Resonance lifetime in Boltzmann-Uehling-Uhlenbeck theory: Observable consequences

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Within the transport Boltzmann-Uehling-Uhlenbeck theory we study the influence of the choice for the Δ-resonance lifetime on pion-nucleus reactions and on heavy-ion collisions at $1A$ GeV. A quite small effect of these modifications on the pion absorption on nuclei is found provided that the absorption probability of the Δ resonance is modified consistently. Observable effects in the case of heavy-ion collisions are demonstrated.

I. INTRODUCTION

It is usually assumed that the lifetime of a resonance is given by its inverse total width $1/\Gamma$. This assumption is widely used in the transport simulations of nuclear collisions in order to describe the decays of various baryonic and mesonic resonances. However, such a picture is not always correct.

Let us consider a resonance as the intermediate state of the two-body (e.g., πN) scattering. The lifetime of the resonance depends crucially on the relation between its width Γ and the energy spread of the incoming particles ΔE [1]: (i) For $\Delta E \ll \Gamma$ (broad resonance) the lifetime is given by the derivative of the phase shift with respect to the center-of-mass (c.m.) energy:

$$\tau = \frac{d\,\delta(E_{\mathrm{c.m.}})}{dE_{\mathrm{c.m.}}}, \tag{1}$$

which is the time delay in the transmission of the scattered wave in the case of scattering with only one partial wave [2,3]. (ii) In the opposite limit $\Delta E \gg \Gamma$ (narrow resonance) the averaging of Eq. (1) can be done over $E_{\mathrm{c.m.}}$ weighted with the cross section which leads to the average lifetime of $1/\Gamma$ [3].

In the Boltzmann-Uehling-Uhlenbeck (BUU) transport theory the colliding particles—by definition—have fixed energies and momenta. Therefore, the *correct* lifetime of a resonance in BUU theory is that of Eq. (1) (see also Ref. [4]). In particular, the lifetime of the $\Delta(1232)$ resonance is given by Eq. (1) with $\delta(E_{\mathrm{c.m.}}) \equiv \delta_{33}(E_{\mathrm{c.m.}})$, where δ_{33} is the phase shift of the πN scattering in the $S=3/2$, $I=3/2$ channel.

The two definitions of the lifetime, the *wrong* one as $1/\Gamma$ and the *correct* one as the phase shift derivative, have a completely different dependence on the c.m. energy [see Fig. 1 and Eqs. (2) and (11) below]: the inverse width decreases monotonically with $E_{\mathrm{c.m.}}$ while the phase shift derivative has a maximum near $m_\Delta = 1.232$ GeV. Especially, at threshold $E_{\mathrm{c.m.}} = m_N + m_\pi = 1.076$ GeV, the inverse width becomes infinitely large, while the phase shift derivative is zero. In view

of the fact that most transport simulations use the wrong $1/\Gamma$ prescription it is interesting to find observable signals sensitive to the different lifetime prescriptions. In the present work—using the BUU model—we study the influence of the Δ resonance lifetime on the pion absorption on nuclei and on some pionic observables from heavy-ion collisions at $1A$ GeV. Note that a quite thorough investigation of the analogous effects within BUU in the Fermi energy domain has been done by Morawetz and coauthors. Their works concern the nonlocal and quasiparticle corrections to the nucleon-nucleon scattering (see Ref. [5], and references therein).

The paper is structured as follows: In Sec. II, the modifications of the Δ-resonance absorption and rescattering probabilities in a nuclear medium are discussed, and the parametrizations of the Δ width and lifetime for transport simulations are chosen. Section III contains numerical BUU results. Conclusions are given in Sec. IV.

II. IN-MEDIUM Δ LIFETIME

Recently Eq. (1) has been generalized in Ref. [6] to the case of the decay of the Δ resonance with multiple final channels, i.e., when the total width Γ contains not only the
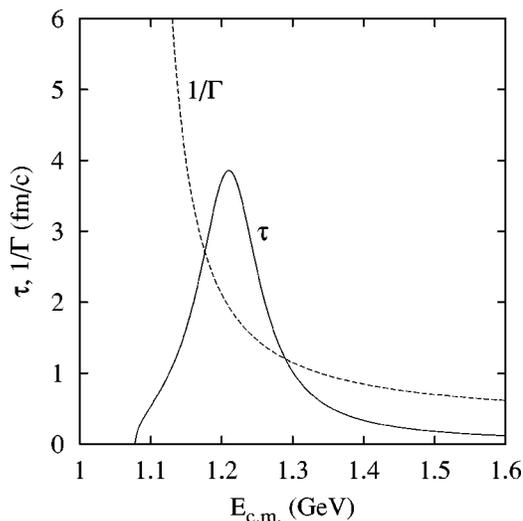
FIG. 1. The inverse width $1/\Gamma$ (dashed line), where Γ is given by Eq. (11), and the lifetime [solid line, Eq. (2)] of the Δ resonance as functions of the total c.m. energy of the pion and nucleon.

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$\Delta \to N\pi$ contribution, but also the absorptional contributions such as, e.g., $\Delta \to N^{-1}NN$. The result of Ref. [6] for the decay time of a uniform bunch of resonances which do not interact with each other [see Eq. (3.3) in Ref. [6]] rewritten here for simplicity in the nonrelativistic approximation is as follows:

$$\tau = \frac{1}{2} A(1-K), \qquad (2)$$

where

$$A(\omega, \mathbf{p}) = \frac{\Gamma(\omega, \mathbf{p})}{[\omega - \mathbf{p}^2/2m - \mathrm{Re}\,\Sigma^+(\omega, \mathbf{p})]^2 + \Gamma^2(\omega, \mathbf{p})/4} \qquad (3)$$

is the spectral function and

$$K \equiv \frac{\partial \mathrm{Re}\,\Sigma^+(\omega, \mathbf{p})}{\partial \omega} + \frac{\omega - \mathbf{p}^2/2m - \mathrm{Re}\,\Sigma^+(\omega, \mathbf{p})}{\Gamma(\omega, \mathbf{p})} \frac{\partial \Gamma(\omega, \mathbf{p})}{\partial \omega} \qquad (4)$$

with the total width $\Gamma(\omega, \mathbf{p}) = -2\,\mathrm{Im}\,\Sigma^+(\omega, \mathbf{p})$. In the case of a resonance gas in vacuum, i.e., when only the $\Delta \to N\pi$ decay channel is open, Eqs. (1) and (2) are equivalent. This can be easily seen by introducing the phase shift (cf. Ref. [2])

$$\delta = \arctan \frac{\Gamma(E_{\mathrm{c.m.}})}{2[E_R(E_{\mathrm{c.m.}}) - E_{\mathrm{c.m.}}]}, \qquad (5)$$

with $E_R = m + \mathrm{Re}\,\Sigma^+(E_{\mathrm{c.m.}})$, and $E_{\mathrm{c.m.}} = m + \omega - \mathbf{p}^2/2m$.

In transport simulations one deals with the partial lifetimes τ_i of resonances with respect to decay into different channels, including absorption and rescattering. If one uses, e.g., $1/\Gamma_{\Delta \to N\pi}$ as the lifetime for the Δ with respect to the $N\pi$ channel, this corresponds to the use of the "standard" cross section $d\sigma_{\Delta N \to NN}/d\Omega$ for the absorption channel. Conversely, if the partial lifetime is changed then the cross section has to be changed accordingly. Notice that the probabilities of the processes with the Δ resonance in the *final* state, such as, e.g., $\pi N \to \Delta$ and $NN \to N\Delta$, are *not* modified. We now assume that the overall lifetime is given by Eq. (2) and define a modified total width $\tilde{\Gamma} \equiv \tau^{-1}$. $\tilde{\Gamma}$ can be decomposed into modified partial widths $\tilde{\Gamma}_i$: $\tilde{\Gamma} = \Sigma_i \tilde{\Gamma}_i$. There is one important aspect: the modified branching ratios, $\tilde{\Gamma}_i/\tilde{\Gamma}$, have to be the same as the original ones, Γ_i/Γ, i.e., $\tilde{\Gamma}_i = \Gamma_i \tilde{\Gamma}/\Gamma = \Gamma_i(\Gamma\tau)^{-1}$. This ensures that the measurable cross sections for multistep processes are correct. To see this we consider the process $NN \to NN\pi$ with a Δ as an intermediate state. For the cross section one obtains $\sigma_{NN \to NN\pi} = \sigma_{NN \to N\Delta} B_{\mathrm{out}}$, where $\sigma_{NN \to N\Delta}$ is the production cross section of the Δ resonance in a nucleon-nucleon collision and $B_{\mathrm{out}} = \Gamma_{\Delta \to N\pi}/\Gamma \overset{!}{=} \tilde{\Gamma}_{\Delta \to N\pi}/\tilde{\Gamma}$ is the outgoing branching ratio. Keeping the branching ratios constant, both quantities, $\sigma_{NN \to N\Delta}$ and B_{out}, are not modified and hence the (experimentally measurable) cross section $\sigma_{NN \to NN\pi}$ is not modified as well.

In the transport simulation the modified partial widths must be treated by the Monte Carlo method in the same way as the usual partial widths. For the Δ resonance, therefore, the probabilities of processes where the Δ resonance is present in the *initial* state are *all* multiplied by the same factor $\tilde{\Gamma}/\Gamma = (\Gamma\tau)^{-1}$ to keep the branching ratios constant (cf. also Ref. [6] for a different derivation):

$$\tilde{\Gamma}_{\Delta \to N\pi} = \Gamma_{\Delta \to N\pi}(\Gamma\tau)^{-1}, \qquad (6)$$

$$\frac{d\tilde{\sigma}_{\Delta N \to NN}}{d\Omega} = \frac{d\sigma_{\Delta N \to NN}}{d\Omega}(\Gamma\tau)^{-1}, \qquad (7)$$

$$\frac{d\tilde{\sigma}_{\Delta N \to \Delta N}}{d\Omega\, dM^2} = \frac{d\sigma_{\Delta N \to \Delta N}}{d\Omega\, dM^2}(\Gamma\tau)^{-1}, \qquad (8)$$

$$\tilde{\Gamma}_{\Delta NN \to NNN} = \Gamma_{\Delta NN \to NNN}(\Gamma\tau)^{-1}, \qquad (9)$$

where $\Gamma_{\Delta \to N\pi}$ is the standard Δ decay width in nuclear matter taking into account the Pauli blocking for the final nucleon, and $d\sigma_{\Delta N \to NN}/d\Omega$ is the usual differential cross section obtained directly as

$$\frac{d\sigma_{\Delta N \to NN}}{d\Omega} = \frac{1}{64\pi^2} \overline{|\mathcal{M}|^2} \frac{p_{NN}}{p_{N\Delta}s} \times \frac{4}{C_{NN}}, \qquad (10)$$

where p_{NN} and $p_{N\Delta}$ are the c.m. momenta of incoming and outgoing particles, respectively, s is the c.m. energy squared, $\overline{|\mathcal{M}|^2}$ is the spin-averaged matrix element squared given by the one-pion exchange model [7], and $C_{NN} = 2\,(1)$ if the final nucleons are identical (different). Analogous modifications have also to be done with the Δ rescattering cross section (8) and with the three-body absorption width (9), whenever the process $\Delta NN \to NNN$ is included in the BUU theory.

Before discussing BUU results we will specify the explicit forms of the Δ resonance width and lifetime. It has been shown in Ref. [8] that the total width of the Δ resonance in nuclear matter is very close to the vacuum decay width. Thus, we take

$$\Gamma \simeq \Gamma_{\Delta \to N\pi}^{vac}(E_{\mathrm{c.m.}}) = \Gamma_0 \left(\frac{q}{q_0}\right)^3 \frac{m_\Delta}{E_{\mathrm{c.m.}}} \frac{\beta_0^2 + q_0^2}{\beta_0^2 + q^2}, \qquad (11)$$

where the parametrization from Ref. [9] is used for $\Gamma_{\Delta \to N\pi}^{vac}$, with $q(E_{\mathrm{c.m.}})$ being the pion momentum in the rest frame of Δ, $q_0 \equiv q(m_\Delta)$, $\Gamma_0 = 0.118$ GeV, and $\beta_0 = 0.2$ GeV. For simplicity we will also neglect the energy and momentum dependence of $\mathrm{Re}\,\Sigma^+$ putting $E_R = \mathrm{const} = m_\Delta$. In this case the lifetime of Eq. (2) reduces to Eq. (1) with δ given by Eq. (5).

Figure 1 shows the dependence of the Δ resonance lifetime on the πN c.m. energy as given by the derivative of the phase shift (solid line) and by the inverse width (dashed line). The shape of the function $\tau(E_{\mathrm{c.m.}})$ is basically dominated by the presence of the spectral function A in Eq. (2). This implies that resonances with mass near the pole mass m_Δ have the longest lifetimes.
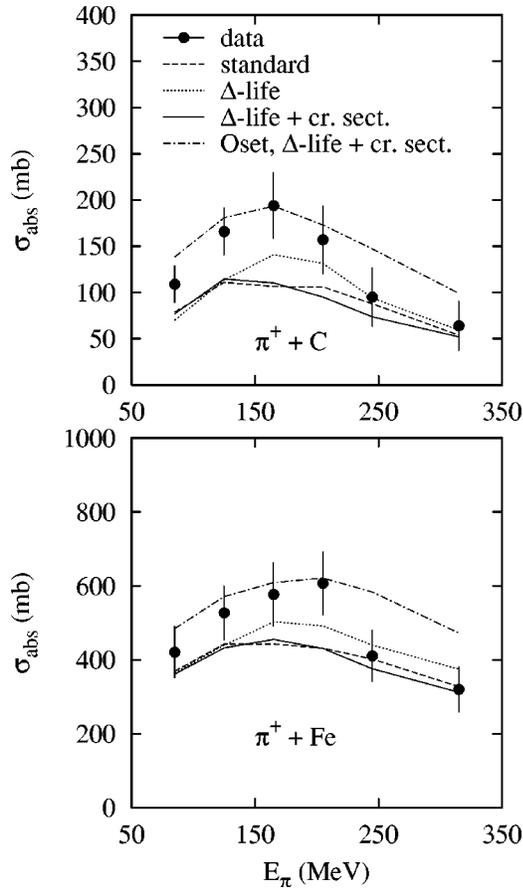
FIG. 2. The beam energy dependence of the π^+ absorption cross section on carbon (upper panel) and on iron (lower panel). Standard BUU calculations (with $1/\Gamma$ Δ lifetime) are represented by the dashed line, while dotted and solid lines show, respectively, the results with the modified lifetime of Eq. (2) and with both modified lifetime and cross sections $\Delta N \rightarrow NN$, $\Delta N \rightarrow \Delta N$ of Eqs.(7) and (8). The dash-dotted line shows the calculations including the three-body absorption contribution (Ref. [12]) (see text for details). Experimental data are from Ref. [11].

III. BUU RESULTS

In the following we will address the question of whether the difference between the lifetime prescriptions could be visible in observable quantities. The numerical calculations have been performed on the basis of the BUU code in the version of Ref. [9] using the soft momentum-dependent mean field (SM) with the incompressibility $K = 220$ MeV. The resonance production/absorption quenching [10] has been implemented in order to reproduce the experimentally measured pion multiplicity in central Au+Au collisions at $1A$ GeV.

First, we have performed a BUU calculation of the pion absorption on nuclei. To this aim we have selected the experimental data from Ref. [11] on reactions $\pi^+ + C$ and $\pi^+ + Fe$ at the pion beam energies $E_\pi = 85$, 125, 165, 205, 245, and 315 MeV. Figure 2 shows the calculated excitation function of the π^+ absorption cross section in comparison with the data. The standard BUU calculation (dashed lines) employing the Δ lifetime $1/\Gamma$ underpredicts the data at the
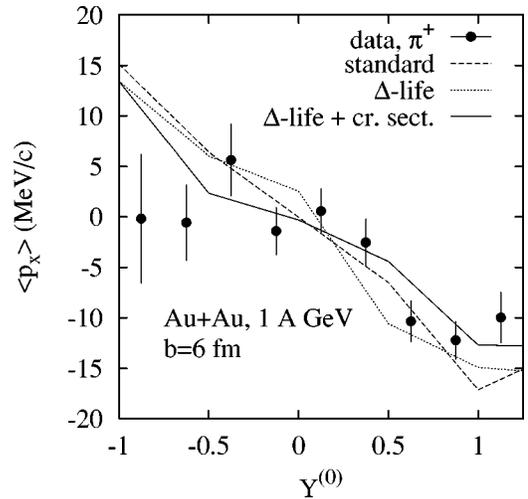
FIG. 3. The π^+ average transverse momentum in the reaction plane vs normalized c.m. rapidity $Y^{(0)} \equiv (y/y_{proj})_{c.m.}$ for the collision Au+Au at $1A$ GeV and $b = 6$ fm. Calculated curves are denoted as in Fig. 2. Data are from Ref. [13].

lower energies. Using the lifetime of Eq. (2) (dotted lines) improves the agreement amplifying the absorption peak at $E_\pi = 150$–200 MeV. The peak corresponds to the beam energy $E_\pi = (m_\Delta^2 - m_\pi^2 - m_N^2)/2m_N - m_\pi = 192$ MeV at which the Δ resonance is excited at the pole mass. However, the peak position is slightly changed by the Fermi motion. The physical reason for the increased absorption with the lifetime of Eq. (2) lies in longer living Δ resonances near the pole mass (see Fig. 1, solid line). This increases the probability that a Δ will be absorbed in the collision with a nucleon. Applying now Eq. (2) for the lifetime and, in addition, modifying the $\Delta N \rightarrow NN$ and $\Delta N \rightarrow \Delta N$ cross sections according to Eqs. (7) and (8) results in the absorption cross section (solid lines) practically indistinguishable from the standard calculation, since the factor $(\Gamma \tau)^{-1}$ in Eq. (7) is less than 1 near the pole mass of the Δ.

The discrepancy between our calculations and the data at $E_\pi < 250$ MeV can be explained by missing the three-body absorption mechanism of the Δ resonance $\Delta NN \rightarrow NNN$. We have performed the calculation including the three-body absorption as parametrized by Oset and Salcedo [12] (see dash-dotted lines in Fig. 2). In this case we show only results with the lifetime of Eq. (2) and the modified absorption and rescattering probabilities in the processes $\Delta N \rightarrow NN$, $\Delta NN \rightarrow NNN$, and $\Delta N \rightarrow \Delta N$ [Eqs. (7)–(9)], since these modifications, when done simultaneously, have only a very small effect on the absorption cross section (cf. solid and dashed lines). We see now a good description at the lower energies, but at higher energies the data are overpredicted somewhat.

The authors of Ref. [4] have observed an influence of the Δ lifetime variations on the K^+ in-plane flow in central Ni+Ni collisions at $1.93A$ GeV. We have studied the π^+ in-plane flow for the system Au+Au at $1A$ GeV and $b = 6$ fm. Figure 3 shows our calculations in comparison with the data from Ref. [13]. The acceptance of the detector [13] for π^+'s is good only at positive c.m. rapidities. This causes the measured $\langle p_x \rangle [Y^{(0)}]$ dependence to be asymmetric with
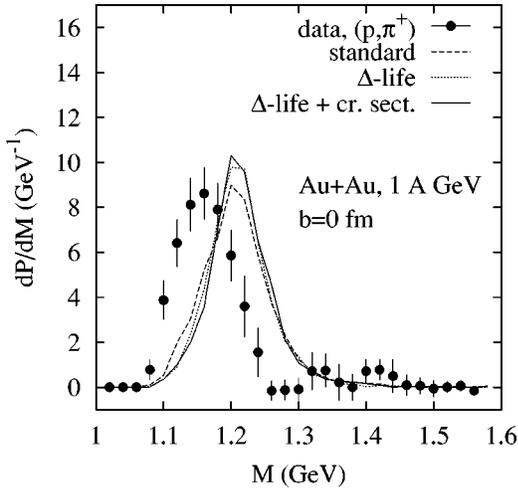
FIG. 4. Invariant mass spectrum of the correlated (p, π^+) pairs for the system Au+Au at $1A$ GeV. Calculated curves are denoted as in Fig. 2. Data are from Ref. [14].

respect to $Y^{(0)} = 0$. At $Y^{(0)} > 0$ all calculations generally agree with data within error bars. However, there is a difference in the flow $[\equiv d\langle p_x \rangle / dY^{(0)}$ at $Y^{(0)} = 0]$ between the calculations: the calculation with the modified lifetime (2) and the modified cross sections (7) and (8) produces less (negative) flow than the standard calculation, while the results with only a modified lifetime (2) are practically the same within statistics with standard ones. Indeed, π^+'s exhibit the antiflow due to the superposition of the shadowing and the Coulomb repulsion from protons. Smaller cross sections $\tilde{\sigma}_{\Delta N \rightarrow NN}$, $\tilde{\sigma}_{\Delta N \rightarrow \Delta N}$ near the Δ pole mass give less shadowing and, therefore, less antiflow.

Figure 4 shows the invariant mass spectrum of the correlated proton-pion pairs from the central collision Au+Au at $1.06A$ GeV in comparison with the data from Ref. [14]. We have extracted the correlated pairs by selecting the proton and pion which are emitted from the same resonance and did not rescatter afterwards (see Ref. [15] for the comparison of this method with the background subtraction technique). The shape of the calculated spectrum well agrees with data, but the peak position is overpredicted by about 50 MeV. We

checked that the peak position is not changed and the spectrum gets slightly wider when taking into account also those pairs, where the nucleon experienced rescattering on other nucleons once or twice. The calculations with the modified lifetime (2) (dotted line) and with modifying both lifetime and cross sections (7) and (8) (solid line) result in somewhat sharper peaks at the invariant mass of 1.2 GeV. This is due to longer living Δ resonances near the pole mass, which reach the late freeze-out stage and then decay to the proton-pion pairs. The Δ resonances propagate now in expanding nuclear matter and, therefore, their absorption is not as effective as in the pion-nucleus reactions. Thus, increasing the lifetime of the Δ resonances near the pole mass does not lead to their increased absorption, unlike the pion-nucleus case.

IV. CONCLUSIONS

We have performed a comparative study of the two choices of the Δ resonance lifetime. Using the inverse width is the standard one. However, this choice is physically *wrong*, since the resonances propagated in BUU theory are not asymptotical plane waves, but intermediate states of the πN scattering. The *correct* choice of the resonance lifetime in BUU theory is the derivative of the phase shift [cf. Eq. (1) or Eq. (2)], which is the delay time of the scattered wave. A consistent transport theory must include also the corresponding modifications of the Δ resonance absorption and rescattering probabilities according to Eqs. (7)–(9). Surprisingly, we have found a quite weak sensitivity of the calculated pionic observables on the lifetime prescription, once the cross sections are modified accordingly, despite the strong difference in the c.m. energy dependence of both lifetimes (Fig. 1). Nevertheless, detailed comparison with more precise data on the pion flow and (p, π) correlations is necessary to confirm the correct choice of the Δ lifetime in nuclear matter.

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