Critical temperature of nuclear matter and fragment distributions in multifragmentation of finite nuclei

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The fragment production in multifragmentation of finite nuclei is affected by the critical temperature of nuclear matter. We show that this temperature can be determined on the basis of the statistical multifragmentation model (SMM) by analyzing the evolution of fragment distributions with the excitation energy. This method can reveal a decrease of the critical temperature that, e.g., is expected for neutron-rich matter. The influence of isospin on fragment distributions is also discussed.

Properties of nuclear matter have been under investigation for several decades (see, e.g., [1]). Besides their general interest for nuclear physics, these studies are very important for our understanding of astrophysical objects, such as neutron stars. The information about nuclear matter in its ground state and at low temperatures is usually obtained as a theoretical extrapolation, based on nuclear models designed to describe the structure of real nuclei.

It is instructive to investigate the thermodynamical properties of neutron-rich matter under extreme conditions of low densities and high temperatures. This situation is expected, for example, at supernova II explosions and during the formation of neutron stars. We believe that the liquid-gas type phase transition is manifested, in this case, in the forms of instabilities leading to fragment production. The models used for extracting nuclear matter properties in the phase transition region should be capable of describing the disintegration of homogeneous matter into fragments. It is also important that they should be tested in nuclear reactions leading to the total disintegration of real nuclei at high excitation energy. The multifragmentation reactions, which started to be investigated experimentally nearly 20 years ago (see reviews [2,3]), are perfectly suited for this purpose.

Naturally, a thermodynamical model involved in this kind of analysis has to include the ingredients necessary for the description of nuclear matter and to provide a good reproduction of experimental data. So far, the statistical multifragmentation model (SMM) [3] satisfies these requirements. The SMM has been designed to describe fragmentation and multifragmentation of excited finite nuclei [4,5]. It includes a liquid-drop approximation for individual fragments which corresponds to the liquid-drop description of nuclear matter [1,6]. This is an essential difference of the SMM from other multifragmentation models, e.g., [7,8], which do not take the nuclear matter properties explicitly into account. Examples of very successful applications of the SMM for the description of different experimental data can be found in [3,9–17]. Furthermore, the descriptions of data with statistical models confirm that multifragmentation of nuclei, despite of being a very fast process, proceeds under a high degree of thermalization.

Details of the SMM can be found in [3], here we concentrate on parts of the model which are important for the following discussion. The model describes the fragment formation at a low-density freeze-out ($\rho \lesssim 1/3\rho_0$, $\rho_0 \approx 0.15$ fm^{-3} is the normal nuclear density), where the nuclear liquid and gas phases coexist. The SMM phase diagram has already been under intensive investigations (see, e.g., Ref. [18]). The liquid-drop approximation suggests that the fragmentation process is accompanied by an increase of the surface of nuclear drops. The surface entropy contributes essentially to the statistical partition sum. We should point out that the surface free energy depends on the ratio of the temperature T to the critical temperature of nuclear matter T_c. In the SMM the surface tension $\sigma(T)$ is given by

$$\sigma(T) = \sigma(0)\left(\frac{T_c^2 - T^2}{T_c^2 + T^2}\right)^{5/4}. \tag{1}$$

This formula is obtained as a parametrization of the calculations of thermodynamical properties of the interface between two phases (liquid and gas) of nearly symmetric nuclear matter, which were performed with the Thomas-Fermi and Hartree-Fock methods by using the Skyrme forces [6]. In addition the scaling properties in the vicinity of the critical point (see [19]) were taken into account. At the critical temperature T_c for the liquid-gas phase transition, the isotherm in the phase diagram has an inflection point. The surface tension vanishes at T_c and only the gas phase is possible above this temperature. We emphasize that our analysis is based on this general effect and that our conclusions will remain qualitatively true in the case of other parametrizations satisfying this condition. This surface effect provides an effective way to study the influence of T_c on fragment production in the multifragment decay of hot finite nuclei.

As was established by numerous studies (see, e.g., [3–5,9,15,16]) the mass (charge) distribution of fragments produced in the disintegration of nuclei evolves with the excitation energy. At low temperatures ($T \lesssim 5$ MeV), there is a so-called U-shape distribution corresponding to partitions with few small fragments and one big residual fragment. This distribution looks like a result of an evaporative emission. At high temperatures ($T \gtrsim 6$ MeV) the big fragments

disappear, and there is an exponential-like fall of the mass distribution with mass number A. In the transition region $T \approx 5-6$ MeV, however, there is a smooth transformation of the first distribution into the second one. The mass distribution of intermediate mass fragments (IMFs, fragments with $A = 5-40$) can be approximated by a power law $A^{-\tau}$ [2,3,20]. The τ parameter decreases with the temperature, goes through the minimum at $T \approx 5-6$ MeV, and then increases again. The small values of τ indicate that the probability for survival of the biggest fragment decreases drastically with the temperature. This behavior may be associated with a phase transition in finite systems. It has been shown in many studies (see, e.g., [3,15,17] and references therein), that there are numerous peculiarities in this region, such as a plateau-like behavior of the caloric curve, large fluctuations of the temperature and of the number of the produced fragments, scaling laws for fragment yields, and other phenomena expected for critical behavior. Therefore, the temperature characterizing these phenomena is sometimes called the critical temperature for finite systems, and τ is considered as one of the critical exponents. The SMM can describe the critical behavior observed in the experiments [15–17]. However, in the present work we use the τ parametrization only for the characterization of shapes of the fragment mass distributions. In order to avoid any confusion with the standard definition of the critical temperature for nuclear matter, we note, in the following, the temperature corresponding to the critical phenomena as a breakup temperature for the disintegration of finite nuclei [5].

The decrease of the surface energy with increasing T/T_c [see formula (1)] influences the fragment production and, therefore, can be observed in the fragment distributions. In this Rapid Communication we show that this effect can be used for the evaluation of T_c by finding the minimum τ parameter τ_{min} and the corresponding temperature T_{min}. The physics behind the phenomenon is quite transparent: If the contribution of the surface energy is rapidly decreasing, a nucleus prefers to disintegrate into small fragments already at low temperatures. Simultaneously, fluctuations of size of the fragments increase considerably. As a result the mass distribution becomes flatter in the transition region, and this leads to a decrease of τ.

The SMM calculations were carried out for the Au nucleus ($A_0 = 197$, $Z_0 = 79$) at different excitation energies and at a freeze-out density of one-third of the normal nuclear density. This choice is justified by the previous descriptions of the experimental data obtained for peripheral collisions [9,12–16]. Below we present results as a function of both the temperature and the excitation energy, since they are related quantities [3].

We have started by using the standard value of the critical temperature implemented in the SMM, $T_c = 18$ MeV. This value is consistent with many theoretical studies [6,21]. In Fig. 1 we show typical mass and IMF charge distributions, $\langle N_A \rangle \sim A^{-\tau}$ and $\langle N_Z \rangle \sim Z^{-\tau_z}$, at an excitation energy $E_x = 7$ MeV/nucleon. One can see from this figure that the extracted τ and τ_z values are very close to each other, since the neutron-to-proton ratio of produced IMFs changes very little within their narrow charge range [9,22]. As seen from Fig. 2,
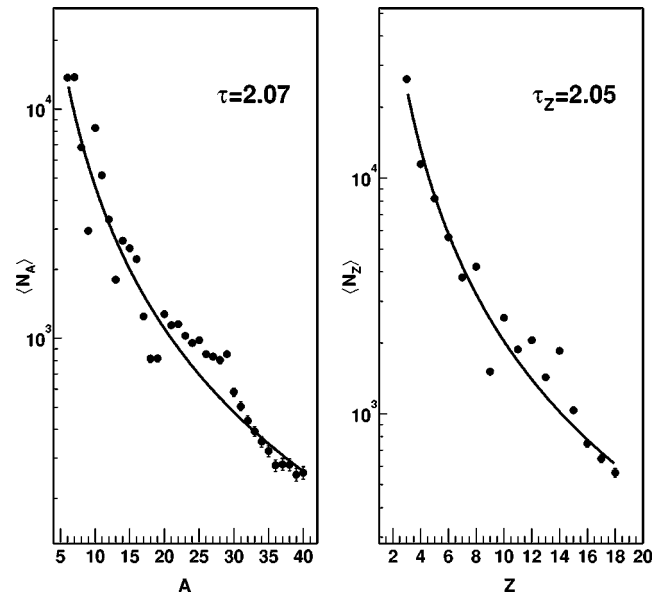

FIG. 1. Average fragment mass and charge yields $\langle N_A \rangle$ and $\langle N_Z \rangle$, after multifragmentation of Au nuclei at an excitation energy of 7 MeV/nucleon. Solids lines are $\sim A^{-\tau}$ and $\sim Z^{-\tau_z}$ fits of the IMF yields.

the dependences of these parameters versus excitation energy are nearly the same. The parameters obtained for primary hot fragments (excited nuclear matter drops) and after their secondary deexcitation (measured cold fragments) are also
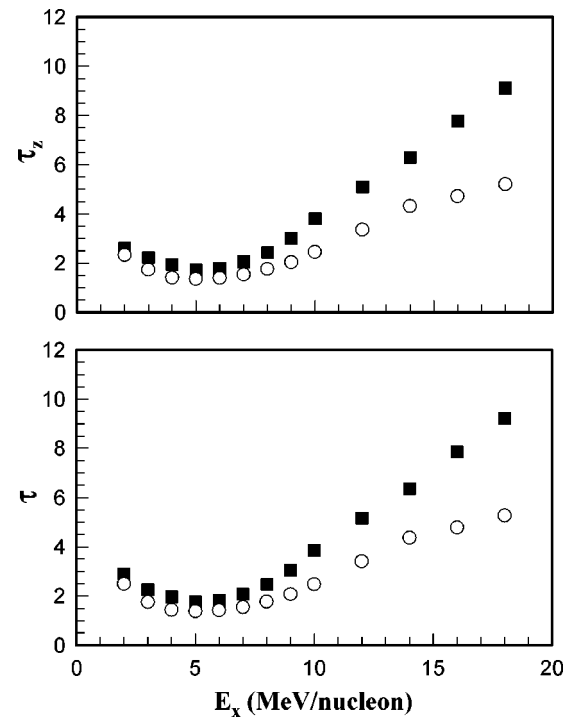

FIG. 2. Evolution of τ_z (top panel) and τ (bottom panel) parameters with the excitation energy E_x of Au sources calculated with the standard SMM parametrization. The open circles are for hot primary fragments and the full squares are for observed cold fragments.
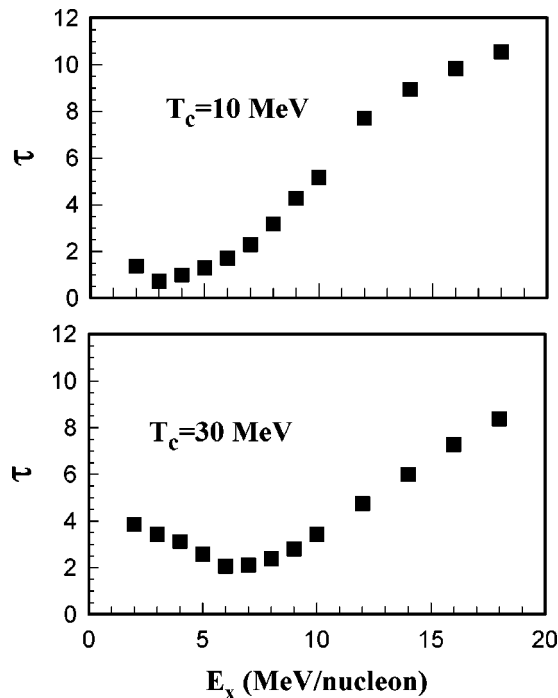
FIG. 3. Evolution of τ parameters in the SMM for cold fragments with the excitation energy E_x. The top panel is for the critical temperature $T_c = 10$ MeV, the bottom panel for $T_c = 30$ MeV.

shown in this figure. One can see that the difference between the two cases is smallest around the minimum τ parameter. Therefore, the τ_{min} point is weakly affected by secondary processes.

The critical temperature reflects the properties of nuclear matter, however, these properties depend on the composition of this matter. For example, T_c tends to decrease for neutron rich matter [23]. As discussed by many authors, see, e.g., [24], the critical temperature can be traced back towards neutron-rich matter by studies of the disassembly of nuclei far from stability. We consider T_c as a free parameter in the SMM and analyze how τ and the breakup temperature can change. In Fig. 3 we show the results for $T_c = 10$ and 30 MeV. In these cases the evolution of τ with the excitation energy is similar to the one shown in Fig. 2. However, the values of T_{min} and τ_{min} are essentially different. This reflects a considerable change of masses for the dominating fragments. These values are plotted in Fig. 4 versus the critical temperature. It is seen that both parameters increase with T_c and that they tend to saturate at $T_c \to \infty$ corresponding to the case of the temperature-independent surface. This behavior is expected, since in this case only the translational and bulk entropies of fragments, but not the surface entropy, influence the probability for the fragment formation.

In the case of neutron-rich matter, the contribution of the symmetry energy increases considerably. It is necessary to take into account the standard dependence of the τ parameter on the isospin of the source, while searching for T_c. We performed SMM calculations of multifragmentation of ^{124}Sn and ^{124}La nuclei, which can be used in experiments [25], and compared them with the results obtained for Au nuclei. This Sn nucleus is nearly as neutron-rich as the gold nucleus,
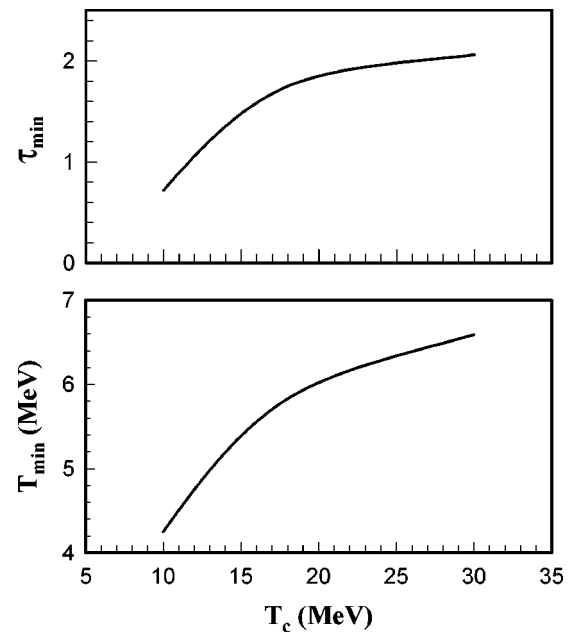

FIG. 4. The minimum of the τ parameter, τ_{min}, for cold fragments (top panel) and the corresponding temperature T_{min} (bottom panel) as function of the critical temperature T_c of nuclear matter.

while the La nucleus is neutron-poor. We have used the same model parameters as for the Au case, with the standard $T_c = 18$ MeV. It is seen from Fig. 5 that the neutron-poor source results in slightly lower microcanonical temperatures in the transition region ($E_x \approx 3 - 5$ MeV/nucleon). In Fig. 5
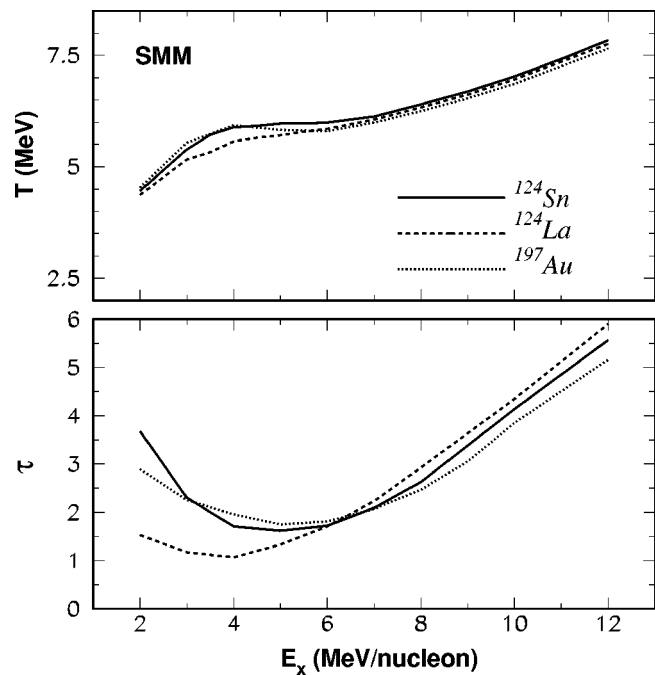

FIG. 5. The temperature (top panel) and τ parameters for cold fragments (bottom panel) versus excitation energy in multifragmentation. The solid, dashed, and dotted lines are SMM calculations performed for sources with different sizes or isospin (see the figure) with the standard $T_c = 18$ MeV.

we show also the evolution of the τ parameter with the excitation energy. The results for Au and Sn are very similar and different from those obtained for the La. One can conclude, that IMF distributions approximately scale with the size of the sources, and that they depend on the neutron-to-proton (N/Z) ratios of the sources. This is because the symmetry energy still dominates over the Coulomb interaction energy for these intermediate-size sources. One can see from Fig. 5 that the source with the lower N/Z ratio leads to smaller τ parameters, i.e., to the flatter fragment distribution. We can explain this as an effect of the isospin (i.e., the symmetry energy) on fragment formation: A high N/Z ratio of the source favors the production of big clusters, since they have a large isospin. Therefore, in the transition region, partitions consisting of small IMFs and a big cluster dominate. This leads to a very prominent U-shape distribution with large τ. When the N/Z ratio is low, the probability for a big cluster to survive is small and the system can disintegrate into IMFs, which have a favorable isospin in this case. The dominant fragment partitions tend to include IMFs of different sizes, and the fragment distribution is characterized by small τ. Since the difference in the τ parameters between the neutron-rich and neutron-poor sources is quite large, it can be easily identified. The evolutions of fragment mass distributions caused by decreasing T_c and changing N/Z ratio can interfere and, therefore, the influence of the critical temperature can be separated only after the comparison of experimental data with calculations.

In view of these theoretical findings it is instructive to demonstrate the possibility of the application of such an approach for the analysis of experimental data. Presently, there are several experimental analyses aimed at the extraction of the critical exponent τ in reactions with Au nuclei [15,17,26]. Those methods are not equivalent to the one suggested above, however, they are related to the fragment distributions, and the critical exponents can be used for an estimation of τ_{min} and T_c. The extracted breakup excitation energies E_x vary within 3.8 to 4.5 MeV/nucleon, while τ is in the range from 2.12 to 2.18. As seen from Figs. 2 and 3, for $T_c \geq 18$, at these excitation energies τ is larger than τ_{min}. We have performed an interpolation of the SMM calculations for the Au sources and found that they fit the experimental values of E_x and τ if T_c is in the range between 18 and 22 MeV. We have also seen from our analysis that τ_{min} is lower than the extracted τ by around 15%. The T_c estimated in this way is very close to the standard SMM parametrization. This conclusion is supported by the analyses of Refs. [15–17] showing that the standard SMM reproduces both the experimental critical exponents and other characteristics of produced fragments. It is interesting that in Ref. [17] a small critical exponent $\tau \approx 1.88$ is reported for multifragmentation of Kr nuclei, which have N/Z ratios lower than Au nuclei. This is an indication of the importance of the isospin effects, as discussed above. It is worth noting that recently a very close critical temperature ($T_c \approx 16.6$ MeV) was extracted from analysis of the break-up ("limiting") temperatures in Ref. [27]. It is also in agreement with the temperature $T_c \approx 20$ MeV obtained in the experiment of Ref. [28]. However, it would be important to identify regular changes of T_c, which needs involving new sources with different isospins.

In summary, we have pointed out that within the SMM the critical temperature of nuclear matter can influence the fragment production in multifragmentation of nuclei through the surface energy. We have suggested that this influence can be observed in the $A^{-\tau}$ parametrization of the fragment yields by finding the minimum τ parameter. In the experiments the measured values of the parameters are consistent with the standard SMM assumption and slightly higher values of the critical temperature, $T_c \approx 18 - 22$ MeV. The SMM predicts that variations of τ_{min} are especially large in the region of low T_c. Therefore, there is a possibility to investigate the decrease of T_c for nuclear matter under extreme conditions, by studying the evolution of τ_{min} and T_{min} in the multifragmentation of finite nuclei. This could be realized in the case of the neutron-rich nuclei delivered by current accelerators with radioactive heavy-ion beams [25]. The isospin of the source influences also the fragment production through the symmetry energy. We have demonstrated how the isospin affects the fragment distributions, that should be taken into account in these studies.

[1] G. Baym, H. Bethe, and C.J. Pethick, Nucl. Phys. **A175**, 225 (1971).

[2] J. Hüfner, Phys. Rep. **125**, 129 (1985).

[3] J.P. Bondorf, A.S. Botvina, A.S. Iljinov, I.N. Mishustin, and K. Sneppen, Phys. Rep. **257**, 133 (1995).

[4] J.P. Bondorf et al., Nucl. Phys. **A443**, 321 (1985).

[5] A.S. Botvina, A.S. Il'inov, and I.N. Mishustin, Sov. J. Nucl. Phys. **42**, 712 (1985).

[6] D.G. Ravenhall, C.J. Pethick, and J.M. Lattimer, Nucl. Phys. **A407**, 571 (1983).

[7] D.H.E. Gross, Rep. Prog. Phys. **53**, 605 (1990).

[8] S.E. Koonin and J. Randrup, Nucl. Phys. **A474**, 173 (1987).

[9] A.S. Botvina et al., Nucl. Phys. **A584**, 737 (1995).

[10] M. D'Agostino et al., Phys. Lett. B **371**, 175 (1996).

[11] C. Williams et al., Phys. Rev. C **55**, R2132 (1997).

[12] S.P. Avdeyev et al., Eur. Phys. J. A **3**, 75 (1998).

[13] W. Trautmann, Adv. Nucl. Dynam. **4**, 349 (1998).

[14] G. Wang et al., Phys. Rev. C **60**, 014603 (1999).

[15] M. D'Agostino et al., Nucl. Phys. **A650**, 329 (1999).

[16] R.P. Scharenberg et al., Phys. Rev. C **64**, 054602 (2001).

[17] B.K. Srivastava et al., Phys. Rev. C **65**, 054617 (2002).

[18] K.A. Bugaev et al., Phys. Rev. C **62**, 044320 (2000).

[19] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon, Oxford, 1980).

[20] A.L. Goodman, J.I. Kapusta, and A.Z. Mekjian, Phys. Rev. C **30**, 851 (1984).

[21] R. Ogul, Int. J. Mod. Phys. E **7(3)**, 419 (1998).

[22] A.S. Botvina and I.N. Mishustin, Phys. Rev. C **63**, 061601(R) (2001).

[23] H. Müller and B.D. Serot, Phys. Rev. C **52**, 2072 (1995).

[24] P. Bonche, S. Levit, and D. Vautherin, Nucl. Phys. **A436**, 265 (1985).

[25] K. Kezzar *et al.*, proposal of experiment S254 at SIS (GSI), http://www-kp3.gsi.de/www/kp3/proposals.html

[26] J.B. Elliott *et al.*, Phys. Rev. Lett. **88**, 042701 (2002).

[27] J.B. Natowitz *et al.*, nucl-ex/0204015, Phys. Rev. Lett. (submitted).

[28] V. A. Karnaukhov *et al.*, Phys. Rev. C (submitted).