PHYSICAL REVIEW C **66**, 051302(R) (2002)

Nuclear level densities within the relativistic mean-field theory

B. Nerlo-Pomorska,[1] K. Pomorski,[1,2] J. Bartel,[2] and K. Dietrich[3]
[1]*Katedra Fizyki Teoretycznej, Uniwersytet Marii Curie-Skłodowskiej, Lublin, Poland*
[2]*Institut de Recherches Subatomiques IN2P3-CNRS and Université Louis Pasteur, Strasbourg, France*
[3]*Physik Department, Technische Universität München, München, Germany*
(Received 19 July 2002; published 27 November 2002)

Self-consistent calculations for excited nuclei are performed in the framework of the relativistic mean-field theory at temperatures between 0 and 4 MeV for 193 spherical even-even nuclei. The temperature dependent macroscopic part of the thermal energy was approximated by a liquid-drop type formula. The average dependence of the single particle level-density parameter *a* on mass and isospin of the nucleus is established.

DOI: 10.1103/PhysRevC.66.051302 PACS number(s): 21.10.Ma, 21.60.−n, 24.60.Dr, 25.70.Jj

The temperature dependence of the internal energy of a nucleus is of special importance for the description of decay modes of compound nuclei formed in heavy-ion collisions [1] and for all kinds of transport theories. Similarly to models based on the extended Thomas-Fermi (ETF) approximation [2,3] of the nuclear many-body problem in connection with Skyrme interactions we observe in the relativistic mean-field theory (RMFT) [4] the nearly exactly quadratic dependence on temperature of the macroscopic part of the nuclear energy.

Using the Strutinsky shell-correction method [5] we can subtract the shell effects from the RMFT self-consistent energy in a similar way as this was done in Ref. [6] for nuclear-structure calculations with the Gogny effective interaction. Estimates obtained in this way for the macroscopic binding energies at zero temperature were used as reference points to observe the change of the average internal energies with temperature. The Strutinsky shell-correction method can also be formulated at finite temperature [7]. We did not attempt to use this prescription here, but rather establish an approximate way to describe the fact that shell effects are washed out when the nuclear temperature increases, so that for $T \geqslant 3$ MeV they have essentially disappeared and the nucleus has become a semiclassical object. Similarly as done in Ref. [3] in the case of Skyrme forces we approximate the change of the RMFT internal energies with temperature by a liquid-drop–like formula.

The temperature dependence of the thermal energy gives some information about the single-particle level-density in the considered nucleus. In our analysis we do not include the effects of the pairing interaction because for nuclear temperatures larger than 1–1.5 MeV the pairing correlations in nuclei are negligibly small, as was shown in Ref. [8] on the basis of shell-model Monte-Carlo calculations. Also the effect of the projection on a given particle number is neglected in our calculations. We conserve only the average number of protons and neutrons in the nucleus when the temperature increases. It was shown, e.g., in Ref. [9] that the energy obtained in such an approximation for higher temperatures differs only slightly from the projected one. We would like also to stress that in our model we obtain only estimates for the single-particle level-density as contributions from the collective degrees of freedom are, by the very nature of the mean-field approach, not included in our calculations.

We give in the following a short overview of our approach and present the numerical results obtained in the RMFT for 193 spherical even-even nuclei. Investigating the temperature dependence of the *average* RMFT nuclear energies we are able to determine the dependence of the level-density parameter *a* on mass and isospin.

The calculations were performed for spherical symmetry within the RMFT with the NL3 set of Lagrangian parameters [4]. The self-consistent procedure includes the resolution of the Dirac equations for the fermions n,p and of the Klein-Gordon equations for the mesons ρ,ω,σ. These equations are solved iteratively until a self-consistent solution is reached.

The self-consistent mean-fields result in single-particle level densities $g_q(\varepsilon), q=\{n,p\}$ which show a significant shell structure at low nuclear excitation energies. At higher temperatures ($T \approx 3$ MeV) the shell effects disappear. The shell structure in the level density can be removed by the Strutinsky prescription [5]. One obtains the average level density $\tilde{g}(\varepsilon)$ by a folding procedure

$$\tilde{g}_q(\varepsilon)=\frac{1}{\gamma\sqrt{\pi}}\int_{-\infty}^{+\infty} f_n\left(\frac{\varepsilon-\varepsilon'}{\gamma}\right) e^{-(\frac{\varepsilon-\varepsilon'}{\gamma})^2} g_q(\varepsilon')\,d\varepsilon'. \quad (1)$$

Here γ is a smoothing parameter and f_n is a polynomial of order $2n$ corresponding to the so-called curvature correction. The shell-correction energy can be obtained by subtracting the Strutinsky smoothed energy $\tilde{E}$ from the sum of single-particle energies $E_{s.p.}$. At zero temperature we have

$$\begin{aligned}\delta E^{(q)}_{\rm shell}(0) &= E^{(q)}_{s.p.}(0)-\tilde{E}^{(q)}(0)\\ &= \sum_\nu 2\varepsilon^{(q)}_\nu - 2\int_{-\infty}^{\tilde{\lambda}_q} \varepsilon\tilde{g}_q(\varepsilon)\,d\varepsilon. \end{aligned} \quad (2)$$

The position of the average Fermi energies $\tilde{\lambda}_q$ is fixed by the condition

$$\mathcal{N}_q = 2\int_{-\infty}^{\tilde{\lambda}_q} \tilde{g}_q(\varepsilon)\,d\varepsilon, \quad \mathcal{N}_q=\{N,Z\} \quad (3)$$

for the particle number. To obtain the shell-correction energy $\delta E_{\rm shell}(T)$ at finite temperature we have taken advantage of the fact that the shell structure is washed out as the nuclear temperature increases. It was shown in Refs. [10,11] that magnitude of the shell-correction energy decreases with temperature by a factor

$$\phi(T)=\frac{\tau}{\sinh\tau}, \qquad \tau=\frac{2\pi^2}{\hbar\omega}T, \tag{4}$$

where $\hbar\omega$ represents the average spacing between two mayor shells which is parametrized in the usual way as $\hbar\omega=41\ \mathrm{MeV}/A^{1/3}$. We then have

$$\delta E_{\rm shell}(T)=\delta E_{\rm shell}(0)\,\phi(T) \tag{5}$$

and this shell-correction energy at finite temperature should be subtracted from the corresponding self-consistent energy to obtain the RMFT estimates of the liquid-drop type energy at finite temperature

$$\widetilde{E}(T)=E_{\rm RMFT}(T)-\delta E^{(n)}_{\rm shell}(T)-\delta E^{(p)}_{\rm shell}(T). \tag{6}$$

It is known that for medium-heavy nuclei the nuclear shell effects disappear at $T\approx 2.5-3$ MeV [12] and we expect that the RMFT results for temperatures above this value are of purely semiclassical nature.

The temperature enters into the calculation of the self-consistent energy via the levels occupation in the form of a Fermi function [12]:

$$n_T(\varepsilon)=\left[1+\exp\left(\frac{\varepsilon-\lambda_q}{T}\right)\right]^{-1}, \tag{7}$$

where the Fermi energy λ_q is found from the particle-number-conservation condition

$$2\sum_{\nu} n_T(\varepsilon_\nu)=\mathcal{N}_q. \tag{8}$$

The average energy of a nucleus obtained self-consistently by solving the RMFT equations with the temperature dependent occupation numbers $n_T(\varepsilon)$ and by the shell-correction procedure described above can be approximated by a parabola

$$E_{\rm RMFT}(T)=\widetilde{E}(T)+\delta E_{\rm shell}(T)\approx\widetilde{E}(0)+aT^2+\delta E_{\rm shell}(T), \tag{9}$$

where a is the average level-density parameter which is related to the entropy S by

$$S(T)=2aT. \tag{10}$$

We have neglected pairing correlations when evaluating the RMFT self-consistent energy so that there is no need to include the pairing correction energy in Eq. (9).

From Eqs. (9) and (10) we can conclude that, except for shell effects, the Helmholtz free energy takes the form

$$\widetilde{F}(T)=\widetilde{E}(T)-TS(T)=\widetilde{E}(0)-aT^2. \tag{11}$$

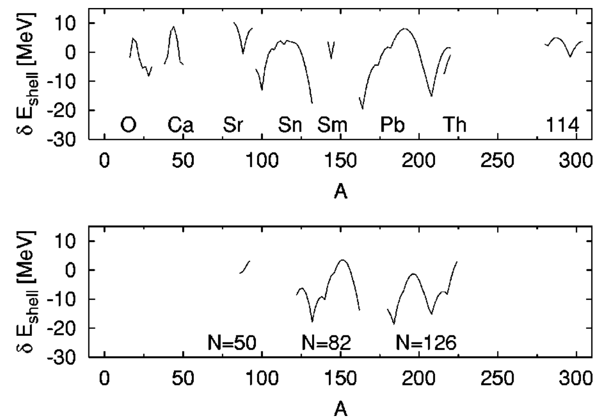

FIG. 1. Strutinsky shell corrections evaluated within the RMFT for some isotopic (top) and isotonic chains (bottom).

The proper estimate of the level-density parameter a, its mass and isospin as well as its deformation dependence is crucial whenever the free energy has to be used, as, e.g., when evaluating fission barriers at finite temperature. Usually, in a rough estimate, this quantity is assumed to be proportional to the mass number

$$a=\frac{A}{n}\ \ \mathrm{MeV}^{-1} \tag{12}$$

and as a crude but quite practical rule one can take $n\approx 10$.

To study the magnitude of the level-density parameter a throughout the periodic table we attempt to give an expression of a as function of the mass number A and the reduced isospin parameter $I=(N-Z)/A$. To this aim we have extracted its values for a very large sample of nuclei ranging from ^{16}O to the region of super-heavy elements by fitting the average energy at finite temperature $\widetilde{E}(T)$ to Eq. (9).

The macroscopic part of the self-consistent energy can be nicely approximated by the liquid-drop type formula [6,13]. The same should be true for the macroscopic part of the excitation energy $\widetilde{E}(T)$ of a hot nucleus [3]. So the most consistent way to approximate the energy (9) would be expressing the parameter a in a liquid-drop–like formula:

$$a=a_{\rm vol}(1-\kappa_{\rm vol}I^2)A+a_{\rm surf}(1-\kappa_{\rm surf}I^2)A^{2/3}+a_{\rm Coul}Z^2A^{-1/3}. \tag{13}$$

Our calculations were performed for 193 spherical even-even nuclei ranging from $Z=8$ to $Z=114$ with several series of isotopic and isotonic chains placed between the neutron and proton drip lines. We have chosen the nuclei having quadrupole moments practically equal to zero, according to Ref. [14]. These are $^{16-24}$O, $^{26-30}$Ne, ^{32}Mg, $^{30-36}$Si, $^{38-50}$Ca, $^{82-90}$Sr, $^{96-140}$Sn, $^{80-84}$Sm, $^{162-220}$Pb isotopes, the isotones $N=50$ with $Z=36-42$, $N=82$ with $Z=40-80$, and $N=126$ with $Z=54-98$ plus 60 other spherical nuclei along the β stability line.

The Strutinsky shell corrections $\delta E_{\rm shell}$ evaluated at zero temperature are presented on the upper and lower part of Fig. 1 for the considered isotopic and isotonic chains, respectively. One notices that the amplitude of $\delta E_{\rm shell}$ exceeds 10

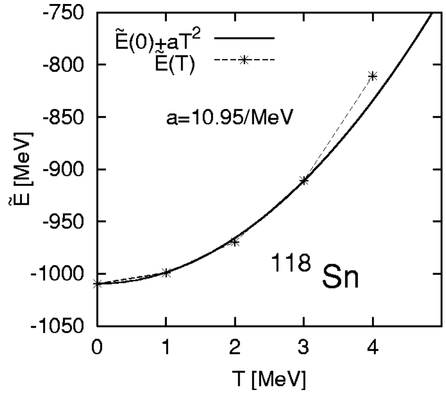

FIG. 2. Temperature dependence of the smooth part of the RMFT energy of ^{118}Sn (stars) and its quadratic approximation (solid line).

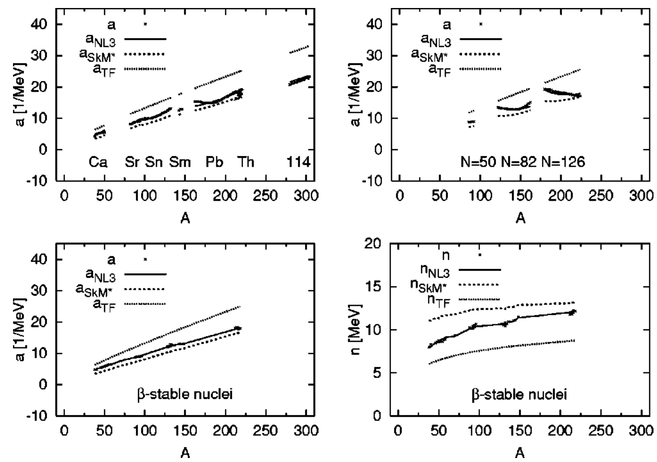

FIG. 3. The average level-density parameter a obtained in the RMFT model (stars) for seven chains of isotopes (upper left), for three chains of isotones (upper right), and for a series of β-stable nuclei (lower left) is compared with the estimates of Ref. [15] for a diffuse Fermi gas (dots) and with the results obtained in [3] within the extended Thomas-Fermi approximation to the SkM* Skyrme interaction (dashed lines). The corresponding factors n of Eq. (12) are given on the lower right part of the figure. Solid lines correspond to the fit made with the LD-like expression (13).

MeV for some nuclei. The temperature dependence of these shell corrections are approximated according to Eq. (5).

For all nuclei the self-consistent RMFT energy $E_{\rm RMFT}(T)$ grows with temperature, but its slope depends strongly on the proton and neutron numbers Z and N of the isotope. This dependence is almost parabolic for temperatures $T \leqslant 3$ MeV when the shell-correction energy is removed from the energies $E_{\rm RMFT}(T)$ by the Strutinsky procedure [Eqs. (1)–(6)]. A typical example of such a dependence is plotted in Fig. 2 for the nucleus ^{118}Sn. The stiffness of the parabola is just the average level-density parameter a in Eqs. (9), (10). As the variation of the nuclear energy as function of the temperature is considerably larger than the shell-correction energy $\delta E_{\rm shell}$ this difference between the full RMFT energies and their semiclassical counterpart would be hardly visible on the figure.

One also notices that the quality of the quadratic fit of the level-density parameter a through Eq. (9) deteriorates considerably when the energies obtained at $T=4$ MeV are included, as opposed to the one where energies only up to $T=3$ MeV are taken into account thus indicating the limits of validity of the low-temperature expansion which is behind Eq. (9).

Using the values of the level-density parameter a obtained through this fit truncated at $T=3$ MeV as an input to Eq. (13) for the 193 nuclei of our sample one obtains following set of coefficients:

$$a_{\rm vol}=0.0126\ \ {\rm MeV}^{-1}, \quad \kappa_{\rm vol}=6.275$$
$$a_{\rm surf}=0.3804\ \ {\rm MeV}^{-1}, \quad \kappa_{\rm surf}=1.101 \tag{14}$$
$$a_{\rm Coul}=0.00014\ \ {\rm MeV}^{-1}.$$

One notices that the volume and Coulomb terms obtained here are considerably smaller than the corresponding values in Ref. [3]. But the average behavior of a as extracted from our RMFT results is very nicely reproduced by our fit to Eq. (13) as can be seen on Fig. 3 which shows the average level-density parameter a obtained in the RMFT model (stars) for seven isotopic (upper lhs) and three isotonic chains (upper rhs) and compares it with the estimates made in Ref. [15] for a diffuse Fermi gas (dots) and with the results of [3] obtained within the extended Thomas-Fermi approximation for the SkM* Skyrme interaction [16] (dashed lines).

The solid lines on Fig. 3 correspond to the fit made with the LD-like formula (13). A similar plot but for the β-stable nuclei is presented in the lower lhs of Fig. 3. It is seen that the RMFT estimates are smaller than the data obtained for the diffuse Fermi gas and slightly larger than those evaluated with the Skyrme force. The average A dependence of the a parameter is similar in the two self-consistent models. The corresponding factors n of Eq. (12) are presented in the lower rhs of the figure.

It might seem astonishing that the level-density parameter a *decreases* with increasing particle number for the $N=126$ isotonic chain (see upper right corner of Fig. 3), since one rather expects that adding to a system more degrees of freedom (more particles) should in general increase the number of levels of that system. It is, however, exactly the point we would like to make here, that a quantity such as the the level-density parameter a can have a strong isospin dependence [see Eq. (12)] which works against this general tendency. A similar behavior is observed when studying the isospin dependence of charge rms radii (see Ref. [17]).

The following conclusions can be drawn from our results:

(i) The temperature dependence of the RMFT internal energies is very closely parabolical for $T \leqslant 3$ MeV.

(ii) The single-particle level density parameter a varies strongly with the mass and isospin of the nucleus.

(iii) The liquid-drop like formula (13) describes the variation of a with A and I quite accurately.

(iv) The factor n in Eq. (12) is not constant as often assumed in a rough estimate but varies between 8 and 16 and depends on mass and isospin of the nucleus.

Our LD-like formula for the average level-density parameter could be quite useful in calculations of the light-particle emission probabilities from hot compound nuclei, where it enters via the Bethe-Wheeler formula [1]. Our results could be also used to estimate the variation of fission-barrier heights with temperature.

Two of us (B.N.P. and K.P.) are very grateful for the nice hospitality extended to us by the Nuclear Theory Groups of the TU Munich and of the IReS in Strasbourg. The work was partially sponsored by the State Committee for Scientific Research under Contract No. 2 P03B 115 19 and the IN2P3-Polish Laboratories Convention, Project No. 99–95.

[1] K. Pomorski, B. Nerlo-Pomorska, A. Surowiec, M. Kowal, J. Bartel, K. Dietrich, J. Richert, C. Schmitt, B. Benoit, E. de Goes Brennand, L. Donadille, and C. Badimon, Nucl. Phys. **A679**, 25 (2000).
[2] M. Brack, C. Guet, and H.-B. Håkansson, Phys. Rep. **123**, 275 (1985).
[3] C. Guet, F. Strumberger, and M. Brack, Phys. Lett. B **205**, 427 (1988).
[4] P. Ring, Prog. Part. Nucl. Phys. **37**, 193 (1996).
[5] V.M. Strutinsky, Nucl. Phys. **A95**, 420 (1967).
[6] M. Kleban, B. Nerlo-Pomorska, J.F. Berger, J. Dechargé, M. Girod, and S. Hilaire, Phys. Rev. C **65**, 024309 (2002).
[7] M. Brack and P. Quentin, Nucl. Phys. **A361**, 35 (1981).
[8] D.J. Dean, S.E. Koonin, K. Langanke, P.B. Radha, and Y. Alhassid, Phys. Rev. Lett. **74**, 2909 (1995).
[9] R. Rossignoli, P. Ring, and N. Dinh Dang, Phys. Lett. B **297**, 9 (1992).
[10] A. Bohr and B. Mottelson, *Nuclear Structure,* Vol. II (Benjamin, New York, 1969).
[11] M. Brack and R. K. Bhaduri, *Semiclassical Physics*, Frontiers in Physics Vol. 97 (Addison-Wesley, Reading, MA, 1997).
[12] M. Brack and P. Quentin, Phys. Lett. **52B**, 159 (1974).
[13] W.D. Myers and W.J. Świątecki, Nucl. Phys. **81**, 1 (1966).
[14] P. Möller, J.R. Nix, W.D. Myers, and W.J. Świątecki, At. Data Nucl. Data Tables **59**, 185 (1995).
[15] J. Töke and W.J. Świątecki, Nucl. Phys. **A372**, 141 (1981).
[16] J. Bartel, P. Quentin, M. Brack, C. Guet, and H.-B. Håkansson, Nucl. Phys. **A386**, 79 (1982).
[17] M. Warda, B. Nerlo-Pomorska, and K. Pomorski, Nucl. Phys. **A635**, 484 (1998).