

Dispersion relation analysis of neutral pion photoproduction and electroproduction at threshold

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Neutral pion photoproduction and electroproduction at threshold is analyzed in the framework of dispersion relations. For this purpose, we evaluate the real threshold amplitudes in terms of Born contributions and dispersion integrals determined by the imaginary parts of the multipoles of the unitary isobar model (MAID) and the phenomenological partial-wave analysis (SAID). The results show considerable cancellations between Born terms and resonance contributions. Good agreement with the data is found for photoproduction. While our dispersion analysis suggests considerable discrepancies for electroproduction, the present state of the experimental multipole analysis at finite Q^2 does not permit drawing conclusions at this time.

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I. INTRODUCTION

Electroproduction and photoproduction of neutral pions near threshold have been a topic of many experimental and theoretical investigations over the past decade. Triggered by surprising results obtained at Saclay [1], the Mainz [2] and Saskatoon [3] groups established that a formerly believed low-energy theorem (LET) [4,5] for S -wave photoproduction was at variance with nature. While the LET predicted a threshold S -wave multipole $E_{0+} = -2.4 \times 10^{-3}/m_\pi$, the experiment yielded $E_{0+} \approx -1.3 \times 10^{-3}/m_\pi$. The discrepancy between the theorem and the experimental data was finally explained by Bernard *et al.* [6] who showed that loop corrections provided nonanalytical terms in the pion mass μ . The flaw of the low-energy theorem was therefore the assumption that the amplitudes would be an analytical function in the pion mass μ , which could be expanded in a Taylor series in the soft-pion limit. In the following years, these calculations were considerably refined by evaluating the S -wave amplitude E_{0+} to order p^4 in the chiral expansion, and the three P -wave amplitudes (E_{1+} , M_{1+} , and M_{1-}) up to order p^3 . While there appear three low-energy constants to that order, two combinations of P -wave amplitudes were found to be independent of these constants. Further work has extended this approach to virtual photons [7].

Recently, a good description of π^0 photoproduction and electroproduction in the threshold region was also found within a meson-exchange dynamical model [8,9]. In particular, the largest contributions to the final-state interaction were shown to come from one-loop charge-exchange rescattering, which lead to a good description of the S -wave multipoles.

The large reduction of the S -wave threshold amplitude was independently obtained using fixed- t dispersion relations [10]. In this approach, the Born terms have to be evaluated at the nucleon pole where the pseudovector and the pseudo-

scalar pion-nucleon coupling are identical. While the result of the old LET was essentially equivalent to the use of pseudovector coupling at threshold, the value of the multipole at the pole position corresponds to pseudoscalar coupling. As a result the Born term to be used in dispersion theory is $E_{0+}(\text{pole}) = -7.9 \times 10^{-3}/m_\pi$, and thus the dispersion integrals over the excited states have to cancel about 80% of the pole term in order to describe the data.

In Ref. [10], the coupled-integral equations were solved using the method of Omnes and Mushkashevili [11]. On the condition that the complex phases of the multipoles are known and with given assumptions for their high-energy behavior, this method allows one to find unique solutions. In practice, however, the phases are known only in the energy region below the two-pion threshold due to the Watson theorem [12]. Extending these calculations to energies above the second resonance region, which coincides with the onset of two-pion production, requires modeling the phases by functions which depend on the pion-nucleon phase shifts and inelasticity parameters. The ansatz for the functional dependence is based on unitarity but by no means unique, and in principle has to be determined by a fit to the data. It is therefore the aim of the present work to extend the energy range of the dispersion analysis by use of the unitary isobar model [13] (called MAID in the following) as an input for the imaginary parts of the multipole amplitudes. At the same time, we want to compare the results obtained by use of MAID with those with the SAID multipoles [14,26]. This allows us to present a qualitative “error band” for the dispersion analysis, which often has been asked for.

Our paper is organized as follows. In Sec. II, we briefly recall the ingredients of dispersion relations at fixed t . The actual calculations are described in Sec. III. In particular, we extend the energy range of the MAID model by including the contributions from all S -, P -, D -, and F -wave resonances with four-star PDG status [25]. As a particularly sensitive test of the extended model, we present predictions of our calculation for threshold production of neutral pions in Sec. IV.

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II. DISPERSION RELATIONS FOR PION ELECTROPRODUCTION

In the present work, we will use fixed- t dispersion relations (DR) to construct the pion electroproduction multipoles (or partial waves) $\tilde{\mathcal{M}}$,

$$\begin{aligned} \text{Re } \tilde{\mathcal{M}}_\alpha(W, Q^2) &= \tilde{\mathcal{M}}_\alpha^{Pole}(W, Q^2) \\ &+ \frac{P}{\pi} \int_{W_{thr.}}^{\infty} dW' \frac{\text{Im } \tilde{\mathcal{M}}_\alpha(W', Q^2)}{W' - W} \\ &+ \frac{1}{\pi} \int_{W_{thr.}}^{\infty} dW' \sum_{\beta} \tilde{\mathcal{K}}_{\alpha\beta}(W, W', Q^2) \\ &\times \text{Im } \tilde{\mathcal{M}}_\beta(W', Q^2), \end{aligned} \quad (1)$$

where α and β are the set of quantum numbers, W is the total c.m. energy of the πN system, and $Q^2 = \mathbf{k}^2 - \omega^2 > 0$ is the four-momentum squared of the virtual photon with three-momentum \mathbf{k} and energy ω . The first term in Eq. (1), $\tilde{\mathcal{M}}_\alpha^{Pole}$, comprises the explicitly known contributions from the pole diagrams with pseudoscalar πNN coupling. The second and third terms are the principal value and regular parts of the dispersion integrals which contain the kernels $\tilde{\mathcal{K}}_{\alpha\beta}$ and the imaginary parts of the multipoles. Both integrals run only over the physical region starting at threshold $W_{thr} = m + \mu$, where m and μ are the nucleon and pion masses, respectively.

The detailed expressions for the kernels and the numerical recipes for their numerical computation are given in Ref. [15] and in the Appendix. The multipoles $\tilde{\mathcal{M}}_\alpha = (\tilde{\mathcal{E}}_{l\pm}, \tilde{\mathcal{M}}_{l\pm}, \tilde{\mathcal{L}}_{l\pm}/\omega)$ are related to the standard Chew-Goldberger-Low-Nambu (CGLN) [16] multipoles $\mathcal{M}_\alpha = (E_{l\pm}, M_{l\pm}, L_{l\pm})$ by the following equations:

$$\begin{aligned} \tilde{\mathcal{E}}_{l+} &= -8\pi \frac{\sqrt{\mathcal{E}_1/\mathcal{E}_2}}{(qk)^l k^2} E_{l+}, \\ \tilde{\mathcal{E}}_{l+1,-} &= -8\pi \frac{\sqrt{\mathcal{E}_2/\mathcal{E}_1} k^2 W}{(qk)^{l+1}} E_{l+1,-}, \\ \tilde{\mathcal{M}}_{l+} &= 8\pi \frac{\sqrt{\mathcal{E}_1/\mathcal{E}_2} W}{(qk)^l} M_{l+}, \\ \tilde{\mathcal{M}}_{l+1,-} &= -8\pi \frac{\sqrt{\mathcal{E}_2/\mathcal{E}_1}}{(qk)^{l+1}} M_{l+1,-}, \\ \tilde{\mathcal{L}}_{l+} &= 8\pi \frac{\sqrt{\mathcal{E}_1/\mathcal{E}_2}}{(qk)^l k^2} L_{l+}, \quad \tilde{\mathcal{L}}_{l+1,-} = 8\pi \frac{\sqrt{\mathcal{E}_2/\mathcal{E}_1}}{(qk)^{l+1}} L_{l+1,-}, \end{aligned} \quad (2)$$

with $\mathcal{E}_{1(2)} = E_{1(2)} + m$, where $E_{1(2)}$ denotes the nucleon c.m. energy in the initial (final) state, $q = |\mathbf{q}|$ and $k = |\mathbf{k}|$ the absolute values of the c.m. pion and photon momenta, respectively, and l the pion orbital momentum.

While the fixed- t DR in the form of Eq. (1) are uniquely defined, the separation into the principal value and regular integral contributions is not unique and depends on the choice of the kinematical factors in Eq. (2). Other kinematical factors, i.e., as used in Refs. [10,17,18], will change the relative contributions of these two integrals and the expressions for the kernels. For example, if we introduce a new set of the multipoles via the relation $\tilde{\mathcal{M}}'_\alpha(W) = \tilde{\mathcal{M}}_\alpha(W)/f_\alpha(W)$ with a certain factor $f_\alpha(W)$, we find the following relation between the new and old kernels:

$$\tilde{\mathcal{K}}'_{\alpha\beta}(W, W') = \frac{f_\beta(W')}{f_\alpha(W)} \tilde{\mathcal{K}}_{\alpha\beta}(W, W') + \delta_{\alpha\beta} \frac{f_\beta(W') - f_\alpha(W)}{f_\alpha(W)(W' - W)}. \quad (3)$$

The different expressions for the kernels given in the literature can be easily checked and compared by use of these relations. For example, we find that at $Q^2 = 0$, the kernels from Refs. [15] and [17] lead to the same result.

For future analysis, it is convenient to rewrite the DR of Eq. (1) in terms of the CGLN multipoles $\mathcal{M}_\alpha = (E_{l\pm}, M_{l\pm}, L_{l\pm}/\omega)$,

$$\begin{aligned} \text{Re } \mathcal{M}_\alpha(W) &= \mathcal{M}_\alpha^{Pole}(W) + \mathcal{M}_\alpha^{Diag}(W) \\ &+ \frac{1}{\pi} \int_{W_{thr.}}^{\infty} dW' \sum_{\beta \neq \alpha} K_{\alpha\beta}(W, W') \text{Im } \mathcal{M}_\beta(W'), \end{aligned} \quad (4)$$

where

$$\begin{aligned} \mathcal{M}_\alpha^{Diag}(W) &= \frac{P}{\pi} \int_{W_{thr.}}^{\infty} dW' \frac{\text{Im } \mathcal{M}_\alpha(W') r_\alpha(W')}{(W' - W) r_\alpha(W)} \\ &+ \frac{1}{\pi} \int_{W_{thr.}}^{\infty} dW' K_{\alpha\alpha}(W, W') \text{Im } \mathcal{M}_\alpha(W'). \end{aligned} \quad (5)$$

The kinematical factor $r_\alpha(W)$ is defined by Eq. (2) with the relation $\tilde{\mathcal{M}}_\alpha(W) = r_\alpha(W) \mathcal{M}_\alpha(W)$, and $K_{\alpha\beta}(W, W') = \tilde{\mathcal{K}}_{\alpha\beta}(W, W') r_\beta(W') / r_\alpha(W)$. One of the advantages of such a representation is that each term in Eq. (4) is individually independent of the choice for the kinematical factor r_α . This statement can be easily proved by use of Eq. (3). Further details about the construction of the kernels are given in the Appendix.

III. CALCULATIONS OF THE DISPERSION INTEGRALS

One of the methods widely used to calculate the dispersion integrals in Eq. (1) or Eqs. (4) and (5) is based on the Watson theorem [12], stating that the phase of pion photo-production and electroproduction is equal to the phase shift of pion-nucleon scattering, $\delta_\alpha(W)$, below the two-pion threshold. Below this threshold, we can therefore use the following relation between the real and imaginary parts of the amplitude:

TABLE I. Model parameters of the nucleon resonances in the proton channels (resonance mass M_R , width Γ_R , pion branching ratio β_π) and corresponding resonance + background values of the imaginary parts of the electric (\mathcal{M}_E) and magnetic multipoles (\mathcal{M}_M) at resonance (in units of $10^{-3}/m_{\pi^+}$) obtained with the MAID2002 and SAID(SM02) solutions. The partial branching ratios for the $S_{11}(1535)$ are assumed to be $\beta_\pi=0.40$, $\beta_\eta=0.50$, and $\beta_{2\pi}=0.10$.

N^*	M_R [MeV]	Γ_R [MeV]	β_π	MAID		SAID	
				\mathcal{M}_E	\mathcal{M}_M	\mathcal{M}_E	\mathcal{M}_M
$P_{33}(1232)$	1232	130	1.0	-0.81	36.85	-0.54	36.01
$P_{11}(1440)$	1440	350	0.70		2.75		2.74
$D_{13}(1520)$	1520	130	0.60	4.56	1.97	5.31	2.18
$S_{11}(1535)$	1520	80	0.40	3.83		3.77	
$S_{31}(1620)$	1620	150	0.25	-1.28		-0.79	
$S_{11}(1650)$	1690	100	0.85	2.45		3.81	
$D_{15}(1675)$	1675	150	0.45	0.10	0.32	0.03	0.25
$F_{15}(1680)$	1680	135	0.70	1.77	1.23	1.80	1.20
$D_{33}(1700)$	1740	450	0.15	-3.54	0.25	-2.83	0.72
$P_{13}(1720)$	1720	250	0.20	0.55	-0.07	0.58	0.02
$F_{35}(1905)$	1905	350	0.10	0.45	0.32	0.40	0.29
$P_{31}(1910)$	1910	200	0.25		0.52		0.83
$F_{37}(1950)$	1950	300	0.20	0.02	1.45	0.04	1.36

$$\text{Im } \mathcal{M}_\alpha(W, Q^2) = \text{Re } \mathcal{M}_\alpha(W, Q^2) \tan \delta_\alpha(W). \quad (6)$$

If we further make an assumption about the high-energy behavior of the multipole phases, we obtain a system of coupled integral equations for $\text{Re } \mathcal{M}_\alpha(W)$. This is the standard method to apply fixed- t dispersion relations to pion photoproduction at threshold and in the $\Delta(1232)$ resonance region, which was successfully used by many authors [10,17–19]. The reliability of this method at low energies ($W < 1400$ MeV) is mainly based on the finding that Eq. (6) can be applied to the important P_{33} multipole, dominated by the $\Delta(1232)$ resonance contribution, with good accuracy up to $W = 1600$ MeV.

Another method to calculate the dispersion integrals is based on isobaric models [20–23] which allow extending the use of fixed- t DR to higher energies. Within these models the imaginary parts of the pion photoproduction and electroproduction multipoles are expressed in terms of background (\mathcal{M}^B) and resonance (\mathcal{M}^R) contributions,

$$\text{Im } \mathcal{M}_\alpha(W, Q^2) = \text{Im } \mathcal{M}_\alpha^B(W, Q^2) + \text{Im } \mathcal{M}_\alpha^R(W, Q^2). \quad (7)$$

In the present work, both parts will be modeled similar to the recently developed unitary isobar model [13] MAID. The imaginary parts from the background appear due to final-state interaction effects for the pions produced by nonresonant mechanisms and contain contributions from both the Born terms (V_α^{Born}) with an energy-dependent mixing of pseudovector-pseudoscalar (PV-PS) πNN coupling and t -channel vector-meson exchanges ($V_\alpha^{\omega, \rho}$),

$$\begin{aligned} \mathcal{M}_\alpha^B(W, Q^2) = & [V_\alpha^{\text{Born}}(W, Q^2) + V_\alpha^{\omega, \rho}(W, Q^2)] [1 \\ & + iT_{\pi N}^\alpha(W)], \end{aligned} \quad (8)$$

where the pion-nucleon scattering amplitude $T_{\pi N}^\alpha = (1/2i) \times [\eta_\alpha \exp(2i\delta_\alpha) - 1]$ is given in terms of the πN phase shifts δ_α and the inelasticity parameters η_α , taken from the analysis of the SAID group [24]. In accordance with Ref. [13] the background contribution depends on five parameters: The PV-PS mixing parameter Λ_m in V^{Born} [see Eq. (12) of Ref. [13]] and four coupling constants in $V^{\omega, \rho}$. Note that in our present work, we do not include hadronic form factors at the ωNN and ρNN vertices.

Following Ref. [13] the resonance contributions are given in terms of Breit-Wigner amplitudes,

$$\mathcal{M}_\alpha^R(W, Q^2) = \bar{\mathcal{A}}_\alpha^R(Q^2) \frac{f_{\gamma R}(W) \Gamma_R M_R f_{\pi R}(W)}{M_R^2 - W^2 - i M_R \Gamma_R} e^{i\phi_R}, \quad (9)$$

where $f_{\pi R}$ is the usual Breit-Wigner factor describing the decay of a resonance R with total width $\Gamma_R(W)$ and physical mass M_R . The main parameters in the resonance contributions are the strengths of the electromagnetic transitions described by the reduced amplitudes $\bar{\mathcal{A}}_\alpha^R(Q^2)$, which have to be extracted from the analysis of the experimental data. In the present work, we extend the previously developed MAID model by including contributions from all S -, P -, D -, and F -wave resonances with four-star PDG status [25]. The addition of new resonances requires performing a new fit. For this purpose, we use the SAID data base [26] for pion photoproduction in the energy range $W_{\text{thr}} < W < 2000$ MeV with 15 700 data points. The resonance parameters and values of $\text{Im } \mathcal{M}$ at the resonance position obtained from the best fit are listed in Table I. We note that in most cases the background contributions to the imaginary parts are less than 10% at the resonance positions. The only exceptions are the channels with the $S_{31}(1620)$, $S_{11}(1650)$, and $P_{31}(1910)$ resonances, for which we find $\text{Im } \mathcal{M}^B = 2.50, 0.85, \text{ and } -1.45$, respec-

tively, in comparison to $\text{Im } \mathcal{M}^R = -1.28, 2.45, \text{ and } 0.52$. Here and in the following, all multipoles are quoted in units of $10^{-3}/m_{\pi^+}$. In the case of the overlapping resonances in the S_{11} proton channel, we find $\text{Im}_p E^{(1/2)}(1535) = 3.32 + 0.14 + 0.37$ and $\text{Im}_p E^{(1/2)}(1650) = 0.43 + 1.17 + 0.85$, where the first and second terms are the contributions from the first and second S_{11} resonance, respectively, and the last terms come from the background contributions.

Alternatively, we calculate the dispersion integrals using the solution SM02 of the SAID multipole analysis [14] (see Table I). Concerning the integration up to infinity, we assume that the multipoles have an asymptotic behavior like $1/W$ for $W \geq 2300$ MeV. This is the minimal power providing convergence for the GDH sum rule [27]. In the threshold region, we introduce the pion mass difference by assuming that the imaginary part of the E_{0+} multipoles is proportional to the π^+ momentum below $W = 1090$ MeV. This assumption is based on the fact that near threshold the main contribution to the imaginary part comes from the coupling with the $\pi^+ n$ channel [9].

IV. RESULTS AND DISCUSSIONS

A. π^0 photoproduction at threshold

The threshold region has traditionally posed a problem to the analysis of π^0 photoproduction within a dispersion-relation approach [17]. This is due mainly to considerable cancellations in the dispersion integrals of Eqs. (4) and (5). As shown in Ref. [10], by solving the integral equations using the Watson theorem, the real part of the $E_{0+}(\pi^0 p)$ threshold multipole obtains surprisingly large contributions from the imaginary parts of higher multipoles which peak at much larger energies. As a result, the high-energy region provides sufficiently large contributions to nearly cancel the nucleon pole term with pseudoscalar πNN coupling, thus leading to agreement with the experimental threshold values.

Similar results are obtained in our present work using fixed- t DR and imaginary parts of the multipoles taken from the MAID model and from the results of the SAID multipole analysis,

$$\begin{aligned} E_{0+}^{thr}(p\pi^0) &= -7.89 + 2.84 + 4.09 - 0.48 - 0.25 + 0.40 \\ &= -1.29 \text{ DR(MAID)}, \end{aligned} \quad (10)$$

$$\begin{aligned} E_{0+}^{thr}(p\pi^0) &= -7.89 + 2.83 + 4.23 - 0.51 - 0.14 + 0.13 \\ &= -1.35 \text{ DR(SAID)}, \end{aligned} \quad (11)$$

where the contributions on the right-hand side are presented, in accordance with Eq. (4), in the following order: the pole term, the diagonal E_{0+} , the kernel terms M_{1+} , M_{1-} , E_{1+} , and the combined kernel contributions of the higher D - and F -wave multipoles. According to Eq. (5), the diagonal E_{0+} contribution can be further divided into the principal-value integral and the regular integral, which contribute $1.23 + 1.61$ using MAID and $1.31 + 1.52$ using SAID solutions. As discussed above, this sum does not depend on the choice for the kinematical factor $r_\alpha(W)$. The individual contribu-

TABLE II. Individual contributions of the D and F multipoles (in units of $10^{-3}/m_{\pi^+}$) to the multipole $E_{0+}^{thr}(\pi^0 p)$ at threshold.

	E_{2-}	M_{2-}	E_{2+}	M_{2+}	E_{3-}	M_{3-}	E_{3+}	M_{3+}
MAID	0.16	-0.16	0.06	0.04	0.34	-0.37	-0.01	0.34
SAID	0.28	-0.51	0.05	0.07	0.37	-0.51	-0.02	0.40

tions from the coupling to the D - and F -wave multipoles are presented in Table II. Taken separately, they are not negligible, but in the sum they nearly cancel and lead to a total value very close to the extracted value of Ref. [28], $E_{0+}^{thr}(p\pi^0) = -1.33 \pm 0.11$.

Figure 1 compares the energy dependence of the E_{0+} amplitude, as obtained, on the one hand, directly from the MAID and SAID solutions (dash-dotted curves) and, on the other hand, by use of the dispersion relations, Eq. (1), with $\text{Im } \mathcal{M}$ as input taken from the MAID and SAID solutions (solid curves). We clearly see the Wigner cusp effect appearing in the DR solutions due to the infinite derivative of $\text{Im } E_{0+}$ (dashed curves) at the charged pion threshold. In the MAID solution (dash-dotted curve), the cusp effect is the result of the strong coupling to the π^+ channel taken into account by the K -matrix approximation [9]. The SAID solution does not include this effect.

Finally, Table III summarizes our results for the threshold S - and P -wave multipoles and compares them to the results of the recent experimental analysis of Ref. [28]. For the P -wave multipoles we list the values of the following linear combinations, $P_1 = 3E_{1+} + M_{1+} - M_{1-}$, $P_2 = 3E_{1+} - M_{1+} + M_{1-}$, and $P_3 = 2M_{1+} + M_{1-}$. In general, the DR results are consistent with the corresponding MAID or SAID solutions and in good agreement with the results of ChPT and the experimental values of Ref. [28]. A large discrepancy remains for the P_3 amplitude, where the theoretical predictions with and without the use of DR are considerably smaller than the experimental value. This may hint at problems in the

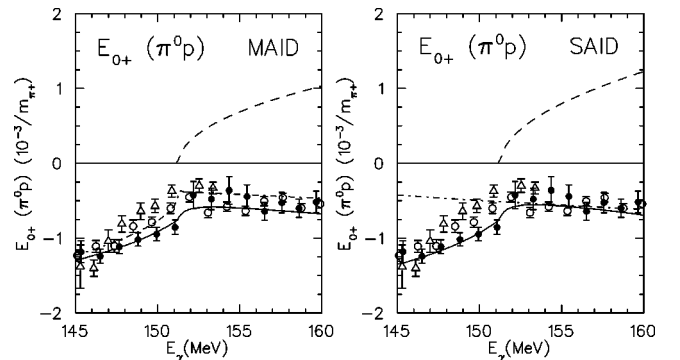


FIG. 1. The E_{0+} multipole for the reaction $\gamma p \rightarrow \pi^0 p$. The dashed and dash-dotted curves show the imaginary and real parts, respectively, as obtained from the MAID2002 (left panel) and SAID solution SM02 with a modified imaginary part as explained in the text (right panel). The solid curves are the predictions for the real parts obtained with the dispersion relations. The data points are the result of the multipole analyses from Ref. [29] (Δ), Ref. [3] (\bullet), and Ref. [28] (\circ).

TABLE III. E_{0+} (in units of $10^{-3}/m_{\pi^+}$) and P_1 , P_2 , and P_3 (in units of $10^{-3}q/m_{\pi^+}^2$) for photoproduction at threshold. The values extracted from the data are taken from the analysis of Ref. [28].

Solutions	E_{0+}	P_1	P_2	P_3
MAID2002	-1.23	9.07	-10.68	7.07
DR(MAID)	-1.29	9.64	-10.29	8.22
SAID SM02		8.79	-11.23	9.60
DR(SAID)	-1.35	9.70	-10.46	8.91
Analysis	-1.33 ± 0.11	9.47 ± 0.33	-9.46 ± 0.39	11.48 ± 0.41

description of the M_{1-} multipole which appears more pronounced in P_3 than in P_1 and P_2 .

B. π^0 electroproduction at threshold

Dispersion relations for pion electroproduction are more involved due to the more complicated structure of the kernels $K_{\alpha\beta}(W, W', Q^2)$. In addition, the transverse multipoles of the virtual photons are also coupled with the longitudinal ones via the kernels. Moreover, we have very limited information about the longitudinal (Coulomb) resonance excitations at finite Q^2 . In the following, we present first calculations for threshold π^0 electroproduction using dispersion relations with the dispersion integrals determined by the MAID model. The longitudinal excitation of the $\Delta(1232)$ and $P_{11}(1440)$ resonances are described as shown in Ref. [13]. For the other resonances we use the ansatz $E_{l\pm}^R = \pm(k/2\omega)(2j+1)L_{l\pm}^R$ as motivated by the pseudothreshold relation [30]. These assumptions lead to the following threshold values for the S -wave multipoles at $Q^2 = 0.1$ (GeV/c) 2 :

$$E_{0+}^{thr}(p\pi^0) = -3.69 + 2.46 + 2.96 - 0.08 = 1.55 \text{ DR(MAID)}, \quad (12)$$

$$\begin{aligned} L_{0+}^{thr}(p\pi^0) &= -3.76 + 0.54 + 1.82 + 0.01 \\ &= -1.41 \text{ DR(MAID)}. \end{aligned} \quad (13)$$

The terms on the right-hand side correspond, in that order, to the contributions of the pole term, the diagonal term, the coupling to the M_{1+} , and the coupling to the higher multipoles. As in the case of real photons, we find that the largest contributions come from the diagonal term and the M_{1+} multipole, which nearly cancel the large contribution of the pole term.

The threshold behavior of the E_{0+} and L_{0+} multipoles at $Q^2 = 0.1$ (GeV/c) 2 is shown in Fig. 2. We point out the much smaller cusp effect in the L_{0+} , as compared to the E_{0+} multipole, due to the smaller imaginary part of L_{0+} . The fixed- t DR results are in good agreement with the results of the analysis of Ref. [32]. On the other hand, the real parts of the E_{0+} and L_{0+} multipoles obtained from the MAID solution, are closer to the results of Refs. [9,33]. However, as discussed in Refs. [9] and [31], the extracted results for the S waves at finite Q^2 strongly depend on the assumptions used for the P -wave contributions. This is especially true for the

E_{0+} multipole. For example, at $Q^2 = 0.1$ (GeV/c) 2 the differences in the P waves used by various groups lead to quite different threshold values for the E_{0+} , namely 1.96 ± 0.33 [32], 2.28 ± 0.36 [9], and 0.58 ± 0.18 [31]. Clearly, these differences in the analyses must be resolved before a comparison with theoretical predictions can be meaningful. Note that we find significant dispersion corrections for both multipoles at finite Q^2 .

Figure 3 shows the Q^2 dependence for several S -wave multipoles and P -wave multipole combinations and compares our results with the results of the analyses of Refs. [31,32]. A number of interesting features emerge. In general, the DR results for the transverse multipoles are consistent with the corresponding MAID solution. For the L_{0+} multipole and the longitudinal P -wave combinations P_4 and P_5 , strong dispersion corrections appear at low Q^2 . Our dispersion results are in agreement with the results from chiral perturbation theory (ChPT) below $Q^2 < 0.05$ (GeV/c) 2 in the case of the E_{0+} multipole and the P_1 combination but differ significantly for the L_{0+} multipole and the amplitudes $P_{23}^2 = (P_2^2 + P_3^2)/2$, $P_4 = 4L_{1+} + L_{1-}$, and $P_5 = L_{1-} - 2L_{1+}$. This may reflect the fact that some of the ChPT low-energy constants were fitted to electroproduction threshold data while the MAID solutions are constrained by data in the resonance sector. Just as in Fig. 2, the experimental points

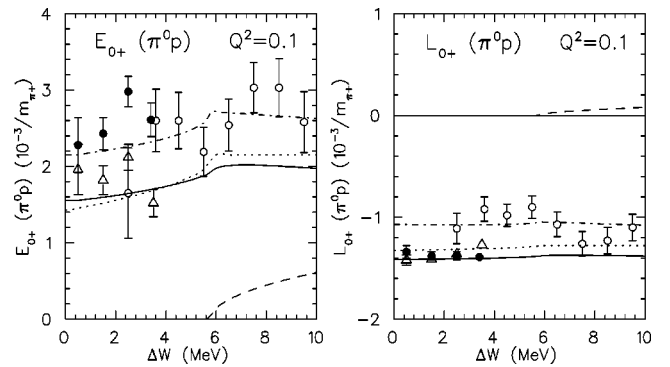


FIG. 2. The E_{0+} (left panel) and L_{0+} (right panel) multipoles for $ep \rightarrow e' \pi^0 p$ at $Q^2 = 0.1$ (GeV/c) 2 as a function of $\Delta W = W - W_{thr}$. The dashed and dash-dotted curves are the imaginary and real parts, respectively, for the the MAID2002 solution. The solid curves are the predictions for the real parts obtained with the dispersion relations. The dotted curves show the results of ChPT [7]. The data points are the result of the analyses from Ref. [33] (\circ), Ref. [32] (\triangle), and Ref. [9] (\bullet).

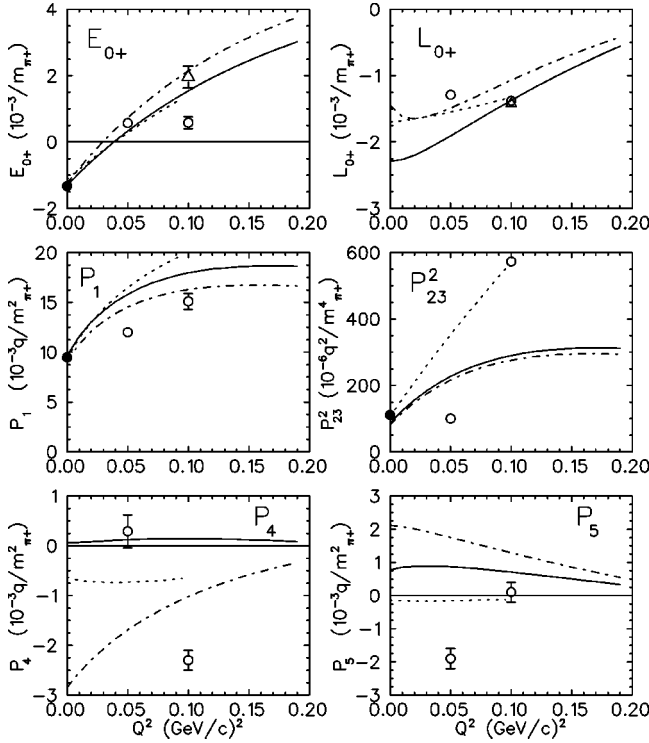


FIG. 3. The S - and P -wave multipoles E_{0+} , L_{0+} , $P_1 = 3E_{1+} + M_{1+} - M_{1-}$, $P_4 = 4L_{1+} + L_{1-}$, $P_5 = L_{1-} - 2L_{1+}$, and $P_{23}^2 = (P_2^2 + P_3^2)/2$ for the reaction $ep \rightarrow e' \pi^0 p$ at threshold as a function of Q^2 . The dash-dotted and solid curves are the MAID2002 solution and the prediction of dispersion relations, respectively. The dotted curves show the results of ChPT [7]. The data points are the results of the analyses from Ref. [32] (Δ), Ref. [31] (\circ), and Ref. [28] (\bullet).

shown have to be understood as the result of different model-dependent analyses techniques.

Finally, we present in Fig. 4 predictions for the quantity $\Delta P_{23}^2 = (P_2^2 - P_3^2)/2$ which determines the sign of the beam

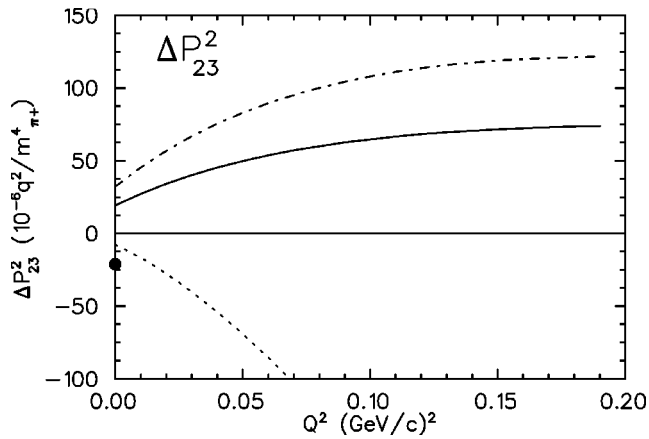


FIG. 4. $\Delta P_{23}^2 = (P_2^2 - P_3^2)/2$ for the reaction $ep \rightarrow e' \pi^0 p$ at threshold as a function of Q^2 . The dashed and solid curves are the MAID2002 solution and the prediction of dispersion relations, respectively. The data point at $Q^2=0$ is the result of the analysis from Ref. [28].

asymmetry, i.e., $\Sigma \sim -\Delta P_{23}^2$. Recent measurements [28] yield a negative value for the ΔP_{23}^2 at $Q^2=0$ and $E_\gamma = 160$ MeV, in rough agreement with ChPT results. However, both the MAID and the DR results are positive at the photon point and become more positive for higher photon virtualities. In contrast, the ChPT results remain negative. Clearly, a measurement of this observable at finite Q^2 is highly desirable.

V. CONCLUSION

Threshold pion photoproduction and electroproduction have been calculated with fixed- t dispersion relations. Unlike previous work for photoproduction following the method of Omnes and Mushkashevili, we have used the imaginary parts of the multipoles of the unitary isobar model MAID and the phenomenological partial-wave analysis SAID as input to calculate the dispersion integrals.

Unitarity, crossing symmetry, Lorentz invariance and gauge invariance are all fulfilled by the dispersion relations. Especially crossing symmetry can only be partially fulfilled in model calculations, even field-theoretical lagrangians violate crossing symmetry when energy-dependent widths for nucleon resonances are introduced. Rather than fitting to the threshold data, we prefer to use dispersion relations whose input are models fitted to data in the resonance region, where more data is available.

For pion photoproduction we obtain very good agreement with the threshold multipoles obtained from experimental analyses. Both the cusp effect and pion-loop effects are well described. The differences between the MAID and SAID inputs play only a minor role, and reveal the small systematic uncertainties in such a dispersion approach. We also find good agreement with the results of ChPT for S and P waves, except for the quantity $P_2^2 - P_3^2$. This discrepancy was already observed in the previous dispersion analysis of Hanstein *et al.* [10] and relates to a very delicate cancellation among two large P -wave amplitudes.

The situation for pion electroproduction reflects a larger uncertainty, both in theory and experiment. Much less data are available which leads to model dependencies in the extraction of the multipoles at finite Q^2 . Since the electroproduction coincidence cross section cannot be completely separated, a model independent analysis as in the photoproduction case is not yet possible, making any comparison with theory difficult. We emphasize that our dispersion theoretical calculation has the advantage that most of the input for the fixed- t dispersion relation comes from the magnetic excitation of the Δ resonance which is very well known even for pion electroproduction. Future experiments will hopefully remove the model dependencies in the extraction of the multipole amplitudes and allow an unambiguous comparison with the predictions from dispersion relations.

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APPENDIX

In this Appendix we give some expressions for the kernel defined by our Eqs. (4) and (5). In the following we essentially use the notations of Ref. [15], and all further equation numbers refer to that reference. The starting point is the dispersion relation (d.r.) for the 6 ($i=1, \dots, 6$) parity conserving helicity amplitudes F_i^{IJ} [given by Eq. (83) of Ref. [15]],

$$\begin{aligned} \text{Re } F_i^{IJ}(W) &= F_i^{IJ(Pole)}(W) + \frac{P}{\pi} \int_{W_{thr.}}^{\infty} dW' \frac{\text{Im } F_i^{IJ}(W') f_i^J(W)}{(W' - W) f_i^J(W')} \\ &+ \frac{1}{\pi} \int_{W_{thr.}}^{\infty} dW' \sum_{J', k} \bar{K}_{ik}^{IJJ'}(W, W') \text{Im } F_k^{IJ}(W'), \end{aligned} \quad (\text{A1})$$

where $J = l_{\pi} \pm \frac{1}{2}$ and I are the total spin and isospin, respectively. The kinematical factor f_i^J is connected with our $r_{\alpha}(\alpha = \{i, J\})$ by the relation $f_i^J(W) = 1/r_{\alpha}(W)$. Since the singularity proportional to $(W' - W)^{-1}$ is already contained in the second term on the right-hand side of Eq. (A1), the kernel $\bar{K}_{ik}^{IJJ'}$ is regular in the physical region. From Eqs. (57) and (58) of Ref. [15] it can be expressed as

$$\begin{aligned} \bar{K}_{ik}^{IJJ'}(W, W') &= \frac{1}{2} \int_{-1}^{+1} dx \sum_{j, j'} W_{ij}^J(x) \mathcal{K}_{jj'}^J(W, W', t) V_{j'k}^{J'}(x') \\ &- \frac{\delta_{ik} \delta_{JJ'} f_i^J(W)}{f_i^J(W') (W' - W)}, \end{aligned} \quad (\text{A2})$$

where the matrices V_{ij}^J and W_{ij}^J only depend on Legendre polynomials and their derivatives as given by Eqs. (20) and (28), respectively, of Ref. [15]. In accordance with Eq. (55) of Ref. [15] the absolute value of the argument $x' = ax + b$ can be larger than unity for $s = W^2 > s' = W'^2$, which allows us to continue the partial wave expansion into the unphysical region. As discussed in Ref. [15] such a generalization should work for photon lab energies up to about 900 MeV.

The matrix \mathcal{K}_{ik}^I can be expressed directly via the D_{ik} and C_{ik} matrices, given in terms of the matrices M_{ik} in Eqs. (42) and (43) of Ref. [15], which project the helicity amplitudes on the invariant amplitudes. In this case we get

$$\begin{aligned} \mathcal{K}_{ik}^I(W, W', t) &= \left\{ \frac{D_{ik}^I(W, W', t)}{s' - s} \right. \\ &\left. + \epsilon^I \frac{C_{ik}^I(W, W', t)}{s' - u} \right\} \frac{2W' h_i(W)}{h_k(W')}, \end{aligned} \quad (\text{A3})$$

where the kinematical factors h_i are given by Eq. (36) of Ref. [15] and ϵ^I is an isospin factor, see Eq. (13). Note that in our definition of the \mathcal{K}_{ik}^I matrix we included the factor $2W'$ arising from $ds' = 2W' dW'$. On the other hand, the \mathcal{K}_{ik}^I matrix can also be obtained from Eqs. (50) and (51) of Ref. [15], where its t dependence is given explicitly. For checking we used both ways and obtained identical results. It follows from Eq. (50) of Ref. [15] that the \mathcal{K}_{ik}^I matrix contains a kinematical singularity at $t = \mu^2$ arising from the A_2 and A_5 invariant amplitudes. This singularity appears only in pion electroproduction and in this case we modify our dispersion relation by adding the subtraction term of Eqs. (101) and (102) of Ref. [15] to the \mathcal{K}_{ik}^I matrix.

As a last step, we derive the dispersion relation for our standard CGLN multipoles \mathcal{M}_{ij} from our Eq. (A1) by use of Eqs. (25) and (33) of Ref. [15], i.e.,

$$\mathcal{M}_{ij} = \sum_j N_{ij}^J F_j^J, \quad F_i^J = \sum_j (N^J)_{ij}^{-1} \mathcal{M}_{jJ}, \quad (\text{A4})$$

where we skipped the isospin index I , as in Ref. [15]. The final result for the CGLN multipoles is the d.r.,

$$\begin{aligned} \text{Re } \mathcal{M}_{\alpha}(W) &= \mathcal{M}_{\alpha}^{Pole}(W) + \frac{P}{\pi} \int_{W_{thr.}}^{\infty} dW' \frac{\text{Im } \mathcal{M}_{\alpha}(W') r_{\alpha}(W')}{(W' - W) r_{\alpha}(W)} \\ &+ \frac{1}{\pi} \int_{W_{thr.}}^{\infty} dW' \sum_{\beta} K_{\alpha\beta}(W, W') \text{Im } \mathcal{M}_{\beta}(W'), \end{aligned} \quad (\text{A5})$$

with $\alpha(\beta) = \{iJ\}(\{kJ'\})$, the kinematical factor $r_{\alpha} = 1/f_i^J$, and the regular kernel

$$\begin{aligned} K_{\alpha\beta}(W, W') &= 2W' \tilde{D}_{ik}^J(W, W') + \sum_{j, j'} N_{ij}^J \bar{K}_{jj'}^{JJ'}(W, W') \\ &\times (N^J)_{j'k}^{-1}. \end{aligned} \quad (\text{A6})$$

The matrix \tilde{D}_{ik}^J is defined by Eqs. (87) and (88) of Ref. [15]. In the case of real photons we have numerically compared this kernel with the kernel of Ref. [17] using our Eq. (3), and found that both kernels are identical.

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