Systematic calculations of the ground state properties of superheavy nuclei

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We investigate the ground state properties of the nuclei with proton number Z=94-110 within the framework of deformed relativistic mean-field (RMF) theory. A systematic comparison between calculated binding energies and experimental data is made. The calculated binding energies are in good agreement with experimental ones. The reliability of the RMF model for superheavy nuclei has been tested by this comparison. The experimental data of alpha-decay energies and half-lives are reasonably reproduced by calculations. Calculated results further show that a prolate deformation is important for the ground state of these nuclei. The properties of some unknown nuclei are predicted and these are useful for future experimental researches of superheavy nuclei.

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I. INTRODUCTION

The possible existence of long-life superheavy elements was predicted in 1960s by nuclear theoreticians. Since then the search of superheavy elements in nature has become an exciting project for scientists. Both physicists and chemists participated in the project for studies on superheavy elements. Various methods are used to identify new elements. After much work was done, it was gradually realized that nuclear synthesis is indeed a reliable way to produce a new element. The elements Z = 105 - 108 were successfully identified by this method and both physicists and chemists agree on the existence of these elements although their half-lives are not very long. Because it is very difficult to detect a few atoms that are produced by nuclear reactions in many days, the progress of synthesizing new elements is slow for a long period. However situations changed recently; more and more large laboratories participated in researches of new elements. During 1995–1996, the elements Z=110-112 were produced by Hofmann *et al.* at GSI in Germany [1-3]. This progress is very quick because the three elements were produced within 2 years. Such a situation speeds up the researches on superheavy nuclei both experimentally and theoretically [4-7]. A breakthrough appeared at Dubna in Russia: the element Z = 114 was produced by Oganessian *et al.* [5,6] in 1999. One year later it was again reported that Z=116was synthesized at Dubna [8]. At present, many large laboratories in the field of nuclear physics focus on the production of new superheavy nuclides [1,5,10,12,9]. Some interesting results are reported very recently. ²⁷⁰Hs was produced at GSI in collaboration with people from PSI [10]. ²⁷⁰110 was also identified at GSI [11]. ²⁵⁹Db was produced at Lanzhou [12]. Another piece of good news on superheavy nuclei is that the experiments on confirmation of elements 111 and 112 were performed in 2000 as these elements were produced again [13]. The present situation shows that current study on superheavy nuclei has been well grounded.

Theoretically there are some studies on superheavy nuclei based on the self-consistent mean-field models or macroscopic-microscopic mass models [14-26]. The Frankfurt group tested the relativistic mean-field (RMF) model for a single nucleus ²⁶⁴Hs [14]. Cwiok et al. investigated the ground state properties of nuclei on the alpha-decay chain ²⁸⁹114 within the framework of the Skyrme-Hartree-Fock-Bogoliubov model [15]. Ren and Toki systematically calculate the properties of superheavy nuclei on the decay chain of Z=110-112 and Z=114 in the RMF model [20]. Shape coexistence is predicted in the ground state of superheavy nuclei and deformation can be an important cause for the stability of superheavy nuclei based on a constraint RMF calculation [20]. Although there are some theoretical calculations, there still exists a gap between experimental data and theoretical calculations. A systematic comparison between theoretical binding energies and experimental data is missing in publications due to the fast growth of this field. This comparison is important to see the global behavior of a model and is also useful for the prediction of unknown nuclei. Another important problem in the study of superheavy nuclei is to calculate the production cross section of superheavy nuclei before experiments. For the prediction of the production cross section of superheavy nuclei, the reliable binding energies must be inputted for reaction calculations. A partial study on this problem was completed by one of us |21|.

In this long paper we report the systematic calculation on the properties of nuclei with proton number Z=94-110 and N=132-172. These cover all heavy nuclei whose binding energies are known ($Z \ge 94$). Now these nuclei are possibly produced in many laboratories. ²⁷⁰Hs and ²⁷⁰110 have been synthesized at GSI [10,11]. After ²⁵⁹Db was observed at Lanzhou in China [12], it is planned to investigate the properties of nuclei with Z=108 experimentally soon. The nuclei in this range also bridge the gap from the known actinium series to the unknown superheavy nuclei. Especially there are some indications that deformations may be important in this range.

This paper is organized in the following way. Section II is the formalism of the RMF model. The numerical results and discussions of even-even nuclei are given in Sec. III. The result of even-odd nuclei is presented in Sec. IV. Section V is a short summary.

II. THE FORMALISM OF THE RELATIVISTIC MEAN-FIELD THEORY

In the RMF approach, we start from the local Lagrangian density for interacting nucleons, σ , ω , and ρ mesons, and photons [27–37]

$$\mathcal{L} = \overline{\Psi} (i \gamma^{\mu} \partial_{\mu} - M) \Psi - g_{\sigma} \overline{\Psi} \sigma \Psi - g_{\omega} \overline{\Psi} \gamma^{\mu} \omega_{\mu} \Psi$$
$$- g_{\rho} \overline{\Psi} \gamma^{\mu} \rho_{\mu}^{a} \tau^{a} \Psi + \frac{1}{2} \partial^{\mu} \sigma \partial_{\mu} \sigma - \frac{1}{2} m_{\sigma}^{2} \sigma^{2} - \frac{1}{3} g_{2} \sigma^{3} - \frac{1}{4} g_{3} \sigma^{4}$$
$$+ \frac{1}{4} c_{3} (\omega_{\mu} \omega^{\mu})^{2} - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} + \frac{1}{2} m_{\omega}^{2} \omega^{\mu} \omega_{\mu} - \frac{1}{4} R^{a\mu\nu} \cdot R^{a}_{\mu\nu}$$
$$+ \frac{1}{2} m_{\rho}^{2} \rho^{a\mu} \cdot \rho_{\mu}^{a} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - e \overline{\Psi} \gamma^{\mu} A^{\mu} \frac{1}{2} (1 - \tau^{3}) \Psi \qquad (1)$$

with

$$\Omega^{\mu\nu} = \partial^{\mu}\omega^{\nu} - \partial^{\nu}\omega^{\mu}, \qquad (2)$$

$$R^{a\mu\nu} = \partial^{\mu}\rho^{a\nu} - \partial^{\nu}\rho^{a\mu}, \qquad (3)$$

$$F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}, \qquad (4)$$

where the meson fields are denoted by σ , ω_{μ} , and ρ_{μ}^{a} and their masses are denoted by m_{σ} , m_{ω} and m_{ρ} , respectively. The nucleon field and rest mass are denoted by Ψ and M. A_{μ} is the photon field which is responsible for the electromagnetic interaction, $e^{2}/4\pi = 1/137$. The effective strengths of the coupling between the mesons and nucleons are, respectively, g_{σ} , g_{ω} , and g_{ρ} . g_{2} and g_{3} are the nonlinear coupling strengths of the σ meson. c_{3} is the self-coupling term of the ω field. The isospin Pauli matrices are written as τ^{a} , τ^{3} being the third component of τ^{a} .

The equations of motion for the fields are easily obtained from the variational principle [27-31,20]. In order to describe the ground state properties of nuclei we need static solution of the above Lagrangian. For this case the meson field and photon fields are assumed to be classical fields and they are time independent (c numbers). The nucleons move in classical fields as independent particles (mean-field approximations). The Dirac field operator can be expanded in terms of single particle wave functions $\Psi = \Sigma_i \phi_i a_i$ where a_i is a particle creation operator [27,28] and ϕ_i is the single particle wave function. For actual calculations, we omit the contribution of the Fermi-sea under no-sea approximations. The sum on single particles states runs on physical bound states, i.e., the occupied shell model states. Symmetries will simplify the calculations considerably. Time reversal symmetry is used and therefore the spacial vector components for $\omega_{\mu}, \ \rho_{\mu}^{a}$, and A_{μ} are zero. Charge conservation guarantees that only the third component of the isovector field (ρ_0^0) survives [27–30]. We denote simply ρ_0^0 as ρ_0 . Finally we have the following Dirac equations for nucleons and the Klein-Gordon equations for meson fields (for details see Refs. [27-30]):

$$\left[-i\,\boldsymbol{\alpha}\nabla + \boldsymbol{\beta}M^{*}(\mathbf{r}) + V(\mathbf{r})\right]\phi_{i}(\mathbf{r}) = \boldsymbol{\epsilon}_{i}\phi_{i}(r), \qquad (5)$$

where the effective mass $M^*(\mathbf{r}) = M + g_\sigma \sigma(r)$. The potential $V(\mathbf{r})$ is a timelike component of a Lorentz vector,

$$V(\mathbf{r}) = g_{\omega}\omega_0(r) + g_{\rho}\tau^a\rho_0^a(\mathbf{r}) + e((1-\tau^3)/2)A_0(r), \quad (6)$$

$$(-\Delta + m_{\sigma}^2)\sigma(\mathbf{r}) = -g_{\sigma}\rho_s(r) - g_2\sigma^2(r) - g_3\sigma^3(r), \quad (7)$$

$$(-\Delta + m_{\omega}^2)\omega_0(\mathbf{r}) = g_{\omega}\rho_v(r) - c_3\omega_0^3(r), \qquad (8)$$

$$(-\Delta + m_{\rho}^2)\rho_0(\mathbf{r}) = g_{\rho}\rho_3(r), \qquad (9)$$

$$-\Delta A_0(\mathbf{r}) = e \rho_p(r), \qquad (10)$$

where ρ_s , ρ_v , and ρ_p are, respectively, the densities of scalar, baryon, and proton. ρ_3 is the difference between the neutron and proton densities. They will be *c* numbers by taking expectation values. Their expressions are as follows:

$$\rho_s(\mathbf{r}) = \sum_{i=1}^{A} \overline{\phi}_i(r) \phi_i(r), \qquad (11)$$

$$\rho_{v}(\mathbf{r}) = \sum_{i=1}^{A} \phi_{i}^{+}(r)\phi_{i}(r), \qquad (12)$$

$$\rho_3(\mathbf{r}) = \sum_{i=1}^{A} \phi_i^+(r) \tau^3 \phi_i(r), \qquad (13)$$

$$\rho_p(\mathbf{r}) = \sum_{i=1}^{A} \phi_i^+(r)((1-\tau^3)/2)\phi_i(r).$$
(14)

Now we have a set of coupled equations for mesons and nucleons and they will be solved consistently by iterations. After a final solution is obtained, we can calculate the binding energies, root-mean-square radii of proton and neutron density distributions, single particle levels, and quadrupole deformation. An axial deformation is assumed in our calculations for superheavy nuclei. The details of numerical calculations are described in Refs. [20,27–30]. The main presentation of this paper is based on deformed RMF calculation. Some constraint RMF results are given as an explanation to the deformed RMF results.

III. NUMERICAL RESULTS AND DISCUSSIONS OF EVEN-EVEN SUPERHEAVY NUCLEI

We carry out RMF calculations with two sets of force parameters, TMA [20] and NLZ2 [16,22]. They are typical forces in the RMF model. The method of harmonic basis expansions [20,29,31–33,35] is used in solving the coupled RMF equations. The number of bases is chosen as $N_f = N_b$ = 20. This space is enough for the calculations here. The inputs of pairing gaps are $\Delta_n = \Delta_p = 11.2/\sqrt{A}$ MeV and this is a standard input in nuclear structure calculations. We do not make any adjustments on the current force parameters or on the pairing gaps. An axial deformation is assumed in

Nuclei	B (MeV)	β_n	$oldsymbol{eta}_p$	Qα	B (MeV)	β_n	β_p	Qα	B(Expt.) (MeV)	$Q_{\alpha}(\text{Expt.})$
²³⁴ Pu	1775.2	0.23	0.25	5.48	1773.8	0.26	0.28	5.50	1774.8	6.31
²³⁶ Pu	1788.6	0.24	0.25	5.16	1787.1	0.28	0.29	5.18	1788.4	5.87
²³⁸ Pu	1801.1	0.25	0.26	4.72	1799.7	0.29	0.29	4.98	1801.3	5.59
²⁴⁰ Pu	1813.7	0.26	0.27	4.45	1811.6	0.29	0.30	4.83	1813.5	5.26
²⁴² Pu	1825.5	0.28	0.28	4.05	1822.9	0.30	0.30	4.25	1825.0	4.98
²⁴⁴ Pu	1836.2	0.27	0.28	4.47	1833.7	0.30	0.30	3.95	1836.1	4.67
²⁴⁶ Pu	1846.4	0.26	0.26	4.29	1843.5	0.30	0.30	4.27		
²⁴⁸ Pu	1856.3	0.25	0.24	3.94	1852.8	0.30	0.30	4.22		
²⁵⁰ Pu	1865.7	0.22	0.21	3.88	1862.1	0.29	0.29	3.74		
²⁵² Pu	1875.1	0.20	0.18	3.56	1871.3	0.28	0.28	3.34		
²⁵⁴ Pu	1884.2	0.19	0.17	3.44	1880.1	0.27	0.27	3.33		
²⁵⁶ Pu	1893.2	0.18	0.16	3.22	1888.2	0.24	0.24	3.68		
²³⁶ Cm	1782.9	0.23	0.25	6.59	1781.0	0.27	0.29	7.04		
²³⁸ Cm	1797.2	0.24	0.25	6.25	1795.5	0.28	0.30	6.55	1796.5	6.62
²⁴⁰ Cm	1811.0	0.25	0.26	5.83	1809.1	0.29	0.31	6.34	1810.3	6.40
²⁴² Cm	1824.2	0.26	0.27	5.49	1822.0	0.30	0.31	6.04	1823.4	6.21
²⁴⁴ Cm	1836.9	0.27	0.28	5.09	1834.4	0.30	0.31	5.50	1835.9	5.90
²⁴⁶ Cm	1848.8	0.27	0.27	5.41	1845.9	0.31	0.31	5.36	1847.8	5.48
²⁴⁸ Cm	1859.5	0.26	0.26	5.00	1856.3	0.31	0.31	5.65	1859.2	5.16
²⁵⁰ Cm	1870.2	0.25	0.25	4.52	1866.3	0.30	0.31	5.49	1869.7	5.17
²⁵² Cm	1880.0	0.24	0.24	4.60	1876.2	0.30	0.30	4.91		
²⁵⁴ Cm	1889.6	0.22	0.21	4.35	1886.0	0.29	0.29	4.42		
²⁵⁶ Cm	1899.1	0.20	0.20	4.23	1895.3	0.27	0.28	4.30		

TABLE I. The binding energies, deformations and alpha-decay energies of even-even Pu and Cm isotopes with TMA and NL-Z2.

calculations. For the details of calculations please see [29,31,20,33].

A. Binding energy of superheavy nuclei

At first let us focus on the global behavior of the RMF model. We calculate the total binding energy (*B*) and the average binding energy of nucleons (B/A) for even-even nuclei on the isotope chain of Z=94-110. The theoretical binding energy, quadrupole deformation parameters of protons and neutrons, and the alpha-decay energy are listed in Tables I–IV, together with the available data of the experimental binding energy and alpha-decay energy. The variation of the average binding energy with nucleon number of some isotopes is drawn in Fig. 1.

The RMF results of Pu and Cm (Z=94,96) isotopes with TMA and NLZ2 are listed in Table I. In Table I the first column is for nuclei. Columns 2–5 correspond to numerical results of TMA and the columns 6–9 correspond to numerical results of NL-Z2. *B* (MeV) is the theoretical binding energy. The symbols β_n and β_p in Table I denote the quadrupole deformations of neutrons and protons, respectively. Further, the symbol Q_{α} (MeV) is the calculated alpha-decay energy are listed in the last two columns of the table. Experimental binding energies are obtained from the nuclear mass table [38] and the experimental alpha-decay energies can be deduced accordingly.

It is seen from Table I that the theoretical binding energies with both TMA and NL-Z2 are very close to the experimental data. The average difference between the theoretical binding energy and experimental one is approximately 1.0 MeV. This shows that the RMF model can give a reliable binding energy for the above nuclei. The relative difference between the theoretical binding energy and the data is approximately 0.06%. This difference is very small. The maximum difference is 3.4 MeV. Because the total binding energy is around 1800 MeV, the maximum relative difference is approximately 0.2%. This is the predicting precision of the RMF model for the binding energy of nuclei near stability. Therefore we can say that the experimental binding energy can be very well reproduced by the RMF model. Especially we do not introduce any adjustments on the pairing strength or on the effective force parameters when we carry out calculations.

When we compare the two sets of RMF results with TMA and NL-Z2 in Table I, we see that the RMF results with TMA overestimate the data a little and the RMF results with NL-Z2 underestimate them a little. The experimental data of neutron deficient nuclei agree well with NLZ2, whereas heavier isotopes agree better with TMA. The experimental data are known in a narrow window which is set by two sets of effective forces in the RMF model. Because this window is narrow, we can use the RMF model to predict the unknown binding energy of nuclei reliably. This will be more

Nuclei	B (MeV)	β_n	$oldsymbol{eta}_p$	Qα	B (MeV)	β_n	$oldsymbol{eta}_p$	Qα	B(Expt.) (MeV)	$Q_{\alpha}(\text{Expt.})$
²⁴² Cf	1818.9	0.25	0.26	6.64	1815.79	0.29	0.30	8.02	1817.3	7.52
²⁴⁴ Cf	1832.9	0.26	0.26	6.40	1829.67	0.30	0.31	7.73	1831.3	7.33
²⁴⁶ Cf	1846.3	0.26	0.27	6.24	1843.07	0.30	0.31	7.23	1844.8	6.86
²⁴⁸ Cf	1859.0	0.26	0.26	6.26	1855.50	0.30	0.31	7.18	1857.8	6.36
²⁵⁰ Cf	1871.0	0.26	0.26	5.73	1866.87	0.30	0.31	7.30	1870.0	6.13
²⁵² Cf	1882.4	0.25	0.26	5.34	1877.75	0.30	0.31	6.89	1881.3	6.22
²⁵⁴ Cf	1892.9	0.25	0.25	5.55	1888.51	0.29	0.30	6.09	1892.1	5.93
²⁵⁶ Cf	1903.0	0.23	0.23	5.36	1899.10	0.28	0.29	5.39		
²⁵⁸ Cf	1912.9	0.22	0.22	5.02	1909.14	0.27	0.28	5.12		
²⁶⁰ Cf	1922.7	0.21	0.21	4.70	1918.53	0.26	0.27	5.03		
²⁶² Cf	1932.2	0.20	0.19	4.60	1927.41	0.25	0.25	4.85		
²⁶⁴ Cf	1941.5	0.15	0.14	4.39	1936.16	0.22	0.23	4.59		
²⁴⁶ Fm	1839.1	0.25	0.26	8.07	1835.8	0.29	0.30	8.25	1837.2	8.37
²⁴⁸ Fm	1853.4	0.26	0.27	7.80	1850.3	0.30	0.31	7.72	1851.6	8.00
²⁵⁰ Fm	1867.0	0.26	0.27	7.61	1863.7	0.30	0.31	7.71	1865.5	7.56
²⁵² Fm	1880.0	0.26	0.26	7.20	1876.1	0.30	0.31	7.74	1878.9	7.15
²⁵⁴ Fm	1892.5	0.26	0.26	6.80	1887.9	0.30	0.30	7.27	1891.0	7.31
²⁵⁶ Fm	1903.7	0.25	0.26	7.00	1899.6	0.29	0.30	6.45	1902.5	7.03
²⁵⁸ Fm	1914.5	0.25	0.25	6.77	1911.0	0.28	0.29	5.80		
²⁶⁰ Fm	1925.0	0.21	0.21	6.22	1921.8	0.28	0.28	5.63		
²⁶² Fm	1935.5	0.21	0.21	5.75	1931.9	0.27	0.28	5.56		
²⁶⁴ Fm	1945.5	0.20	0.20	5.51	1941.2	0.26	0.27	5.64		
²⁶⁶ Fm	1955.2	0.17	0.16	5.28	1950.2	0.23	0.25	5.52		
²⁶⁸ Fm	1964.6	0.14	0.14	5.15	1959.1	0.21	0.23	5.34		

TABLE II. The binding energies, deformations and alpha-decay energies of even-even Cf and Fm isotopes with TMA and NL-Z2.

evident when we see other tables and Fig. 1.

Figure 1 demonstrates the variation of the average binding energy (B/A) with nucleon number for Pu, Cm, Cf, Fm, No, Rf (Z=94-104) isotopes. Figure 1 can be divided into three parts. Figure 1(a) is the results for Pu and Cm isotopes (the left part of Fig. 1). Figure 1(b) is the results for Cf and Fm isotopes (the middle part of Fig. 1). Figure 1(c) is the results for No and Rf isotopes (the right part of Fig. 1). Two sets of the theoretical average binding energies are represented by hollow triangles and hollow squares. We connect each set of theoretical results by a curve. For an isotope chain, the upper curve is the RMF result with TMA and the lower curve is that with NLZ2. Experimental average binding energies are denoted by black points. It is seen from Fig. 1 that the experimental points lie between two theoretical curves. This global agreement between the model and the data is very impressive. Therefore we can use the RMF model to predict the unknown binding energy reliably. This can be useful for future experimental study.

In Table II we list the RMF results of Cf and Fm isotopes with TMA and NL-Z2 forces. Similar notations to Table I are used. The theoretical binding energies are very close to the data. The theoretical value is approximately 0.1% off. The good agreement between model and data is seen again. The maximum difference is 0.2%. This is rather good for the model. When we compare numerical results with two sets of forces, it is concluded again that the RMF results with TMA overestimate the data a little and the RMF results with NL-Z2 underestimate the data a little. So we shows again that the RMF model is reliable and can be used to estimate the binding energy of unknown nuclei. This conclusion can be seen more clearly in Fig. 1(b). The experimental average binding energies lie between two theoretical curves.

For the nuclei on the isotope chain of No and Rf, the available experimental data of binding energies are less than those of Cf and Fm. In order to make a detailed comparison with available information, we also list the estimated binding energy from Audi *et al.* [38] and denote it with a symbol #. The numerical results are given in Table III and Fig. 1(c). For binding energies it is seen that the theoretical results follow experimental data well when nuclei becomes heavier. Previous discussions on Tables I and II are suitable for Table III and we do not repeat here. The previous conclusions also hold true.

For heavy nuclei with proton number Z=106 (Sg), 108 (Hs), and 110, only the binding energies of ²⁶⁰Sg and ²⁶⁴Hs are known. The calculated binding energy and other quantities are listed in Table IV. The agreement for binding energies is still nice even for heavy nuclei with Z=106 and Z=108.

Aside the detailed comparison on theoretical binding energies and experimental data, we conclude that the RMF model can reproduce all available data well. In the following

Nuclei	B (MeV)	β_n	eta_p	Qα	B (MeV)	β_n	β_p	Qα	B(Expt.) (MeV)	$Q_{\alpha}(\text{Expt.})$
²⁵² No	1873.2	0.26	0.26	8.57	1870.7	0.30	0.31	7.86	1871.3	8.55
²⁵⁴ No	1887.2	0.26	0.27	8.11	1884.1	0.30	0.31	7.82	1885.6	8.27
²⁵⁶ No	1900.7	0.26	0.27	7.68	1897.0	0.30	0.31	7.38	1898.6	8.58
²⁵⁸ No	1912.9	0.26	0.27	7.91	1909.6	0.29	0.30	6.68	1911.1#	8.20#
²⁶⁰ No	1924.6	0.26	0.26	7.49	1921.7	0.29	0.30	6.18	1923.1#	7.70#
²⁶² No	1935.8	0.20	0.21	7.01	1933.1	0.28	0.29	6.17	1934.7#	7.32#
²⁶⁴ No	1946.8	0.20	0.20	6.56	1944.0	0.27	0.28	6.09		
²⁶⁶ No	1957.5	0.20	0.20	6.25	1953.8	0.26	0.28	6.41		
²⁶⁸ No	1967.8	0.18	0.18	6.05	1963.0	0.25	0.26	6.49		
²⁵⁶ Rf	1892.6	0.25	0.25	8.85	1890.7	0.30	0.31	8.26	1890.7	8.95
²⁵⁸ Rf	1907.0	0.26	0.26	8.53	1904.5	0.30	0.31	7.94	1904.6#	9.25#
²⁶⁰ Rf	1920.1	0.26	0.27	8.84	1917.9	0.29	0.30	7.41	1917.9#	9.00#
²⁶² Rf	1932.7	0.21	0.21	8.48	1930.8	0.29	0.30	6.98	1930.8#	8.60#
²⁶⁴ Rf	1944.9	0.21	0.21	8.00	1943.0	0.28	0.29	7.01	1943.2#	8.27#
²⁶⁶ Rf	1956.7	0.21	0.21	7.37	1954.6	0.27	0.29	6.87	1955.2#	7.78#
²⁶⁸ Rf	1968.1	0.20	0.20	6.98	1964.9	0.27	0.28	7.39		
²⁷⁰ Rf	1979.0	0.19	0.19	6.86	1974.5	0.26	0.28	7.61		
272 Rf	1989.0	0.17	0.17	7.04	1984.4	0.32	0.32	6.95		
274 Rf	1998.8	0.14	0.15	6.90	1993.8	0.34	0.34	6.66		
²⁷⁶ Rf	2008.9	0.12	0.12	6.54	2003.7	0.16	0.18	5.71		

TABLE III. The binding energies, deformations and alpha-decay energies of even-even No and Rf isotopes with TMA and NL-Z2.

we will discuss the RMF results on quadrupole deformations, on alpha decay energy and on alpha decay half-life.

B. Quadrupole deformation in the ground state of heavy nuclei

It was considered for many years that superheavy nuclei may be spherical. It is still unknown whether it is right. Here we see available experimental information on deformation of heavy nuclei and theoretical predictions on the ground state deformation of these nuclei.

The experimental quadrupole deformation parameters of ^{238,240,242,244}Pu are known [40] and their values are approximately $\beta_2 = 0.29$. The quadrupole deformation parameters of above nuclei from TMA are $\beta_p = 0.26-0.29$ and those values from NL-Z2 are $\beta_p = 0.29-0.30$ (Table I). The agreement between theoretical quadrupole deformation and experimental one is very satisfying.

The measured quadrupole deformation parameters of 244,246,248 Cm are $\beta_2 = 0.30$ [40]. Two sets of theoretical ones from TMA and NL-Z2 (Table I) are $\beta_p = 0.26-0.28$ and $\beta_2 = 0.31$, respectively. A good agreement is also obtained for Cm isotopes.

For Cf isotopes the quadrupole deformation parameters are 0.30 for 250,252 Cf [40]. They are close to two sets of theoretical values 0.26 and 0.31 (Table II).

Recently the deformation of ²⁵⁴No is measured to be $\beta_2 = 0.27 \pm 0.02$ [41]. It is in good agreement with the calculated values $\beta_p = 0.27$ and $\beta_p = 0.31$ (Table III). This good agreement strengthens the predicting power of the RMF model on nuclear deformation. It is also the reason of the appearance of the deformed subshell at N = 152.

For Rf isotopes there is no experimental datum on their deformation. Two sets of theoretical values on deformation agree well for lighter isotopes but a difference gradually appears with the increase of neutron number. The difference becomes large for some heavier nuclei near N = 172. It is not fully clear why this phenomenon happens. We try to give two reasonable explanations on this in the following. The current force parameters of the RMF model are established by fitting the data of nuclei near stability. So there may be a difference when they are extended to neutron-rich nuclei. Some isospindependent factors may be in charge of this difference (such as the coupling strength of the ρ meson and asymmetry energy of nuclear matter). Another cause is that complex phenomenon of nuclear structure can appear with the increase of neutron number. There exist some solutions which have very close energies but with different deformations. For example, there are two solutions with very close energies in ²⁷⁴Rf when NL-Z2 is used. One solution corresponds to the ground state in Table III (B = 1993.8, $\beta_N = 0.34$, $\beta_p = 0.34$). Another solution corresponds to a very low excited state with a binding energy B = 1993.6 MeV and $\beta_N = 0.20, \beta_p = 0.22$. This is shape coexistence and it occurs when neutron number is close to N=172. It could lead to a difference of deformations when different forces are used. It is interesting to study this problem in detail in future.

For heavy nuclei with Z=106 (Sg), 108 (Hs), and 110 (Table IV), experimental deformation is unknown. Some models predicted their quadrupole deformations are around 0.2–0.3 [23,25,26]. The RMF model with two set of typical forces predicts there are prolate deformations in these nuclei.

TABLE IV. The binding energies, deformations and alpha-decay energies of even-even Sg, Hs and Z = 100 isotopes with TMA and NL-Z2.

Nuclei	B (MeV)	β_n	$oldsymbol{eta}_p$	Q_{α}	B (MeV)	β_n	$oldsymbol{eta}_p$	Qα	B(Expt.) (MeV)	$Q_{\alpha}(\text{Expt.})$
²⁶⁰ Sg	1911.9	0.25	0.26	9.08	1909.0	0.30	0.31	10.02	1908.96	9.93
²⁶² Sg	1925.9	0.25	0.26	9.38	1923.4	0.29	0.30	9.45		
²⁶⁴ Sg	1939.3	0.22	0.23	9.03	1937.3	0.29	0.30	8.92		
²⁶⁶ Sg	1952.4	0.22	0.22	8.57	1950.5	0.28	0.29	8.67		8.66
²⁶⁸ Sg	1965.1	0.21	0.21	8.08	1963.0	0.27	0.28	8.31		
²⁷⁰ Sg	1977.2	0.20	0.21	7.78	1974.0	0.26	0.28	8.91		
²⁷² Sg	1988.7	0.19	0.19	7.71	1984.1	0.25	0.27	9.05		
274 Sg	1999.3	0.18	0.18	7.93	1994.3	0.23	0.25	8.46		
²⁷⁶ Sg	2009.5	0.16	0.16	7.86	2004.7	0.19	0.20	7.99		
²⁷⁸ Sg	2019.8	0.13	0.14	7.35	2015.2	0.16	0.18	6.95		
²⁶⁰ Hs	1899.0	0.25	0.26	9.96	1895.9	0.29	0.30	11.15		
²⁶² Hs	1915.3	0.25	0.25	9.59	1911.5	0.28	0.29	11.08		
²⁶⁴ Hs	1930.2	0.24	0.25	9.98	1926.6	0.28	0.29	10.68	1926.72	10.54
²⁶⁶ Hs	1944.5	0.24	0.24	9.74	1941.4	0.28	0.29	10.30		10.18
²⁶⁸ Hs	1958.4	0.22	0.22	9.15	1955.6	0.27	0.28	9.96		
²⁷⁰ Hs	1971.8	0.21	0.22	8.90	1969.2	0.26	0.28	9.54		9.30
²⁷² Hs	1984.6	0.20	0.21	8.79	1981.1	0.26	0.27	10.23		
²⁷⁴ Hs	1996.6	0.19	0.20	8.89	1992.1	0.24	0.26	10.16		
²⁷⁶ Hs	2007.8	0.18	0.19	9.23	2003.3	0.20	0.21	9.13		
²⁷⁸ Hs	2018.3	0.17	0.17	9.32	2014.4	0.18	0.20	8.17		
²⁸⁰ Hs	2029.2	0.23	0.23	8.63	2025.5	0.16	0.18	7.47		
²⁶⁸ 110	1946.7	0.23	0.24	11.77	1943.9	0.26	0.27	11.02		
²⁷⁰ 110	1961.4	0.22	0.22	11.37	1958.9	0.26	0.26	10.78		10.97
²⁷² 110	1975.7	0.21	0.22	11.06	1973.3	0.25	0.26	10.58		
²⁷⁴ 110	1989.4	0.20	0.21	10.67	1986.1	0.24	0.24	11.45		
²⁷⁶ 110	2002.5	0.19	0.20	10.36	1998.7	0.21	0.22	10.65		
²⁷⁸ 110	2014.6	0.18	0.19	10.36	2010.9	0.20	0.21	9.52		
²⁸⁰ 110	2026.0	0.17	0.18	10.08	2022.8	0.18	0.19	8.81		

The parameter of quadrupole deformation (β_n and β_p) is approximately 0.2-0.3 for nuclei studied here. A constraint RMF calculation is carried out for nuclei 264,270 Hs [32]. The variation of the energy of ^{264,270}Hs with quadrupole deformation parameter is plotted in Fig. 2. The minimum in Fig. 2 corresponds to a ground state solution of a nucleus. Figure 2(a) is the result for ²⁶⁴Hs where TMA force is used. There is a prolate deformation in the ground state of ²⁶⁴Hs. This confirms that the numerical results in Table IV are correct. Figure 2(b) is the results for ²⁷⁰Hs. Here the number of the bases is chosen as $N_f = N_b = 18$ for saving computational time and NLZ2 is used. It is seen clearly that there exists a minimum around $\beta_2 = 0.27$. This corresponds to the ground state of ²⁷⁰Hs in Table IV. By the way it is known from macroscopic-microscopic calculations that hexadecupole deformation may be important for the stability of nuclei in this mass range. However it is not sure whether it is right up to now. For the moment there are no experimental data on the hexadecupole deformation. According to previous information on medium nuclei, quadrupole deformation is the most important one in all modes of deformations. Hexadecupole deformation may be important if there is a very small quadrupole deformation or it may play a role with the quadrupole deformation together. We have not calculated the parameter of hexadecupole deformation in this paper. It is interesting to calculate it in future studies.

C. Alpha-decay energy and half-life in ground states

Because many of superheavy nuclei are identified by the observation of a series of alpha decays, alpha decay plays a key role for the study of superheavy nuclei. It is also expected that other information such as deformation and isomers can be extracted by the measurement of different branching ratio of alpha decays. In this section we concentrate on alpha decays that occurred in the ground states of even-even nuclei. This is the most important branch of alpha decays in even-even nuclei.

Theoretical alpha-decay energies can be calculated by the binding energy difference of parent nucleus and daughter nucleus. They are listed in columns 5 and 9 of Tables I–IV. The experimental decay energies are taken from nuclear mass table and are listed in the last column.

For the alpha-decay energies it is seen from Tables I–IV that the theoretical value is close to the experimental one.



FIG. 1. The comparison of theoretical and experimental average binding energy (B/A) for Pu, Cm, Cf, Fm, No, Rf isotopes. This includes all available data of binding energy for even-even nuclei on these isotope chains. It is interesting to note that the experimental data lie between two sets of the RMF results. The TMA and NL-Z2 forces are used in calculations.

The theoretical value is only 0.1–0.8 MeV off in many cases. Usually the difference of theoretical binding energy and experimental one is more than 1 MeV for a heavy nucleus. Considering that the alpha-decay energy is the subtraction of two big quantities (the subtraction of the binding energies of parent and daughter nuclei), the RMF results on the decay energies is good. Therefore the RMF model can be used for predictions of the alpha-decay energies.

The variation of alpha-decay energies with nucleon number for some isotopes is plotted in Fig. 3. In Fig. 3, the black points are for experimental data. The two sets of RMF results are denoted by hollow triangle and hollow square and each set is connected by a curve. For the nuclei on the isotope chain of Pu and Cm [Fig. 3(a)], the RMF model slightly underestimates the data but the deviation is less than 1.0 MeV. For Cf isotopes, the data is between two sets of RMF results [the lower part of Fig. 3(b)] and theoretical results follow experimental trend well. For Fm isotopes, the deviation between model and data is less than 0.6 MeV [the upper part of Fig. 3(b)]. Figure 3(c) shows the variation of the decay energy with nucleon number for No and Rf isotopes. The RMF model slightly underestimates the data. We should stress here that above results are obtained without any adjustments on the force parameters or on the pairing strength



FIG. 2. The variation of the energy of ^{264,270}Hs with quadrupole deformation. The points are numerical results and they are connected by solid lines. A minimum appears around the deformation parameter 0.25 in (a) and it corresponds to the deformation of the ground state for ²⁶⁴Hs. A minimum appears around the deformation parameter 0.27 in (b) and it corresponds to the deformation of the ground state for ²⁷⁰Hs.



FIG. 3. The variation of the alpha-decay energy of Pu, Cm, Cf, Fm, No, Rf isotopes with nucleon number (A). The two sets of theoretical results are connected by solid curves.

in the RMF model.

At present experimental physicists hope that theoretical physicists can give more accurate value on alpha decay energies. They consider that the deviation of theoretical and experimental alpha-decay energy should be less than 0.5 MeV in order to arrange experimental setup of new nuclides suitably. It is interesting to emphasize that it is difficult to keep the deviation less than 0.5 MeV in mean-field models without any adjustments. If people introduce some artificial adjustments on pairing strengths, it is possible to give a more



FIG. 4. The comparison of theoretical half-life and experimental one for Pu, Cm, Cf, Fm, No, Rf isotopes. This includes available ground-state half-lives of alpha decays for even-even nuclei on these isotope chains. It is interesting to note that two sets of theoretical results are close to experimental data.

Nuclei	B (MeV)	β_n	$oldsymbol{eta}_p$	Qα	B (MeV)	β_n	β_p	Qα	B(Expt.) (MeV)	$Q_{\alpha}(\text{Expt.})$
²³³ Pu	1768.276	0.23	0.24		1766.821	0.26	0.27			
²³⁵ Pu	1781.921	0.24	0.25	5.388	1780.527	0.27	0.28	5.341	1781.020	6.000
²³⁷ Pu	1795.072	0.25	0.25	4.912	1793.534	0.29	0.29	5.057	1794.275	5.750
²³⁹ Pu	1807.626	0.26	0.26	4.546	1805.724	0.29	0.30	4.937	1806.922	5.245
²⁴¹ Pu	1819.687	0.27	0.28	4.212	1817.320	0.30	0.30	4.568	1818.697	5.140
²⁴³ Pu	1830.988	0.28	0.28	4.190	1828.401	0.30	0.30	4.020	1830.041	4.754
²⁴⁵ Pu	1841.310	0.27	0.27	4.503	1838.695	0.30	0.30	4.076		
²⁴⁷ Pu	1851.429	0.26	0.26	4.051	1848.166	0.30	0.30	4.331		
²⁴⁹ Pu	1861.032	0.24	0.24	3.937	1857.450	0.30	0.29	3.979		
²⁵¹ Pu	1870.435	0.21	0.19	3.589	1866.712	0.29	0.28	3.489		
²⁵³ Pu	1879.637	0.19	0.18	3.537	1875.737	0.28	0.27	3.278		
²⁵⁵ Pu	1888.687	0.18	0.17	3.320	1884.172	0.26	0.26	3.495		
²⁵⁷ Pu	1897.555	0.17	0.16	3.167	1892.284	0.23	0.22	3.523		
²³⁵ Cm	1775.502	0.23	0.24	6.507	1773.506	0.26	0.28	7.214	1773.610	7.200
²³⁷ Cm	1790.110	0.24	0.25	6.466	1788.335	0.28	0.29	6.786	1788.530	6.800
²³⁹ Cm	1804.194	0.25	0.26	6.027	1802.412	0.29	0.30	6.415	1802.860	6.460
²⁴¹ Cm	1817.691	0.26	0.27	5.681	1815.610	0.30	0.31	6.224	1816.386	6.185
²⁴³ Cm	1830.686	0.27	0.28	5.240	1828.264	0.30	0.31	5.760	1829.049	6.169
²⁴⁵ Cm	1842.771	0.27	0.28	5.216	1840.251	0.30	0.31	5.369	1841.369	5.624
²⁴⁷ Cm	1853.980	0.26	0.26	5.308	1851.218	0.31	0.31	5.483	1852.984	5.353
²⁴⁹ Cm	1864.908	0.26	0.26	4.702	1861.336	0.30	0.31	5.659		
²⁵¹ Cm	1875.199	0.25	0.25	4.530	1871.250	0.30	0.30	5.216		
²⁵³ Cm	1884.816	0.23	0.23	4.516	1881.104	0.29	0.29	4.646		
²⁵⁵ Cm	1894.404	0.21	0.20	4.331	1890.684	0.28	0.28	4.328		
²⁵⁷ Cm	1903.846	0.20	0.19	4.091	1899.668	0.27	0.27	4.369		
²⁵⁹ Cm	1913.114	0.19	0.18	3.873	1908.197	0.24	0.24	4.275		

TABLE V. The binding energies, deformations, alpha-decay energies of odd-A Pu and Cm isotopes with TMA and NL-Z2.

accurate value on alpha-decay energies. But in this case people should explain why the pairing strengths change.

Another very difficult task is to predict the alpha-decay half-life of nuclei reliably. Here we use the Viola-Seaborg formulas to calculate the half-life according to theoretical decay energy. Their expressions are given in the following:

$$\log(T_{\alpha}) = (aZ+b)(Q_{\alpha})^{-1/2} + (cZ+d) + h_{\log}, \quad (15)$$

where T_{α} is given in second and Q_{α} in MeV, and Z is the proton number of the parent nucleus. This formula is usually used to estimate the half-life of alpha decays by the decay energies [23,39]. The constants in this expression have been determined as a = 1.66175, b = -8.5166, c = -0.20228, d = -33.9069, $h_{\log}=0.0$ for even-even nuclei. These values are obtained by fitting the experimental data of middle and heavy nuclei [23,24,26,39].

Experimental half-life of alpha decays is also taken from Audi *et al.* [38]. Theoretical half-life is calculated according to the Viola-Seaborg formulas [23,39]. The numerical results for half-life is plotted in Fig. 4. The ratio between theoretical half-life and experimental one is less than 10^4 in many cases. This is acceptable in theory. It is well known that the half-life is very sensitive to the decay energy. A small difference on decay energy will lead to a large difference of half-life. For

example, the difference of decay energy 0.5 MeV can induce a difference of half-life 10⁵ times or more. Up to now we do not have a good model to calculate the alpha-decay energy and binding energy very accurately. This leads to that we cannot reproduce experimental half-life accurately. In future it is useful to develop theoretical models for improving the calculations of binding energies.

IV. NUMERICAL RESULTS AND DISCUSSIONS OF SOME ODD-A SUPERHEAVY NUCLEI

After we discuss the ground state properties of even-even nuclei, we present some numerical results for odd-N nuclei of above even-Z isotopes. We call these nuclei as odd-A nuclei for convenience. For a detailed calculation of odd-A nuclei, the vector current of the ω meson and the blocking effect of the last odd nucleon should be taken into account. Especially some special quantities may be sensitive to these effects. In this paper we are interested in the average properties of nuclei such as the binding energy, alpha-decay energy, quadrupole deformation. These quantities are not sensitive to the vector current of the ω meson and the blocking effect of the last odd nucleon. In the following we omit these effects in order to simplify the calculations for superheavy nuclei. Without this simplification the calculations for super-

TABLE VI. The binding energies, deformations, alpha-decay energies of odd-A Cf and Fm isotopes with TMA and NL-Z2.

Nuclei	B (MeV)	β_n	$oldsymbol{eta}_p$	Q_{α}	B (MeV)	β_n	$oldsymbol{eta}_p$	Qα	B(Expt.) (MeV)	$Q_{\alpha}(\text{Expt.})$
²³⁹ Cf	1796.657	0.23	0.25	7.145	1793.824	0.27	0.29	7.982	1794.090	7.810
²⁴¹ Cf	1811.635	0.24	0.25	6.775	1808.668	0.29	0.30	7.967	1809.160	7.660
²⁴³ Cf	1825.974	0.25	0.26	6.520	1822.774	0.29	0.30	7.938	1823.760	7.390
²⁴⁵ Cf	1839.711	0.26	0.27	6.280	1836.459	0.30	0.31	7.451	1837.426	7.256
²⁴⁷ Cf	1852.710	0.26	0.27	6.276	1849.406	0.30	0.31	7.158	1850.818	6.527
²⁴⁹ Cf	1865.035	0.26	0.26	6.036	1861.308	0.30	0.31	7.243	1863.370	6.295
²⁵¹ Cf	1876.800	0.26	0.26	5.480	1872.332	0.30	0.31	7.186	1875.104	6.176
²⁵³ Cf	1887.812	0.25	0.26	5.396	1883.136	0.30	0.30	6.500	1886.081	6.124
²⁵⁵ Cf	1897.939	0.24	0.25	5.560	1893.854	0.29	0.29	5.696	1896.740	5.720
²⁵⁷ Cf	1907.956	0.22	0.22	5.160	1904.195	0.28	0.28	5.209		
²⁵⁹ Cf	1917.846	0.21	0.21	4.858	1913.919	0.27	0.27	5.065		
²⁶¹ Cf	1927.554	0.20	0.20	4.592	1923.012	0.26	0.26	4.956		
²⁶³ Cf	1936.802	0.16	0.15	4.612	1931.788	0.23	0.24	4.709		
²⁶⁵ Cf	1946.039	0.13	0.12	4.283	1940.485	0.21	0.22	4.532		
²⁶⁷ Cf	1955.386	0.10	0.09	3.986	1949.010	0.19	0.20	4.427		
²⁴³ Fm	1816.667	0.23	0.24	8.290	1813.335	0.28	0.29	8.789	1813.700	8.690
²⁴⁵ Fm	1831.706	0.24	0.25	8.229	1828.422	0.29	0.30	8.546	1829.030	8.440
²⁴⁷ Fm	1846.361	0.26	0.27	7.913	1843.143	0.29	0.30	7.931	1843.860	8.190
²⁴⁹ Fm	1860.285	0.26	0.27	7.726	1857.079	0.30	0.31	7.680	1857.910	7.810
²⁵¹ Fm	1873.588	0.26	0.26	7.422	1869.965	0.30	0.31	7.741	1871.689	7.425
²⁵³ Fm	1886.368	0.26	0.26	6.967	1882.017	0.30	0.31	7.591	1884.469	7.197
²⁵⁵ Fm	1898.247	0.26	0.26	6.853	1893.760	0.29	0.30	6.872	1896.159	7.241
²⁵⁷ Fm	1909.124	0.25	0.26	6.988	1905.371	0.29	0.29	6.065	1907.513	6.864
²⁵⁹ Fm	1919.699	0.23	0.24	6.540	1916.482	0.28	0.29	5.672	1918.540	6.490
²⁶¹ Fm	1930.267	0.21	0.21	5.989	1926.901	0.27	0.28	5.594		
²⁶³ Fm	1940.575	0.21	0.21	5.571	1936.642	0.26	0.27	5.577		
²⁶⁵ Fm	1950.396	0.18	0.17	5.458	1945.670	0.25	0.26	5.642		
²⁶⁷ Fm	1959.953	0.16	0.15	5.149	1954.682	0.22	0.24	5.406		
²⁶⁹ Fm	1969.390	0.12	0.12	4.949	1963.535	0.20	0.22	5.250		

heavy nuclei will become very complicated.

We calculate the ground state properties of odd-A nuclei for Pu, Cm, Cf, Fm, No, Rf isotopes using the deformed RMF model. The numerical results are listed in Tables V–VII and Fig. 5.

Table V shows the ground state properties of Pu and Cm isotopes. The same symbols as in the preceding tables are used. It is seen from Table V that the experimental binding energies are very well reproduced by the model. The deviation between theory and experiment is approximately 1-2 MeV. This is very small as compared with the total binding energy which is as high as 1700-1900 MeV. The calculated alpha-decay energy is in good agreement with the experimental data. The deviation for decay energies is less than 1 MeV. There are also quadrupole deformations in the ground state of these nuclei. This is in agreement with the theoretical result for even-even nuclei. In order to see the global behavior of the variation of the binding energies, we plot the average binding energy of Pu and Cm isotopes in Fig. 5(a). It is noted again that the experimental data is between two sets of theoretical results. So the RMF model can also be used for

the prediction of the binding energy of odd-A nuclei.

For odd-A nuclei on the chain of Cf, Fm, No, Rf isotopes, the variation of the average binding energy with nucleon number is plotted in Figs. 5(b) and 5(c). It is seen again that the RMF model is very reliable for the binding energy of these nuclei. The numerical results of odd-A Cf, Fm, No, Rf isotopes are also listed in Tables VI and VII. Previous conclusions on even-even nuclei still hold true for odd-A nuclei here. We do not repeat them.

V. SUMMARY

We have investigated the structure of nuclei with proton number Z=94-110 in the RMF model without any adjustment on current force parameters or on pairing gaps. The global agreement between the theoretical results and various data is very impressive. The experimental data of binding energy are known in a narrow window which is formed by two sets of RMF results. The experimental data of neutron deficient nuclei agree well with the force NL-Z2, whereas heavier isotopes agree better with TMA. This is useful for a

TABLE VII. The binding energies, deformations, alpha-decay energies of odd-A No and Rf isotopes with TMA and NL-Z2.

Nuclei	B (MeV)	β_n	$oldsymbol{eta}_p$	Q_{α}	B (MeV)	β_n	$oldsymbol{eta}_p$	Qα	B(Expt.) (MeV)	$Q_{\alpha}(\text{Expt.})$
²⁴⁹ No	1851.210	0.25	0.25	8.796	1848.598	0.29	0.31	8.124		
²⁵¹ No	1865.983	0.26	0.26	8.678	1863.583	0.30	0.31	7.860	1863.280	8.890
²⁵³ No	1880.224	0.26	0.26	8.361	1877.526	0.30	0.31	7.853	1877.770	8.440
²⁵⁵ No	1894.042	0.26	0.27	7.846	1890.614	0.30	0.31	7.651	1891.540	8.445
²⁵⁷ No	1906.909	0.26	0.27	7.759	1903.271	0.29	0.30	7.046	1904.310	8.450
²⁵⁹ No	1918.726	0.26	0.27	7.821	1915.691	0.29	0.30	6.369	1916.550	7.910
²⁶¹ No	1930.150	0.21	0.21	7.274	1927.524	0.28	0.29	6.147	1928.320	7.490
²⁶³ No	1941.293	0.20	0.20	6.706	1938.652	0.27	0.29	6.130	1939.760	7.080
²⁶⁵ No	1952.202	0.20	0.20	6.365	1949.011	0.27	0.28	6.190		
²⁶⁷ No	1962.702	0.19	0.19	6.173	1958.392	0.26	0.27	6.550		
²⁶⁹ No	1972.686	0.17	0.17	6.010	1967.669	0.23	0.25	6.301		
²⁴⁹ Rf	1838.383	0.24	0.24	9.916	1835.625	0.29	0.31	9.262		
²⁵¹ Rf	1854.611	0.24	0.25	9.546	1852.493	0.30	0.31	8.570		
²⁵³ Rf	1870.189	0.24	0.25	9.321	1868.555	0.30	0.31	8.343		
²⁵⁵ Rf	1885.261	0.24	0.25	9.022	1883.587	0.30	0.31	8.296	1882.270	9.300
²⁵⁷ Rf	1899.863	0.25	0.26	8.661	1897.689	0.30	0.31	8.137	1896.820	9.250
²⁵⁹ Rf	1913.673	0.26	0.27	8.669	1911.218	0.29	0.31	7.696	1910.730	9.110
²⁶¹ Rf	1926.433	0.21	0.22	8.776	1924.428	0.29	0.30	7.143	1923.800	8.810
²⁶³ Rf	1938.815	0.21	0.21	8.211	1937.012	0.28	0.30	6.979	1936.400	8.450
²⁶⁵ Rf	1950.806	0.21	0.21	7.644	1948.887	0.28	0.29	6.937	1948.740	7.880
²⁶⁷ Rf	1962.457	0.20	0.21	7.136	1959.910	0.27	0.29	7.042		
²⁶⁹ Rf	1973.598	0.19	0.20	6.904	1969.708	0.27	0.28	7.603		
²⁷¹ Rf	1984.098	0.18	0.18	6.904	1979.489	0.31	0.32	7.203		
²⁷³ Rf	1993.907	0.16	0.16	7.079	1989.108	0.33	0.33	6.861		
²⁷⁵ Rf	2003.899	0.13	0.13	6.627	1998.675	0.17	0.19	6.260		



FIG. 5. The comparison of theoretical and experimental average binding energy (B/A) for odd-A Pu, Cm, Cf, Fm, No, Rf isotopes. This includes all available data of binding energy for odd-A nuclei on these isotope chains. It is interesting to note that the experimental data lie between two sets of the RMF results. The TMA and NL-Z2 forces are used in calculations.

good estimate of the binding energies and decay properties on unknown nuclei. The RMF results show that there is prolate deformation in these nuclei and deformation plays an important role for the ground state of these nuclei. The theoretical deformation parameters agrees well with experimental data. The theoretical alpha-decay energy also agrees well with the data. For alpha-decay half-life the ratio between experimental data and theoretical ones is less than 10^5 in many cases. This is a first systematic calculation on superheavy nuclei based on the RMF model and is also a complete comparison with various experimental data such as binding energies, deformation, alpha-decay energies, and half-lives. This complete comparison is necessary to test the global behavior of a model for a new mass range such as superheavy elements. The merits and drawbacks of a model can be seen

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clearly. This will be helpful for further developments of the model.

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