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Quadrupole and octupole softness in the $N=Z$ nucleus ^{64}Ge

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Quadrupole and octupole softness in the even-even $N=Z$ nucleus ^{64}Ge is studied on the spherical shell model basis. We carry out the shell model calculation using the pairing plus quadrupole (QQ) plus octupole (OO) interaction with monopole corrections. It is shown that ^{64}Ge is an unstable nucleus with respect to both the quadrupole and octupole deformations, which is consistent with the previous discussions predicting the γ softness and octupole instability. It is demonstrated that the proton-neutron part Q_pQ_n of the QQ interaction is important for the γ softness or triaxiality.

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Heavy $N=Z$ nuclei with $A=56\sim80$ show strong shape variations such as prolate shape, oblate shape, prolate-oblate shape coexistence, and γ softness, depending on the mass number. These nuclei lie in transitional regions from spherical shape (e.g., ^{56}Ni [1]) to strong prolate deformation (e.g., ^{80}Zr [2]). The $N=Z=32$ nucleus $^{64}_{32}$Ge$_{32}$ is known to be a typical example showing γ-soft structure in $N=Z$ proton-rich unstable nuclei, according to theoretical calculations based on the mean-field approximation [3]. The calculations predict probable γ instability in the ground state, and triaxiality in the excited states, i.e., the quadrupole deformation $\beta_2\sim0.22$ and $\gamma\sim27°$.

Deformed shell model calculations predict that the nucleon numbers 34, 56, 88, and 134 are strongly octupole driving in nuclei where the Fermi surface lies near single-particle levels with $\Delta l=\Delta j=3$ [4]. We can expect that nuclei with $N=Z$ near the octupole magic numbers exhibit an especially strong octupole effect, because neutrons and protons contribute cooperatively. This does not necessarily mean a permanent octupole deformation. The level pattern of negative-parity states in ^{64}Ge is not a rotational one [3], and the sequence of the 3^-, 5^-, and 7^- levels is irregularly spaced. Thus, ^{64}Ge is an $N=Z$ proton-rich unstable nucleus manifesting a soft structure with respect to quadrupole and octupole deformations, from experimental and theoretical evidences. In fact, inclusion of the γ deformation improves the $E2$ transitions of negative-parity states in ^{68}Ge [5].

The spherical shell model approach could be more appropriate for describing various aspects of nuclear structure. It is desirable to add the $g_{9/2}$ orbital to the full pf shell ($f_{7/2},p_{3/2},f_{5/2},p_{1/2}$) for studying both positive and negative parity states of ^{64}Ge, but the shell model calculation in this space is impractical at present because of the huge dimension. So we restrict the model space to the $p_{3/2}$, $f_{5/2}$, $p_{1/2}$, and $g_{9/2}$ orbitals, and carry out the shell model calculation with the recently developed shell model code [6]. There are few effective shell-model interactions [7] in this model space.

Recently, an extended $P+QQ$ force [8] was applied to the $f_{7/2}$-shell nuclei. This interaction is schematic but works remarkably well. The conventional $P+QQ$ force was first suggested by Bohr and Mottelson, and widely used by Kisslinger and Sorensen, Baranger and Kumar, and many authors. Unlike the original application to heavy nuclei, the extended $P+QQ$ interaction is isospin-invariant [9]. In this Rapid Communication, we introduce the octupole-octupole (OO) force into the extended $P+QQ$ force model to describe negative-parity states. This interaction is quite useful for studying not only the γ softness but also the octupole instability mentioned above. The QQ and OO forces are the long-range and the deformation-driving part of the effective interaction. Contrary to this, the monopole pairing force can be associated with short-range force, and restores the spherical shape. Thus, the competitions among the QQ, OO, and monopole pairing forces are expected to be important for shape transitions of quadrupole and octupole deformations in ^{64}Ge. The $P+QQ$ force with the OO interaction will be suitable for studying the monopole pairing, quadrupole, and octupole correlations. Recently, the fp shell model calculation [10] with the FPD6 interaction [11] has been performed in ^{64}Ge as a test case for quantum Monte Carlo diagonalization (QMCD) method. The projected shell model calculation [14] has been performed in ^{64}Ge modifying the standard Nilsson parameters.

Since protons and neutrons in the $N=Z$ nuclei occupy the same levels, one would expect strong proton-neutron (p-n) interactions [15]. In particular, one of the most interesting questions in the study of nuclear structure is what roles the p-n interaction play in the nuclear deformation. The long-range p-n isoscalar ($T=0$) interaction between valence nucleons has been suggested to be a source of the nuclear deformation [16]. On the other hand, the isoscalar QQ interaction used in the $P+QQ$ force model has very strong p-n component Q_pQ_n, which gives rise to nuclear quadrupole deformation [17]. The Q_pQ_n interaction is expected to be important for quadrupole collectivity in ^{64}Ge. The rotational behavior of $T=0$ and $T=1$ bands in the odd-odd $N=Z$ nucleus ^{62}Ga is recently studied using the spherical shell model and the cranked Nilsson-Strutinsky model [18].

In order to study the octupole correlation, let us introduce an isoscalar octupole interaction H_{OO} with the force strength χ_3 to the extended $P+QQ$ model [8] with monopole corrections $H_{\rm m}^{\rm corr}$:

$$H=H_0+H_{P_0}+H_{P_2}+H_{QQ}+H_{OO}+H_{\mathrm{m}}^{\mathrm{corr}}=\sum_{\alpha} \varepsilon_a c_{\alpha}^{\dagger} c_{\alpha}$$

$$-\sum_{J=0,2} g_J \sum_{M\kappa} P_{JM1\kappa}^{\dagger} P_{JM1\kappa} - \frac{1}{2}\sum_{M} \chi_2 :Q_{2M}^{\dagger} Q_{2M}:$$

$$-\frac{1}{2}\sum_{M} \chi_3 :O_{3M}^{\dagger} O_{3M}: + H_{\mathrm{m}}^{\mathrm{corr}}, \qquad (1)$$

where ε_a is a single-particle energy, $P_{JMT\kappa}$ is the pair operator with angular momentum J and isospin T, and Q_{2M} (O_{3M}) is the isoscalar quadrupole (octupole) operator. Due to the isospin-invariance, each term of the above Hamiltonian includes $p-n$ components, which play important roles in $N=Z$ nuclei.

We carried out shell model calculations in a model space restricted to the $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, and $1g_{9/2}$ orbitals (called pfg-shell henceforth). The model assumes a closed $^{56}_{28}\mathrm{Ni}_{28}$ core and does not allow for core breaking. The neutron single-particle energies of $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, and $1g_{9/2}$ in this pfg-shell region can be read from the low-lying states of ^{57}Ni, because the low-lying states of ^{57}Ni are well characterized as pure single-particle levels when ^{56}Ni is a closed shell core. The adopted single-particle energies relative to the $2p_{3/2}$ are $\varepsilon_{p3/2}=0.0$, $\varepsilon_{f5/2}=0.77$, $\varepsilon_{p1/2}=1.11$, and $\varepsilon_{g9/2}=3.70$ in MeV [19]. Since the above Hamiltonian is assumed to be an isospin-invariant, the proton single-particle energies are taken as the same values as the neutron single-particle energies. The force strengths of the extended $P+QQ$ interaction are taken so as to reproduce the energy levels of low-lying states in ^{64}Ge as follows:

$$g_0=0.426(42/A), \quad g_2=0.274(42/A)^{5/3},$$

$$\chi_2=\chi_2^0(42/A)^{5/3}/b^2=0.567(42/A)^{5/3}/b^2, \qquad (2)$$

$$\chi_3=\chi_3^0(42/A)^2/b^6=0.275(42/A)^2/b^6,$$

where g_0, g_2, χ_2, and χ_3 are the monopole pairing, quadrupole-pairing, QQ, and OO force strengths, respectively. We adopt the harmonic-oscillator range parameter $b \sim A^{-1/3}$, the effective charge $e_p=1.50e$ for proton and $e_n=0.50e$ for neutron. We adjust phenomenologically force strengths of several monopole corrections so as to approximately reproduce the low-lying energy levels of ^{64}Ge. These force strengths can also reproduce the low-lying energy levels of $^{58-66}$Ni, $^{60-64}$Zn, ^{66}Ge, and ^{68}Se.

In Fig. 1, calculated energy spectra are compared with experimental data for ^{64}Ge. Two side bands are shown in addition to the ground-state band, i.e., positive-parity band on the band head 2^+ and negative-parity band on the band head 3^-. The calculations reproduce the observed three bands at good energies. The agreement between theory and experiment for the ground-state band up to spin $I=8$ is good. The calculated $B(E2)$ value between the ground state and the first excited $I=2^+$ state is $B(E2;2_1^+\rightarrow 0_1^+)=245.3$ (e^2 fm^4) corresponding to the quadrupole deformation $\beta \sim 0.2$. This value is consistent with the predictions of $\beta \sim 0.22$ by Möller and Nix [20] and of $\beta \sim 0.22$ by Ennis *et al.* [3], and is comparable to the experimental data 12 W.u. of ^{66}Ge nucleus.

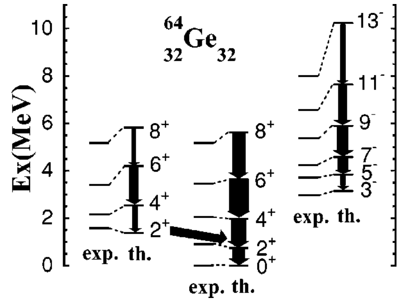

FIG. 1. Comparison of experimental and calculated energy levels of ^{64}Ge. The arrows designate $E2$ transitions with the calculated $B(E2)$ values indicated by their widths.

The calculated occupation numbers of the $p_{3/2}$, $f_{5/2}$, $p_{1/2}$, and $g_{9/2}$ orbitals in the ground-state band are 3.6, 3.2, 0.6, and 0.6 on the average, respectively. More than four nucleons are excited from the unperturbed configuration $(p_{3/2})^8$. The full fp shell model calculation [10] with the FPD6 interaction [11] using the QMCD method has recently been performed for low-lying $I=0_1^+$, 2_1^+, 2_2^+, and 4_1^+ states of positive parity in ^{64}Ge. The FPD6 calculation predicts the deformation $\beta_2 \sim 0.28$ which is somewhat larger than the other predictions $\beta_2 \sim 0.22$, and gives the triaxiality $\gamma \sim 27°$ which is consistent with the others predictions. The FPD6 interaction seems too strong to yield appropriate collectivity [12], due to its drawback [13]. This can be seen from the occupation numbers of $f_{7/2}$, $p_{3/2}$, $f_{5/2}$, and $p_{1/2}$ which are 15.1, 2.6, 5.5, and 0.8, respectively. Two more nucleons are jumping to the orbitals above $p_{3/2}$ as compared with our result. The stronger collectivity caused by the FPD6 interaction is probably attributed to the mixture of the three orbitals ($f_{7/2}$, $p_{3/2}$, and $f_{5/2}$) due to the large matrix elements between ($f_{7/2}$,$p_{3/2}$) and $f_{5/2}$. This is the reason why $B(E2;2_1^+\rightarrow 0_1^+)=5\times 10^2(e^2\ \mathrm{fm}^4)$ obtained in Ref. [10] is almost twice that of ours 245.3(e^2 fm^4).

The ratio of excitation energies for the 4_1^+ state and the 2_1^+ state gives additional information with respect to the shape. This ratio is 3.33 for a rigid rotor, 2.0 for a pure vibrator, and ~ 2.4 for a γ-soft nucleus. Therefore, the ratio 2.65 in the calculation indicates γ softness. We can also see the γ-soft nature in $E2$ transitions. The $B(E2)$ value of $2_2^+ \rightarrow 2_1^+$ transition is larger than $B(E2;2_2^+\rightarrow 0_1^+)$, and the ratio $B(E2;2_2^\dagger\rightarrow 2_1^\dagger)/B(E2;2_2^\dagger\rightarrow 0_1^\dagger)$ is ~ 27, corresponding to $\gamma \sim 26°$ in the Davydov model [21]. The present result is in agreement with the triaxiality estimated from the experimental data and the other theoretical models, though it is difficult for the shell model calculation to definitely discuss γ softness and triaxiality.

On the other hand, the low-lying negative-parity states with $I=3^-$, 5^-, 7^-, and 9^- are nicely reproduced. The octupole force newly introduced to our model plays an essential role in describing the negative-parity states. The value of $B(E2;5^-\rightarrow 3^-)$ is small, while the $B(E2)$ values between higher states above 3^- are large. This indicates a different

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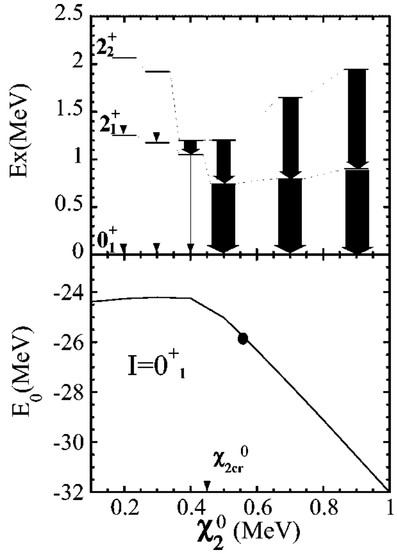

FIG. 2. The excitation energies of the first and second excited 2^+ states in the upper figure and the 0^+ ground-state energy in the lower figure, as a function of the quadrupole force strength. The arrows indicated by their widths designate $E2$ transitions with the calculated $B(E2)$ values.

structure of 3^- from the higher negative-parity states. The same is known from the occupation numbers of $p_{3/2}$, $f_{5/2}$, $p_{1/2}$, and $g_{9/2}$ orbitals. Their values of the $I=3^-$ state are, respectively, 4.0, 1.2, 0.6, and 2.2, while the average values of the negative-parity states above 3^- are 3.4, 2.7, 0.5, and 1.4. More nucleons are jumping from the unperturbed configuration $(p_{3/2})^7(g_{9/2})^1$ in the 3^- state as compared with the states above 3^-. The $I=3^-$ state is mainly a coherent mixture of neutron and proton $(p_{3/2}f_{5/2}g_{9/2})_{3^-}$ configurations. The present result is consistent with the calculations by Petrovici and Faessler [5]. In Fig. 1, the calculated 11^- and 13^- levels are considerably higher than the observed excitation energies. This may suggest that excitation from the $f_{7/2}$ orbital in ^{56}Ni core should be taken into account for these states.

Figure 2 shows the excitation energies of the first and second 2^+ states and the 0^+ ground-state energy as a function of the quadrupole force strength χ_2^0. The other force strengths are fixed to the values of Eq. (2). We can see a level crossing of the first excited 2_1^+ state and the second excited 2_2^+ state near the crossing strength $\chi^0_{2\mathrm{cr}}\sim 0.45$ MeV. When the quadrupole force strength is apart from $\chi^0_{2\mathrm{cr}}$, the second excited $I=2_2^+$ state rises abruptly in energy, while the first excited $I=2_1^+$ state does not change largely. This means that γ softness or triaxiality is realized beyond the crossing point. The ground-state energy rapidly decreases beyond this crossing point, while it is almost constant for $0<\chi_2^0<0.45$ MeV.

There seems to be a critical point of phase transition with respect to the quadrupole correlation. Indeed the $B(E2;2_1^+\to 0_1^+)$ values dramatically increase when the quadrupole force strength goes beyond the critical point. The force strength adopted in Fig. 1 is $\chi_2^0=0.567$ MeV denoted by a solid circle in the lower part of Fig. 2, and its position lies in the deformed region but near the critical point. We can also see that the $B(E2;2_2^+\to 2_1^+)$ value is large for $\chi_2^0>0.45$ MeV. This is consistent with the γ softness or the triaxiality discussed above.

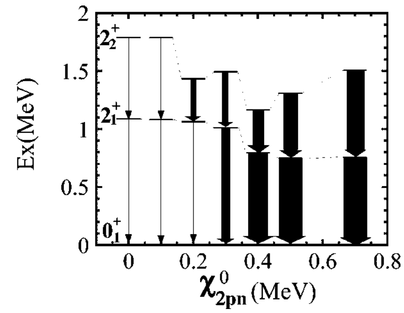

FIG. 3. Excitation energies of the first and second excited 2^+ states as a function of the p-n quadrupole force strength χ^0_{2pn}. The arrows designate $E2$ transitions with the calculated $B(E2)$ values indicated by their widths.

As mentioned earlier, it is very interesting to study roles of the proton-neutron part of the QQ force (Q_pQ_n). We make the study by varying the strength of Q_pQ_n (χ^0_{2pn}) and keeping the other force strengths of Eq. (2). This is effective in seeing the dependence of quadrupole deformation on Q_pQ_n, though the Hamiltonian without the isovector type of QQ force stops beeing isospin invariant. Figure 3 shows the excitation energies of the first and second 2^+ states and the $B(E2)$ values as a function of χ^0_{2pn}. The first $I=2_1^+$ state is flat in energy with respect to the force strength χ^0_{2pn}. However, the second excited $I=2_2^+$ state strongly depends on χ^0_{2pn}, and becomes lowest near $\chi^0_{2pn}\sim 0.35$ MeV. In the isoscalar QQ force, χ^0_{2pn} is equal to the magnitude of proton-proton (pp) and neutron-neutron (nn) force strengths due to the isospin-invariance, i.e., $\chi^0_{2pn}=\chi^0_{2pp}=\chi^0_{2nn}$. The force strength $\chi^0_{2pn}=0.567$ MeV which is adopted in Fig. 1 leads to the large $B(E2)$ value $B(E2;2_1^+$

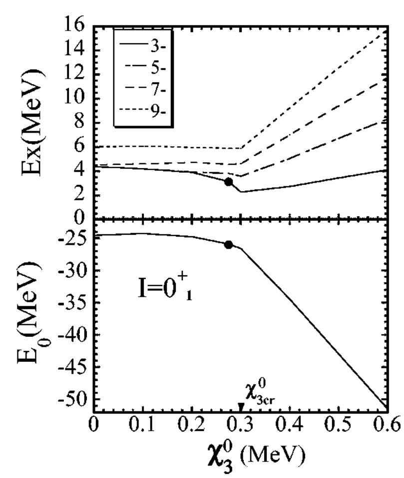

FIG. 4. Excitation energies of the negative-parity states (in the upper figure) and the 0^+ ground-state energy (in the lower figure) as a function of the octupole force strength. The solid circles show the energies for the adopted force strength.

$\rightarrow 0_1^+) = 245.3$ (e^2 fm^4) corresponding to the quadrupole deformation $\beta \sim 0.2$, as mentioned above. We can see, however, that $B(E2;2_1^+ \rightarrow 0_1^+)$ is very small for $0 < \chi_{2pn}^0 < 0.35$ MeV. This suggests that a phase transition occurs near the critical point $\chi_{2pn\text{cr}}^0 \sim 0.35$ MeV. The $B(E2)$ value of the $2_2^+ \rightarrow 2_1^+$ transition also becomes large for $\chi_{2pn}^0 > 0.35$ MeV, which results in the expected triaxiality $\gamma \sim 26°$ when $\chi_{2pn}^0 = 0.567$ MeV. Thus, the $Q_p Q_n$ interaction plays an important role for the γ softness or the triaxiality in ^{64}Ge.

Let us lastly study the negative-parity states as a function of the octupole force strength in Fig. 4. The other force strengths are fixed to those of Eq. (2). As mentioned above, the $I = 3_1^-$ state is very collective, and the excitation energy of the $I = 3_1^-$ state decreases as the octupole force strength increases until $\chi_3^0 \sim 0.3$ MeV, and increases as it goes beyond this point. The other negative-parity states have an insignificant dependence with respect to the force strength χ_3^0 until the critical point, and increase for $\chi_3^0 > 0.3$. The ground-state energy is almost constant for $0 < \chi_{3\text{cr}}^0 < 0.3$ MeV and decreases quickly when going beyond this critical point. It seems that a phase transition occurs near the critical force strength $\chi_{3\text{cr}}^0 \sim 0.3$ MeV. The octupole force strength $\chi_3^0 = 0.275$ MeV adopted in Fig. 1 is very close to the critical point $\chi_{3\text{cr}}^0$. Thus, ^{64}Ge seems to be near an octupole instability.

In summary, we have studied quadrupole and octupole correlations in the even-even $N = Z$ nucleus ^{64}Ge by means of spherical shell model calculations. The $P + QQ$ force model including octupole interaction and monopole corrections, which is schematic but realistic, is adopted for describing the quadrupole and octupole correlations. It is shown that ^{64}Ge is an unstable nucleus with respect to both the quadrupole and octupole deformations. The present results reveal that the *p-n* QQ interaction ($Q_p Q_n$) induces an onset of quadrupole deformation and γ softness. It can be expected to play an important role in the prolate-oblate shape coexistence of the neighboring even-even $N = Z$ nucleus ^{68}Se, which has been recently observed [22,23].

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