Nonadditivity in nuclear moments of inertia and interference between blocking effects

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The experimentally observed obvious nonadditivity in the nuclear moment of inertia and angular momentum alignment of multiquasiparticle rotational bands, both in normally deformed and superdeformed nuclei, presents a serious challenge to the traditional BCS treatment for nuclear pairing. It is shown that the systematically observed nonadditivity mainly comes from the destructive interference between blocking effects, and can be satisfactorily accounted for by the particle-number conserving method, in which the blocking effects are treated consistently and exactly.

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Since the suggestion of nuclear superconductivity by Bohr and co-workers [1,2], the nuclear pairing correlation is usually treated using the BCS method [3–5] and the concept of the quasiparticle (qp) turned out to be very convenient and useful. There is no doubt that the BCS theory had achieved great success in the description of the superconductivity of metals. However, when it is applied to treat nuclear pairing correlation, along with its benefit, it brings some defects which should be considered seriously [6–11]. There have been some serious challenges presented to the BCS description for nuclear structure.

One defect of the BCS approximation is the particle-number nonconservation. Because the number of nucleons in a nucleus ($\sim 10^2$), particularly the number of valence nucleons (~ 10) which dominate the properties of low-lying excited states, is very limited, some conclusions drawn from the BCS approximation turned out to be incorrect. A famous example is that in all self-consistent solutions to the cranked Hartree-Fock-Bogliubov equation, a pairing collapse is found [12], but calculation with particle-number projection before variation showed that the gap parameter decreases very slowly with rotational frequency and no sharp phase transition is found [13]. Another problem related to particle-number nonconservation is that there exists an extensive spurious state in the BCS approximation [8].

The most serious defect of the BCS treatment is that it is unable to treat the blocking effect consistently. According to the BCS theory of pairing, the moment of inertia (MOI) of a 1-qp band in an odd-A nucleus should be larger than those of qp-vacuum bands in neighboring even-even nuclei by a factor of about 15%, and for a 2-qp band a 30% increase is expected [2,14]. However, experiments show that a lot of 1-qp bands at low spin in normaly deformed (ND) rare-earth nuclei have moments of inertia nearly identical to that of the qp-vacuum band in neighboring even-even nuclei. To date this remains an open problem [15] and it seems hopeless to be explained in the BCS formalism. Experiments also show that there exists a large fluctuation of the odd-even difference in MOI $\delta J/J$ [2,16]; i.e., while for some ND bands in the rare-earth nuclei, $\delta J/J \sim 0$ (the so-called identical bands), $\delta J/J$ may be quite large for some ND bands, e.g., $\delta J/J \approx 130\%$ for ^{151}Dy([642]5/2) and ^{150}Dy(g.s.b.).

Another serious problem is that the BCS picture leads to additivity in some physical quantities (qp energy, MOI, etc.). However, it has been established experimentally that the additivity in nuclear MOI and angular momentum alignment $i(\omega)$, in general, fails for ND nuclei and superdeformed (SD) bands in the $A \sim 190$ region [8,17–20]. In Ref. [19], the experimental alignments of the odd-odd SD bands in ^{192}Tl and the neighboring odd-proton band in ^{191}Tl and odd-neutron band in ^{191}Hg are compared and an obvious nonadditivity in $i(\omega)$ is found, which is attributed to the residual qp interaction. In the PNC treatment there is no particle-quasiparticle transformation, thus no residual qp interaction. We will show that, while the alignment $i(\omega)$ itself is attributed to the Pauli blocking effects, the nonadditivity in $i(\omega)$ comes from the destructive interference between Pauli blocking effects. In Ref. [18] the Pauli blocking effects and a number of related questions are discussed in detail; e.g., why the behavior of $J^{(2)}$ of SD bands in the $A \sim 190$ region is so similar at higher frequency, but at low frequency there exists a large difference. In the PNC treatment it has been shown that at low frequency the Pauli blocking effect on pairing is crucial to account for various odd-even differences and their large fluctuation, but with increasing ω, while the Coriolis antipairing effect becomes more and more important, the blocking effects on pairing gradually weaken, thus the odd-even difference gradually disappears.

In Refs. [18,19] and in earlier related papers, the experimental and theoretical $[i(\mu)+i(\nu)-i(\mu\nu)]$ were compared. In this paper, the ratios of $i(\mu)+i(\nu)$ to $i(\mu\nu)$, rather than their differences, are investigated. Assume the kinematic MOI of rotational bands based on the qp vacuum state $|0\rangle\rangle$, 1-qp states $\alpha_\mu^+|0\rangle\rangle$ and $\alpha_\nu^+|0\rangle\rangle$, and 2-qp state $\alpha_\mu^+\alpha_\nu^+|0\rangle\rangle$ are denoted by J_0, $J(\mu)$, $J(\nu)$, and $J(\mu\nu)$, respectively, it can be shown that in the BCS approximation [8],

$$R(\mu,\nu)=\frac{[J(\mu)-J_0]+[J(\nu)-J_0]}{J(\mu\nu)-J_0}=1 \qquad (1)$$

or equivalently, the angular momentum alignments of qp bands relative to the qp vacuum band, $i(\mu)=\omega[J(\mu)-J_0]$, are additive. Similar relations hold for multi-qp

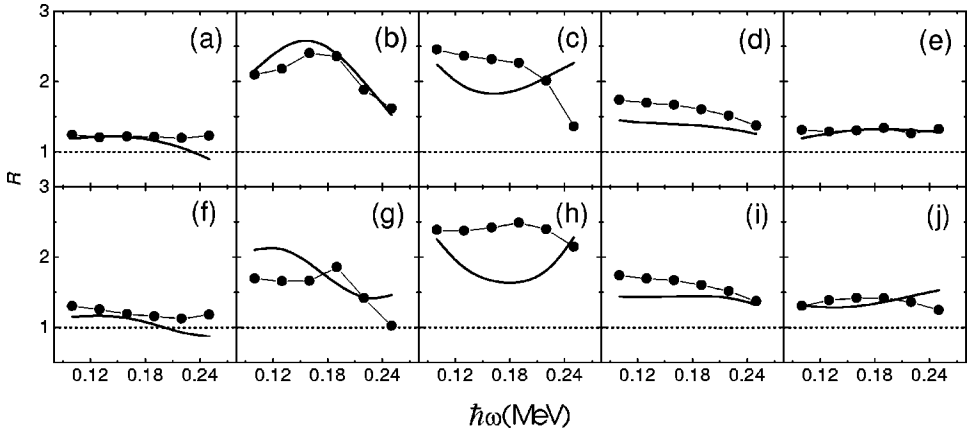

FIG. 1. The comparison between experimental and calculated R ratios [see Eq. (1)] for three 2-qp bands, a 4-qp band both in ^{172}Hf and ^{174} Hf, and two 3-qp bands in ^{173}Hf. (a) ^{172}Hf ($K^{\pi}=6^{-}$), (b) ^{172}Hf ($K^{\pi}=6^{+}$), (c) ^{172}Hf ($K^{\pi}=8^{-}$), (d) ^{172}Hf ($K^{\pi}=14^{+}$), (e) ^{173}Hf ($K^{\pi}=19/2^{+}$), (f) ^{174}Hf ($K^{\pi}=6^{-}$), (g) ^{174}Hf ($K^{\pi}=6^{+}$), (h) ^{174}Hf ($K^{\pi}=8^{-}$), (i) ^{174}Hf ($K^{\pi}=14^{+}$), and (j) ^{173}Hf ($K^{\pi}=23/2^{-}$). The experimental and calculated R are plotted by solid circles and lines, respectively. The corresponding BCS R ratios are equal to 1 and denoted by dot lines.

bands. However, all the available experimental data of multi-qp bands both in ND and SD nuclei show that $R_{\rm exp}$ obviously deviates from 1. In this paper we will show that the systematically observed nonadditivity both in ND and SD nuclei can be satisfactorily accounted for in the PNC treatment of the cranked shell model.

The cranked shell model Hamiltonian with pairing interaction is

$$H_{CSM}=H_{SP}-\omega J_x+H_P=H_0+H_P, \qquad (2)$$

where $H_0=H_{SP}-\omega J_x=\Sigma_i h_0(\omega)_i$, $h_0(\omega)=h_{Nilsson}-\omega j_x$ is the one-body part of H_{CSM}, $h_{Nilsson}$ is the Nilsson Hamiltonian, $-\omega J_x$ the the Coriolis interaction, and H_P is the pairing interaction including both the monopole and quadrupole pairing interactions. In the PNC calculation, first, $h_0(\omega)$ is diagonalized to obtain cranked Nilsson orbitals. Then, H_{CSM} is diagonalized in a sufficiently large cranked many-particle configuration space to obtain accurate solutions to the yrast and low-lying eigenstates. For the details of the PNC method, see Refs. [21–24].

The calculated results for some typical multi-qp bands by the PNC method are given in Figs. 1–3. In Fig. 1 are shown

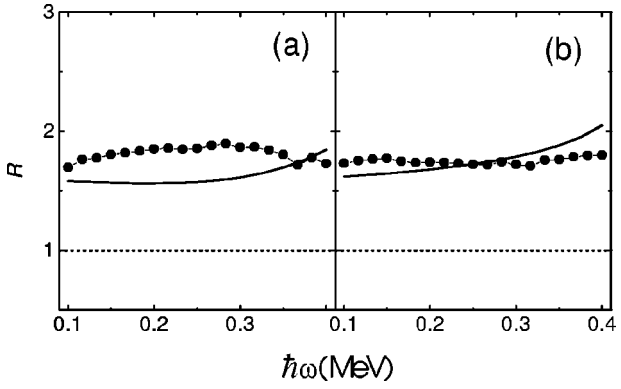

FIG. 2. The same as Fig. 1, but for the 2-qp SD bands ^{194}Hg(2, 3). (a) ^{194}Hg(2), (b) ^{194}Hg(3).

three low-lying excited 2-qp bands and one 4-qp band both in ^{172}Hf [25] and ^{174}Hf [26], $K^{\pi}=6^{-}(\nu^2[633]7/2\otimes[512]5/2)$, $6^{+}(\pi^2[404]7/2\otimes[402]5/2)$, $8^{-}(\pi^2[404]7/2\otimes[514]9/2)$, $14^{+}(\pi^2[404]7/2\otimes[514]9/2\otimes\nu^2[633]7/2\otimes\nu[512]5/2)$ and two 3-qp bands in ^{173}Hf, $K^{\pi}=19/2^{+}(\pi^2[404]7/2\otimes[402]5/2\otimes\nu[633]7/2)$, $23/2^{-}(\pi^2[404]7/2\otimes[514]9/2\otimes\nu[633]7/2)$. The ^{172}Hf(g.s.b.) is taken as the qp vacuum (reference) band. The 1-qp bands are ^{173}Hf($\nu[633]7/2$), ^{173}Hf($\nu[512]5/2$), $^{171}_{71}$Lu($\pi[404]7/2$), ^{171}Lu($\pi[402]5/2$) and ^{171}Lu($\pi[514]9/2$) [27,28]. In our calculation for these ND bands, the deformation and Nilsson parameters (κ,μ) are taken from the Lund systematics [29,30] and the pairing strength is determined by the experimental odd-even differences in binding energies and bandhead moments of inertia as in Ref. [24]. In Fig. 1, a significant deviation of the experimental R ratios from 1 is found in the observed frequency range ($\hbar\omega<0.25$ MeV). In general, the deviation of the R ratio from 1 may also come from a possible deformation difference between the qp vacuum and various multi-qp states. However, for the most stable ND rare-earth nuclei, such as ^{172,173,174}Hf, the deformation change may be of minor importance and the significant nonadditivity mainly comes from the destructive interference between Pauli blocking effects. It is seen that the experimental R ratios are satisfactorily reproduced by the PNC calculation with no free parameters.

One might expect that additivity should hold better for SD bands because of their larger and more stable deformation and weaker effective pairing interaction [19]. In Fig. 2 is shown the analysis for the 2-qp signature partner SD bands, ^{194}Hg(2) ($\alpha=0$) and ^{194}Hg(3) ($\alpha=1$) [31]. The ^{192}Hg(1) [32] is taken as the reference and the 1-qp bands are ^{193}Hg($2a$) ($\alpha=+1/2$, $\nu[512]5/2$), ^{193}Hg($2b$) ($\alpha=+1/2$, $\nu[624]9/2$), and ^{193}Hg(3) ($\alpha=-1/2$, $\nu[624]9/2$) [33]. In the calculation, the Nilsson parameters (κ, μ) are taken from the Lund Systematics [29] and the deformation parameters are taken from Ref. [30]. Because the spins of ^{194}Hg(3) were established experimentally, the large number of $J^{(1)}(\omega)$

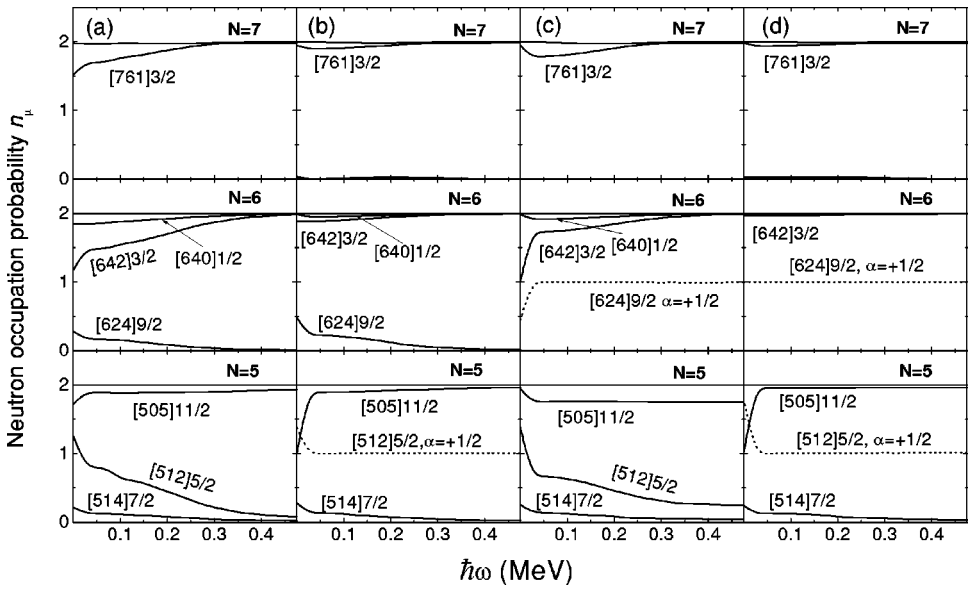

FIG. 3. The occupation probability n_μ of cranked neutron orbital μ (including both signatures $\alpha=\pm 1/2$) in each major shell ($N=5,6,7$) near the Fermi surface is plotted by a solid line. n_μ of the blocked orbital is denoted by a dotted line. (a) ^{192}Hg(1), (b) ^{193}Hg($2a$), (c) ^{193}Hg($2b$), and (d) ^{194}Hg(3).

($\hbar\omega \sim 0.10$–0.40 MeV) can be extracted accurately from the experimental intraband γ transition energies E_γ's, which are used to determine the pairing interaction strengths. The dimension of the CMPC space is about 900 for protons and 1000 for neutrons, and in this case the effective pairing interaction strengths (in units of MeV) are G_0(monopole) $=0.300$, G_2(quadrupole) $=0.065$ for protons and $G_0=0.205$, $G_2=0.003$ for neutrons. It is seen that for the 2-qp signature partner SD bands ^{194}Hg(2,3), the experimental R ratios are reproduced quite well by the PNC calculation. Considering the constancy of deformation along the same SD band, the approximate constancy of both the experimental and calculated R ratios implies that the significant deviation of R from 1 mainly comes from Pauli blocking effects.

More careful analysis shows that the nonadditivity in 2-qp bands can be understood by considering the destructive interference between blocking effects. For example, for the 1-qp bands ^{193}Hg($2a$) ($\alpha=+1/2$) and ^{193}Hg($2b$) ($\alpha=+1/2$), the separate blocking effect of orbitals $\nu[512]5/2$ and $\nu[624]9/2$ on the corresponding MOI is manifested sufficiently, whereas for the 2-qp band ^{194}Hg(3) ($\alpha=1$) the blocking effects of $\nu[512]5/2$ and $\nu[624]9/2$ cancel each other to a certain extent, which is manifested in the difference in the structure of CMPC space and the corresponding neutron occupation probability n_μ for the 2-qp band comparing with those for two 1-qp bands (see Fig. 3). As a result, $[J(\pi^2[512]5/2\otimes[624]9/2)-J_0]<[J(\nu[512]5/2)+J(\nu[624]9/2)-2J_0]$ and $R>1$.

The influence of blocking effects is clearly exhibited in the occupation probabilities n_μ's of each orbitals, which in turn affect the gap parameter. In Fig. 3 are shown the neutron occupation probabilities of each orbital near the Fermi surface for four SD bands: the qp vacuum band ^{192}Hg(1), 1-qp bands ^{193}Hg($2a$) and ^{193}Hg($2b$), and 2-qp band ^{194}Hg(3). It is seen that the neutron occupation for ^{192}Hg(1) obviously deviates a complete degenerate Fermi distribution due to the relatively strong pairing interaction in the qp vacuum state. In Figs. 3(b) and 3(c) are shown the n_μ's for ^{193}Hg($2a$) and ^{193}Hg($2b$), in which is observed a moderate pairing reduction due to the separate blocking effect of $\nu[512]5/2(\alpha=+1/2)$ and $\nu[624]9/2(\alpha=+1/2)$. Therefore, in Fig. 3(b), except $n_\mu \approx 1$ for the blocked orbital ($\nu[512]5/2,\alpha=+1/2$), some transitions with a certain probability from orbitals below the Fermi surface ([761]3/2, [642]3/2, [505]11/2, etc.) to those above the Fermi surface (624]5/2, [514]7/2, etc.) are observed. The situation of ^{193}Hg($2b$) is similar. As for the 2-qp band ^{194}Hg(3)($\alpha=1$) [Fig. 3(d)], due to the double blocking of orbitals $\nu[512]5/2(\alpha=+1/2)$ and $\nu[624]9/2(\alpha=+1/2)$, a significant reduction of pairing is found and the distribution almost tends to a complete Fermi distribution (neutron gap parameter $\Delta\sim 0$), except the two blocked levels. A more important fact is that because of the interference between the blocking effects of orbitals [512]5//2 and [624]9/2, the n_μ's for ^{193}Hg($2a$) and ^{193}Hg($2b$) are quite different from those in the 2-qp band ^{194}Hg(3), i.e., for these unblocked levels, the average of n_μ for ^{193}Hg($2a$) and ^{193}Hg($2b$) is different from that for ^{194}Hg(3). The change in the occupation probabilities of each orbital mentioned above implies that the corresponding neutron gap parameters $\Delta(^{192}\mathrm{Hg}(1))>\Delta(^{193}\mathrm{Hg}(2a))\sim\Delta(^{193}\mathrm{Hg}(2b))>\Delta(^{194}\mathrm{Hg}(3))$.

In summary, one of the serious challenges to the BCS description for nuclear structure, the nonadditivity in nuclear MOI widely observed both in ND and SD nuclei, can be accounted for satisfactorily by the PNC treatment for nuclear pairing, in which the blocking effects are treated consistently and exactly. It is noted that, while the odd-even differences in MOI's themselves are mainly due to the blocking effects, the nonadditivity in moments of inertia and angular momentum alignment of multi-qp bands in stable ND and SD nuclei mainly comes from the destructive interference between blocking effects.

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