Resolving the large-N_c nuclear potential puzzle

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The large-N_c nuclear potential puzzle arose because three- and higher-meson exchange contributions to the nucleon-nucleon potential did not automatically yield cancellations that make these contributions consistent with the general large-N_c scaling rules for the potential. Here it is proposed that the resolution to this puzzle is that the scaling rules only apply for energy-independent potentials while all of the cases with apparent inconsistencies were for energy-dependent potentials. It is shown explicitly how energy-dependent potentials can have radically different large-N_c behavior than an equivalent energy-independent one. One class of three-meson graphs is computed in which the contribution to the energy-independent potential is consistent with the general large-N_c rules even though the energy-dependent potential is not.

I. INTRODUCTION

The nature of the nucleon-nucleon interaction is at the heart of nuclear physics. One traditional picture of nuclear interactions at low energies is that they are mediated via meson exchange. From the QCD perspective one can envision the quarks and gluons organizing themselves into hadrons and then the baryons interact amongst themselves via the exchange of mesons. It is not immediately clear how one can test this picture of how nucleon-nucleon interactions emerge from QCD since we have no *a priori* method for deriving these interactions directly from QCD. In Ref. [3] it was pointed out that large-N_c QCD can provide some insight into the issue. Since the meson-exchange picture of the nucleon-nucleon interaction if valid is justified on rather generic grounds, it should be expected to hold at any N_c greater than unity and hence should hold for large-N_c. However, as noted in Ref. [3] it is by no means obvious that the meson-exchange picture is in fact consistent with large-N_c counting rules. In particular, there is a threat that while a one meson-exchange description yields nucleon-nucleon interactions which are consistent with the large N_c counting rules [4,5], multiple-meson exchange graphs yield interactions which are not. A consistent large-N_c description requires a cancellation of all of these dangerous graphs. It was shown in Ref. [3] that for all two-meson exchange such cancellations do in fact occur, provided that the large-N_c scaling rules for the nucleon-nucleon interaction are interpreted as applying for a nucleon-nucleon potential for use in a Schödinger or Lippmann-Schwinger equation (as opposed to a kernel in a four dimensional Bethe-Salpeter-type equation). This result seems to support the traditional meson-exchange picture for nucleon-nucleon potentials. Clearly, this support would be stronger if the cancellations seen for two-meson exchange also happen for general multiple-meson exchanges.

Unfortunately, in Ref. [6] it was shown that the extension of the techniques of Ref. [3] does not automatically lead to the types of cancellations seen for two-meson exchanges. Thus in the absence of some type of conspiracy leading to such cancellations there appears to be no way to justify the meson-exchange picture from large-N_c QCD. The result is

puzzling—why should such cancellations occur for two-meson exchange for multiple channels and at the same time fail for three-or-higher meson exchange? The purpose of this note is to resolve this puzzle. As will be argued below, there is a reorganization of the analysis that yields precisely the type of conspiracy needed to resolve the puzzle.

To begin with let us consider the background to the problem. Large-N_c QCD [1,2] has proven to be a powerful tool to learn about qualitative and semiquantitative features of hadronic physics. In principle one may hope that it will also provide important insights into nuclear physics. The possible implications of large-N_c QCD for nuclear interactions were already evident in Witten's original paper on baryons in large-N_c QCD. Witten pointed out that (i) the nucleon-nucleon interaction is characteristically of order N_c, (ii) that nucleon-nucleon scattering observables had no smooth limit as $N_c \to \infty$ if the nucleon momenta were of order N_c^0, and (iii) for momenta of order N_c, a relativistic time-dependent Hartree formalism is appropriate and has a smooth large-N_c limit. In practice, no such time-dependent Hartree calculations have been carried out, although recently [7] it was shown what type of observables are calculable in principle in this framework and the spin and isospin dependence of these observables was deduced.

The problem of nucleon-nucleon interactions for momenta of order N_c^0 is of real significance. Kaplan and Manohar [4] following the work of Kaplan and Savage [5] have argued that useful information about the nucleon-nucleon potential can be extracted in this regime. In particular, they suggest that the nucleon-nucleon potential can be associated with quark-line connected diagrams between two color-singlet clusters of N_c quark lines. This is motivated by Witten's Hartree analysis. Combining this with the known contracted SU(4) spin-flavor symmetry of two flavor QCD [8–12] for the coupling to each cluster, this was used to deduce that the large-N_c scaling behavior for the various spin and isospin contributions to the potential. The leading scaling behavior is given by

$$V_{I=J} \sim N_c ; \quad V_{I \neq J} \sim N_c^{-1} , \tag{1}$$

where the subscript indicates the quantum numbers of the exchange in the t channel. It is worth noting that nucleon-nucleon potentials which are fit to scattering data have a pattern which is consistent with Eq. (1) in the sense that the components which are order N_c in Eq. (1) are characteristically significantly larger than those which are of order $1/N_c$ [4,13]. Recently there have also been attempts to study this regime from the perspective of effective field theory [14].

The issue of consistency is studied in the following way. First one supposes that there exists some hadronic field theory whose masses and couplings scale with N_c according to the standard large-N_c rules [15]. It was pointed out in Refs. [3,13] that the large-N_c scaling rules of meson-baryon couplings given by the contracted SU(4) spin-flavor symmetry of two flavor QCD [8–10] yield a one-meson exchange potential and will satisfy Eq. (1). At two-meson exchange the key issue is that both the box graph and the crossed-box graph are formally of order N_c^2 and hence individually cannot be consistent with Eq. (1). It is straightforward to show, however, that the box graph can be decomposed into two parts—a contribution arising from the nucleon poles and a contribution arising from the meson poles when the graph is evaluated via contour integration. The contribution arising from the nucleon poles can be shown to be of exactly the same form as an iterate of the one-meson exchange potential in a Lippmann-Schwinger equation. Thus the nucleon pole contribution to the graph will be picked up when solving the Schrödinger or Lippmann-Schwinger equation and must not be included as part of the potential in order to avoid double counting. The meson-pole contributions to the box graph are retardation effects. The full contribution to the potential from these graphs is the sum of the retardation part of the box graph with the crossed box.

In Ref. [3], it was shown that for all types of two-meson exchange there are cancellations between these two graphs so that the sum is consistent with Eq. (1). These cancellations were highly nontrivial since a number of different spin and isospin structures must all cancel. For example, two-pion exchange, for which both the box and crossed-box contributions are of order N_c^2, contribute to, among other things, the isoscalar central potential, which is of order N_c and requires cancellations of at least relative order $1/N_c$ and to the isovector central potential, which is of order $1/N_c$ and requires cancellations of a least relative order $1/N_c^3$, in order to maintain consistency. By explicitly checking meson exchanges for all relevant spin and isospin couplings it was seen that all of the ''dangerous'' contributions that were inconsistent with Eq. (1) canceled to the degree necessary to ensure consistency. This demonstration required explicit use of the contracted SU(4) algebra which in turn implied that intermediate Δ states have to be kept as explicit degrees of freedom in the potential model to obtain consistency.

The analysis of Ref. [3] clearly helps justify the meson exchange picture of nucleon-nucleon forces. However, no general theorem was proved. Rather all of the relevant cases for two-meson exchange were individually tested. Clearly, one's confidence in the generality of the result would increase if a number of examples of three- and higher-meson exchange show the same type of cancellations seen in two-meson exchange. An analysis of certain multimeson exchange graphs was done in Ref. [6]. The diagrams involved can get quite complicated. Accordingly, it was necessary to establish some bookkeeping rules. The basic method used was in many ways analogous to that used in Ref. [3]: First one identifies all two-baryon irreducible Feynman diagrams as contributing to the potential. Next one considers the two baryon-reducible graphs and notes that such graphs all contain parts that have two baryon propagators between interactions. These propagators are then expressed as the sum of two parts—one where one of the nucleons is on-shell, and the remainder. Next, one makes use of the fact the two baryon propagators with one baryon on-shell is identical to the propagator in the context of a Lippmann-Schwinger equation (up to relativistic corrections that are suppressed by $1/N_c$). Thus these contributions will be included when iterating the Lippmann-Schwinger equation the potential is identified as coming from the two-baryon reducible graphs as the full graph minus the contributions arising from two propagators between interactions with one baryon on shell.

This organization of the full problem into a potential and then its iteration via a Lippmann-Schwinger or Schrödinger equation has a number of virtues. First, provided the problem is nonrelativistic, the scattering amplitude obtained by such a procedure correctly reproduces the sum of all Feynman diagrams for on-shell scattering. Second, this procedure is well suited to the study of ladder and crossed-ladder graphs including various noncommuting couplings since the non-Abelian generalization of the eikonal formula of Refs. [16,17] can be implemented for the sum of these graphs. The non-Abelian generalization of the eikonal formula expresses the sum of all meson and crossed-meson lines entering a single baryon in terms of commutators multiplying δ function which correspond to an on-shell baryon. In the context of summing ladder and crossed-ladder contributions to baryon-baryon scattering it is straightforward to see which of these on-shell contributions correspond to iterates of the Lippmann-Schwinger equation.

When this organizing principle was implemented for multimeson exchanges, however, it was found that the cancellations needed for the potentials to maintain consistency with Eq. (1) did not occur. This was seen for two classes of graphs that were considered. One class was the sum of ladders and crossed ladders with noncommutating couplings. The most dangerous type was where the meson coupled to baryon in a vector-isovector manner. The issue there was the emergence of contributions in which one nucleon was on-shell but which were not iterates of the potential as defined above. The generalized eikonal formula [16,17] implies that these yield contributions to the potential given in terms of commutators. Although the commutators typically yield a $1/N_c^2$ suppression and may be expected to induce a commutator on the other nucleon leg for another factor of $1/N_c^2$ if the ladder has six rungs or higher, the explicit $N_c^{1/2}$ associated with each meson-nucleon vertex overwhelms the suppression and an inconsistency with Eq. (1) is the result. A simpler case where an inconsistency can be seen is the case of three scalar-isoscalar meson exchange between nucleons where two of the mesons couple to one of the nucleons in a seagull-type vertex as
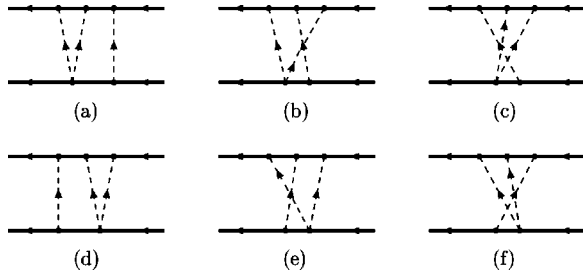
FIG. 1. Three scalar-isoscalar meson exchange diagrams with one seagull-type coupling.

shown in Fig. 1. Again when the part representing an iterate of the potential using the organizing principle discussed above is removed, the remaining contribution which contributes to the potential itself does not cancel and is of order N_c^2, in violation of Eq. (1). This inconsistency is the large-N_c potential puzzle.

In the remainder of this paper a resolution to the puzzle will be discussed. In the following section, it will be suggested that the heart of the problem lies in the organizing principle discussed above which has the feature that the potential obtained have explicit energy dependence. Following this a toy problem will presented to show how energy dependence can alter the N_c counting of a potential. Next a new organizing principle is suggested which yields energy independent potentials. In the final section it is shown explicitly that this new procedure, when applied to the case of three scalar-isoscalar meson exchange where two of the mesons couple to one of the nucleons in a seagull-type vertex (of the type in Fig. 1), yields a potential consistent with Eq. (1).

II. ENERGY DEPENDENCE

In Ref. [6] various possible resolutions to the puzzle were suggested. One possibility was that necessary cancellations might happen naturally and generically (without any special conspiracies involving coupling constants of different mesons) with a different organization of the problem. However, no plausible reorganization was suggested. The purpose of the present paper is to argue that this is the correct resolution of the problem and to provide the needed reorganization. It will then be shown explicitly that with this reorganization, the contributions to the potential of the three-meson exchange graphs of Fig. 1 indeed give rise to the cancellations needed for the consistency with Eq. (1).

The key to this reorganization is the realization that the separation of the contributions into a potential and its iterates in a Lippmann-Schwinger equation is not unique if one allows energy-dependent potentials: there are an infinite number of ways to distribute the energy dependence between explicit energy dependence of the potential and energy dependence arising from iteration which yield identical scattering amplitudes. Of course, one often thinks of potentials as being energy independent. However, energy-dependent potentials arise naturally when one suppresses explicit inclusion of degrees of freedom and in the present problem the final nucleon-nucleon potential suppresses explicit meson

degrees of freedom that are in the underlying problem. The procedure used in Ref. [6] described above to isolate the potential from its iterates produces such an energy-dependent potential. Of course, at a fundamental level, there is nothing wrong with energy-dependent potentials for use in a Schrödinger equation provided they correctly predict the physical scattering amplitudes, and the procedure used in Ref. [6] should reproduce the scattering amplitude. The notion that the elimination of energy dependence to establish consistency may remind one of the situation in NR QCD (nonrelativistic QCD) where field redifntions are used [18].

Since the correct physical amplitude is obtained, the fact that the procedure used in Ref. [6] produces an energy-dependent potential might seem completely innocuous. However, the key point is that the large-N_c scaling rules in Eq. (1) is *not* for a physical quantity which can be directly measured; rather it is for the potential. Moreover, it is possible for an energy-dependent potential to have radically different N_c scaling behavior from an energy-independent potential which is physically equivalent.

III. A TOY PROBLEM

To see how energy dependence can alter the N_c scaling consider the following simple example: Begin with a simple energy-dependent potential operator,

$$\tilde{V}(E) = \tilde{V}_0 + \tilde{V}_1(E - \hat{p}^2/M_N), \tag{2}$$

where the tilde is used to distinguish the potential from an equivalent energy-independent one; $\tilde{V}_0$ and $\tilde{V}_1$ have no energy dependence (so that all energy dependence is explicitly given), $\hat{p}$ denotes the relative momentum operator, and $M_N/2$ is the reduced mass. The potential is not explicitly Hermitian but this is not significant for the present purpose, which is merely illustrative. This non-Hermitian form is used since it is the simplest model which demonstrates the key point. The scattering amplitude T is obtained via the Lippmann-Schwinger equation, which as an operator equation can be represented as

$$T = V + V G_{LS} T = V + V G_{LS} V + V G_{LS} V G_{LS} V + \cdots$$

$$\text{with } G_{LS} = (E - \hat{p}^2/M_N + i\epsilon)^{-1}. \tag{3}$$

Now suppose that we have another potential V which is energy independent and has the same on-shell scattering amplitude as in Eq. (2). By comparing the iterates of Eq. (3) for the two potentials, it is straightforward to see that

$$V = \tilde{V}_0 + \tilde{V}_1\tilde{V}_0 + \tilde{V}_1\tilde{V}_1\tilde{V}_0 + \cdots \sum_{j=0,\infty} \tilde{V}_1^j \tilde{V}_0. \tag{4}$$

The products of the $\tilde{V}_1$ and $\tilde{V}_0$ emerge in V from iterates of the Lippmann-Schwinger equation for $\tilde{V}$; the $(E - \hat{p}^2/M_N)$ factors accompanying $\tilde{V}_1$ when multiplied by G_{LS} give unity and thus correspond to an uniterated term in the Lippmann-Schwinger equation for V. Also, the $(E - \hat{p}^2/M_N)$ operators annihilates the on-shell external state and thereby eliminates

many terms in the sum. Thus we see explicitly that energy dependence reshuffles what goes into the potential and what is obtained via iteration. For the present context this is significant in terms of the N_c behavior. From the form of Eq. (4), it is easy to see that the energy-dependent potential $\tilde{V}$ and the energy-independent potential V cannot both generically be of order N_c. If $\tilde{V}$ is generically of order N_c (i.e., both $\tilde{V}_0$ and $\tilde{V}_1$ are order N_c), then we see that V contains all powers of N_c. Conversely, if V and $\tilde{V}_1$ are each of order N_c, then $\tilde{V}_0$ contains all powers of N_c.

IV. A NEW ORGANIZING PRINCIPLE

We see from the preceding example that energy-dependent potentials need not have the same N_c dependence as energy-independent ones. The large-N_c scaling rules of Eq. (1) were derived by associating the quark-line connected diagrams with "the potential." This association is somewhat heuristic and it is not immediately obvious whether it is supposed to apply for energy-dependent or energy-independent potentials. It is a reasonable hypothesis, however, that it applies to energy-independent potentials. Thus it is plausible that the failure of the analysis of Ref. [6] to reproduce the N_c scaling rules for Eq. (1) is because the separation into a potential and its iterates used in the analysis does not ensure that the potential is energy independent. To see why the potential so derived can depend on energy, consider the algorithm used in the analysis to isolate the potentials. One calculates Feynman diagrams and subtracts off all contributions coming from places where one nucleon out of a pair that separates interacting subdiagrams is on-shell. This removes terms that look like Lippmann-Schwinger equation iterates. The issue is simply that the remaining subdiagrams may, themselves, be energy dependent.

To test whether energy-independent potentials from multiple meson exchange are consistent with the rules of Eq. (1), we must first extract the contributions from energy-independent potentials from various meson exchanges graphs. The basic algorithm is similar to the one used in Ref. [6] with one modification. The potential contributions from two-particle irreducible diagrams will be taken as the static limit (i.e., zero energy limit) of the diagram. For two-particle reducible graphs, one subtracts off all contributions coming from places where one nucleon out of a pair that separates interacting subdiagrams is on-shell, and where the remaining subdiagrams are replaced by their static limits.

It is worth noting at this point that at the level of two-meson exchange, the organizing principle of Ref. [6] and the present one both give results consistent with Eq. (1). The reason for this is quite simple. The two only differ in the treatment of the energy dependence of subdiagrams. In the case of two-meson exchange, the subdiagrams are single meson exchanges which do have nontrivial energy dependence due to the poles in the meson propagators. However, when the external lines are on-shell, the two methods are identical; both methods have the same cancellation between retardation effects in the box diagram and the full crossed box. Of course, one can distinguish between the two-meson ex-

change in the two methods if one or the other external line goes off-shell. However, external line can go off-shell only inside larger graphs and the lowest order one can distinguish between the two approaches is at the level of three-meson exchange.

V. TESTING THE N_c SCALING OF ENERGY-INDEPENDENT POTENTIALS

The modification suggested in the previous section greatly complicates the analysis. For involved cases, such as ladders and crossed ladders with many rungs, it may require a considerable effort to test the consistency of Eq. (1) since the non-Abelian generalization of the eikonal formula can no longer be implemented in a straightforward way. However, the case of three scalar-isoscalar meson exchange between nucleons where two of the mesons couple to one of the nucleons in a seagull-type vertex represented in Fig. 1 is tractable. The diagrams in Fig. 1 include all contributions which has the seagull attached to one of the nucleon lines. There is an identical contribution which have the seagull attached to the other line. It is useful to group the diagrams into two sets, (a), (b), and (c) as one group and (e), (f), and (g) as the other. The cancellations needed to get consistency with Eq. (1) can be shown to occur separately in these groups. Now let us consider the contribution from the first three diagrams. The amplitude is given by

$$
i\mathcal{A}_{a,b,c} = g_{1m}^4 g_{2m} \int \prod_{j=1}^{3} \frac{d^3 \mathbf{k}_j}{(2\pi)^3} (2\pi)^3 \delta^{(3)}\left(\sum_{j=1}^{3} \mathbf{k}_j - \mathbf{q} \right)
$$
$$
\times \int \prod_{j=1}^{3} \frac{d\omega_j}{2\pi} (2\pi)\delta\left(\sum_{j=1}^{3} \omega_j - q_0 \right)\left(\prod_{j=1}^{3} D(k_j^2) \right)
$$
$$
\times G(p-k_1)[G(\tilde{p}+k_1)G(\tilde{p}+k_1+k_2)+G(\tilde{p}
$$
$$
+k_2)G(\tilde{p}+k_1+k_2)+G(\tilde{p}+k_2)G(\tilde{p}+k_2+k_3)],
$$
(5)

where boldfaced indicates three vectors, and nonboldfaced four vectors, with $k_j = (\omega_j, \mathbf{k}_j)$; g_{1m} and g_{2m} are the one-meson and two-meson coupling constants; the initial four-momenta of the two nucleons are p and $\tilde{p}$; the four-momentum transfer is q; $D(k) \equiv (k^2 - m_m^2 + i\epsilon)^{-1}$ is the meson propagator; and the fermion propagator is denoted by G. For simplicity we will work in the center of mass frame with $p = (\mathbf{p}^2/(2M_n), \mathbf{p})$ and $\tilde{p} = (\mathbf{p}^2/(2M_n), -\mathbf{p})$. In fact, the graph as written is ultraviolet divergent. We will assume that it is regulated by some short distance physics which acts to cut off the momentum integrals. The details of how this is done is irrelevant for what follows, provided the same cutoff procedure is used for all of the graphs.

Let us note the N_c dependence of the various inputs to this expression: $g_{1m} \sim N_c^{1/2}$, $g_{2m} \sim N_c^0$ and $M_N \sim N_c$; the meson mass in the propagators D is of order N_c^0. Thus there is an overall prefactor of N_c^2 coming from the coupling constants; it is this factor which must somehow be canceled up to relative order N_c^{-1} in order to get consistency with Eq. (1).

The kinematic regime of interest is intrinsically nonrelativistic since $\boldsymbol{p} \sim N_c^0$ while $M_N \sim N_c^1$. In this regime it is legitimate to replace the full fermion propagator by

$$G(p) = \frac{1}{p_0 - \boldsymbol{p}^2/(2M_N) + i\epsilon} \frac{1+\gamma_0}{2} [1 + \mathcal{O}(1/N_c)], \quad (6)$$

where the $1/N_c$ corrections come from the nonrelativistic reduction. The propagator contains a recoil correction. This will be a $1/N_c$ correction everywhere except in the vicinity of the propagator's pole. This pole correction is relevant only for the piece which looks like a Lippmann-Schwinger equation iterate and hence we will drop the recoil correction everywhere except for this one contribution. These recoilless propagators depend only on the energy and are given by $(p_0 + i\epsilon)^{-1}$.

A tedious calculation yields

$$i\mathcal{A}_{a,b,c} = g_{1m}^4 g_{2m} \int \prod_{j=1}^{3} \frac{d^3\boldsymbol{k}_j}{(2\pi)^3} (2\pi)^3 \delta^{(3)}\left(\sum_{j=1}^{3} \boldsymbol{k}_j - \boldsymbol{q} \right)$$

$$\times \int \prod_{j=1}^{3} \frac{d\omega_j}{2\pi} (2\pi) \delta\left(\sum_{j=1}^{3} \omega_j \right)$$

$$\times D(k_1) [\delta(p_0 - \omega_1 + (\boldsymbol{p}+\boldsymbol{k}_1)^2/(2M_N)) G(\tilde{p}+k_1)]$$

$$\times D(k_2) D(k_3) \frac{1}{\omega_1 + \omega_2 + i\epsilon} + \mathcal{O}(N_c), \quad (7)$$

where the $\mathcal{O}(N_c)$ corrections come from using nonrelativistic propagators and neglecting recoil. The combination in square brackets can easily be seen to be the delta function fixing ω_1 times the Lippmann-Schwinger propagator, $G_{LS}(\boldsymbol{p},E) = 1/(E - \boldsymbol{p}^2/M_N + i\epsilon)$.

Note that the delta function restricts ω_1 to being of order N_c^{-1} for these kinematics. The expression in curly braces is sensitive to the infrared kinematics and depends on the fact that $\omega_1 \neq 0$. However, for all parts of the expression, setting ω_1 to zero will only induce an error of relative size $(1/N_c)$. Substituting zero for ω_1 in all parts of Eq. (7) except for the factor in square brackets gives

$$i\mathcal{A}_{a,b,c} = \int \frac{d^3\boldsymbol{k}_1}{(2\pi)^3} V_1(\boldsymbol{k}_1) G_{LS}(\boldsymbol{k}_1,E) V_2(\boldsymbol{q}-\boldsymbol{k}_1) + \mathcal{O}(N_c),$$

$$V_1(\boldsymbol{k}) = \frac{g_{1m}^2}{\boldsymbol{k}^2 + m_m^2},$$

$$V_2(\boldsymbol{k}) = g_{1m}^2 g_{2m} \int \prod_{j=1}^{3} \frac{d^3\boldsymbol{k}_j}{(2\pi)^3} \int \prod_{j=2}^{3} \frac{d\omega_j}{2\pi} (2\pi)^3 \delta^{(3)}\left(\sum_{j=2}^{3} \boldsymbol{k}_j \right.$$

$$\left. - \boldsymbol{k} \right) (2\pi) \delta\left(\sum_{j=2}^{3} \omega_j \right) D(k_2) D(k_3) \frac{1}{\omega_2 + i\epsilon}. \quad (8)$$

Note that V_1 is a static one-meson exchange potential while V_2 is the static potential for the exchange of two mesons with one seagull vertex. Thus the form of Eq. (8) is precisely part of an iterate of the Lippmann-Schwinger equation, with V_1 as the first iteration and V_2 the second. The other ordering comes from the graphs (e), (f), and (g). Since the part of $i\mathcal{A}_{a,b,c}$ which is of order N_c^2 is a static potential iterate, one concludes the contributions of these graphs to the potential is necessarily of order N_c or less and hence is consistent with the N_c scaling rules of Eq. (1).

Note that had one followed the organization of Ref. [6] there would have been a remaining contribution to the potential of order N_c^2. From this we see an explicit example where the energy-dependent potentials extracted from a set of Feynman graphs associated with multiple-meson exchange using the algorithm of Ref. [6] are inconsistent with the large-N_c counting rules of Refs. [5,4] while energy-independent ones are consistent. Thus we see the large-N_c nuclear potential puzzle has been resolved for this set of diagrams. Although we have not explicitly calculated other multiple-meson exchange graphs such as the sum of ladders and crossed-ladders considered in Ref. [6] due to their complexity, it is highly plausible that consistency would be obtained for the energy-dependent potentials for such cases.

In summary, it has been shown that energy-dependent potentials can have different N_c scaling behavior than energy-independent potentials. It has been argued that the N_c scaling rules of Eq. (1) should apply only to energy-independent potentials. With these energy-dependent potentials one expects consistency. From this perspective, the inconsistency of the three- and higher-meson exchange potentials in Ref. [6] with Eq. (1) can be understood as arising from the fact that the analysis used in that work yielded energy-dependent potentials. This interpretation is highly plausible given the explicit demonstration of consistency for the case of the energy-independent potential associated with three-scalar-meson exchange of the type seen in Fig. 1 despite the fact that the energy-dependent potential was obtained using the methods of Ref. [6]

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