Boson-mapping-based extension of the random-phase approximation in a three-level Lipkin model

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Working within an exactly solvable three level model a boson Hamiltonian is defined via a mapping procedure and its expansion truncated at four-boson terms. The resulting spectrum is found in good agreement with the exact one. We discuss an extension of the random-phase approximation (RPA) based on this boson formalism. Nonlinear RPA-type equations are constructed and solved iteratively. The new solutions gain in stability with respect to the RPA ones. We perform diagonalizations of the boson Hamiltonian in restricted spaces; approximate spectra exhibit an improved quality with increasing the size of these spaces. Special attention is addressed to the problem of the anharmonicity of the spectrum.

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I. INTRODUCTION

The most commonly used microscopic approach for the study of collective vibrational states in many-fermion systems is the random-phase approximation (RPA) [1]. In this theory the lowest collective excitations result from the action of phonon operators $Q_\nu^\dagger$ on a state $|\mathrm{RPA}\rangle$ which is defined by the condition that $Q_\nu|\mathrm{RPA}\rangle=0$. This state represents the ground state of the system. It is a distinctive feature of RPA that multiphonon states, i.e., states obtained by repeated actions of phonon operators on the ground state, are eigenstates of the Hamiltonian with energies forming a harmonic spectrum. The existence of states which can be approximately described as corresponding to the multiple excitation of low-lying and/or high-lying phonons is well established in atomic nuclei. However, deviations from the harmonic picture are also observed and their influence on several processes has been analyzed [2].

In a standard derivation of the RPA equations a crucial point is represented by the so called quasiboson approximation (QBA). This is a rather crude approximation which causes the operators $Q_\nu^\dagger$ to behave as boson operators in spite of their (composite) fermionic structure. Overcoming this approximation has represented the starting point of many attempts aiming at improving RPA [3–22]. One of the line of research in such a context has been based on a reformulation of the whole theory in a boson formalism [16–22]. In other words, the operators $Q_\nu^\dagger$ have been defined from the beginning in terms of true boson operators and all the fermion operators of interest have been replaced by their boson images via a mapping procedure. The RPA-type equations that one constructs in this formalism depend on the degree of expansion of the boson Hamiltonian. Truncating this expansion at the lowest order, i.e. at two-boson terms only, gives the boson counterpart of RPA. Including higher-order terms in the boson image of the Hamiltonian provides a natural way to reach a higher level of approximation. In addition to that, the inclusion of these terms has another important effect: it leads to a coupling among multiphonon states. States which result from a diagonalization in a m-phonon space are therefore superpositions of zero-, one-, . . . , m-phonon states. Such a diagonalization is expected to lead to a further improved degree of approximation as well as to cause anharmonicities in the spectrum.

Calculations in this boson formalism have been performed in the recent past for atomic nuclei [19,20] considering a Hamiltonian truncated at four-boson terms and diagonalizing it in the space of one- and two-phonon states. The resulting anharmonicities have not been found large especially in ^{208}Pb. In particular, the anharmonicity associated with states whose main component is a double giant resonance has been found of the order of a few hundred keV. This is certainly related to the fact that RPA gives a good description of giant resonances especially in heavy closed shell nuclei.

In metallic clusters the dipole plasmon is a strongly collective state which corresponds to the oscillation of the delocalized electrons of the cluster against the positively charged ions. The experimental evidence for states corresponding to the double excitation of the plasmon has not been confirmed [23]. From the theoretical point of view the situation is also quite unclear. On the one hand, in Ref. [24] a purely harmonic spectrum for the multiple excitation of the plasmon has been predicted. On the other hand, by using the same approach as in Refs. [19,20] huge anharmonicities in the two-plasmon states have been found [25]. An important difference with respect to the case of atomic nuclei is that in metallic clusters the two-body interaction is very long ranged. This is probably the main reason why the RPA ground state is very different from the Hartree-Fock one and the RPA backward amplitudes are quite large. Of course, this may cause that the same level of truncation in the boson expansion is not adequate both in the case of nuclei and of metal clusters.

In principle, configuration mixing calculations can give a clear, model independent, indication on the existence of such

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two-phonon states and on their degree of anharmonicity. Unfortunately, since the states one looks for are quite high in energy, the number of configurations required to get stable results is huge. In Ref. [26] such a study has been performed for a very simple case: two interacting electrons moving in a uniform positive charge distribution. This is a kind of precursor of a metal cluster in the jellium approximation and allows for a numerically exact calculation. Important deviations from the harmonic limit have been found. More specifically, in addition to an almost perfectly harmonic vibrational band based on the ground state, other states appear which have a much lower degree of harmonicity.

In this paper we will analyze the anharmonicities present in the low-lying spectrum of a three-level solvable model [27] by providing at the same time an interesting test for a boson-mapping-based extension of RPA. We will shed some light on the limits of the approach adopted in Refs. [20] and [25]. The analysis we are going to present is very similar to that made in Ref. [21] where a two-level model was considered and the parameters were adjusted in such a way to mimick the multiple excitation of giant resonance. Of course, the three-level model is richer. In particular, since there are two single particle states above the Fermi surface (particle states) and one below (hole), two different elementary p-h configurations and, correspondingly, two different phonons can be excited. Therefore, one can better simulate the situation encountered in nuclei which generally present one high-lying and one low-lying collective modes for each multipolarity. Also, matrix elements of the interaction connecting a particle-hole state with a two-particle one can be included in a natural way as well as those involving four-particle states or four-hole states. These terms are present in a generic two-body interaction and are very important since the former couple states having numbers of phonons differing by one while the latter couple states having the same number of phonons.

The paper is organized as follows. In Sec. II we will describe the model and analyze the anharmonicities of its exact excitation spectrum. In Sec. III we will introduce the boson-mapping technique and construct the image of the fermion Hamiltonian. In Sec. IV we will present an extension of RPA and show the results obtained by diagonalizing the boson hamiltonian. Finally, in Sec. V we will draw some conclusions.

II. THE MODEL AND THE EXACT SPECTRUM

The model [27] consists of three 2Ω-fold degenerate single-particle shells which are occupied by 2Ω particles. Therefore, in the absence of interaction, the lowest level is completely filled while the others are empty. This lowest state represents the ''Hartree-Fock'' (HF) state of the system and is denoted by $|0\rangle$. A single-particle state is specified by a set of quantum numbers (j,m), where j stands for the shell $(j=0,1,2)$ and m specifies the 2Ω substates within the shell. The creation and annihilation operators of a fermion in a state (j,m) are defined by $a^{\dagger}_{jm}$ and a_{jm}, respectively.

Let us define the operators

$$K_{ij} \equiv \sum_{m=1}^{2\Omega} a^{\dagger}_{im} a_{jm} \quad (i,j=0,1,2). \tag{2.1}$$

These operators satisfy the commutation relations

$$[K_{ij},K_{kl}] = \delta_{jk} K_{il} - \delta_{il} K_{kj} \tag{2.2}$$

thus forming a U(3) algebra. With the additional constraint that fixes the total number of particles, the operators K become the generators of the algebra SU(3).

The Hamiltonian of the model is written in terms of the generators K_{ij} only and contains up to two-body interactions. Its most general form is

$$\begin{aligned} H_F = & \sum_{i=1,2} \epsilon(i) K_{ii} + \sum_{i,j=1,2} V_0(i,j) K_{i0} K_{0j} \\ & + \sum_{i,j=1,2} V_1(i,j)(K_{i0}K_{j0} + K_{0j}K_{0i}) \\ & + \sum_{i,j,k=1,2} V_2(i,j,k)(K_{i0}K_{jk} + K_{kj}K_{0i}) \\ & + \sum_{i,j,k,l=1,2} V_3(i,j,k,l) K_{ij} K_{kl} + V_4 K_{00} K_{00}, \end{aligned} \tag{2.3}$$

with real coefficients. The eigenstates and the eigenvalues of H_F can be constructed either by using the properties of the algebra SU(3) or by diagonalizing H_F in the following space:

$$F = \left\{ |n_1 n_2\rangle = \frac{1}{\sqrt{\mathcal{N}_{n_1 n_2}}} (K_{10})^{n_1} (K_{20})^{n_2} |0\rangle \right\}_{0 \le n_1 + n_2 \le 2\Omega}, \tag{2.4}$$

where $\mathcal{N}_{n_1 n_2}$ are normalization factors.

We simplify the calculations by assuming the coefficients of the interaction terms independent of the levels, i.e., $V_0(i,j)=V_0$, $V_1(i,j)=V_1$, $V_2(i,j,k)=V_2$, $V_3(i,j,k,l)=V_3$, and proportional to one parameter χ which is expressed in units of energy. We have chosen all the coefficients negative, assuming in this way that all the interation terms are attractive. We set the first two coefficients V_0 and V_1, which involve only particle-hole excitations, equal to a common value $-\chi$. For the remaining coefficients we have considered smaller values: in a first case they have been set all equal to $-\chi/10$ and in a second case they have been doubled. We have chosen the energies of the three levels equal to 0, ϵ and 2.5ϵ where ϵ is expressed in units of energy. Therefore the two sets of parameters used in the calculations are

$$\begin{aligned} & \epsilon(0)=0, \quad \epsilon(1)=\epsilon, \quad \epsilon(2)=2.5\epsilon, \\ & V_0=-\chi, \quad V_1=-\chi, \quad V_2=-\chi/10, \\ & V_3=-\chi/10, \quad V_4=-\chi/10 \end{aligned} \tag{2.5}$$

and

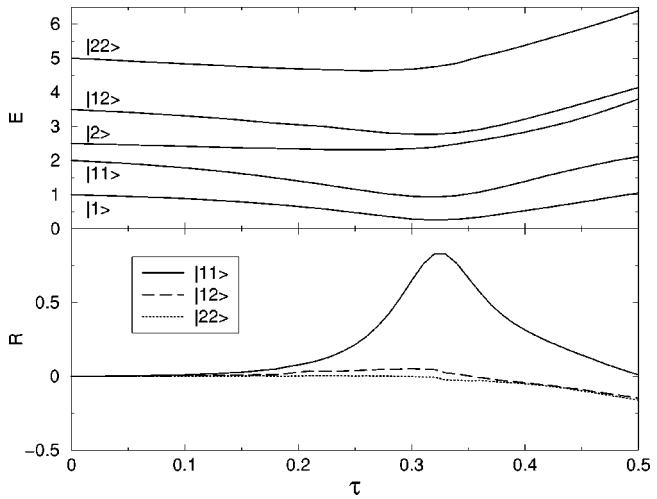

FIG. 1. Top: exact excitation energies in units of ϵ for the one-phonon and two-phonon states as function of the strength τ for the set of parameters (2.5). Bottom: ratios $R_{\nu_1\nu_2}$, Eq. (2.7), for the two-phonon states and the same set of parameters.

$$\epsilon(0)=0, \quad \epsilon(1)=\epsilon, \quad \epsilon(2)=2.5\epsilon,$$

$$V_0=-\chi, \quad V_1=-\chi, \quad V_2=-\chi/5,$$

$$V_3=-\chi/5, \quad V_4=-\chi/5. \tag{2.6}$$

For both sets of parameters we have chosen $2\Omega=20$.

We show in the upper part of Figs. 1 and 2 some excitation energies calculated by diagonalizing the fermion hamiltonian H_F in the space F, Eq. (2.4), with the two sets of parameters (2.5) and (2.6), respectively, as function of the parameter $\tau \equiv 2\Omega\chi/\epsilon$. We report only the energies of those states which are pure $1p-1h$ and $2p-2h$ states at $\tau=0$; in this way, we simplify our analysis looking only at those states that in RPA are pure one-phonon ($|\nu\rangle$) and two-phonon ($|\nu_1\nu_2\rangle$) states. One can observe that the two reported spectra are slightly different. In the second case, the stronger attractive interaction terms in the three-particle–one-hole (V_2), four particle (V_3), and four hole (V_4) channels have the general effect (except for the state $|2\rangle$) of pushing the excitation energies towards higher values. We observed that the results do not depend on the sign of V_2, due to the structure of the matrix to be diagonalized. Changing the signs of the interaction terms V_3 and V_4 we got a lowering of the excitation energies; in particular, the term with strength V_3 starts to be important at high values of the strength τ, while the term with strength V_4 is important already at low values of τ. This is related to the fact that the four hole term affects the ground state energy also when the correlations are absent or small, while the other term is effective only when the correlations are well developed.

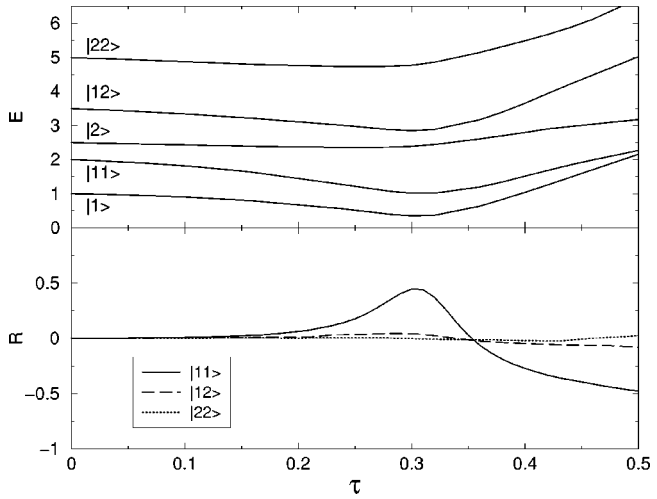

FIG. 2. Same as in Fig. 1 for the set of parameters (2.6).

Although not clearly visible at a first glance, the spectra of Figs. 1 and 2 show marked anharmonicities. In order to quantify these, in the lower part of Figs. 1 and 2 we show, for the two sets of parameters, respectively, the following ratios:

$$R_{\nu_1\nu_2}=\frac{E_{\nu_1\nu_2}-(E_{\nu_1}+E_{\nu_2})}{E_{\nu_1}+E_{\nu_2}}, \tag{2.7}$$

where by E_ν and $E_{\nu_1\nu_2}$ we mean the exact excitation energies of the states $|\nu\rangle$ and $|\nu_1\nu_2\rangle$. One notices a well different behavior of the plotted ratios. In both figures the ratio R is very small for the states $|12\rangle$ and $|22\rangle$ showing that the corresponding exact states can be quite well described as pure two-phonon states for all the considered values of the parameter τ. On the contrary, the ratio R_{11} changes rapidly with increasing τ showing stronger anharmonicities for the state $|11\rangle$. Thus, for both sets of parameters the spectrum is found to exhibit some levels with a clear harmonic nature and other levels which do not display this nature and whose anharmonicities depend on the strength of the interaction terms of the Hamiltonian. These results resemble to some extent those of Ref. [26] where a realistic two-electron system was examined. The existence of anharmonicities represents an evident limit to the harmonic picture of RPA that cannot be a good approximation to reproduce the multiphonon spectrum of the model.

III. THE BOSON MAPPING

Let us define the space

$$B\equiv\left\{ |n_1n_2)=\frac{1}{\sqrt{n_1!n_2!}}(b_1^\dagger)^{n_1}(b_2^\dagger)^{n_2}|0) \right\}_{0\leqslant n_1+n_2\leqslant 2\Omega}, \tag{3.1}$$

where the operators $b_i^\dagger$ obey the standard boson commutation relations

$$[b_i,b_j^\dagger]=\delta_{ij}, \quad [b_i,b_j]=0 \tag{3.2}$$

and $|0)$ is the vacuum of the b_i's operators. A one-to-one correspondence exists between the states of F and B, the boson operators $b_i^\dagger$ playing the role of the excitation operators K_{i0} and the boson vacuum $|0)$ replacing the HF state $|0\rangle$.

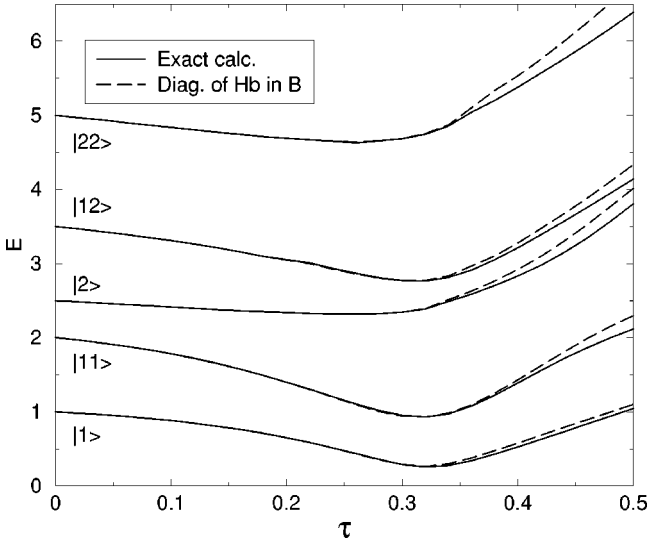

FIG. 3. Comparison between the exact one-phonon and two-phonon spectrum and the corresponding one obtained by diagonalizing H_B in the whole boson space B for the set of parameters (2.5). The energies are expressed in units of ϵ.

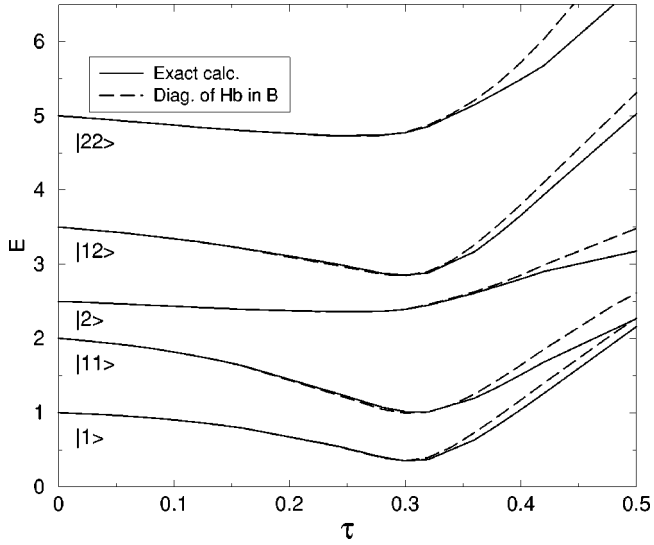

FIG. 4. Same as in Fig. 3 for the set of parameters (2.6).

The mapping procedure to construct boson images of fermion operators is the same discussed in previous works (see, for instance, Ref. [18]) and, due to the orthonormality of both sets of states $|n_1,n_2\rangle$ and $|n_1,n_2)$, it is simply based on the requirement that corresponding matrix elements in F and B be equal. Therefore, the procedure is of Marumori type. We refer to Refs. [18,28] for more details. Here, we simply say that, in correspondence with the Hamiltonian H_F (2.3), we introduce a hermitian boson Hamiltonian H_B which contains up to four-boson terms and whose general form is

$$H_B=\alpha+\sum_i \beta_i(b_i^\dagger+\text{H.c.})+\sum_{ij}\gamma_{ij}b_i^\dagger b_j + \sum_{i\leqslant j}\phi_{ij}(b_i^\dagger b_j^\dagger+\text{H.c.})+\sum_{i\leqslant j}\sum_k \epsilon_{ijk}(b_i^\dagger b_j^\dagger b_k+\text{H.c.}) + \sum_{i\leqslant j}\sum_{k\leqslant l}\delta_{ijkl}b_i^\dagger b_j^\dagger b_k b_l + \sum_{i\leqslant j\leqslant k}\sum_l \rho_{ijkl}(b_i^\dagger b_j^\dagger b_k^\dagger b_l+\text{H.c.}), \tag{3.3}$$

with the coefficients depending on the parameters $\epsilon(i), V_0, V_1, V_2, V_3, V_4$ of Eq. (2.3).

For the two sets of parameters, Eqs. (2.5) and (2.6), we show in Figs. 3 and 4, respectively, the comparison between the exact excitation energies and those obtained by diagonalizing H_B in the space B in order to check the validity of the four-boson truncation of the boson Hamiltonian. We can observe that the agreement between the two spectra is rather good for both sets of parameters. Differences between boson and fermion spectra start to be significant only for rather large values of τ ($\gtrsim 0.35$) and are more evident for the set (2.6). This means that in the chosen range of values of τ the considered truncation of the Hamiltonian is sufficient. We have also verified that the inclusion of an additional five-boson term of the type $b^\dagger b^\dagger b^\dagger b b$ considerably reduces the above differences in the spectra but we have nevertheless preferred to keep the boson Hamiltonian in the form (3.3) since this more closely reflects a realistic case (where the evaluation of the five-boson term would likely become rather difficult and would therefore be avoided).

In the next section we will show and comment some results obtained by diagonalizing H_B in restricted spaces, containing up to two, three, and four bosons. The quality of the results will be judged by comparing them with those obtained by diagonalizing H_B in the full boson space B. We will denote the latter as reference spectrum.

IV. EXTENSION OF RPA AND DIAGONALIZATION OF H_B IN RESTRICTED SPACES

The calculation in the full bosonic space B is not feasible for realistic many-body systems and one has to resort to restricted spaces where only states containing up to a certain number of bosons are considered. In Fig. 5 we compare the reference spectrum for the set of parameters (2.5) with those obtained by limiting the number of bosons to two, three, and four. These calculations correspond to diagonalizations in spaces constructed by acting with two-particle–two-hole, three-particle–three-hole, and four-particle–four-hole excitation operators on the Hartree-Fock state $|0\rangle$. We see that the results obtained by limiting the number of bosons to two are very poor even for those states which at zero interaction strength are pure 1 boson states. Enlarging the space the quality of the results improves. However, even for the largest space it is satisfactory only for strength $\tau\leqslant 0.25$. One may expect that by introducing collective bosons (phonons) the agreement becomes better.

Let us now introduce the operators

$$Q_\nu^\dagger=\sum_i (X_i^{(\nu)}b_i^\dagger - Y_i^{(\nu)}b_i), \tag{4.1}$$

and let the state $|\Psi_0)$ satisfy the condition

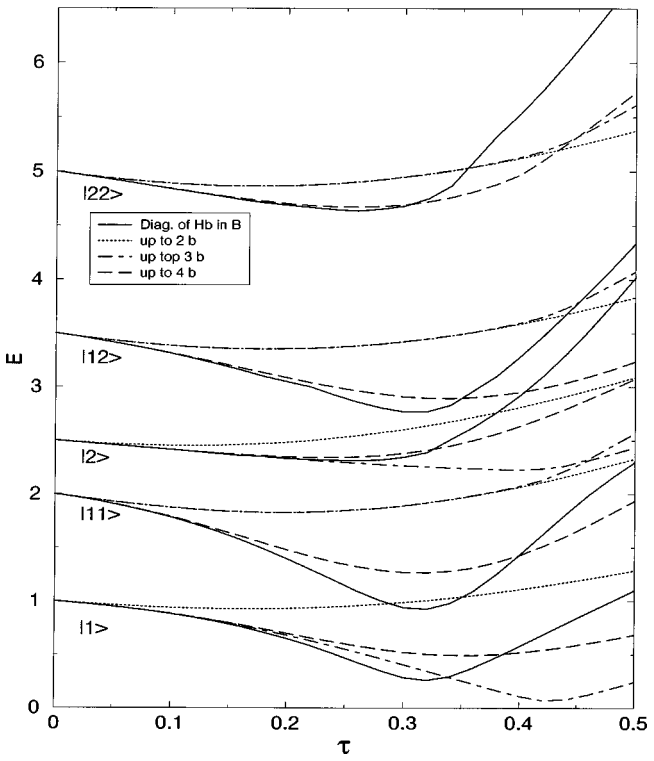

FIG. 5. Comparison between the reference spectrum and those obtained by limiting the bosonic space up to two, three, and four bosons b for the set of parameters (2.5). The energies are expressed in units of ϵ.

$$Q_\nu|\Psi_0)=0. \tag{4.2}$$

By using the equations of motion method [29] one finds that the amplitudes X and Y are solutions of

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}\begin{pmatrix} X^{(\nu)} \\ Y^{(\nu)} \end{pmatrix}=\omega^{(\nu)}\begin{pmatrix} X^{(\nu)} \\ Y^{(\nu)} \end{pmatrix}, \tag{4.3}$$

where

$$A_{ij}=(\Psi_0|[b_i,[H_B,b_j^\dagger]]|\Psi_0), \tag{4.4}$$

$$B_{ij}=-(\Psi_0|[b_i,[H_B,b_j]]|\Psi_0). \tag{4.5}$$

As anticipated in Sec. I, the form of Eqs. (4.3) is strictly related to the degree of truncation of the boson Hamiltonian. In the hypothesis that H_B contains up to two-boson terms, the double commutators in Eqs. (4.4) and (4.5) are just numbers which, therefore, are also the values of the matrices A and B. This is the simplest case which can be realized in this formalism and represents the boson counterpart of the standard RPA. One limitation of RPA is that it collapses at a given strength of the interaction and imaginary energies are found. This problem is not present in spherical closed shell nuclei, while it shows up in other many-body systems as, for example, in metallic clusters. This degree of approximation can be improved by introducing a Hamiltonian with higher-order terms such as, for instance, Eq. (3.3). These terms originate on one hand from those parts of the fermionic

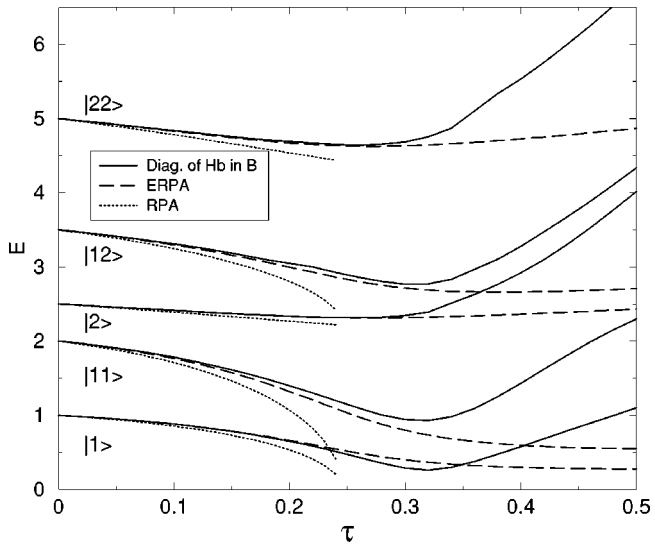

FIG. 6. Excitation energies of the states $|\nu\rangle$ and $|\nu_1\nu_2\rangle$ calculated within RPA (dotted lines) and ERPA (dashed lines) compared with the corresponding reference states (full lines) for the set of parameters (2.5). The energies are in units of ϵ.

Hamiltonian, proportional to V_2, V_3, and V_4, which do not enter in the RPA equations. On the other hand, they take into account some corrections to the violation of the Pauli principle. In this case the double commutators are operators. In order to calculate their expectation values in $|\Psi_0)$, as required in Eqs. (4.4) and (4.5), one can express the operators b and $b^\dagger$ in terms of Q and $Q^\dagger$, by reversing Eq. (4.1) (and its adjoint) and using the orthonormality conditions

$$\delta_{\nu\nu'}=\sum_{ij}\,(X_i^\nu X_j^{\nu'}-Y_i^{\nu'}Y_j^\nu). \tag{4.6}$$

This procedure gives, however, matrices A and B which depend on the X and Y amplitudes and, consequently, equations of motion (4.3) which are nonlinear. In what follows this nonlinear extension of RPA will be called ERPA [16].

Having determined the amplitudes X and Y within RPA or ERPA, one can express the Hamiltonian H_B in terms of the operators Q and $Q^\dagger$. In the case of RPA, namely, when the boson Hamiltonian Eq. (3.3) is truncated at two-boson terms only, H_B can be rewritten simply as

$$H_B=E_0+\sum_\nu\,\omega^{(\nu)}Q_\nu^\dagger Q_\nu, \tag{4.7}$$

where $\omega^{(\nu)}$ are the energies solutions of the RPA equations (4.3). This Hamiltonian obviously does not mix states with different phonon numbers and so its eigenstates are pure zero-, one-, . . . , m-phonon states. For a higher-level truncation in the boson Hamiltonian, such as for instance, that of Eq. (3.3), H_B acquires instead the more general form

$$\begin{aligned} H_B=&E_0+H_{10}(Q^\dagger+\text{H.c.})+H_{11}Q^\dagger Q+H_{20}(Q^\dagger Q^\dagger+\text{H.c.})\\ &+H_{21}(Q^\dagger Q^\dagger Q+\text{H.c.})+H_{30}(Q^\dagger Q^\dagger Q^\dagger+\text{H.c.})\\ &+H_{22}Q^\dagger Q^\dagger QQ+H_{31}(Q^\dagger Q^\dagger Q^\dagger Q+\text{H.c.})\\ &+H_{40}(Q^\dagger Q^\dagger Q^\dagger Q^\dagger+\text{H.c.}), \end{aligned} \tag{4.8}$$

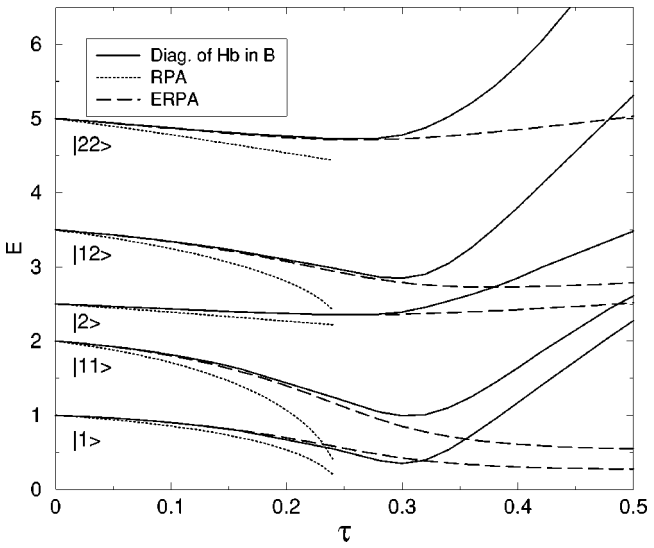

FIG. 7. Same as in Fig. 6 for the set of parameters (2.6).

where the coefficients H_{ij} are functions of X and Y (for simplicity, we have dropped all the indices). Also in this case, as in RPA, the term H_{20} as well as the nondiagonal terms H_{11} vanish, as can be easily shown using the fact that the amplitudes X and Y are solutions of the ERPA equations. However, the remaining terms of Eq. (4.8) mix different multiphonon states so that the eigenstates of the full Hamiltonian become combinations of these states. This fact introduces an evident difference with RPA since the energies which result from the

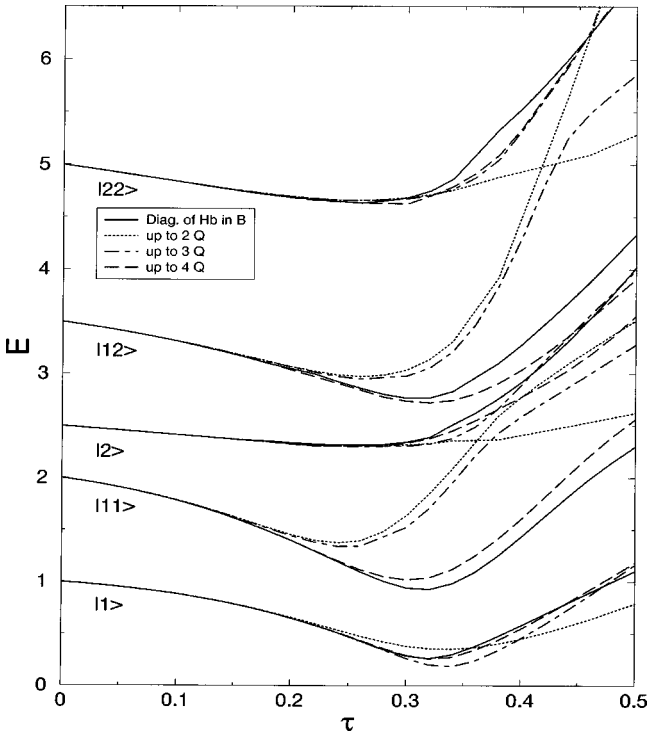

FIG. 8. Comparison between the reference spectrum and those obtained by diagonalizing H_B in the spaces containing up to two, three and four ERPA phonons for the set of parameters (2.5). The energies are expressed in units of ϵ.

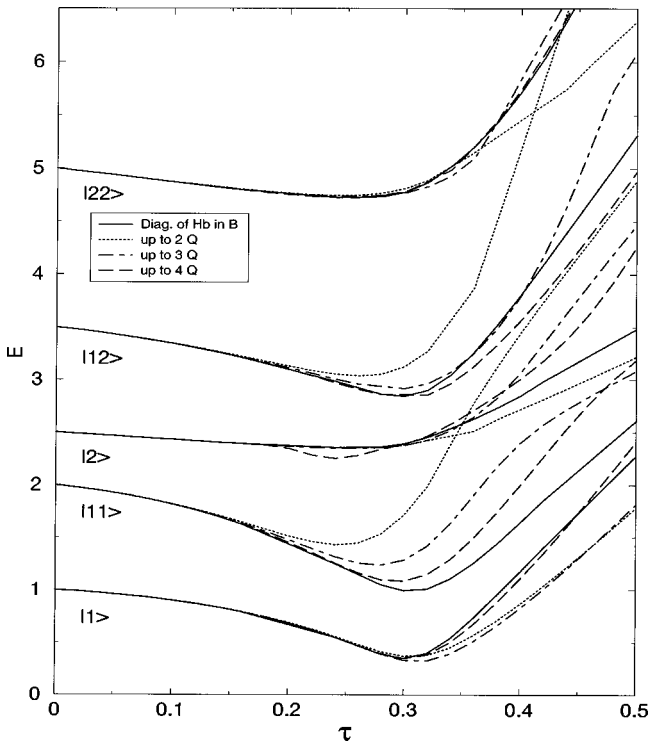

FIG. 9. Same as in Fig. 8 for the set of parameters (2.6).

ERPA equations are not eigenvalues of the boson Hamiltonian in the phonon space as it is in the case of RPA (where they provide the excitation energies of the one-phonon eigenstates).

With the chosen parameters RPA collapses at $\tau=0.24$. On the contrary, we found real solutions of the ERPA equations of motion in the whole considered strength range. Moreover, the so obtained energies for the states $|1\rangle$ and $|2\rangle$ are in good agreement with the reference ones for strength up to $\tau=0.3$. This shows that these states can be quite well described as pure one-phonon states. However, we want to stress that this does not imply a harmonic spectrum. Indeed, as already shown in Figs. 1 and 2, anharmonicities are present for $\tau \geqslant 0.2$. We show in Figs. 6 and 7, for the two sets of parameters (2.5) and (2.6), respectively, the excitation energies of the states $|\nu\rangle$ and $|\nu_1 \nu_2\rangle$ calculated within RPA and ERPA. They are compared with the corresponding reference states.

In Figs. 8 and 9 we show the spectra obtained by diagonalizing H_B in spaces containing up to two, three, and four ERPA phonons. By comparing them with the reference results (shown as full lines) one sees that the agreement improves by enlarging the space and is satisfactory in the whole range of the interaction strength when the mixing of states up to four phonons is taken into account. It is worthwhile noting that the results with the phonons Q are much better than those obtained with the bosons b. The comparison is shown in Fig. 10 in the case of the largest spaces and with the parameters (2.5).

V. CONCLUSIONS

In this paper we have analyzed some of the lowest excited states of the spectrum of a solvable three-level model,

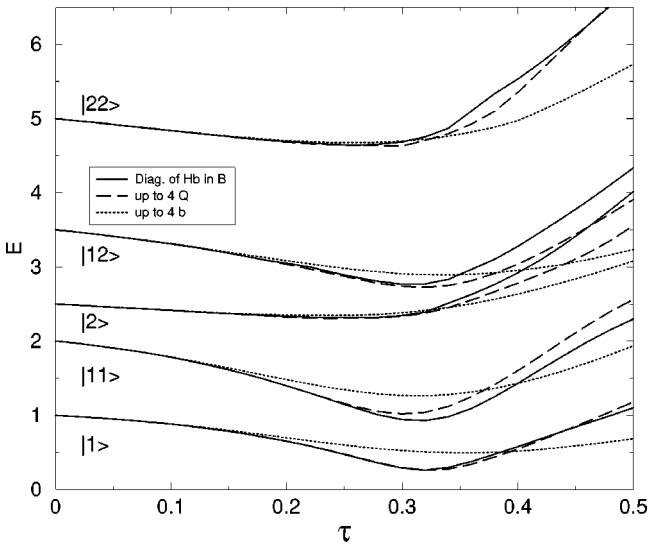

FIG. 10. Comparison between the reference spectrum, that obtained by limiting the boson space up to four bosons *b* and that obtained by limiting the space up to four ERPA phonons for the set of parameters (2.5). The energies are in units of ϵ.

namely, those that in RPA would be described as one- and two-phonon states. We have worked in a boson formalism. As a preliminary step we have constructed a boson image of the fermion Hamiltonian whose expansion has been truncated at four-boson terms. The procedure followed in such a derivation has been of Marumori type. The quality of the boson Hamiltonian has been tested by comparing its eigenvalues with the exact ones. Within the considered range of variation of the interaction strength the agreement between fermion and boson energies has always been found rather good.

By making use of this boson Hamiltonian we have introduced an extension of the RPA. The resulting equations of motion have the same form as the RPA ones but are nonlinear. They have been solved iteratively. The new solutions have gained in stability with respect to the RPA ones and, in particular, around the RPA collapse point the new energies have exhibited a good agreement with the exact ones. This extension of RPA, while introducing corrections to the Pauli principle violations present in RPA, naturally leads to a Hamiltonian which mixes states with a different number of phonons. We have performed diagonalizations in spaces containing up to two-, three-, and four-phonon states and observed an improved quality of the approximate spectra with increasing the size of the spaces.

Special attention has also been addressed to the problem of the anharmonicity of the spectrum. This has been found relevant for the state which, in RPA, is described as the double excitation of the lowest one-phonon state. On the contrary, for the other states, a less pronounced anharmonicity has been found. These findings agree with those of Ref. [26] and point to the necessity of considering together all the possible elementary excitations of a many-body system when discussing the anharmonicity of its spectrum.

[1] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer, New York, 1980).
[2] M. N. Harakeh and A. van der Woude, *Giant Resonances* (Clarendon, Oxford, 2001).
[3] J. Dukelsky and P. Schuck, Nucl. Phys. **A512**, 466 (1990).
[4] D. Janssen and P. Schuck, Z. Phys. A **339**, 43 (1991).
[5] A. Klein, N. R. Walet, and G. Do Dang, Nucl. Phys. **A535**, 1 (1991).
[6] D. Karadjov, V. V. Voronov, and F. Catara, Phys. Lett. B **306**, 166 (1993).
[7] F. Catara, N. Dinh Dang, and M. Sambataro, Nucl. Phys. **A579**, 1 (1994).
[8] J. Dukelsky and P. Schuck, Phys. Lett. B **387**, 233 (1996).
[9] F. Catara, G. Piccitto, M. Sambataro, and N. Van Giai, Phys. Rev. B **54**, 17 536 (1996).
[10] A. A. Raduta, C. M. Raduta, A. Faessler, and W. A. Kamiński, Nucl. Phys. **A634**, 497 (1998).
[11] F. Catara, M. Grasso, G. Piccitto, and M. Sambataro, Phys. Rev. B **58**, 16 070 (1998).
[12] F. Krmpotić, E. J. V. de Passos, D. S. Delion, J. Dukelsky, and P. Schuck, Nucl. Phys. **A637**, 295 (1998).
[13] F. Šimkovic, A. A. Raduta, M. Veselsky, and A. Faessler, Phys. Rev. C **61**, 044319 (2000).
[14] M. Grasso and F. Catara, Phys. Rev. C **63**, 014317 (2001).
[15] N. Dinh Dang and A. Arima, Phys. Rev. C **62**, 024303 (2000).
[16] D. Beaumel and Ph. Chomaz, Ann. Phys. (N.Y.) **213**, 405 (1992).
[17] M. Sambataro and F. Catara, Phys. Rev. C **51**, 3066 (1995).
[18] M. Sambataro and J. Suhonen, Phys. Rev. C **56**, 782 (1997).
[19] F. Catara, Ph. Chomaz, and N. Van Giai, Phys. Lett. B **233**, 6 (1989).
[20] E. G. Lanza, M. V. Andres, F. Catara, Ph. Chomaz, and C. Volpe, Nucl. Phys. **A613**, 445 (1997).
[21] C. Volpe, Ph. Chomaz, M. V. Andres, F. Catara, and E. G. Lanza, Nucl. Phys. **A647**, 246 (1999).
[22] M. Sambataro and N. Dinh Dang, Phys. Rev. C **59**, 1422 (1999).
[23] R. Schlipper, R. Kusche, B. von Issendorf, and H. Haberland, Phys. Rev. Lett. **80**, 1194 (1998); (unpublished).
[24] K. Hagino, Phys. Rev. B **60**, R2197 (1999).
[25] F. Catara, Ph. Chomaz, and N. Van Giai, Phys. Rev. B **48**, 18 207 (1993).
[26] F. Catara and M. Sambataro, J. Phys.: Condens. Matter **13**, L705 (2001).
[27] S. Y. Li, A. Klein, and R. M. Dreizler, J. Math. Phys. **11**, 975 (1970).
[28] M. Sambataro, Phys. Rev. C **60**, 064320 (1999).
[29] D. J. Rowe, *Nuclear Collective Motion* (Methuen, London, 1970).