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Expanding the nonlinear coupling constant in the derivative coupling model and the analysis of the effective nucleon mass

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In light of the derivative coupling model, we make an expansion of the nonlinear coupling constant, and calculate its effective nucleon mass at different orders. The varying characteristic of the effective nucleon mass at these orders is discussed and its physical meaning is interpreted. We find that the mechanism of phase transition at high temperature in the Walecka model is suppressed in the derivative coupling model.

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In recent years, the properties of hadronic matter at finite temperature have remained an active area of investigation [1–3]. The quantum hadrodynamics model (QHD), proposed by Walecka, has proved to be a successful model in this area [4]. In this model, the interaction between nucleons is mediated by the exchange of σ and ω mesons. The σ meson is simulated as a long-range attraction, while the ω meson is simulated as a short-range repulsion. This simple model accurately describes the properties of nuclear matter and finite nuclei. For example, it gives the binding energy and saturation density of nuclear matter, and it explains the noncentral spin-orbit splitting of finite nuclei in a more natural way. It also predicts that there is a liquid-gas phase transition of nuclear matter at low temperature ($T<20$ MeV). However, this model has its shortcomings. One of them is that the effective nucleon mass in nuclear matter at moderately high density and/or temperature becomes very small, or even negative if Δ particles are included [5]. In order to avoid this problem, Zimanyi and Moszkowski have proposed the derivative coupling model (also called the ZM model) [6]. In this model, they have introduced a nonlinear effective scalar coupling constant between nucleon and σ meson. This modification has corrected the defect of effective nucleon mass in the Walecka model. Recently, variations of the ZM models have already been applied to investigate many physical problems, such as, multilambda matter properties [7], neutron star [8], Δ excited nuclear matter [9] as well as some thermodynamical properties of nuclear matter [1,10].

In the study of nuclear matter at finite temperature, an important problem is how the effective nucleon mass changes in medium. By applying the ZM model, many authors have discussed the temperature and density dependence of the effective nucleon mass [1,11]. Although the ZM model has improved the effective nucleon mass, it is still not clear how the nonlinear coupling constant changes the effective nucleon mass. The main purpose of this Rapid Communication is to discuss the mechanism of how the effective nucleon mass has been modified by expanding the nonlinear coupling constant and study them order by order.

Since the ZM model has been discussed in detail in past literature [1,6,11], here we just write down its Lagrangian [6],

$$\mathcal{L}=\bar{\psi}i\gamma_\mu\partial^\mu\psi-\bar{\psi}(M-m^*g_\sigma\sigma)\psi-g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu+\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma-m_\sigma^2\sigma^2), \tag{1}$$

in which

$$m^*=\left[1+\frac{g_\sigma\sigma}{M}\right]^{-1}; \tag{2}$$

ψ, σ, and ω are the fields of nucleon, σ, and ω mesons, respectively. M is the nucleon mass and $F_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu$. After the mean field approximation, $\sigma\rightarrow\bar{\sigma}$, $\omega_\mu\rightarrow\delta_{\mu0}\bar{\omega}_0$, and through finite temperature field theory, we could derive the partition function Z of the system. Considering the thermodynamic relation $p=T\ln Z$, we obtain the pressure

$$p=\frac{4T}{(2\pi)^3}\int d^3k[\ln(1+e^{-\beta(E^*-\mu^*)})+\ln(1+e^{-\beta(E^*+\mu^*)})] +\frac{1}{2}m_\omega^2\bar{\omega}_0^2-\frac{1}{2}m_\sigma^2\bar{\sigma}^2, \tag{3}$$

in which $E^*=\sqrt{k^2+M^{*2}}$, and

$$M^*=M-m^*g_\sigma\bar{\sigma}, \tag{4}$$

$$\mu^*=\mu-g_\omega\bar{\omega}_0. \tag{5}$$

T is the temperature, μ is the chemical potential. Through the use of Gibbs's relation for thermodynamic equilibrium, one can get the self-consistency equations of $\bar{\sigma}$ and $\bar{\omega}_0$ by maximizing p with respect to $\bar{\sigma}$ and $\bar{\omega}_0$, respectively [12]. So we can get

$$\frac{\partial M^*}{\partial\bar{\sigma}}\rho+m_\sigma^2\bar{\sigma}^2=0, \tag{6}$$

$$\bar{\omega}_0=\frac{g_\omega}{m_\omega^2}\rho_B, \tag{7}$$

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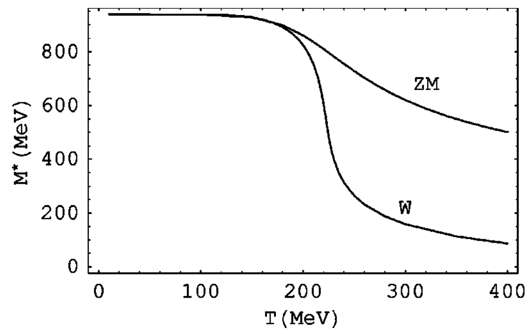

FIG. 1. The effective nucleon mass as a function of *T* for ZM and Walecka models at $\mu=0$.

in which ρ_B is the net baryon density; ρ is the scalar density which is related to the total baryon density,

$$\rho=\frac{4}{(2\pi)^3}\int d^3k\frac{M^*}{E^*}(n_k+\overline{n}_k), \tag{8}$$

$$\rho_B=\frac{4}{(2\pi)^3}\int d^3k(n_k-\overline{n}_k), \tag{9}$$

where n_k and $\overline{n}_k$ stand for the Fermi-Dirac distribution for baryons and antibaryons, respectively. These equations could be solved self-consistently to determine the effective nucleon mass M^*. As is known, $m^*=(1+g_\sigma\overline{\sigma}/M)^{-1}$, here for small $g_\sigma\overline{\sigma}/M$, we could expand m^* in power series. Then from Eq. (4) we can get

$$M^*=M-g_\sigma\overline{\sigma}+\frac{(g_\sigma\overline{\sigma})^2}{M}-\frac{1}{2!}\frac{(g_\sigma\overline{\sigma})^3}{M^2}+\cdots. \tag{10}$$

So Eqs. (6) and (7) could be solved up to different orders of $\overline{\sigma}$ at certain *T* and μ. Here we only discuss the temperature dependence of M^* under the case of $\rho_B=0$. So only Eq. (6) need be solved at given *T*. For coupling constant, we set $g_\sigma=7.62$ and $g_\omega=6.41$ [1]. Then we could plot the curves of M^* versus *T* at different orders.

It is obvious that, when M^* is expanded to the first order, the case corresponds to the Walecka model, while at the infinite order it corresponds to the ZM model. So the ZM model is preferable to the Walecka model mainly because it accommodates for the higher orders of $\overline{\sigma}$. In Fig.1, in the Walecka model, M^* suddenly drops to a very low value at moderately high temperature, while in the ZM model, M^* declines in a smooth way and will not drop very low [1]. When M^* is expanded to the second order, M^* falls monotonically and smoothly with temperature as shown in Fig. 2. While at moderately high temperature, it goes up and separates with the curve of the ZM model. When it is expanded to the third order, again, M^* will drop to a very low value, but in a nonmonotonical way. When M^* is expanded to higher orders, we find that the varying characteristic of M^* will switch between these two varying types. That is to say, when M^* is expanded to the even order, M^* will drop slowly, and will approach the curve of the ZM model at the infinite order; when expanded to the odd order, M^* will drop to a very low value in a nonmonotonical way, however, at the infinite order it will also approach the curve of the ZM model. At the same time, the nonmonotonically falling part disappears. This is shown in Fig. 3.

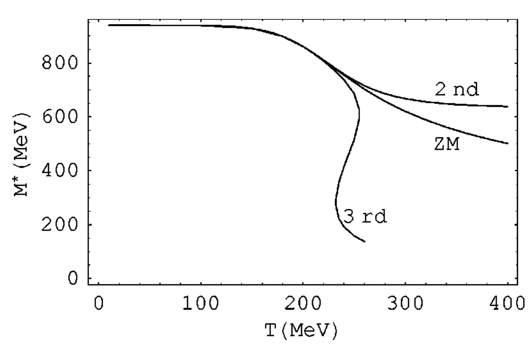

FIG. 2. The effective nucleon mass as a function of *T* at $\mu=0$. "2nd" stands for expanding to the second order; "3rd" means expanding to the third order. The curve between them corresponds to the ZM model.

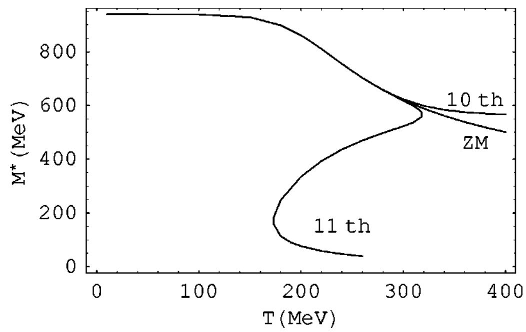

FIG. 3. The effective nucleon mass as a function of *T* at $\mu=0$. "10th" stands for expanding to the tenth order; "11th" means expanding to the eleventh order. The curve between them corresponds to the ZM model.

Now we are in a position to discuss the related physics. When M^* is expanded to the first order, from Eq. (10), $\overline{\sigma}$ will increase as temperature, increases while M^* decreases. At certain high temperatures, $\overline{\sigma}$ gets large very quickly and leads M^* suddenly drop to a very low value. When M^* is expanded to the second order, it will decrease smoothly. This means the second order of $\overline{\sigma}$ plays the role which prevents $\overline{\sigma}$ increase so quickly at certain high temperatures. So the effect of the second order is opposite to that of the first order. And the third order will be opposite to the second order, and so on. Now let us take a look at the interaction Lagrangian of σ meson and nucleon,

$$\mathcal{L}_{int}=g_\sigma\sigma\overline{\psi}\psi-\frac{(g_\sigma\sigma)^2}{M}\overline{\psi}\psi+\frac{1}{2!}\frac{(g_\sigma\sigma)^3}{M^2}\overline{\psi}\psi-\cdots. \tag{11}$$

If we only consider the first order, it is a Yukawa coupling and results in an attractive force between nucleons. As mentioned above, the effect of the second order is opposite to that of the first order. It shows a repulsive force. Again the

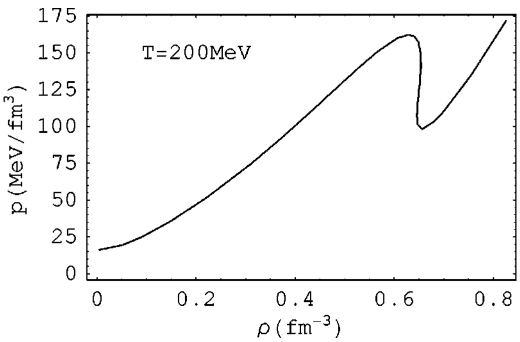

FIG. 4. The pressure p as a function of the scalar density ρ at $T = 200$ MeV when expanding to the third order.

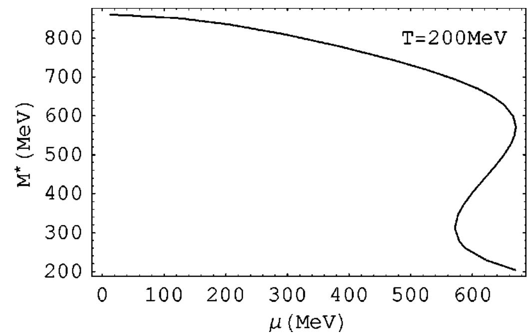

FIG. 5. The effective nucleon mass as a function of μ at $T = 200$ MeV when expanding to the third order.

third order gives the attractive force, and so on. From the view of the interaction between nucleons through σ field, we can see that the two different characteristics of M^* are the result of two forces, attraction and repulsion, competing with each other. If the attraction dominates, the effective mass will drop very low. If the repulsion dominates, the effective mass will be lifted up and drop slowly. When it goes to the infinite order, the final result corresponds to the ZM model.

It is important to note that when M^* is expanded to the odd order, M^* always drops in a nonmonotonical way. As is known from the Walecka model, at high temperature, there is also a first order phase transition from high density to low density [3], and it has also been shown that at the two phases coexistent area, the self-consistence equation of effective nucleon mass has multiple solutions which correspond to the effective nucleon mass at high density phase and low density phase, respectively. Here the nonmonotonical decline of M^* suggests there is a phase transition at high temperature. But this nonmonotonically falling part is greatly pushed downward at high orders and finally disappears in the ZM model. This fact can explain why there is no phase transition in the ZM model at high temperature. It is because this phase transition mechanism has been suppressed and is not manifested. In order to make it clear, we pay attention to Fig. 2. When M^* is expanded to the third order of $\overline{\sigma}$, it is shown that, at high temperature, M^* drops nonmonotonically with temperature. This means there is a phase transition. Let $T = 200$ MeV, then solving Eqs. (6) and (7) self-consistently and together with the Eq. (3), we could plot the isotherm of pressure versus the scalar density and the effective nucleon mass as a function of chemical potential as shown in Figs. 4 and 5. Here we can see this phase transition at high temperature is analogous to that in the Walecka model. As in higher orders, this phase transition will be suppressed. Finally, when it approaches the ZM model, it disappears.

In summary, in this Rapid Communication, through expanding the nonlinear coupling constant in the ZM model, we discuss the characteristics of the effective nucleon mass with variations in temperature. We find there are two effects, attraction and repulsion, in the expanding terms, and their competing results in two varying characteristics of M^*. The final result is the case of the ZM model. From our discussion, it is also seen that the mechanism of phase transition at high temperature that occurs in the Walecka model is completely suppressed in the ZM model. There is no phase transition in the ZM model at high temperature.

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