# Multiplicity moments and hard processes in relativistic heavy ion collisions

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The normalized multiplicity moments and their relation with soft and hard processes in relativistic heavy ion collisions are analyzed in a general two-component model. It is found that the strong fluctuations in the binary collision number  $N_c$  in minimum-bias events can enhance the hard component, especially for the higher order moments. This enhancement cannot be effectively described by modifying the participant number in the one-component model.

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# I. INTRODUCTION

The relativistic heavy ion collisions at SPS and RHIC may be the only way to create the extreme conditions necessary to produce a new state of matter-quark-gluon plasma in the laboratory [1,2]. One can attempt to understand the energy density achieved in the collisions by studying the multiplicity and transverse energy distributions through hydrodynamic models [3]. At SPS energies, the global quantities, such as average multiplicity, multiplicity distribution, and rapidity distribution, can be well described by soft processes only, namely, by the number of participant nucleons only [4]. However, at RHIC energies, the measured pseudorapidity density normalized per participant pair for central Au-Au collisions shows that 70% more particles are produced than at SPS [5,6]. This indicates that the yield of particles created by hard scattering processes becomes important at RHIC [1,7]. One can decompose the multiplicity at a fixed impact parameter into a soft component and a hard component as [1,6,8]

$$n = aN_p + bN_c, \qquad (1)$$

where  $N_p$  and  $N_c$  are the participant number and binary collision number, respectively.

However, the two-component expression (1) can be effectively described by a simple power-law form

$$n = c N_n^{\alpha}, \quad \alpha > 1, \tag{2}$$

which is then similar to that measured at SPS [9]. A natural question then arises: Can one find other global observables which are more sensitive to the hard processes than the multiplicity itself, and which cannot be effectively described in the models with only soft processes?

As is well known, the multiplicity moments are important characteristics in multiparticle production. The properties of the multiplicity distribution can be completely described by the normalized moments

$$C_i = \frac{\langle n^i \rangle}{\langle n \rangle^i}, \quad i = 2, 3, \dots$$
(3)

In Ref. [10]  $C_i$  were investigated at SPS energies with a general wounded nucleon model. It was found that the nor-

malized multiplicity moments are independent of the concrete behavior of elementary nucleon-nucleon collisions, but dominated by the normalized participant moments

$$C_i \simeq C_{ip} = \frac{\langle N_p^i \rangle}{\langle N_p \rangle^i},\tag{4}$$

provided that the colliding nuclei are not too light.

In this paper we investigate how the hard processes change the normalized multiplicity moments. We extend the study in Ref. [10] to include the hard component. We will focus on the sensitivity of  $C_i$  to the colliding energy, nuclear geometry, and especially to the geometry fluctuations.

# **II. MULTIPLICITY MOMENTS**

If the average multiplicity distribution of each soft source is  $g_p(n_p)$ , and the average multiplicity distribution of each hard source is  $g_c(n_c)$ , the multiplicity distribution of an *AB* collision is the supposition of the contributions of  $N_p$  soft sources and  $N_c$  hard sources:

$$G_{N_{p},N_{c}}(n) = \sum_{\substack{n_{p}^{(1)}, \dots, n_{p}^{(N_{p})} \\ n_{c}^{(1)}, \dots, n_{c}^{(N_{c})}}} \delta \left( n - \sum_{i=1}^{N_{p}} n_{p}^{(i)} - \sum_{j=1}^{N_{c}} n_{c}^{(j)} \right) \prod_{i=1}^{N_{p}} g_{p}(n_{p}^{(i)}) \prod_{j=1}^{N_{c}} g_{c}(n_{c}^{(j)}).$$
(5)

It will be seen later that the results in this paper are not concerned with the concrete form of  $g_p(n_p)$  and  $g_c(n_c)$ . Taking into account all the processes with different  $N_p$  and  $N_c$ , the observed multiplicity distribution is

$$P(n) = \sum_{N_p, N_c} p(N_p, N_c) G_{N_p, N_c}(n),$$
(6)

where  $p(N_p, N_c)$  is the distribution function of  $N_p$  and  $N_c$ .

At fixed impact parameter b, the nuclear geometry of soft and hard processes is expressed in terms of  $N_p(b)$  and  $N_c(b)$ [11], respectively,

$$N_{p}(b) = \int d^{2}\mathbf{s} [T_{A}(\mathbf{s})(1 - e^{-\sigma_{N}T_{B}(\mathbf{b}-\mathbf{s})}) + T_{B}(\mathbf{b}-\mathbf{s})(1 - e^{-\sigma_{N}T_{A}(\mathbf{s})})],$$
$$N_{c}(b) = \int d^{2}\mathbf{s}\sigma_{N}T_{A}(\mathbf{s})T_{B}(\mathbf{b}-\mathbf{s}),$$
(7)

where  $\sigma_N$  is the nucleon-nucleon inelastic cross section, and  $T_A(\mathbf{s})$  and  $T_B(\mathbf{b}-\mathbf{s})$  are the local participant densities in the plane orthogonal to the collision axis defined as

$$T_{A}(\mathbf{s}) = \int dz \rho_{A}(\mathbf{s}, z),$$
$$T_{B}(\mathbf{b} - \mathbf{s}) = \int dz \rho_{B}(\mathbf{b} - \mathbf{s}, z).$$
(8)

Since  $N_c(b)$  calculated with Eq. (7) is a monotonic function of  $N_p(b)$ , the distribution of  $N_p(b)$  and  $N_c(b)$ ,  $p(N_p(b), N_c(b))$ , is just the distribution of  $N_p(b)$ , which can be obtained from Eq. (7) as

$$p(N_p(b), N_c(b)) \sim \frac{b}{dN_p(b)/db}.$$
(9)

When *b* is fixed, the stochastic variables  $N_b$  and  $N_c$  in Eq. (6) still have fluctuations around  $N_p(b)$  and  $N_c(b)$ . For simplicity, we use the Gaussian distribution to describe the stochastic fluctuations,

$$p(N_p|N_p(b)) = \frac{1}{\sqrt{2\pi\sigma_p^2(b)}} e^{-[N_p - N_p(b)]^2/2\sigma_p^2(b)},$$
$$p(N_c|N_c(b)) = \frac{1}{\sqrt{2\pi\sigma_c^2(b)}} e^{-[N_c - N_c(b)]^2/2\sigma_c^2(b)}, \quad (10)$$

with the variances

$$\sigma_p^2(b) = a_p N_p(b),$$
  
$$\sigma_c^2(b) = a_c N_c(b),$$
 (11)

where  $a_p$  and  $a_c$  are constants. Considering both the geometry fluctuations (9) and the stochastic fluctuations (10), the probability of the stochastic variables  $N_p$  and  $N_c$  in Eq. (6) is given by

$$p(N_{p}, N_{c}) = \sum_{N_{p}(b) \ge N_{min}} p(N_{p}|N_{p}(b))p(N_{c}|N_{c}(b))p(N_{p}(b), N_{c}(b)),$$
(12)

where we have used the minimum participant number  $N_{min}$  to select events.  $N_{min}=2$  means minimum-bias events and very large  $N_{min}$  corresponds to central events.

We introduce now generating functions [10,12]  $F(\theta), f_p(\theta)$ , and  $f_c(\theta)$  for the whole system and the elementary soft and hard sources,

$$F(\theta) \equiv \sum_{n} \theta^{n} P(n) = \sum_{N_{p}, N_{c}} p(N_{p}, N_{c}) [f_{p}(\theta)]^{N_{p}} [f_{c}(\theta)]^{N_{c}},$$

$$f_{p}(\theta) \equiv \sum_{n_{p}} \theta^{n_{p}} g_{p}(n_{p}),$$

$$f_{c}(\theta) \equiv \sum_{n_{c}} \theta^{n_{c}} g_{c}(n_{c}), \quad -1 \leq \theta \leq 1.$$
(13)

Differentiating Eq. (13) with respect to  $\theta$  and making use of the relations

$$F(\theta)|_{\theta=1} = f_{p}(\theta)|_{\theta=1} = f_{c}(\theta)|_{\theta=1} = 1,$$

$$\frac{\partial}{\partial \theta} F(\theta)|_{\theta=1} = \langle n \rangle,$$

$$\frac{\partial}{\partial \theta} f_{p}(\theta)|_{\theta=1} = \langle n_{p} \rangle,$$

$$\frac{\partial}{\partial \theta} f_{c}(\theta)|_{\theta=1} = \langle n_{c} \rangle,$$

$$\frac{\partial^{2}}{\partial \theta^{2}} F(\theta)|_{\theta=1} = \langle n(n-1) \rangle,$$

$$\frac{\partial^{2}}{\partial \theta^{2}} f_{p}(\theta)|_{\theta=1} = \langle n_{p}(n_{p}-1) \rangle,$$

$$\frac{\partial^{2}}{\partial \theta^{2}} f_{c}(\theta)|_{\theta=1} = \langle n_{c}(n_{c}-1) \rangle, \dots, \qquad (14)$$

we derive the multiplicity moments  $\langle n^i \rangle$  in terms of the elementary soft and hard moments  $\langle n^i_p \rangle$  and  $\langle n^i_c \rangle$  and the nuclear geometry moments  $\langle N^i_p \rangle, \langle N^i_c \rangle$ , and  $\langle N^j_p N^j_c \rangle$ ,

$$\langle n \rangle = \langle N_p \rangle \langle n_p \rangle + \langle N_c \rangle \langle n_c \rangle,$$
  

$$\langle n^2 \rangle = (\langle N_p^2 \rangle - \langle N_p \rangle) \langle n_p \rangle^2 + \langle N_p \rangle \langle n_p^2 \rangle + (\langle N_c^2 \rangle - \langle N_c \rangle) \langle n_c \rangle^2 + \langle N_c \rangle \langle n_c^2 \rangle + 2 \langle N_p N_c \rangle \langle n_p \rangle \langle n_c \rangle,$$
  

$$\langle n^3 \rangle = (\langle N_p^3 \rangle - 3 \langle N_p^2 \rangle + 2 \langle N_p \rangle) \langle n_p \rangle^3 + 3 (\langle N_p^2 \rangle - \langle N_p \rangle) \langle n_p \rangle \times \langle n_p^2 \rangle + \langle N_p \rangle \langle n_p^3 \rangle + (\langle N_c^3 \rangle - 3 \langle N_c^2 \rangle + 2 \langle N_c \rangle) \langle n_c \rangle^3 + 3 (\langle N_c^2 \rangle - \langle N_c \rangle) \langle n_c \rangle \langle n_c^2 \rangle + \langle N_c \rangle \langle n_c^3 \rangle + 3 (\langle N_p^2 N_c \rangle - \langle N_p N_c \rangle) \langle n_p \rangle^2 \langle n_c \rangle + 3 (\langle N_p N_c^2 \rangle - \langle N_p \rangle) \langle n_p \rangle \langle n_c \rangle^2 + 3 \langle N_p N_c \rangle \langle n_p^2 \rangle \langle n_c \rangle + 3 \langle N_p N_c \rangle \langle n_p \rangle \langle n_c^2 \rangle, \dots, \qquad (15)$$

with the definitions of the moments,

$$\langle n^i \rangle = \sum_n n^i P(n) / \sum_n P(n),$$
  
$$\langle n_p^i \rangle = \sum_{n_p} n_p^i g(n_p) / \sum_{n_p} g(n_p),$$
  
$$\langle n_c^i \rangle = \sum_{n_c} n_c^i g(n_c) / \sum_{n_c} g(n_c),$$

$$\langle N_{p}^{i} N_{c}^{j} \rangle = \sum_{N_{p}, N_{c}} N_{p}^{i} N_{c}^{j} p(N_{p}, N_{c}) / \sum_{N_{p}, N_{c}} p(N_{p}, N_{c}).$$
 (16)

With the known multiplicity moments, the normalized moments  $C_i = \langle n^i \rangle / \langle n \rangle^i$  can be expressed as an expansion in the inverse number of average participants  $1/\langle N_p \rangle$ ,

$$C_{i} = \frac{\left\langle \left( \frac{N_{p}}{\langle N_{p} \rangle} + \frac{N_{c}}{\langle N_{c} \rangle} x \right)^{i} \right\rangle}{(1+x)^{i}} + O\left( \frac{1}{\langle N_{p} \rangle} \right), \quad (17)$$

where the average ratio of the hard to soft component

$$x = \frac{\langle N_c \rangle \langle n_c \rangle}{\langle N_p \rangle \langle n_p \rangle} \tag{18}$$

depends on the elementary nucleon-nucleon dynamics and the nuclear geometry. If we do not consider peripheral interactions alone,  $\langle N_p \rangle, \langle N_c \rangle \ge 1$ , we can then consider only the zeroth order in the expansion (17). In this case, only the average ratio of the hard to soft component remains, the other dynamics of elementary soft and hard processes hidden in  $\langle n_p^i \rangle$  and  $\langle n_c^i \rangle$  with i > 2 is washed away by the nuclear geometry.

When the hard contribution can be neglected, namely,  $x \rightarrow 0$ , the normalized multiplicity moments are just the normalized participant moments

$$C_i = C_{ip} = \frac{\langle N_p^i \rangle}{\langle N_p \rangle^i}.$$
(19)

This is the case discussed in Ref. [10] at SPS energies.

# III. NUCLEAR GEOMETRY AND ENERGY DEPENDENCE OF HARD CONTRIBUTION

Let us first determine the soft and hard components  $\langle n_p \rangle$ and  $\langle n_c \rangle$  in elementary nucleon-nucleon collisions. To this end, we compare the average multiplicity with the experimental data for central Au-Au collisions. Since we did not introduce rapidity dependence in our discussion, we consider only the central rapidity region where the data show a plateau structure. By comparing the average participant number  $\langle N_p \rangle$ , the average multiplicity per participant pair,

$$\frac{\langle n \rangle}{0.5 \langle N_p \rangle} = 2 \langle n_p \rangle (1+x), \tag{20}$$

and the multiplicity for  $P\overline{P}$ ,

TABLE I. The ratio x of the hard to soft component and the geometry parameter  $N_{min}$  determined from the comparison with the data of central Au-Au collisions at RHIC.

1	$\overline{0.5\langle N_p \rangle}$	$N_{min}$	<i>x</i> 0.32
330	2.47	297	
343	3.24	323	0.58
344	3.78	326	0.74
-	343	330         2.47           343         3.24	330         2.47         297           343         3.24         323

$$\langle n_{P\bar{P}} \rangle = 2 \langle n_p \rangle + \langle n_c \rangle, \qquad (21)$$

with the experimental data [5,13] in the central rapidity region  $|\eta| < 1$  at RHIC and the parametrization of the  $P\bar{P}$  data [14],

$$\langle n \rangle_{P\bar{P}} = 2.5 - 0.25 \ln s + 0.023 \ln^2 s,$$
 (22)

we can determine at different energies the average ratio x and the minimum participant number  $N_{min}$  which is used to select centrality in calculating geometry moments. Using a Woods-Saxon distribution

$$\rho_A(\mathbf{r}) = \frac{\rho_0}{1 + e^{(r - R_A)/a}}, \quad \int d^3 \mathbf{r} \rho_A(\mathbf{r}) = A, \quad (23)$$

with the parameters a = 0.53 fm,  $R_A = 1.1A^{1/3}$  fm for <sup>197</sup>Au, taking  $\sigma_N = 37$  mb at  $\sqrt{s} = 56A$  GeV ( $\sigma_N = 41$  mb for  $\sqrt{s}$ = 130,200 A GeV) [8], and choosing the constants  $a_p$  and  $a_c$ in the variances of the Gaussian distributions (10) to be  $a_p$  $=a_c=1$ , the two parameters are shown in Table I. We see that at RHIC energies, x < 1, the soft component is still more important than the hard component. In our numerical calculations, when we change the variance parameters  $a_p$  and  $a_c$ from 0.1 to 1, the ratio x determined using Eqs. (20)-(22)remains almost a constant, but  $N_{min}$  increases with increasing  $a_p$  and  $a_c$ . Taking  $a_p = a_c = 0.1$ ,  $N_{minb}$  is 289, 314, and 316 corresponding to colliding energy  $\sqrt{s}$  = 56,130, and 200 GeV. The change in  $N_{min}$  will certainly lead to a considerable change in the moments  $\langle N_n^i N_c^j \rangle$ , especially for the minimum-bias events, but our numerical calculations show that the normalized moments  $C_i$  and  $C_{ip}$  are not sensitive to the variance parameters  $a_p$  and  $a_c$ .

The influence of nuclear geometry is twofold: The average numbers  $\langle N_p \rangle$  and  $\langle N_c \rangle$  and the fluctuations of  $N_p$  and  $N_c$  around their average values. For central collisions the average numbers  $\langle N_p \rangle$  and  $\langle N_c \rangle$  are huge, but the fluctuations are small. This can be seen clearly in Table I where  $\langle N_p \rangle \geq 330$  and  $291 \leq N_p(b) \leq N_p(b=0)$ . From the Gaussian distributions (10), when  $N_p(b)$  and  $N_c(b)$  vary in such a narrow region, the stochastic variables  $N_p$  and  $N_c$  fluctuate only around this small region. For minimum-bias events the average numbers are relatively small, but the fluctuations are the maximum.

The multiplicity  $\langle n \rangle$  is only related to the average values  $\langle N_p \rangle$  and  $\langle N_c \rangle$ . When the hard contribution vanishes, the average multiplicity is proportional to  $\langle N_p \rangle$ . The hard con-

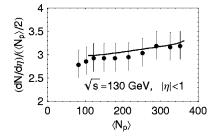


FIG. 1. The centrality dependence of the average multiplicity normalized to per participant pair and its comparison with the RHIC data.

tribution reflected in the ratio x leads to an extra  $\langle N_c \rangle$  dependence. The centrality dependence of the average multiplicity per participant pair (20) can be calculated by changing the minimum participant number  $N_{min}$  from 2 to  $N_p(b=0)$ . In Fig. 1 it is compared with the data in the central rapidity region  $|\eta| < 1$  for the central Au–Au collisions at  $\sqrt{s} = 130A$  GeV [15]. The extra geometry dependence induced by the hard component is weak.

Since  $\langle N_p^i \rangle = \langle (N_p / \langle N_p \rangle)^i \rangle \langle N_p \rangle^i$  and  $\langle N_c^i \rangle = \langle (N_c / \langle N_c \rangle)^i \rangle \langle N_c \rangle^i$ , the multiplicity moments  $\langle n^i \rangle$  for  $i \ge 2$  are associated with both the average numbers  $\langle N_p \rangle$  and  $\langle N_c \rangle$  and the fluctuations in  $N_p$  and  $N_c$ . From Eq. (17) the normalized moments  $C_i$  depend on the fluctuations and the average ratio x of the hard to soft component. Figure 2 shows the centrality dependence is very weak. Therefore, the behavior of the normalized moments  $C_i$  is mainly controlled by the fluctuations in  $N_p$  and  $N_c$ . Let us first consider the limit of no fluctuations,  $N_p = \langle N_p \rangle, N_c = \langle N_c \rangle$ . In this limit,

$$p(N_p) = \delta_{N_p \langle N_p \rangle}, \qquad (24)$$

we have

$$C_i = C_{ip} = 1.$$
 (25)

In this case there is no difference between the twocomponent and one-component model. Although fluctuations around the average numbers always exist, and it is difficult to choose events with the same impact parameter *b*, namely, with the same  $N_p(b)$  and  $N_c(b)$ , in experiments, for very central collisions with large  $\langle N_p \rangle$  and  $\langle N_c \rangle$ ,  $N_p$  and  $N_c$  fluctuate in a narrow region, the case is then similar to the above limit.

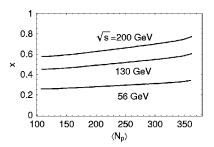


FIG. 2. The energy and centrality dependence of the average ratio x of the hard to soft component.

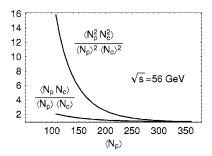


FIG. 3. The centrality dependence of the geometry fluctuations.

The fluctuations grow up when the minimum participant number  $N_{min}$  decreases from its maximum value  $N_p(0)$ . Figure 3 shows the centrality dependence of the fluctuations  $\langle N_p^i N_c^j \rangle / \langle N_p \rangle^i \langle N_c \rangle^j$ . As the orders *i* and *j* are not too small, the fluctuations are very strong for minimum-bias events.

In order to see the contribution from the hard processes, we define the ratio of the normalized moments with and without consideration of the hard component,

$$r_i = \frac{C_i}{C_{ip}}.$$
(26)

The centrality and energy dependence of  $r_i$  is shown in Fig. 4. While there is no remarkable difference between  $C_{ip}$  and  $C_i$  in central collisions, the big fluctuations in  $N_p$  and  $N_c$  in minimum-bias events enhance the hard contribution, and this enhancement becomes more and more important when the colliding energy increases. At  $\sqrt{s} = 200A$  GeV, the hard contribution to  $C_5$  is larger than 50%.

In order to see the contribution of the fluctuations of the stochastic variables  $N_p$  and  $N_c$  around  $N_p(b)$  and  $N_c(b)$  at a fixed impact parameter *b*, we recalculated the ratio  $r_i$  without considering the Gaussian distributions (10). From the comparison shown in Fig. 5, the difference between with and without stochastic fluctuations lies mainly in minimum-bias events.

To search for nonstatistical fluctuations in the distributions of secondary particles in high energy interactions, the method of factorial moments was introduced by Bialas and Peschanski [16]. The pseudorapidity bin  $\delta \eta$  dependence of the factorial moments  $F_i$  in relativistic heavy ion collisions at SPS can be well described by a linear fit [17]

$$\ln F_i = \beta_i + \alpha_i (-\ln \delta \eta). \tag{27}$$

The rapidity bin independent intersect  $\beta_i$  can be calculated in our global model. When the hard contribution is neglected at SPS energies, we have to the zeroth order of  $1/\langle N_n \rangle$ ,

$$\beta_i = \ln F_{iP},$$

$$F_{iP} = \frac{\langle N_p (N_P - 1) \cdots (N_P - i + 1) \rangle}{\langle N_P \rangle^i}.$$
(28)

Table II shows the comparison between our calculation with  $N_{min} = 10$  and the experimental data [17] for O-Ag/Br. The calculation agrees with the data reasonably well. Since we did not consider the rapidity dependence of the soft and hard

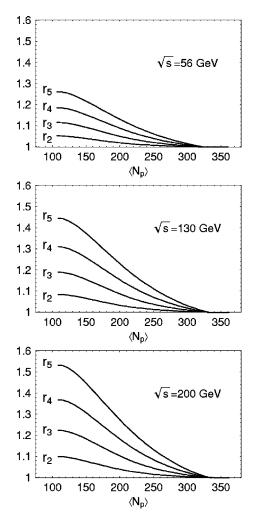


FIG. 4. The ratio of the two-component to one-component normalized moment as a function of the centrality.

sources in our global model, we cannot distinguish different rapidity bins, and therefore we cannot calculate the interesting slope parameter  $\alpha_i$ . Its calculation depends on the details of the distributions of  $N_p$  and  $N_c$  in momentum space.

### IV. COMPARISON WITH THE EFFECTIVE MODEL WITHOUT THE EXPLICIT HARD COMPONENT

The effect of the hard scattering processes on the average multiplicity can be effectively described in the one-

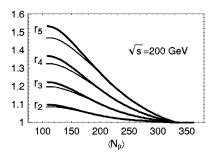


FIG. 5. The centrality dependence of  $r_i$  with (thick lines) and without (thin lines) stochastic fluctuations (10).

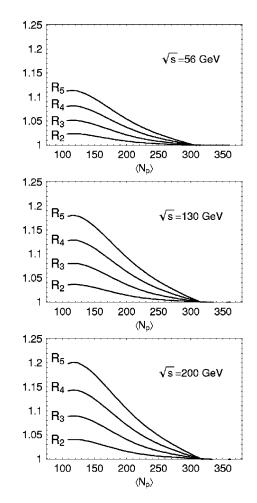


FIG. 6. The ratio of the two-component to effective onecomponent normalized moment as a function of the centrality.

component model by modifying the participant number [9],

$$N_c \rightarrow 0, \quad N_p \rightarrow N_p^{\alpha}, \quad \alpha > 1.$$
 (29)

By comparing the average multiplicity  $\langle n \rangle = \langle N_p^{\alpha} \rangle \langle n_p^{eff} \rangle$  with the RHIC data listed in Table I, we can determine the power  $\alpha$  and the average contribution of each effective soft source  $\langle n_p^{eff} \rangle$ . Corresponding to the colliding energy  $\sqrt{s} = 56$ , 130, 200 A GeV, we have  $\alpha = 1.04$ , 1.07, and 1.08, respectively.

In the effective one-component model, the normalized moments are just the effective participant moments,

$$C_i^{eff} = \frac{\langle N_p^{\alpha i} \rangle}{\langle N_p^{\alpha} \rangle^i},\tag{30}$$

TABLE II. The intercept parameter  $\beta_i$  for O-Ag/Br. The data are taken from the KLM collaboration at  $E_{lab}$  = 200A GeV and for the pseudorapidity interval  $\Delta \eta$  = 0.5–5.5.

i	2	3	4	5	6
Data	0.183	0.496	0.902	1.377	1.903
Model	0.190	0.501	0.877	1.282	1.691

when the peripheral interactions are not considered alone. While the contribution of the hard processes to the average multiplicity through the average binary collision number  $\langle N_c \rangle$  can be equivalently expressed by increasing the average participant number from  $\langle N_p \rangle$  to  $\langle N_p^{\alpha} \rangle$ , the fluctuations in  $N_c$ cannot be effectively included in the fluctuations in  $\langle N_p^{\alpha} \rangle$ . This can be seen clearly in Fig. 6, which shows the ratio

$$R_i = \frac{C_i}{C_i^{eff}} \tag{31}$$

as a function of the centrality for Au-Au collisions. From the comparison with Fig. 4,  $R_i < r_i$ , the fluctuations in  $N_c$  are partly included in the fluctuations in the effective participant number  $N_p^{\alpha}$ . However, the difference between the two-component model and the effective one-component model is still remarkable in minimum-bias events, especially for the higher order moments and at high energies.

#### **V. CONCLUSIONS**

The huge average participant number  $\langle N_p \rangle$  and binary collision number  $\langle N_c \rangle$  in relativistic heavy ion collisions make it difficult to extract dynamic information on hard processes from the geometry background. Different from the multiplicity moments  $\langle n^i \rangle$ , which depend on both the average numbers  $\langle N_p \rangle$  and  $\langle N_c \rangle$  and the fluctuations in  $N_p$  and

 $N_c$  strongly, the normalized moments  $C_i = \langle n^i \rangle / \langle n \rangle^i$  have only weak  $\langle N_p \rangle$  and  $\langle N_c \rangle$  dependence, and are mainly associated with the fluctuations in  $N_p$  and  $N_c$ . Therefore, the geometry background for  $C_i$  is not so complicated as that for  $\langle n^i \rangle$ .

We have investigated the normalized moments  $C_i$  in the frame of a general two-component model. When the hard component can be neglected at SPS energies,  $C_i$  are completely determined by the geometry fluctuations, the dynamics is totally washed away. When the hard processes become important at RHIC energies, the average ratio of the hard to soft component depends on the centrality weakly, and  $C_i$  are dominated by the fluctuations. For central collisions where the fluctuations are weak,  $C_i$  approach 1, the dynamic information cannot be seen in  $C_i$ . However, the big fluctuations in minimum-bias events allow us to see clearly the difference between the models with and without the hard component.

While the average effect of the hard processes can be effectively described in the one-component model by modifying the participant number, we have found that the fluctuations in the binary collision number cannot be fully included in the fluctuations in the effective participant number.

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