# Nucleon elastic scattering potentials: Energy and isospin dependence

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Volume integrals of the real potentials derived from proton elastic scattering studies have been calculated for data available from the lowest to the highest energy. Because of the spread of the volume integrals at low energies, an average of volume integrals in each 1 MeV bin was calculated. These average volume integrals show a logarithmic dependence on the beam energy. A similar analysis of neutron potentials shows that the proton energy dependence can be applied to neutron data. The data for proton scattering from Ca isotopes at 1044 MeV were analyzed in terms of the optical model, and an isospin component of the potential was determined. The derived isospin potential is compared with those obtained for proton and neutron scattering in previous investigations.

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## I. INTRODUCTION

Elastic scattering of nucleons from nuclei has been studied for several decades. The primary goal of these studies was to determine the nucleon-nucleus interaction. The interaction between a nucleon and the nucleus is a many-body problem, and the potential has many components due to different mechanisms of nuclear reactions. However, the optical model potential has been found to provide an acceptable macroscopic, phenomenological description of the interaction. It essentially reduces the highly complicated description of the many-body nucleon-nucleus system to the solution of the Schrödinger equation with a complex mean-field potential. It provides good fits to the differential cross section angular distributions for a wide range of target nuclei at different energies. In many instances the optical model analyses have provided interesting physics information through the energy, target-mass, and isospin dependence of the derived parameters.

Accurate data for the elastic scattering of protons from a large sample of nuclei are available at many bombarding energies up to 60 MeV [1–5]. Most of these data have been analyzed in terms of the standard nuclear optical model parametrization employing a real and an imaginary potential. Generally, the Woods-Saxon form (defined in Sec. II) was used for both potentials. Occasionally, the derivative of these form factors was used when the scattering is not too sensitive to the interior of the nucleus. Many investigators analyzed individual sets of data and obtained a linear dependence of the real potentials on the incident proton energies. However, because these low-energy protons do not penetrate deep into the nucleus, the derived potentials are ambiguous: different sets of potentials that are similar in the surface region provide equally good fits to the data. Because of these ambiguities in the potentials, the parameters obtained by different authors are not consistent with each other, and the resulting uncertainty in interpolation between energies and nuclear masses precludes the determination of reliable systematic trends of the parameters.

With the advent of higher-energy beams, single-energy proton elastic scattering studies have been made at 100 MeV [6], 156 MeV [7], 185 MeV [8], 200–500 MeV [9], 800 MeV [10] and 1044 MeV [11]. A comprehensive analysis over a limited energy range of 80–180 MeV proton elastic scattering from several targets [12] was also carried out. Most of these data were analyzed in a consistent and uniform manner in terms of a local optical model potential with Woods-Saxon form factors. Because of their higher energies, these data required the use of relativistic kinematics and a relativistic extension of the Schrödinger equation. At these energies, the the Coulomb repulsion is relatively weak and the nucleus is fairly transparent to the incident proton. Thus the proton is able to sample interior regions of the nucleus, resulting in a significant reduction in the ambiguities of the potentials.

Investigators were then able to derive more reliable systematics of the potentials. Of particular interest was the energy dependence of the potentials. The optical model potentials have two basic sources for their energy dependence. First is the intrinsic energy dependence, which is derived from the dispersion relation. Second, the proton-nucleus potential is nonlocal. The Fourier transform of this nonlocal potential to an equivalent local potential naturally leads to an energy dependence. The optical model energy dependence is reflected by the variation of the derived parameters with energy. In many analyses, particularly at low energies, the dependence of the strength $V$ is considered. However, because of the correlations between $V$ and the Woods-Saxon geometry parameters $r_0$ and $a_0$, these individual parameters can exhibit spurious energy dependences. On the other hand, it has been found that the volume integral of the potential $J_R$ is a well-defined quantity, free of parameter correlations [5,13,14], and thus would provide a reliable measure of the energy dependence of the optical potential. Van Oers *et al.* [8] obtained a linear energy dependence of the real potential volume integrals up to 60 MeV for six target nuclei and a different linear energy dependence from 160 to 200 MeV for $^{40}$Ca and $^{208}$Pb. This inconsistency was then resolved by determining a logarithmic energy dependence, which provided a good description of $p+{}^{12}$C, $^{16}$O, $^{27}$Al, $^{40}$Ca, $^{208}$Pb data from 10 MeV to 1000 MeV [15]. It had the form $J_R(E)=J_R(0)-\beta \ln E$ with $J_R(0)=850$–$930$ MeV fm$^3$ and $\beta=142$–$156$ MeV fm$^3$. The results gave a zero crossing of the real potential, from attractive to repulsive, at about 500 MeV. Nadasen *et al.* [12] also obtained a logarithmic energy dependence for the energy range of 80–180 MeV, with $J_R(0)=815$ MeV fm$^3$ and $\beta=120$ MeV fm$^3$, which gave an extrapolated zero crossing value of 890 MeV. This global analysis was extended to a maximum energy of 1 GeV (Ref. [16]) by including newer scattering data at 200, 300, 400, and 500 MeV from TRIUMF [9], as well as the older data at 800 and 1044 MeV.

Extensive investigations of neutron elastic scattering have been carried out up to 24 MeV [17]. The first major analysis of differential cross sections was carried out by Bjorklund and co-workers [18] for neutron scattering at energies of 4, 7, and 14 MeV. Perey and Buck [19] obtained good fits to data up to 24 MeV using a nonlocal potential. Becchetti and Greenlees [5] performed the first global analysis of neutron elastic scattering up to 24 MeV. During the 1970s, accurate neutron elastic scattering cross sections over wide angular ranges on several target nuclei were measured up to 26 MeV at Ohio University [20], and global optical model potentials were derived. DeVito *et al.* [21] measured neutron elastic scattering from $^{40}$Ca at 30 and 40 MeV. Additional high-quality measurements over wide angular ranges for energies from 8 to 14 MeV were made at TUNL [22,23]. However, because these low-energy neutrons sample only the extreme surface region of the nucleus, it was virtually impossible to determine unambiguous optical model potentials. The derived volume integrals range from $\sim 300$ to $\sim 700$ MeV fm$^3$. The global analyses of Becchetti and Greenlees [5], Rapaport [20], and Varner *et al.* [24] show some agreement between their derived volume integrals with only about 10% differences for the heaviest target nuclei. However, all these analyses obtained linear energy dependences of the volume integrals because of the narrow energy range of their investigations. Volume integrals derived from measurements of total cross sections for five target nuclei from 100 to 150 MeV [25] seem to favor a logarithmic energy dependence.

It became apparent that because different investigators analyzed different sets of data, a number of curious, and in some cases inconsistent, features arose from the various analyses. What was lacking was a complete single analysis over the entire range of energies. We have therefore carried out and presented in this paper a global review of all nucleon elastic scattering studies up to 1 GeV. Section II describes the optical model parameter selection and analysis procedure. A new optical model analysis of proton scattering from Ca isotopes at 1 GeV for the determination of the isospin potential is presented in Sec. III. The energy dependence of the potentials is derived in Sec. IV. Section V contains the results and conclusions of the investigation.

## II. OPTICAL MODEL PARAMETER SELECTION AND PROCEDURE

The initial real central potential parameters were taken from the compilation of Perey and Perey [17], which lists potential parameters derived from proton elastic scattering studies up to 1975. More values were obtained from the works of Perey [26], Becchetti and Greenlees [5], Van Oers [8], Alkhazov *et al.* [27], Kwiatkowski and Wall [6], Igo *et al.* [28], Nadasen *et al.* [12], Woo *et al.* [29], and Hutcheon *et al.* [9]. Most of these optical model analyses have been carried out with the real central potentials of the Woods-Saxon form:

$$V(r)=V_0/\{1+\exp[(r-r_0A_t^{1/3})/a_0]\},$$

where $V_0$, $r_0$, and $a_0$ define the strength and shape of the potential. Values for $V_0$, $r_0$, and $a_0$ were derived from the analyses of elastic scattering data. The Woods-Saxon potential is a spherically symmetric potential that resembles the shape of the nuclear matter distribution. The potential parameters $V_0$, $r_0$, and $a_0$ are not completely independent. They correlate with each other, resulting in continuous ambiguities between them. An increase or decrease in one parameter can be compensated by changes in the other two, resulting in equally good fit to a set of the scattering data. However, two quantities have been found to be free of ambiguities. One is the root-mean-square radius, $r_{\langle rms\rangle}$, which is the radius of the nucleus averaged over the potential. Greenlees, Pyle, and Tang [14] showed that combinations of different radii ($\sim$19% variation) and diffuseness ($\sim$55% variation) parameters that provide acceptable fits to the data give $r_{\langle rms\rangle}$ values that are within 3% of each other. The $r_{\langle rms\rangle}$ basically gives the size of the nucleus, and is thus of no interest in the present study. The other is the real potential volume integral $J_{\mathrm{R}}$, which is the spatial integral of the potential, weighted by the strength. This quantity defines the total effective potential for the interaction of the proton with a nucleus at a particular energy. The potential volume integral is not subjected to the $V$, $r_0$, $a_0$ continuous ambiguity. It has been found that all the different sets of $V$, $r_0$, and $a_0$ parameters that provide equally good fits to a set of data have the same volume integral [5,13,14].

Comparison of analyses of proton elastic scattering from different targets showed that the potential volume integral was proportional to the mass of the target nucleus. This is expected if the potential and nucleon (mass) density distributions have essentially similar radial shapes. Therefore a quantity largely independent of the target mass, namely, the reduced volume integral $J_{\mathrm{R}}/A$ where $A$ is the mass number

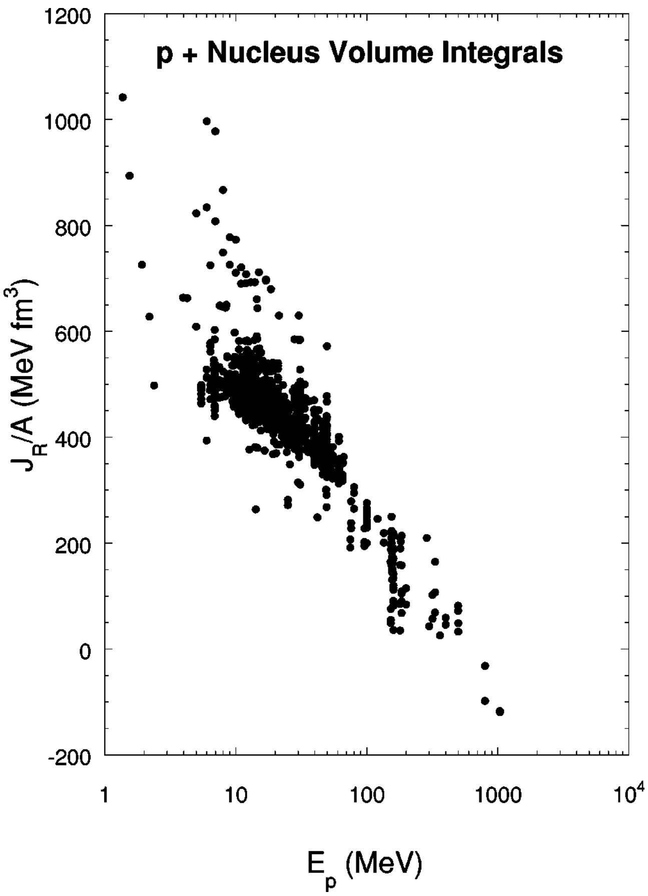


FIG. 1. Proton volume integrals versus energy.

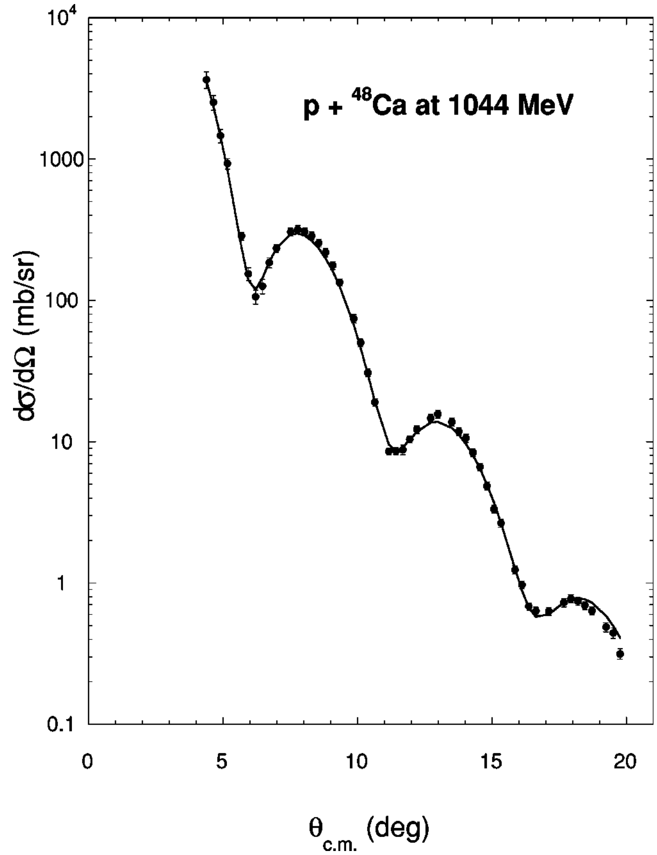


FIG. 2. $p+{}^{48}$Ca differential cross sections (dots) and OM calculations (solid line).

of the target, has been defined. This quantity provides a basis for the determination of the systematics of the potential across the entire Periodic Table (except for very light few-nucleon systems). The reduced volume integrals of the real potentials calculated using parameters from all available studies of proton elastic scattering, are shown in Fig. 1. They will be discussed in Sec. IV.

## III. ANALYSIS OF 1044-MeV PROTON ELASTIC SCATTERING FROM Ca ISOTOPES

The original analysis of 1044-MeV proton elastic scattering from four Ca isotopes [11] was based on the Glauber theory with the purpose of determining neutron and nuclear matter distributions. Thus this study did not provide the volume integrals required for the present investigation. We have therefore carried out an optical model analysis of these data in a formalism consistent with those of lower-energy data. The optical model potential of conventional form containing a Coulomb term, a complex nuclear central term, and a complex nuclear spin-orbit term was used. We used the relativistic extension of the Schrödinger equation with relativistic kinematics [30]. The Woods-Saxon form factors were used for the potentials. Since the proton has a spin of 1/2, it is advisable to include a spin-orbit potential in the analysis, since otherwise systematics in the other components of the potential could be distorted. However, only differential cross section data are available at 1044 MeV. In order to include polarization data, needed to fix the spin-orbit potential, we resorted to the 800-MeV analyzing power measurements [28] for all four nuclei. We do not expect the polarization to change much between 800 MeV and 1 GeV. Therefore we transformed the 800-MeV data to 1 GeV by changing the angles of the data by means of the equivalence of momentum transfer, i.e., $2k\sin(\theta/2)$ values are equal at both 800 MeV and 1 GeV ($k$ is the wave number of the incident proton). This procedure was deemed adequate for determining a spin-orbit potential at 1 GeV, which is sufficiently realistic to constrain ambiguities in the central potential that would otherwise arise from the analysis of cross section data alone.

The code SNOOPY8 [30] was used to carry out the analyses. The starting parameters were obtained from the extrapolations of lower-energy studies. For each angular distribution, first single-parameter searches were carried out on all twelve parameters. The optimized fit parameters were then used to carry out all combinations of two-parameter searches. The number of search parameters was continually increased until searches were made on combinations of six parameters. This provided very good fits to the data. An attempt was made to improve the fits obtained by allowing the normalization of the cross section data to vary, but only $^{44}$Ca preferred a normalization different from unity. Figures 2 and 3 show the results for $^{48}$Ca differential cross section and polarization data. The dots represent the data with the error bars indicated. The solid lines show the optical model (OM) fits.

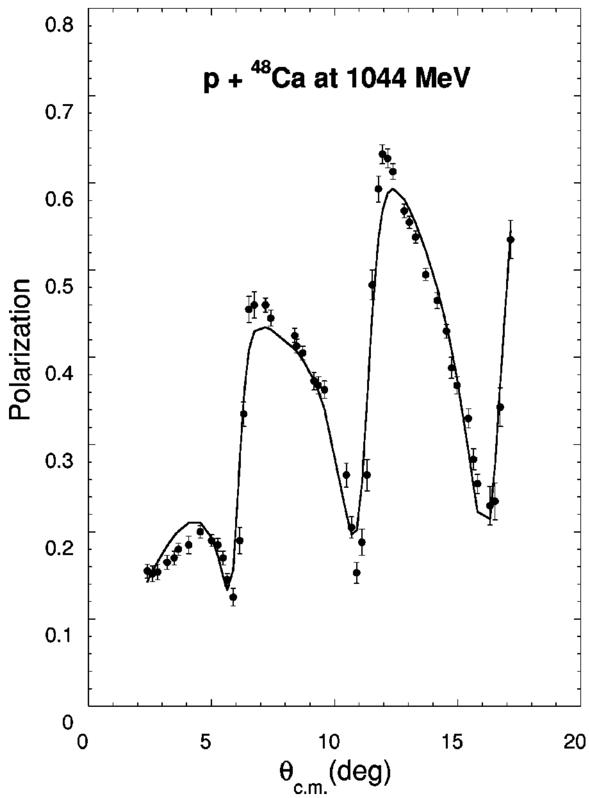


FIG. 3. $p+{}^{48}$Ca analyzing powers (dots) and OM calculations (solid line).

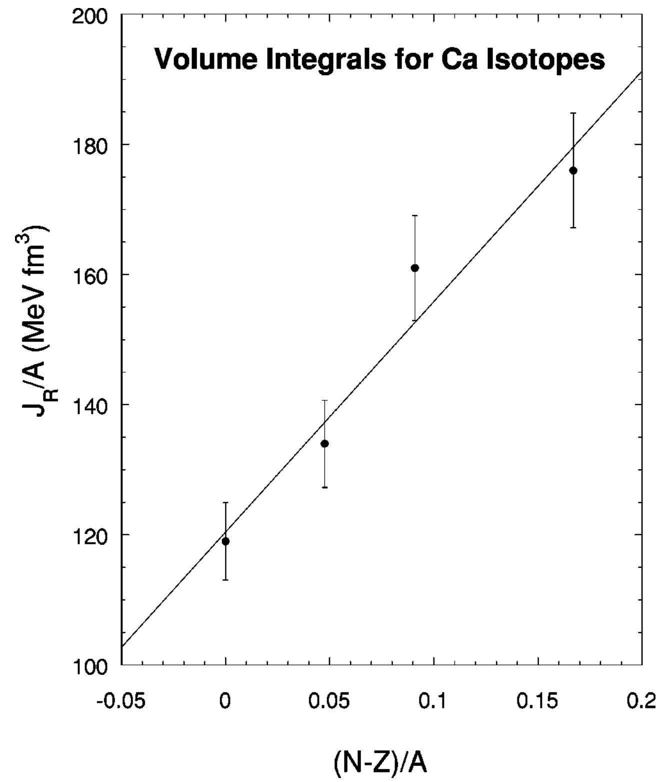


FIG. 4. Volume integrals for Ca isotopes at 1044 MeV versus $(N-Z)/A$. The solid line is a linear fit to the data.

The derived central potential volume integrals are positive, indicating that the repulsive component of the nuclear force dominates at this energy. The volume integrals steadily increased in going from $^{40}$Ca to $^{48}$Ca. This variation is due to the isospin component of the nuclear potential. Several authors have considered the existence of an isospin component in the central nuclear potential [31]. Lane [32] explicitly showed that the volume integral of the isospin component of the potential is given by $J_S(N-Z)/A$, where $J_S$ is the coefficient of the symmetry term. Figure 4 shows the plot of the volume integrals (after subtracting the Coulomb correction term $V_c=0.4Z/A^{1/3}$) as a function of $(N-Z)/A$. The linear relationship between $J_R/A$ and $(N-Z)/A$ provides a value of $J_S=350\pm35$ MeV fm$^3$. This value agrees well with the values 200–400 obtained by Becchetti and Greenlees [5], and $300\pm100$ by Perey [26], but is much higher than the value of $120\pm40$ obtained by Kwiatkowski and Wall [6]. The isospin effect arises from nucleon-nucleon interactions. For proton elastic scattering, neutron-rich nuclei have a positive isospin term, that remains essentially constant with energy. Since the total central potential decreases with energy, the isospin component becomes relatively more important as energy increases. In fact, for $^{48}$Ca at 1044 MeV, we find that almost one-third of the potential is due to the isospin effect. This can be understood in terms of the fundamental nucleon-nucleon forces. It is well known that, because of the existence of the triplet state, the proton-neutron interaction is three times as strong as the proton-proton and neutron-neutron interaction. Thus the potential for proton scattering from a neutron-rich nucleus is strongly enhanced.

We calculated the isospin coefficient $J_S$ for neutron scattering using the 131-MeV potential parameters of Schneider and Cormack [25], and obtained a value of 190 $\pm80$ MeV fm$^3$. The values of $J_S$ in literature vary widely, ranging from $\sim100$ [33,34] to as high as $\sim470$ [35–37]. In a global analysis of neutron scattering, Rapaport [20] obtained an energy-dependent isospin coefficient of the form $J_S=J_S(0)-\alpha E$. The values of $310\pm80$, $234\pm80$, and 110 $\pm25$ for $J_S(0)$ have been obtained from different sets of data. The global analysis of nucleon elastic scattering by Varner *et al.* [24] gives a value of $110\pm10$ MeV fm$^3$. It is clear that the neutron isospin potential needs to be determined unambiguously. This can be done by measuring high-energy neutron scattering from target nuclei having a range of values of $(N-Z)/A$.

## IV. ENERGY DEPENDENCE OF THE REAL VOLUME INTEGRAL

Several theoretical attempts have been made to derive the energy dependence of the empirical real potential. Brueckner *et al.* [38] proposed a description of the dispersive nature of nuclear matter, which gave the correct magnitude of the potential at zero energy. Lipperheide and Schmidt [39] used the dispersion integral, but the real potential was too strong at high energies and did not change sign as indicated by the scattering data. The nonlocal energy-independent potential of Perey and Buck [19], extended in energy by Engelbrecht and Fiedeldey [40], was in reasonable agreement with experimental results up to about 150 MeV. Passatore noted strong disagreement between the experimentally determined potentials and those calculated from the dispersion relation for the energy range 100–500 MeV [41], but the slope agreed with the empirical results above 500 MeV [42]. Using both the intrinsic energy dependence resulting from the dispersion relation and that due to nonlocality, he reformulated the calculations to obtain a logarithmic energy dependence of the potentials up to 1 GeV. As shown below, our results confirm Passatore's predictions.

All the calculated volume integrals $J_R/A$ were ordered in terms of increasing energy. Since these are the reduced volume integrals, they should be largely independent of the target mass. Therefore no consideration was given to the mass of the target in determining the systematics of these volume integrals. The volume integrals determined from all known proton elastic scattering studies are plotted as a function of beam energy in Fig. 1. It is observed that the values at low energies have a large spread, ranging from $\sim 200$ to $\sim 1000$ MeV fm$^3$. This is basically due to the fact that the incident proton cannot get into the nuclear interior because of Coulomb repulsion effects and a short nuclear mean free path. Thus the proton only samples the surface region of the nucleus. Therefore it is not clear which of these potentials represents the true mean-field interaction between the proton and the nucleus.

As the beam energy increases, the spread of the volume integrals decreases. This is a consequence of the ability of the proton to sample a larger region of the nucleus and experience almost the total average nuclear potential. As this trend continues, the derived volume integrals fall within a cone-shaped region, converging towards single values at energies above 100 MeV. Even the single values at the higher energies are not always completely consistent with each other. These differences may be due to different methodologies of analysis, as well as differences in scattering data, particularly in the absolute cross section normalization. However, there is no *a priori* reason not to accept the results of any of the studies.

The large spread in the low-energy values precludes an accurate determination of the energy dependence, unless one bins the data over some appropriate energy interval to determine an average volume integral for each energy bin. Since investigations were carried out in small energy intervals at low energies, we averaged all results in 1-MeV intervals. For energies below 10 MeV, nuclear structure effects, core polarization, compound nuclear scattering, and other reaction machanisms influence and mask the assumed pure potential scattering. Thus the empirical values of $J_R/A$ do not necessarily reflect the actual energy dependence of the real central potential. Therefore we decided to omit results below 10 MeV in the determination of the energy dependence. The average volume integral at each energy is plotted as a function of beam energy in Fig. 5. There is still some spread in these values, particularly at the lower energies. However, the overall pattern of the data clearly indicates a logarithmic dependence of the volume integrals on the incident energy. We made a least squares fit to the data, which provides a dependence of the volume integrals on incident energy of the form

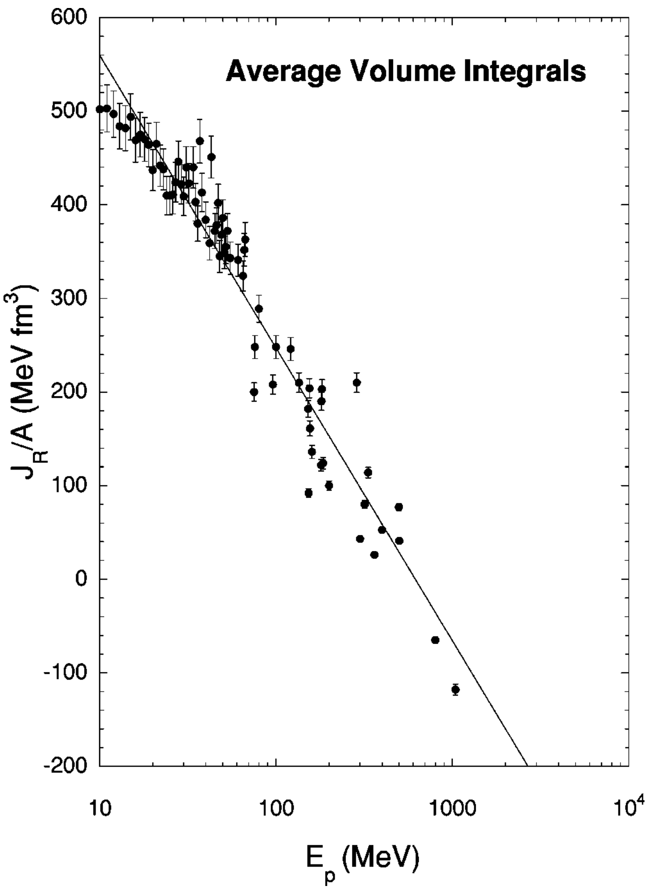


FIG. 5. Volume integrals for proton elastic scattering averaged over 1-MeV bins. The solid line is a logarithmic fit to the data.

$$J_R(\mathrm{E}) = J_R(0) - \beta \ln E,$$

with $J_R(0) = 872 \pm 44$ MeV fm$^3$ and $\beta = 136 \pm 7$ MeV fm$^3$. The zero crossing of the real potential is at $600 \pm 60$ MeV, in good agreement with the earlier results of phenomenological optical model studies [16] and impulse-approximation calculations [43].

We have also calculated real potential volume integrals using parameters of all available neutron elastic scattering studies. These are plotted as a function of energy in Fig. 6. The values for $E \geqslant 10$ MeV coincide well with the proton data. We averaged the volume integrals in 1-MeV intervals, and a least squares fit of the data for energies $\geqslant 10$ MeV gave a logarithmic energy dependence with $J_R(0) = 773 \pm 39$ MeV fm$^3$ and $\beta = 120 \pm 6$ MeV fm$^3$, with a zero crossing at $630 \pm 60$ MeV. This is shown as the solid line in Fig. 6. It compares well with the proton energy dependence. There is only $\sim 10\%$ difference in the slope and the zero crossing is essentially the same. Thus it seems appropriate to apply the proton potentials for neutron studies.

## V. SUMMARY AND CONCLUSION

We have carried out a global analysis of all available potential parameters for proton elastic scattering from the known studies at all energies. The volume integral of the real potential was calculated for each parameter set. These vol-

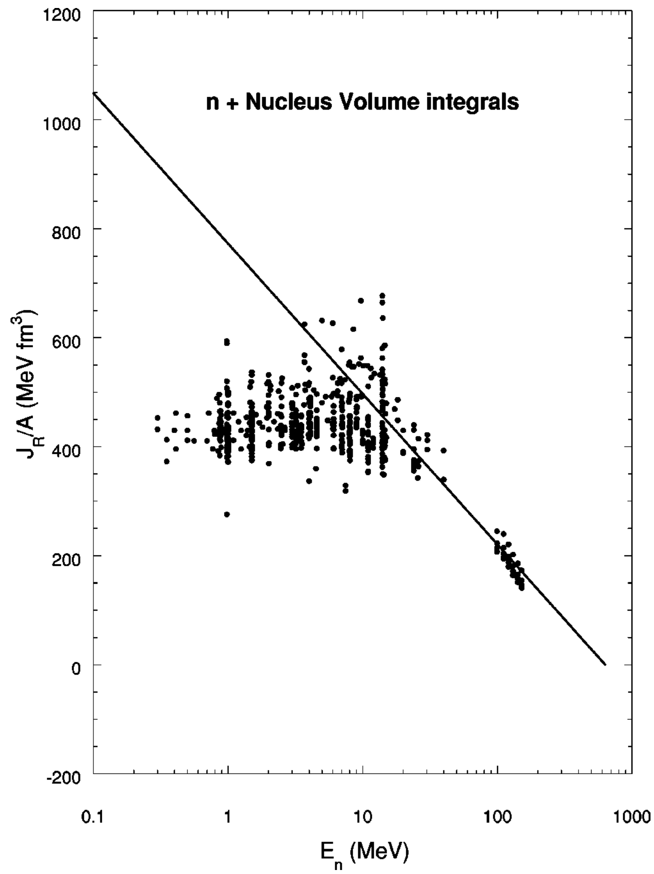


FIG. 6. Volume integrals for neutron elastic scattering. The solid line is the energy dependence derived using data at $E \geqslant 10$ MeV.

ume integrals, averaged over 1-MeV bins, show a logarithmic dependence on the proton energy from the lowest energies to 1 GeV. The derived relationship between the volume integrals and the energy describes the real part of the proton-nucleus interaction as a function of the beam energy. It may thus be concluded that the attractive mean field dominates the proton-nucleus interaction at low energies. As the energy increases, the repulsive nucleon-nucleon interaction increases in importance. In the energy region around 600 MeV, the two effects balance each other and the net real potential goes to zero. Beyond this region, the repulsive component of the potential dominates. Analyses using more flexible radial shapes for the nuclear potential than the simple Woods-Saxon form considered here indicate [16] that the change from attraction to repulsion occurs first in the nuclear interior at a much lower proton energy (between 200 and 300 MeV), while the nuclear surface potential remains weakly attractive up to 800 MeV. From an utilitarian point of view, this study provides an universal formulation of the proton-nucleus real potential at any energy up to 1 GeV for all target nuclei. Thus the *p*-nucleus potentials required for global reaction studies can be derived from this formulation. By assuming reasonable radius, $r_0$, and diffuseness, $a_0$, parameters, one can determine the strength $V$ from the correct volume integral for a particular target nucleus at the appropriate energy. Because of the overlap of the neutron volume integrals above 10 MeV with those of protons, the proton formulation can also be used for neutron reaction studies.

We have also carried out optical model analyses of 1044-MeV proton elastic scattering from Ca isotopes. This provided an asymmetry potential of the form $J_S(N-Z)/A$ for Ca isotopes at 1044 MeV. The value of $\sim 350$ MeV fm$^3$ for the isospin coefficient $J_S$ is in agreement with the values determined at lower energies. In the determination of proton potentials for reaction studies, it is important to include the asymmetry potential for neutron-rich nuclei. Therefore, the volume integral of the real potential should be increased by $J_S(N-Z)/A$. Values of $J_S$ ranging from $\sim 300$ MeV fm$^3$ to $\sim 400$ MeV fm$^3$ at 1 GeV may be appropriate. The neutron volume integrals for neutron-rich nuclei should be decreased by $J_S(N-Z)/A$.

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