Three-body approach to the K^-d scattering length in particle basis

A. Bahaoui,[1] C. Fayard,[2] T. Mizutani,[3] and B. Saghai[4]
[1]*Université Chouaib Doukkali, Faculté des Sciences, El Jadida, Morocco*
[2]*Institut de Physique Nucléaire de Lyon, IN2P3-CNRS, Université Claude Bernard, F-69622 Villeurbanne cedex, France*
[3]*Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061*
[4]*Service de Physique Nucléaire, DSM/DAPNIA, CEA/Saclay, F-91191 Gif-sur-Yvette cedex, France*
(Received 12 July 2002; published 6 November 2002)

We report on the first calculation of the scattering length A_{K^-d} based on a relativistic three-body approach where the $\bar{K}N$ coupled channel two-body input amplitudes have been obtained with the chiral SU(3) constraint, but with isospin symmetry breaking effects taken into account. Results are compared with a recent calculation applying a similar set of two-body amplitudes, based on the fixed center approximation, and for which we find significant deviations from the three-body results. Effects of the deuteron D-wave component, pion-nucleon, and hyperon-nucleon interactions are also evaluated.

DOI: 10.1103/PhysRevC.66.057001 PACS number(s): 11.80.Gw, 13.75.Jz

While the threshold behavior of the K-nucleon system has been found to be simple, the corresponding one for the $\bar{K}$-nucleon ($\bar{K}N$) system is quite complicated as its threshold is above those for the πY ($Y \equiv \Lambda, \Sigma$) channels to which it couples strongly [1]. In addition, it also couples to the below-threshold $\Lambda(1405)$ resonance. Moreover, this topic has suffered from years of persistent ambiguity in the sign of the real part of the K^-p scattering length a_{K^-p}: the sign from the scattering data is opposite to the one from the kaonic hydrogen atomic data. Even under these circumstances, a few three-body calculations on the K^-- deuteron scattering length A_{K^-d} were performed with different degrees of refinement, by always disregarding the controversial kaonic hydrogen constraint on $Re(a_{K^-p})$ [2–7]. Some of these works were devoted primarily to calculations of the mass and momentum distributions such as $m(\pi Y)$, in the breakup reactions $K^-d \to \pi NY$, so the K^-d scattering length was, to some extent, a by-product [4,5]. Calculations required various two-body amplitudes as input, the most important of which was the coupled $\bar{K}N$, and πY channels. Those amplitudes were derived from *ad hoc* rank 1 separable potentials with (energy independent) strengths, and ranges in the form factors determined by fit to the low energy K^-p scattering data. On the average the thus obtained values for A_{K^-d} were centered around $\approx(-1.5+i1.0)$ fm. Due to the very restricted quantity and quality of the data and to the lack of sound theoretical guidance (apart from isospin symmetry) on the form of the potentials, along with the then troubled $\mathrm{Re}(a_{K^-p})$, it appeared meaningless to continue this theoretical endeavor any further. So the investigation in the subject became dormant. One very important finding, however, was that the iterative solution for K^-d did diverge; hence solving the three-body equations without truncation became a must.

Recently, there has been a steady progress in effective low energy hadronic methods such as chiral perturbation theory [8,9]. This advance, as well as the new K^-p data, has created a renewed interest in physics with low energy kaons, to the extent that there have even been discussions on extracting the kaon-nucleon σ terms, which are expected to provide important information on chiral symmetry breaking, strangeness content of the nucleon, etc. [10–12]. Note that both a_{K^-p} and A_{K^-d} are vital ingredients in this respect.

On the experimental side, the long-standing sign puzzle in a_{K^-p} was finally resolved by the KEK x-ray measurement in the kaonic hydrogen [13]. The extracted scattering length is $a_{K^-p} = (-0.78 \pm 0.15 \pm 0.03) + i(-0.49 \pm 0.25 \pm 0.12)$ fm. Though the sign of the real part is now settled, one clearly needs a more accurate value, particularly for its imaginary part. With this in mind, remeasuring this quantity along with extracting A_{K^-d} from kaonic atom experiments is underway in the DEAR experiment at DAΦNE; see, e.g., Ref. [10]. This should, in principle, allow for an extraction of the scattering length a_{K^-n} (see, e.g., Ref. [14]).

The interest in improving the calculation of A_{K^-d} may be witnessed in two recent publications. First, Deloff [15] compared the results of old generation multichannel three-body calculations [5] with a simplified three-body result, keeping only K^-p, K^-n, and NN(deuteron) input (all in the S wave), and with the fixed center approximation (FCA) applied to the simplified three-body model. Here the positions of the proton and neutron in the target deuteron were frozen at a certain separation, while the $\bar{K}N$ amplitudes were replaced by their scattering lengths. The K^-d amplitude was then obtained algebraically as a function of the proton-neutron separation. To include the effect of the Fermi motion partially, its expectation value over the separation was calculated with the deuteron wave function. (This leads to the results called *FCA-integ* in Ref. [15]). Second, Kamalov *et al.* [16] performed yet another FCA calculation, but with an essential difference: the input $\bar{K}N$ potentials for the S-wave amplitudes were obtained at $O(1/f^2)$, the lowest order in the SU(3) chiral Lagrangian, which couples the pseudoscalar meson octet and $1/2^+$ baryons octet [17]. Only two free parameters were involved: the best fit to the data was found with a cutoff in the momentum integration at $p_{max} = 630$ MeV, and with an effective meson decay constant f only 15% larger than the physical pion decay constant: $f_\pi = 93$ MeV, putting the value of f between f_π and f_K. With the *hadron physical masses* resulting from the isospin symmetry breaking, which the authors called the *physical* (or *particle*) *basis* as compared with the *isospin basis*, the obtained amplitudes for the coupled $\bar{K}N$, πY, and ηY channels allow one to reproduce the existing low energy data quite well (see Ref. [17]). The $\Lambda(1405)$ resonance was also generated as a bound state below the K^-p and $\bar{K}^\circ n$ thresholds. This approach is in sharp contrast to the models mentioned earlier, in which the only constraint on the parameters was the χ^2 fit to the available data. (note, however, that improved models of this type exist with SU(3) constraints on the relative strengths of the potentials [18]).

Here we have chosen to employ a strategy similar to the one in Ref. [17] for determining the essential part of the input to the three-body equations, and we solve them exactly. In this way, we will be able not only to provide the best theoretical value for A_{K^-d} to date, to our knowledge, but also to test the reliability of the FCA, the effect of the NN (deuteron), πN and YN interactions, etc., on this quantity, as investigated in Ref. [15] within the old scheme.

We have introduced two distinct sets of potentials that are slightly different from the one in Ref. [17]. The main reasons for this are (i) to check the sensitivity of A_{K^-d} to the two-body input, and (ii) to embody them in our current investigations on the finite energy K^-d scattering including the three particle final states like πNY, for which the momentum integration must be done along a rotated line in the complex plane. For this objective, instead of truncating the integration at p_{max}, the potentials should have a smooth cut-off by form factors. Following closely Eqs. (1) to (9) of [17], the first set of potentials (OS1) is expressed, using the isospin notation, as

$$V(I)_{ij} = -\frac{1}{4f^2} C^I_{ij} g(p_i)(\epsilon_i + \epsilon_j) g(p_j), \tag{1}$$

where p_i and ϵ_i are the magnitude of the center-of-mass momentum and the corresponding meson energy in the ith channel, respectively. The SU(3) coupling coefficients are $C^{I=0}_{ij} \equiv D_{ij}$ and $C^{I=1}_{ij} \equiv F_{ij}$, as defined in Tables II and III of Ref. [17]. The form factor has been chosen as

$$g(p) = \frac{\beta^2}{p^2 + \beta^2} \tag{2}$$

for all coupled channels. A fit to the data with comparable quality to Ref. [17] has been reached with $\beta = 870$ MeV and $f = 1.20 f_\pi$. The second set of potentials (OS2) introduces the possible SU(3) breaking effect in the coupling strengths, such that its form is identical to the one for OS1, except that it is now multiplied by an extra coefficient b^I_{ij}. By performing a standard statistical fit to the data, we have obtained $\beta = 865$ MeV and $f = 1.16 f_\pi$. The values of the SU(3) breaking coefficients all stay within 20% around unity; see Table I. Note that, unlike in Ref. [18], the radiative capture $K^-p \to \gamma Y$ has not been investigated. Overall, the fit to data by these two interactions and the one in Ref. [17] are just about the same: differences may be examplified in terms of the scattering lengths shown in Table II. All of them have been evaluated at the K^-p threshold ($=1432$ MeV): beware the discussion below regarding the value of the threshold at which these quantities are calculated. As compared with experiment [13], both the real and imaginary parts of a_{K^-p} given by all models adopted are found within $2\sigma_{stat}$ of the central values. The extra parameters in OS2 make the results somewhat distinct from the two other models. The symmetry breaking effect in the mass of the hadron isospin multiplets on the scattering lengths is quite visible, especially on the real parts; see Table II. (In the limit of isospin symmetry, one has $a_p = a_n{}^\circ$ and $a_{ex} = a_p - a_n$). Finally, we should note that, just as in Ref. [17], we have also retained the ηY channels to obtain a reasonable fit to some data like the $\pi\Sigma$ mass spectrum.

TABLE I. SU(3)-symmetry breaking coefficients b^I_{ij} ($\equiv b^I_{ji}$) for model OS2.

I=0	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	I=1	$\bar{K}N$	$\pi\Sigma$	$\pi\Lambda$	$\eta\Sigma$
$\bar{K}N$	0.93	1.19	0.84	$\bar{K}N$	1.07	1.20	0.83	1.07
$\pi\Sigma$		0.87	0	$\pi\Sigma$		0.81	0	0
$\eta\Lambda$			0	$\pi\Lambda$			0	0
				$\eta\Sigma$				0

Another major two-body input for the three-body equations is the NN interaction in the deuteron channel. We have mostly adopted the rank 1 relativistic potential constructed in Ref. [19], hereafter called model A. The parameters were fitted to the static properties of the deuteron, with D-state percentage value $P_D = 6.7\%$, and to the monopole charge form factor up to ~ 6 fm^{-1} [this parametrization was denoted as SF(6.7) in Ref. [19]]. In order to study in a forthcoming paragraph the dependence on deuteron description, we have also considered two other models. One, hereafter called model B, is the relativized version of the model elaborated in Ref. [20], based on a separable representation of the Paris potential, and with $P_D = 5.77\%$ (denoted PEST1 in Ref. [20]). The other (model C) is a relativistic interaction including only the S-state component.

In our three-body calculation, we first retain, in addition to the deuteron channel, the two-body $\bar{K}N$ t matrices only: for the elastic K^-p, K^-n, $\bar{K}^\circ n$, and charge exchange $K^-p \leftrightarrow \bar{K}^\circ n$, which is in line with Ref. [16]. It turns out that, with only these two-body channels for K^-d at threshold,

TABLE II. $\bar{K}N$ scattering lengths (in fm) calculated at $W = M_{K^-} + M_p$ in the particle basis with models OS1 and OS2. The values in the last column have been evaluated by Ramos [17] at the same energy. a_p, a_n, a^0_n, and a_{ex} are the scattering lengths for elastic K^-p, K^-n, $\bar{K}^\circ n$, and charge exchange $K^-p \leftrightarrow \bar{K}^\circ n$, respectively.

	OS1	OS2	Oset-Ramos
a_p	$-1.04 + i\,0.83$	$-0.71 + i\,0.92$	$-1.01 + i\,0.95$
a_n	$0.57 + i\,0.45$	$0.71 + i\,0.69$	$0.54 + i\,0.53$
a^0_n	$-0.60 + i\,0.89$	$-0.23 + i\,0.97$	$-0.52 + i\,1.05$
a_{ex}	$-1.37 + i\,0.48$	$-1.16 + i\,0.39$	$-1.29 + i\,0.48$

TABLE III. K^-d scattering length (in fm) calculated in the particle basis, using the FCA and Faddeev approaches. Model A is used for the deuteron. The calculations in the last column have been performed by us with the Oset-Ramos $\bar{K}N$ scattering lengths given in Table II.

FCA	OS1	OS2	Oset-Ramos
el. only	$-1.32+i\,1.10$	$-1.09+i\,1.41$	$-1.36+i\,1.26$
charge ex.	$-0.83+i\,0.82$	$-0.64+i\,0.35$	$-0.63+i\,0.69$
total	$-2.15+i\,1.92$	$-1.73+i\,1.76$	$-1.99+i\,1.95$
Faddeev			
el. only	$-1.70+i\,1.31$	$-1.41+i\,1.48$	$-1.68+i\,1.33$
charge ex.	$-0.29+i\,0.34$	$-0.27+i\,0.18$	$-0.24+i\,0.25$
total	$-1.99+i\,1.65$	$-1.68+i\,1.66$	$-1.92+i\,1.58$

effectively there is no other strong branch cut along the real axis in the momentum integration in the three-body equations, so no contour rotation into the complex plane is needed for integration, and even a sharp cutoff may be imposed. Thus with the two-body input from Ref. [17], we were able to find the exact solution to the three-body equations *without* making the FCA as adopted in Ref. [16]. Table III summarizes our calculations in the *particle basis*, namely, the FCA which we adopt to characterize the on-shell contribution, and the three-body Faddeev calculation. The result with the amplitudes from Ref. [17] is presented in the column labeled "Oset-Ramos," along with our own sets OS1 and OS2. For later discussions we have separated the results into (i) the pure elastic case, i.e., with K^- multiple scattering on the proton and neutron; (ii) the total contribution; and (iii) the charge exchange contribution, which is the difference between the values in (ii) and (i).

As for the FCA, we see in Table III that the results for all three $\bar{K}N$ models are more or less a reflection of the differences in the scattering lengths in Table II. Now there is a bit of trouble in the present situation: near the threshold the K^-p and $\bar{K}^\circ n$ elastic amplitudes and $K^-p \leftrightarrow \bar{K}^\circ n$ charge exchange amplitudes all vary rapidly due to the proximity of the $\Lambda(1405)$ resonance. In fact, the minimum of the real part of the K^-p amplitude is found to be located slightly below the K^-p threshold ($W_{th}=1432$ MeV); see, e.g., Fig. 9 of Ref. [17]. In addition, the threshold is slightly different for each physical $\bar{K}N$ channel, except in the limit of exact isospin symmetry. So, depending on the threshold energy adopted in determining the different scattering lengths for use in the FCA, the resulting *on-shell* contribution to A_{K^-d} has been found to vary up to at least 20% for its real part, while its imaginary part was relatively stable. It may be useful to remark that this strong variation in the present FCA result is due to the violation of Bég's theorem. (Bég's theorem [21] states that, "if the ranges of interactions for the projectile and target constituents between two successive collisions do not overlap, the projectile-target interaction is described entirely by the on-shell properties of the two-body input." This theorem is relevant to the reactions studied here due to the fact that the deuteron is very loosely bound.)

TABLE IV. K^-d scattering length (in fm) calculated with models OS1 and OS2 in the physical basis, with different deuteron models.

Model	A	B	C
OS1	$-1.99+i\,1.65$	$-1.97+i\,1.52$	$-1.98+i\,1.31$
OS2	$-1.68+i\,1.66$	$-1.68+i\,1.55$	$-1.69+i\,1.33$

The finite lifetime of the $\Lambda(1405)$ causes its propagation, hence, non-overlapping of the interaction ranges does not materialize.

Now we wish to underline a significant finding of the present work: as one can see in Table III, the Faddeev results for all three models are closer to each other than in the FCA, with $\mathrm{Re}(A_{K^-d})$ for OS2 only about 15% different from the values given by the other two models. By comparing the three-body result and its FCA version for a given set of $\bar{K}N$ interactions, there is a noticeable difference which may be regarded as due to off-shell effects. Particularly, the effect of the charge exchange scattering $K^-p \leftrightarrow \bar{K}^\circ n$ in the multiple scattering process has been found to be grossly overestimated in the FCA. This is because the $\bar{K}^\circ n$ channel has a higher threshold than that of K^-p. The constant scattering length approximation adopted in the FCA ignores this aspect. Within the FCA, the situation gets even worse with the *isospin basis*, in which the two thresholds are identical, see, e.g., Table II of Ref. [16]. To make clear the threshold effects within the Faddeev approach, we have calculated A_{K^-d} in the *isospin basis*, using deuteron model A. The results, $(-1.76+i2.91)$ fm for OS1 and $(-1.37+i2.68)$ fm for OS2, compared with the values in the last line of Table III, show significant differences, especially for the imaginary part.

Next we have checked the dependence on the deuteron models mentioned above. First, we compare the results obtained with the two parametrizations including the *D*-state component, namely, models A and B. As shown in Table IV, the difference in A_{K^-d} is found mostly in the imaginary part, but is only within a few percent. But when a simple $^3S_1$-wave model is used (model C), this difference grows to be about 20%, as seen in the third column of Table IV. However, the real part appears quite stable. The short range part of the deuteron wave function should be responsible for this difference. Hence one needs to retain a realistic deuteron model with the $^3D_1$ component.

We then want to check the claim in Ref. [15] that the FCA is rather reliable with respect to the full three-body result. In fact, by comparing the rows for *FCA-integ* and *Faddeev* in Table II of Ref. [15], the author seems to be right: the two methods provide almost identical imaginary parts, while the FCA tends to underestimate the magnitude of the real part slightly. This is just opposite to what was reported above; see Table III. Eventually, we solved this apparent puzzle: by taking a pure *S*-wave deuteron and also by excluding the charge exchange contribution in the $\bar{K}N$ input to the three-body equations, we found that the exact and FCA solutions present very similar values for the imaginary part, but that the latter underestimates the real part by about 30%. In fact this is how the author of Ref. [15] performed his calculation, and the characteristic of the outcome was just the same: main difference in the real part. Then, once the charge exchange contribution is introduced, we find that the trend changes considerably. We found further that by introducing a realistic deuteron model including the *D*-state component, even the result without charge exchange process does not satisfy the finding of Ref. [15]. Hence we conclude that the FCA is not as reliable as claimed in Ref. [15].

Finally, we need to check the effects due to the πN and YN interactions, which have been excluded so far from our two-body input: they introduce the $\pi(YN)$ and $Y(\pi N)$ states in the three-body equations, where particles outside the parentheses are the spectators. To evaluate these contributions, we have used the P_{33} πN and *S*-wave YN interactions from Refs. [6,7]. Our preliminary results show effects smaller than 5%, so the semiquantitative estimate of Ref. [16] seems justified.

To summarize, the main item of interest reported here consists of the elaboration of a relativistic three-body coupled channel approach of the K^-d scattering length, embodying isospin symmetry breaking effects, thus allowing one to show the crucial role played by opening threshold effects for different two- and three-body channels. Moreover, the two-body $\bar{K}N$ interactions have been constructed in line with the recent chiral perturbation formalisms. Starting from a study of K^-p scattering length, reproducing the data well enough, we have investigated the sensitivity of A_{K^-d} to various input ingredients. The obtained values agree with each other within $\pm 20\%$, leading to

$$A_{K^-d} \approx (-1.8 + i1.5) \text{ fm}. \tag{3}$$

Here our approach embodied elastic and inelastic $\bar{K}N$ channels in the three-body formalism. To go further, we are in the process of including all other relevant inelastic channels, such as πY and ηY. How one may extract the scattering length a_{K^-n} from the experimental values of a_{K^-p} and A_{K^-d}, is another question under study. A more extensive account will be reported in a forthcoming paper.

A.B. and T.M. wish to thank IPN/Lyon for the kind hospitality extended to them in the course of this enterprise. A.B. also thanks CEA/Saclay for a generous six months of hospitality. We are indebted to A. Ramos, R. Machleidt, A. Olin, and E. Oset for their kind help in several issues related to the present work.

[1] J.M. Eisenberg and D.S. Koltun, *Theory of Meson Interactions with Nuclei* (Wiley, New York, 1980); A.B. Martin, Nucl. Phys. **B179**, 33 (1981).

[2] J.H. Hetherington and L.H. Schick, Phys. Rev. **137**, B935 (1965); **165**, B1647 (1967).

[3] L.H. Schick and B.F. Gibson, Z. Phys. A **288**, 307 (1978).

[4] G. Toker, A. Gal, and J.M. Eisenberg, Nucl. Phys. **A362**, 405 (1982).

[5] M. Torres, R.H. Dalitz, and A. Deloff, Phys. Lett. B **174**, 213 (1986).

[6] A. Bahaoui, C. Fayard, G.H. Lamot, and T. Mizutani, Nucl. Phys. **A508**, 335c (1990).

[7] A. Bahaoui, Ph.D. thesis, Université Claude Bernard Lyon-1, 1990.

[8] U.-G. Meissner, Rep. Prog. Phys. **56**, 903 (1993).

[9] G. Ecker, Prog. Part. Nucl. Phys. **35**, 1 (1995).

[10] C. Guaraldo and B. Lauss, Nucl. Phys. News **11**, 20 (2001).

[11] A. Olin and T.-S. Park, Nucl. Phys. **A691**, 295 (2001).

[12] P.M. Gensini, hep-ph/9804344.

[13] M. Iwasaki *et al.*, Phys. Rev. Lett. **78**, 3067 (1997); T.M. Ito *et al.*, Phys. Rev. C **58**, 2366 (1998).

[14] R.C. Barrett and A. Deloff, Phys. Rev. C **60**, 025201 (1999).

[15] A. Deloff, Phys. Rev. C **61**, 024004 (2000).

[16] S.S. Kamalov, E. Oset, and A. Ramos, Nucl. Phys. **A690**, 494 (2001).

[17] E. Oset and A. Ramos, Nucl. Phys. **A635**, 99 (1998); A. Ramos (private communication).

[18] P.B. Siegel and B. Saghai, Phys. Rev. C **52**, 392 (1995); T.S.H. Lee, J.A. Oller, E. Oset, and A. Ramos, Nucl. Phys. **A643**, 402 (1998).

[19] N. Giraud, C. Fayard, and G.H. Lamot, Phys. Rev. C **21**, 1959 (1980).

[20] J. Haidenbauer and W. Plessas, Phys. Rev. C **30**, 1822 (1984).

[21] M.A.B. Bég, Ann. Phys. (N.Y.) **13**, 110 (1961).