the 4^- state (right). In the upper panels the τ curves are displayed. The middle panels depict the data corresponding to the numerator of the DDCM equation, and the bottom panels illustrate the decay curves and their derivatives.

The comparison of all the new results with previous data [14,15], which is given in Table I, generally shows a good agreement within the experimental errors for the lifetimes determined in this work. Reliable values for the 2_1^+, 4_1^+, and 2_1^- states in ^{46}V had not been published and the lifetime of the 5_2^+ state had not been measured before. Only the value for the 3^- state in ^{46}Ti differs significantly from the previous one, which had been deduced from a Doppler-shift attenuation measurement [16], where the limited knowledge of the stopping power might have distorted the result. For most lifetime values the precision has been improved by a factor or 2 or more.

The reduced transition probabilities resulting from our precision lifetime data are used for comparison with shell model calculations and corresponding data on the isospin partners of ^{46}V. In Figs. 3 and 4 the levels of interest of ^{46}V and ^{46}Ti, respectively, are depicted. Table I gives the experimental $B(E\lambda)$ values together with the results of the shell model calculations described in Refs. [1] and [4], respectively.

The 2^+ state in ^{46}V at 915 keV level energy is interpreted as the isobaric $T = 1$ analog of the 2^+ state at 889 keV in ^{46}Ti. Their corresponding $B(E2;2_1^+ \rightarrow 0_1^+)$ values agree well within the experimental errors. Neglecting the Coulomb interaction or isospin breaking parts of the strong force, the level energies and the transition probabilities within the isospin triplet are expected to be constant. So far our data show no significant difference, due to the aforementioned interactions.

In Refs. [1,6] the experimental ^{46}V data are compared with shell model calculations, using the KB3 residual interaction and effective charges $e_p = 1.5e$ and $e_n = 0.5e$. They agree well with experimental branching and multipole mixing ratios. Our new experimental transition probabilities are in good agreement with these calculations, too, except for the 2_1^+ and 7_1^+ states, where the calculated values are too low. In ^{46}Ti the comparison with the calculation described in Ref. [4] shows that indeed both calculated values are considerably smaller. In addition, it is worth mentioning that the calcula-

TABLE I. Adopted lifetimes τ_{expt} and $B(E\lambda)$ values of analyzed transitions in ^{46}V and ^{46}Ti in comparison with shell model calculations. Lifetime values τ_{ref} from Refs. [14–16] are given in column six. Calculated values for ^{46}Ti are taken from Ref. [4] and marked with an asterisk. For completeness an upper limit for the lifetime of the 4^+, $T=1$ state is also given (taken from Ref. [5]).

	E_x (keV)	J_i^π, T_i	E_γ (keV)	J_i^π, T_i	τ_{expt} (ps)	τ_{ref} (ps)	Mult. σl	$B(E\lambda)$ values (e^2fm^{2l}) Expt.	Theor.
^{46}V	915	2_1^+, 1	915	0_1^+, 1	6.8(8)	9.0(23) [14]	$E2$	188(22)	143
	1179	4_1^+, 0	378	3_1^+, 0	460(60)	510(120) [14]	$M1+E2$	230(30)	234
	1224	5_1^+, 0	423	3_1^+, 0	895(15)	900(110) [14]; 610(200) [15]	$E2$	67(1)	65
	1366	2_1^-, 0	373	1_1^+, 0	950(20)	1400(600) [14]	$E1$	$7.5(13)\times10^{-6}$	
			451	2_1^+, 1			$E1$	$1.3(7)\times10^{-6}$	
	1603	7_1^+, 0	379	5_1^+, 0	985(20)	1080(170) [15]	$E2$	106(2)	62
	1725	5_2^+, 0	349	3_2^+, 0	800(250)		$E2$	146(46)	158
			501	5_1^+, 0			$E2$	5(2)	17
			546	4_1^+, 0			$E2$	2(1)	10
	2054	4_2^+, 1	1139	2_1^+, 1		$\leqslant$0.264 [5]	$E2$	$\geqslant$130	187
^{46}Ti	889	2_1^+, 1	889	0_1^+, 1	7.63(7)	7.68(22) [15]	$E2$	193(2)	115*
	2010	4_1^+, 1	1121	2_1^+, 1	2.00(15)	2.34(14) [15]	$E2$	231(17)	154*
	3058	3_1^-, 1	97	2_2^+, 1	31.3(5)	10(3) [16]	$E1$	$2.1(7)\times10^{-3}$	
			1049	4_1^+, 1			$E1+M2$	$1.6(1)\times10^{-5}$	
	3441	4_1^-, 1	383	3_1^-, 1	106.6(5)	95(6) [15]	($M1/E2$)	698(38)	
			1432	4_1^+, 1			$E1$	$0.52(6)\times10^{-6}$	
	4662	6_1^-, 1	1221	4_1^-, 1	1.5(3)	2.0(6) [15]	$E2$	129(33)	

tions both in Ref. [6] and in Ref. [4] use the same interaction and the same effective charges as the one described here.

Considering the results for the $2_1^+\to0_1^+$ transitions within the $A=46$ isospin triplet, the following picture is achieved. The $B(E2;2_1^+\to0_1^+)$ values within the isobaric $T=1$ multiplet provide an important benchmark for the theoretical relation between $\Delta T=0$ $E2$ transition matrix elements, which is given by

$$\langle J,T,T_z\|\mathbf{T}(E2)\|J-2,T,T_z\rangle=S(J)+V(J)T_z\,. \qquad (2)$$

The coefficients $S(J)$ and $V(J)$ depend on the isoscalar and isovector components of the Hamiltonian, respectively, and $\mathbf{T}(E2)$ is the $E2$ transition operator:

$$\mathbf{T}(E2)=\sum_{\rho=p,n}e_\rho\mathbf{T}_\rho(E2), \qquad (3)$$

where e_ρ are the effective nucleon quadrupole charges and $\mathbf{T}_\rho(E2)=\Sigma_i(r_i^\rho)^2\mathbf{Y}_2(\theta_i^\rho,\phi_i^\rho)$. For the most symmetric lowest $T=1$ states ($J_i^\pi=2_1^+,4_1^+,6_1^+,\ldots$) in the isospin triplet nuclei the $V(J)/S(J)$ ratio is positive, and thus, one can write

$$\langle J\|E2\|J-2\rangle(^{46}\mathrm{V})=S(J), \qquad (4)$$

$$\langle J\|E2\|J-2\rangle(^{46}\mathrm{Ti})=S(J)-V(J). \qquad (5)$$

From Eq. (5) it follows that the $B(E2;2_1^+\to0_1^+)$ value for ^{46}Ti has to be smaller than for ^{46}V. This is supported by the shell model results, illustrated in Fig. 5 and yielding

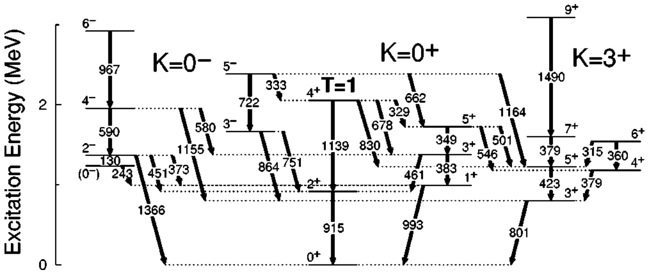

FIG. 3. Partial level scheme of ^{46}V taken from Refs. [1,6].

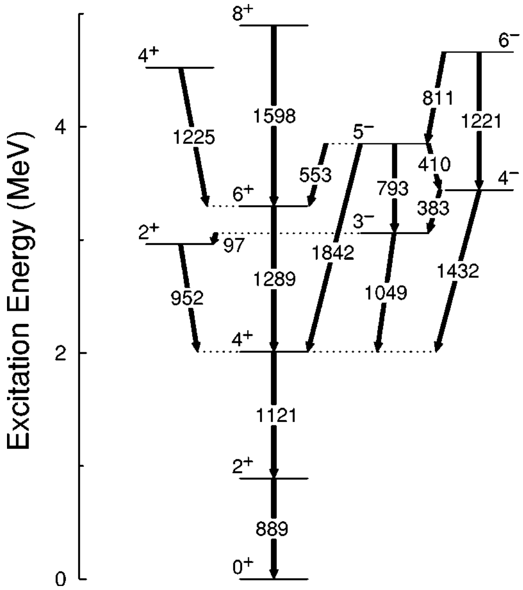

FIG. 4. Partial level scheme of ^{46}Ti taken from Refs. [15,17].

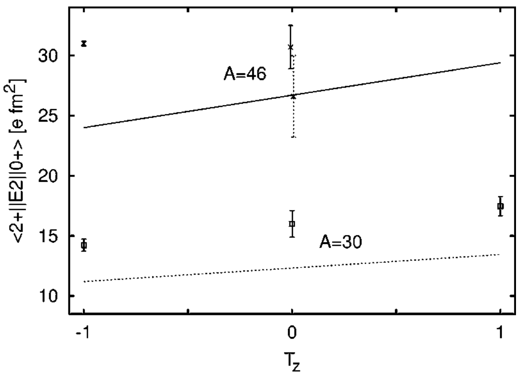

FIG. 5. Reduced matrix elements $\langle 2_1^+||E2||0^+\rangle$ of the $A=46$ isospin triplet versus T_z, denoted by crosses ($\times$). For comparison the old ^{46}V value (dotted error bar) from Ref. [14], values for the corresponding isospin triplet at $A=30$, denoted by boxes ($\square$), and shell model values (lines) for $A=30{,}46$ according to Refs. [18,19] are given.

$V(2)/S(2)=0.1$. According to Ref. [18], the experimental values for the $T_i=T_f=1$, $2_1^+\rightarrow 0_1^+$ transitions are compared with shell model values. Corresponding experimental and calculated values for the isospin triplet of 2_1^+ states at $A=30$ comprising the nuclei ^{30}Si, ^{30}P, and ^{30}S, taken from [19], are also given. The previously measured $B(E2;2_1^+\rightarrow 0_1^+)$ value of $137(35)e^2\mathrm{fm}^4$ for ^{46}V [14] gave a ratio of $V(2)/S(2)=-0.2$ in qualitative disagreement with the theory, despite its nice agreement with the theoretical value of 143 $e^2\mathrm{fm}^4$. In contrast with this former value, the new $B(E2;2_1^+\rightarrow 0_1^+)$ value presented in this work gives a ratio close to zero in much better agreement with the shell model ratio, reconciling the experimental trend with the predicted positive slope in Fig. 5.

However, the shell model calculations obviously underestimate the $B(E2;2_1^+\rightarrow 0_1^+)$ values in both nuclei ^{46}V and ^{46}Ti. This shows, that the collectivity of the 2_1^+ state is not properly reproduced in the calculations and might be considered as an indication for the importance of the ^{40}Ca core breaking even for the lowest yrast states.

Referring to $E1$ transitions in ^{46}V, the $2^-,T=0\rightarrow 2_1^+,T=1$ decay (451 keV) to the level at 915 keV is of special interest. It is of allowed $\Delta T=1$ nature, whereas the competing 373 keV transition to the $1^+,T=0$ state at 993 keV is isospin forbidden. Based on the branching ratios, the relative transition strengths had been determined, showing that the forbidden $\Delta T=0$ transition was enhanced by a factor of 6 [1]. Our measurement gives absolute $B(E1)$ values and the comparison shows that the forbidden transition remains comparable to known $E1$ strengths, which range from 0.5 to about 5.4×10^{-6} $e^2\mathrm{fm}^2$ for other $\Delta T=0$ transitions in this nucleus [6]. Values of the order of magnitude of 10^{-6} $e^2\mathrm{fm}^2$ are at the lower limit of what might be expected for $E1$ transitions in $N\neq Z$ nuclei of this mass region, whereas a strong $E1$ transition has typically 10^{-4} $e^2\mathrm{fm}^2$. Therefore one can conclude that the isospin allowed transition is hindered with an $E1$ strength coming down to a forbidden one. As a possible explanation of the puzzle of this very weak allowed $\Delta T=1$ transition, it had been argued [18] that the isospin mixing might be larger than expected. But this can be excluded by our absolute $B(E1)$ strengths.

This puzzle can easily be understood by taking into account the quadrupole deformation of $\beta\approx 0.28$, deduced from the new $B(E2;2_1^+\rightarrow 0^+)$ value. The deformation of ^{46}V had already been considered in previous works [5,20,21], where the $K^\pi=0^+$ $(T=0,1)$, $K^\pi=3^+$ $(T=0)$, and $K^\pi=0^-$ $(T=0)$ bands were identified. It was found that the 2_1^+, $T=1$ and 1^+, $T=0$ states belong to the $K^\pi=0^+$ band, whereas the 2_1^-, $T=0$ state belongs to the $K^\pi=0^-$ band (see Fig. 3). Within the rotational model [22] one obtains for $E1$ matrix elements between states of the $K=0$ bands:

$$\langle K^\pi=0^-,T=0,I_i\|\mathbf{T}(E1)\|K^\pi=0^+,T=1,I_f\rangle=\sqrt{2I_f+1}\langle I_f010|I_i0\rangle\langle K^\pi=0^-,T=0|\mathbf{T}(E1)|K^\pi=0^+,T=1\rangle, \tag{6}$$

with the intrinsic $E1$ matrix element and the Clebsch-Gordan coefficient $\langle I_f010|I_i0\rangle$. If I_i and I_f are both even or both odd, the Clebsch-Gordan coefficient will vanish, which means $\langle I_f=2010|20\rangle=0$. Subsequently, the $2_1^+,T=1\rightarrow 2^-,T=0$ transition within the $K^\pi=0^+$ band is forbidden by collective model selection rules despite its isovector character. Therefore the whole $E1$ strength originates from the small admixtures of $K\neq 0$ components of both states. Since K is a good quantum number for the lowest states in ^{46}V [21], the $B(E1;2_1^-,T=0\rightarrow 2_1^+,T=1)$ value has to be very small, being comparable to the one for the $2_1^-,T=0\rightarrow 1^+,T=0$ transition, which is isospin forbidden, but allowed by collective model selection rules.

In summary, the present work improves our knowledge of the absolute transition probabilities of two members of the $A=46$ isospin multiplet, which are accessible by means of RDDS measurements. The new precise lifetime data agree well with shell model calculations for positive parity states of ^{46}V, and the experimental $B(E2;2_1^+\rightarrow 0^+)$ values confirm the trend predicted from the isospin symmetry within the isobaric $T=1$ triplet. Furthermore, in ^{46}V a first comparison of isospin allowed and forbidden $E1$ transitions is given, and the puzzle of a weak allowed $\Delta T=1$ transition is clarified. Nevertheless, in this nucleus, further lifetime data on other $E1$ transitions, especially allowed ones, are needed for a more general comparison.

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