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Coulomb-nuclear interference in the breakup of ^{11}Be

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Within a theory of breakup reactions formulated in the framework of the post-form distorted wave Born approximation, we calculate contributions of the pure Coulomb and the pure nuclear breakups as well as those of their interference terms, to a variety of cross sections in breakup reactions of the one-neutron halo nucleus ^{11}Be on a number of target nuclei. In contrast to the assumption often made, the Coulomb-nuclear interference terms are found to be non-negligible in case of exclusive cross sections of the fragments emitted in this reaction on medium mass and heavy target nuclei. The consideration of the nuclear breakup leads to a better description of such data.

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Projectile breakup reactions have played a major role in probing the structure of neutron rich light radioactive nuclei [1]. Features of the breakup data such as strongly forward peaked angular distributions and extremely narrow parallel momentum distributions of the fragments [1–3] have contributed in a major way in confirming the existence of a novel structure, called neutron halo [4], in some of these nuclei. The data on breakup studies of radioactive nuclei have been increasing rapidly both in quality and quantity [1–3,5]. In majority of them both the Coulomb and nuclear breakup effects as well as their interference terms are likely to be significant. However, in many analyses of the experimental data on halo breakup reactions the latter term has not been included [6–9].

Therefore, a theoretical treatment of breakup reactions of radioactive nuclei, where Coulomb, nuclear, and their interference terms are treated consistently on an equal footing, is an important requirement in efforts to extract the information about the structure of light exotic nuclei from the experimental data. For breakup reactions of light stable isotopes, such a theory has been formulated within the post-form distorted wave Born approximation (DWBA) [10] where this reaction is treated as a direct process in which the incoming projectile breaks up instantaneously in the nuclear and Coulomb fields of the target. Even though this theory has been remarkably successful in describing the light ion breakup data [10], its application to calculations of breakup of heavier projectiles and at higher beam energies is not reliable as it uses the simplifying approximation of a zero-range (ZR) interaction (see, e.g., Ref. [11]) between constituents of the projectile. The ZR approximation is inapplicable to cases where the internal orbital angular momentum of the projectile is different from zero. Recently, an extended version of this theory, where the ZR approximation is avoided, has been used to investigate the pure Coulomb breakup of one- and two-neutron halo nuclei [12,13].

Pure Coulomb and pure nuclear breakups of halo nuclei have been studied in several different approaches [3,14–16]. On the other hand, in Ref. [17], the pure nuclear and the pure Coulomb breakup as well as their interference terms have been treated on the same footing in a study of ^{8}B breakup within a reaction model that describes breakup as an excitation of the projectile to a two-body continuum state. The corresponding T matrix is written in terms of the prior-form DWBA where interactions between the fragments and the target are treated in first order. With this approximation, the prior-form DWBA is no longer equivalent to its post-form counterpart [10]. The prior DWBA can be regarded as the first iteration of the solutions of a coupled channels problem (e.g., the coupled discretized continuum channels equations). In breakup studies of both the stable isotopes [18] and halo nuclei [19,20], it is shown that the prior DWBA is insufficient to describe the data; higher-order coupling effects of the breakup channels are found to be important in both the cases. For example, the prior DWBA results for ^{8}B breakup at low beam energies, as shown in Ref. [17], are changed completely by the higher-order effects [19]. For the higher beam energy ($\approx$50 MeV/nucleon) case studied in Ref. [17], it is expected [21] that coupled channels effects would be noticeable for angles beyond 5°, while in the region below this they may be relatively weaker.

Contributions of the Coulomb and nuclear breakups as well as those of their interference terms have also been calculated within models [22,23] where the time evolution of the projectile in coordinate space is described by solving the time dependent Schrödinger equation, treating the projectile-target (both Coulomb and nuclear) interaction as a time dependent external perturbation. These calculations use the semiclassical concept of the motion of the projectile along a trajectory. While in Ref. [22] no perturbative approximation has been made in calculations of the breakup cross section, the Coulomb breakup amplitudes have been calculated in the first order perturbation theory in Ref. [23].

In this paper, we present calculations for the breakup of the one-neutron halo nucleus ^{11}Be within the post-form DWBA theory of the breakup reactions that includes consistently both Coulomb and nuclear interactions between the projectile fragments and the target nucleus to all orders, but treats the fragment-fragment interaction in first order. This is an extension of the theory presented in Ref. [12] which was able to describe only the pure Coulomb breakup of such nuclei. As in Ref. [12], finite range effects are included within the local momentum approximation (LMA) [24]. The full ground state wave function of the projectile of any orbital angular momentum structure enters into this theory. It can treat the Coulomb and nuclear breakups as well as their interference terms consistently within a single setup. Since this theory uses the post-form scattering amplitude, the breakup contributions from the entire continuum corresponding to all the multipoles and the relative orbital angular momenta between the valence nucleon and the core fragment are included in it. Furthermore, it can account for the post-acceleration effects in a unique way [25]. Within this theory, we investigate here the role of the nuclear and the Coulomb-nuclear interference (CNI) terms in breakup reactions of the halo nucleus ^{11}Be.

We consider the elastic breakup reaction $a+t\rightarrow b+c+t$, in which the projectile a ($a=b+c$) breaks up into fragments b and c in the Coulomb and nuclear fields of a target t. Unlike the assumption made in Ref. [12], both fragments can be charged. The triple differential cross section for this reaction is given by

$$\frac{d^3\sigma}{dE_b d\Omega_b d\Omega_c}=\frac{2\pi}{\hbar v_a}\rho(E_b,\Omega_b,\Omega_c)\sum_{\ell m}|\beta_{\ell m}|^2, \tag{1}$$

where $\rho(E_b,\Omega_b,\Omega_c)$ is the appropriate three-body phase space factor (e.g., see Ref. [12]), v_a is the velocity of a, and ℓ is the orbital angular momentum for the relative motion of b and c in the ground state of a. The amplitude $\beta_{\ell m}$ is defined as

$$\begin{aligned}\hat{\ell}\beta_{\ell m}(\mathbf{k}_b,\mathbf{k}_c;\mathbf{k}_a) &= \int d\mathbf{r}_1 d\mathbf{r}_i \chi_b^{(-)*}(\mathbf{k}_b,\mathbf{r})\chi_c^{(-)*}(\mathbf{k}_c,\mathbf{r}_c)V_{bc}(\mathbf{r}_1)\\ &\quad\times u_\ell(r_1)Y_{\ell m}(\hat{r}_1)\chi_a^{(+)}(\mathbf{k}_a,\mathbf{r}_i),\end{aligned} \tag{2}$$

with $\hat{l}\equiv\sqrt{2\ell+1}$. In Eq. (2), functions χ_i represent the distorted waves for the relative motions of various particles in their respective channels with appropriate boundary conditions. Arguments of these functions contain the corresponding Jacobi momenta and coordinates. $V_{bc}(\mathbf{r}_1)$ represents the interaction between b and c, and $u_\ell(r_1)$ represents the radial part of the corresponding wave function in the ground state of a. The position vectors satisfy the relations: $\mathbf{r}=\mathbf{r}_i-\alpha\mathbf{r}_1$, $\mathbf{r}_c=\gamma\mathbf{r}_1+\delta\mathbf{r}_i$, with $\alpha=(m_c/m_a)$, $\delta=[m_t/(m_b+m_t)]$, and $\gamma=(1-\alpha\delta)$. It may be noted that Eq. (1) uses full three-body kinematics and it can readily be used to analyze the coincidence breakup data (see, e.g., Ref. [26]) of halo nuclei, which are now becoming available with the advent of the secondary beams of sufficiently high intensity.

To facilitate an easier computation of Eq. (2), which involves a six-dimensional integral with the integrand having a product of three scattering waves that exhibit an oscillatory behavior asymptotically, we perform a Taylor series expansion of the distorted waves of particles b and c about $\mathbf{r}_i$ and write

$$\chi_b^{(-)}(\mathbf{k}_b,\mathbf{r})=e^{-i\alpha\mathbf{K}_b\cdot\mathbf{r}_1}\chi_b^{(-)}(\mathbf{k}_b,\mathbf{r}_i), \tag{3}$$

$$\chi_c^{(-)}(\mathbf{k}_c,\mathbf{r}_c)=e^{i\gamma\mathbf{K}_c\cdot\mathbf{r}_1}\chi_c^{(-)}(\mathbf{k}_c,\delta\mathbf{r}_i). \tag{4}$$

We now employ the LMA [27,24], the attractive feature of which is that it leads to the factorization of Eq. (2) into two terms, each involving a three-dimensional integral. In the LMA, the magnitudes of momenta $\mathbf{K}_j$ are taken as $K_j(R)=\sqrt{(2m_j/\hbar^2)[E_j-V_j(R)]}$, where m_j is the reduced mass of the j-t system, E_j is the energy of particle j relative to the target in the c.m. system, and $V_j(R)$ is the potential between j and t at a distance R. As is shown in Ref. [12], the magnitude of $K(R)$ remains constant for $R>10$ fm for the reaction under investigation in this paper. Due to the peripheral nature of breakup reactions, this region contributes maximally to the cross section. Therefore, we have taken a constant magnitude for K_j evaluated at $R=10$ fm for all the values of the associated radial variable. Furthermore, we checked that the results of the calculations are almost independent of the choice of the direction of the local momentum. Hence, we take the direction of $\mathbf{K}_j$ to be the same as that of the asymptotic momentum $\mathbf{k}_j$. A detailed discussion of the validity of the LMA, as applied to the reaction under investigation here, can be found in Refs. [12,28].

Substituting Eqs. (3) and (4) into Eq. (2) and introducing the partial wave expansion of the distorted waves and carrying out the angular momentum algebra, we get

$$\begin{aligned}\hat{l}\beta_{lm}&=\frac{(4\pi)^3}{k_a k_b k_c \delta}i^{-l}Y_{lm}(\hat{Q})Z_\ell(Q)\sum_{L_a L_b L_c}(i)^{L_a-L_b-L_c}\hat{L}_b\hat{L}_c\\ &\quad\times\mathcal{Y}^{L_b}_{L_c}(\hat{k}_b,\hat{k}_c)\langle L_b 0 L_c 0|L_a 0\rangle\mathcal{R}_{L_b,L_c,L_a}(k_a,k_b,k_a),\end{aligned} \tag{5}$$

where

$$\begin{aligned}\mathcal{Y}^{L_b}_{L_c}(\hat{k}_b,\hat{k}_c)&=\sum_M(-)^M\langle L_b M L_c\\ &\quad -M|L_a 0\rangle Y_{L_b M}(\hat{k}_b)Y^*_{L_c M}(\hat{k}_c),\end{aligned}$$

$$Z_\ell(Q)=\int_0^\infty r_1^2 dr_1 j_l(Qr_1)u_l(r_1)V_{bc}(r_1),$$

$$\mathcal{R}_{L_b,L_c,L_a}=\int_0^\infty\frac{dr_i}{r_i}f_{L_a}(k_a,r_i)f_{L_b}(k_b,r_i)f_{L_c}(k_c,\delta r_i).$$

In Eq. (5), Q is the magnitude of vector $\mathbf{Q}=\gamma\mathbf{K}_c-\alpha\mathbf{K}_b$. Functions f appearing in the radial integrals $\mathcal{R}_{L_a,L_b,L_c}$ are the radial parts of the distorted wave functions χ's of Eq. (2). These are calculated by solving the Schrödinger equation with appropriate optical potentials, which include both the Coulomb and nuclear terms. The slowly converging integrals $\mathcal{R}_{L_b,L_c,L_a}$ can be handled effectively by using the complex plane method [29].

This theory can be used to calculate breakups of both the neutron and proton halo nuclei. Generally, the maximum value of the partial waves L_a, L_b, L_c must be very large in order to ensure the convergence of the partial wave summations in Eq. (5). However, for the case of the one-neutron halo nuclei, one can make use of the following method to include summations over infinite number of partial waves. We write $\beta_{\ell m}$ as

TABLE I. Optical potential parameters for the ^{10}Be-target interaction. Radii are calculated with the $r_j t^{1/3}$ convention.

system	V_r (Mev)	r_r (fm)	a_r (fm)	W_i (Mev)	r_i (fm)	a_i (fm)
^{10}Be-^{197}Au	400	2.08	0.9	76.2	1.52	0.38
^{10}Be-^{44}Ti	70	2.5	0.5	10.0	1.5	0.50
^{10}Be-^{9}Be	100	2.6	0.5	18.0	2.6	0.50

$$\beta_{\ell m}=\sum_{L_i=0}^{L_i^{max}} \hat{\beta}_{\ell m}(L_i)+\sum_{L_i=L_i^{max}}^{\infty} \hat{\beta}_{\ell m}(L_i), \tag{6}$$

where $\hat{\beta}$ is defined in the same way as Eq. (5) except for the summation sign, and L_i corresponds to L_a, L_b, and L_c. If the value of L_i^{max} is chosen to be appropriately large, the contribution of the nuclear field to the second term of Eq. (6) can be neglected and we can write

$$\sum_{L_i=L_i^{max}}^{\infty} \hat{\beta}_{\ell m}(L_i)\approx\sum_{L_i=0}^{\infty} \hat{\beta}_{\ell m}^{Coul}(L_i)-\sum_{L_i=0}^{L_i^{max}} \hat{\beta}_{\ell m}^{Coul}(L_i), \tag{7}$$

where the first term on the right hand side is the pure Coulomb breakup amplitude, that for the case where one of the outgoing fragments is uncharged can be expressed analytically in terms of the Bremsstrahlung integral (see, e.g., Ref. [12]). Therefore, only two terms, with reasonable upper limits, are required to be evaluated by the partial wave expansion in Eq. (6).

The wave function $u_\ell(r)$ appearing in the structure term Z_ℓ has been calculated by adopting a single particle potential model. The ground state of ^{11}Be was assumed to have a $2s_{1/2}$ valence neutron coupled to the ^{10}Be(0^+) core with a binding energy of 504 keV and a spectroscopic factor of 0.78. The corresponding single particle wave function was constructed by assuming the neutron-^{10}Be interaction of a central Woods-Saxon type. For a given set of radius and diffuseness parameters (1.15 fm and 0.5 fm, respectively [12]), the depth of this potential was searched so as to reproduce the ground state binding energy. The neutron-target optical potentials used by us were extracted from the global set of Bechhetti-Greenlees (see, e.g, Ref. [30]), while those used for the ^{10}Be+ target ([30,31]) system are shown in Table I. Following Ref. [22], we have used the sum of these two potentials for the ^{11}Be-target channel. We found that values of L_i^{max} of 500 for Au and Ti and 150 for the Be provided very good convergence of the corresponding partial wave expansion series [Eq. (6)].

In Fig. 1, we show our results for the neutron angular distributions ($d\sigma/d\Omega_n$) for the reaction as mentioned in the corresponding figure caption. Our calculations are in good agreement with the experimental data [6] (shown by solid circles) for all the three targets. For the Be target, $d\sigma/d\Omega_n$ is governed solely by the nuclear breakup effects at all the angles. The pure Coulomb breakup contributions are down by at least an order of magnitude at the forward angles and by two to three orders of magnitude at the backward angles. The CNI terms are also small in this case. On the other hand, for Ti and Au targets the Coulomb terms are dominant at the forward angles while the nuclear breakup effects are important at larger angles. Magnitudes of the CNI terms vary with angle; for many forward angles they almost coincide with those of the nuclear breakup while at the backward angles they are closer to the pure Coulomb breakup contributions. Signs of these terms also change with the neutron angle; a feature common to all the three targets. It is clear that the interference terms are not negligible for Ti and Au targets at the forward angles. For $\theta_n \leqslant 10°$, the magnitudes of the CNI contributions are similar to those of the pure nuclear terms, leading to a better description of the data in this region.

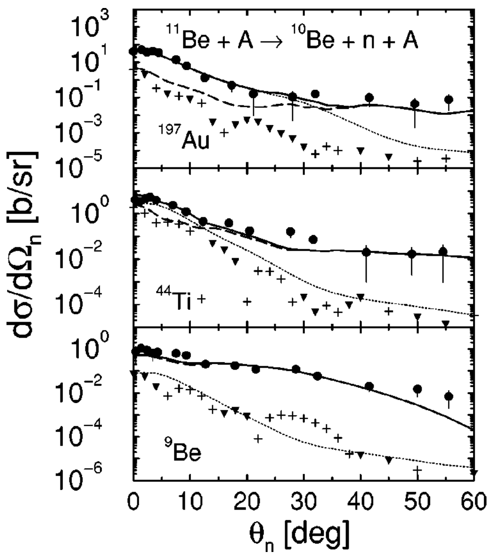

FIG. 1. Neutron angular distribution for the breakup reaction ^{11}Be+A→^{10}Be+n+A at the beam energy of 41 MeV/nucleon. The dotted and dashed lines represent the pure Coulomb and nuclear contributions, respectively, while their coherent sums are shown by the solid lines. The plus signs and the inverted solid triangles represent the magnitudes of the positive and negative interference terms, respectively. The data are taken from Ref. [6].

In Fig. 2, we compare the results of our calculations with the data (taken from Ref. [3]) for the relative energy spectrum of the fragments (neutron and ^{10}Be) emitted in the breakup of ^{11}Be on a ^{208}Pb target at the beam energy of 72 MeV/nucleon. The optical potential parameters, in this case, were taken to be the same as those used for the gold target. We note that while the pure Coulomb contributions dominate the cross sections around the peak value, the nuclear breakup is important at the larger relative energies. This is attributed to the different energy dependence of the two contributions [22]. The coherent sum of the Coulomb and nuclear contributions provides a good overall description of the experimental data. The nuclear and the CNI terms are necessary to explain the data at larger relative energies. Despite the peripheral nature of the reaction, nuclear interactions between the projectile and the target may become possible due to the extended nature of the ^{11}Be wave function. This is the reason for the failure of the pure Coulomb DWBA calculations [12] in describing properly the cross sections in this region.

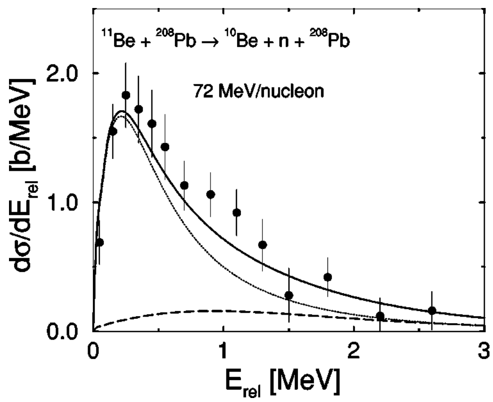

FIG. 2. The differential cross section as a function of the relative energy of the fragments (neutron and ^{10}Be) in the breakup reaction of ^{11}Be on a ^{208}Pb target. Various curves have the same meaning as that in Fig. 1. The data are taken from Ref. [3].

The effect of the interference terms is small (of the order of 2–8 %) on the total breakup cross section. It is constructive for the Be and Ti targets and destructive for the Au target. Therefore, the role of the CNI terms in the total breakup cross section is dependent on the target nucleus.

In summary, we have developed a complete quantal formulation for investigating the breakup reactions of the halo nuclei within the framework of the post-form distorted wave Born approximation, where the pure Coulomb, the pure nuclear, as well as their interference terms are treated consistently within the same framework. Our theory takes into account both the Coulomb and nuclear parts of the fragment-target interactions to all orders, while the interaction between the fragments is treated in first order. It may be mentioned that the higher-order dynamical polarization processes that become important at lower energies in the Coulomb dissociation of proton halo nuclei [32] may not have been treated properly in our theory. However, this effect does not play any role for the Coulomb dissociation of the neutron halo nuclei, which is the subject of study in this paper. Nevertheless, the lack of proper knowledge of the appropriate optical potentials, particularly in the halo projectile-target channel, is a source of uncertainty in our calculations, which is indeed the case for all reaction studies of halo nuclei where the distorted waves in the projectile-target channel are required.

As a first numerical application of this theory, we studied the breakup of the one-neutron halo nucleus ^{11}Be on several target nuclei. We calculated the angular distributions of the neutron fragment emitted in breakup reactions of this nucleus on Be, Ti, and Au targets at the beam energy of 41 MeV/nucleon. The results of our calculations are in good agreement with the available data for all the three targets. We find that for medium mass and heavy target nuclei, the neutron angular distributions are dominated by the nuclear and the Coulomb breakup terms at larger and smaller angles, respectively. Contributions of the Coulomb-nuclear interference terms are non-negligible. They can be as big in magnitude as the pure nuclear or the pure Coulomb breakup and have a negative or positive sign depending upon the angle and energy of the outgoing fragments. For these targets, the interference terms help in better description of the trends of the experimental data even at smaller angles. Similarly, the data on the relative energy spectra of the fragments (neutron and ^{10}Be) emitted in breakup of ^{11}Be on a Pb target at the beam energy of 72 MeV/nucleon cannot be described properly by considering only the pure Coulomb breakup mechanism; inclusion of the nuclear and Coulomb-nuclear interference terms is necessary. In most of the previous studies of this reaction, these terms were neglected. Therefore, the exclusive halo breakup data on medium mass and the heavy target nuclei need to be analyzed more accurately than what has been done so far.

More results on the comparison of calculations performed within this theory and the halo breakup data, particularly on the momentum distribution of fragments, will be presented elsewhere. Work is under way on the calculations of the breakup amplitude [Eq. (2)] without making the local momentum approximation (which is computationally a very involved problem) so that the question of the validity of this approximation can be addressed more rigorously.

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