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Axial vector form factor of nucleons in a light-cone diquark model

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The nucleon axial vector form factor is investigated in a light-cone quark spectator diquark model, in which Melosh rotations are applied to both the quark and vector diquark. This model gives a very good description of available experimental data and the results have very little dependence on the parameters of the model. The relation between the nucleon axial constant and the anomalous magnetic moment of nucleons is also discussed.

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Since the momenta of quarks within a hadron are of the same order as the masses of quarks, fully relativistic quark models are required in order to describe hadron transition processes. The light-cone quantization provides such a fully relativistic treatment with many unique properties [1]. It is well known that the matrix elements of local operators such as the electromagnetic and weak currents have exact representation in terms of light-cone wave functions of Fock states [2]. If one chooses the special frame [3] $q^+=0$ for the spacelike momentum transfer and takes the matrix elements of plus components of currents, the contribution from pair creation or annihilation is forbidden and the matrix elements of spacelike currents can be expressed as overlaps of light-cone wave functions with the same number of Fock constituents. Therefore, the light-cone frame is well suited for the description of electromagnetic and weak transition processes.

In previous papers, a light-cone quark spectator diquark model was proposed in order to investigate the nucleon spin problem [4–6]. This model is based on the assumption that deep inelastic scattering is well described by the impulse approximation picture of the quark-parton model [7], in which the incident lepton scatters incoherently off a quark in the nucleon, with the remaining nucleon constituents treated as quasiparticle spectators to provide the remaining nucleon quantum number. In fact, in the quark spectator diquark form, some nonperturbative effects between the two spectator quarks or other nonperturbative gluon effects in the nucleon can be effectively taken into account by the mass of the diquark spectator. After taking into account Melosh rotation effects, this model is in good agreement with experimental data of polarized deep inelastic scattering, and the mass difference between the scalar and vector spectators reproduces the up and down valence quark asymmetry [4–6].

Recently, based on the impulse approximation, this model was extended to study the electromagnetic form factors of nucleons and the results agree with experiment [8]. After applying Melosh rotations to both quark and spectator vector diquark, the difference between the scalar and vector diquarks breaks the SU(6) symmetry of nucleon wave functions and reproduces the correct electromagnetic nucleon properties, especially in the case of the neutron. It is natural to extend the light-cone quark diquark model to study other transition processes, such as the nucleon axial decay transition process which is important in the study of the structure of nucleons [9]. This is clearly a nontrivial extension, since we are going now into the weak interactions domain.

The nucleon axial vector form factor $G_A(Q^2)$ is defined by

$$\langle P',S'|A_a^\mu(0)|P,S\rangle=\overline{u}(P',S')\bigg[G_A(Q^2)\gamma^\mu + G_P(Q^2)\frac{q_\mu}{2M}\bigg]\gamma_5\frac{\tau_a}{2}u(P,S), \quad (1)$$

where $A_a^\mu(0)$ is the axial vector current, $q^\mu=(P'-P)^\mu$ is four-momentum transfer, $Q^2=-q^2$, $u(P,S)$ is the nucleon spinor, and τ_a is the isospin matrix with the Cartesian index a. In the light-cone frame, the plus component of the axial vector current reads

$$\frac{\overline{u}(k',\uparrow)}{\sqrt{k'^+}}\gamma^+\gamma_5\frac{u(k,\uparrow)}{\sqrt{k^+}}=-\frac{\overline{u}(k',\downarrow)}{\sqrt{k'^+}}\gamma^+\gamma_5\frac{u(k,\downarrow)}{\sqrt{k^+}}=2, \quad (2)$$

and in these calculations we choose the Drell-Yan assignment [3]:

$$q=(q^+,q^-,\vec{q}_\perp)=\left(0,\frac{-q^2}{P^+},\vec{q}_\perp\right),$$

$$P=(P^+,P^-,\vec{P}_\perp)=\left(P^+,\frac{M^2}{P^+},\vec{0}_\perp\right), \quad (3)$$

thus we have

$$G_A(Q^2)=\left\langle P',\uparrow\left|\frac{A^+(0)}{2P^+}\right|P,\uparrow\right\rangle. \quad (4)$$

Thus, similar to the electromagnetic operators, the matrix element of axial vector currents can be expressed in the light-

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cone formalism as overlaps of light-cone wave functions with the same number of Fock constituents as

$$G_A(Q^2)=\sum_a \int \frac{d^2\vec{k}_\perp dx}{16\pi^3} \times\sum_j \tau_j\lambda_j\psi_a^{\uparrow\star}(x_i,\vec{k}'_{\perp i},\lambda_i)\psi_a^{\uparrow}(x_i,\vec{k}_{\perp i},\lambda_i), \quad (5)$$

where τ_j and λ_j are the isospin and helicity of the struck constituents, $\psi_a^{\uparrow}(x_i,\vec{k}'_{\perp i},\lambda_i)$ is the light-cone Fock expansion wave function; and λ_i, x_i, and $\vec{k}_{\perp i}$ are the spin projections along the quantization z direction, light-cone momentum fractions, and relative momentum coordinates of QCD constituents, respectively. Here, for the final state light-cone wave function, the relative momentum coordinates are

$$\vec{k}'_{\perp i}=\vec{k}_{\perp i}+(1-x_i)\vec{q}_\perp \quad (6)$$

for the struck quark and

$$\vec{k}'_{\perp i}=\vec{k}_{\perp i}-x_i\vec{q}_\perp \quad (7)$$

for each spectator.

In this work we study the nucleon axial vector form factor based on the light-cone quark spectral diquark model [8], in which the Melosh rotation is applied to both quark and diquark, explicitly

$$\chi_T^{\uparrow}=w[(k^+ +m)\chi_F^{\uparrow}-k^R\chi_F^{\downarrow}],$$
$$\chi_T^{\downarrow}=w[(k^+ +m)\chi_F^{\downarrow}-k^L\chi_F^{\uparrow}], \quad (8)$$

for quarks [10], and

$$V_T^1=w^2[(k^+ +m)^2V_F^1-\sqrt{2}(k^+ +m)k^RV_F^0+k^{R2}V_F^{-1}],$$
$$V_T^0=w^2[\sqrt{2}(k^+ +m)k^LV_F^1+2\{(k^0+m)k^+ -k^Rk^L\}V_F^0 - \sqrt{2}(k^+ +m)k^RV_F^{-1}],$$
$$V_T^{-1}=w^2[k^{L2}V_F^1+\sqrt{2}(k^+ +m)k^LV_F^0+(k^+ +m)^2V_F^{-1}], \quad (9)$$

for vector diquarks [11]. Here, χ_T and χ_F are instant and light-cone spin-$\frac{1}{2}$ spinors, V_T and V_F are the instant and light-cone spin-1 spinors respectively, $w=[2k^+(k^0+m_q)]^{-1/2}$, $k^{R,L}=k^1\pm ik^2$, and $k^+=k^0+k^3$. And the details of the quark-diquark model can be found in Ref. [8]. Therefore, according to Eq. (5), we have

$$G_A(Q^2)=3\int \frac{d^2k_\perp dx}{16\pi^3}w'_qw_q\Bigg\{\frac{1}{9}\sin^2\theta\{-[(k'^+_q+m_q) \times(k_q^+ +m_q)-k'^L_\perp k^R_\perp]O_{V^{0,0}}+\sqrt{2}[(k'^+_q+m_q)k^L_\perp + (k_q^+ +m_q)k'^L_\perp]O_{V^{0,1}}+\sqrt{2}[(k'^+_q+m_q)k^R_\perp + (k_q^+ +m_q)k'^R_\perp]O_{V^{1,0}}+2[(k'^+_q+m_q)(k_q^+ +m_q) - k'^R_\perp k^L_\perp]O_{V^{1,1}}\}\varphi_V(x,\vec{k}'_\perp)\varphi_V(x,\vec{k}_\perp)+\cos^2\theta[(k'^+_q + m_q)(k_q^+ +m_q)-k'^L_\perp k^R_\perp]\times\varphi_S(x,\vec{k}'_\perp)\varphi_S(x,\vec{k}_\perp)\Bigg\}, \quad (10)$$

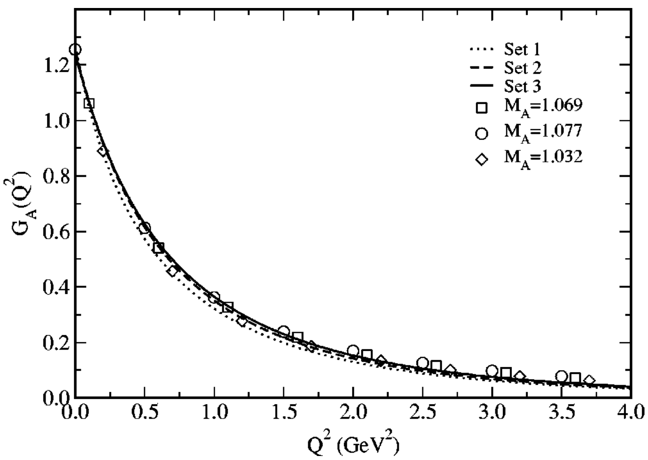

FIG. 1. Axial form factor of nucleons. The experimental data are from [17,18].

where O_V come from the Melosh rotation of vector diquarks, and φ is the momentum space wave function which is assumed to be a harmonic oscillator wave function (the Brodsky-Huang-Lepage prescription [12]).

Following Ref. [8], we calculate the nucleon axial vector form factor using three different sets of parameters in order to show its dependence on the difference between the scalar and vector diquarks. This is shown in Fig. 1. As in Ref. [13], the experimental data are assumed to be the dipole form

$$G_A(Q^2)=\frac{g_A}{(1+Q^2/M_A^2)^2}, \quad (11)$$

where the axial constant $g_A=1.2670(35)$ is from the most recent review by Particle Data Group [14], and M_A is the axial mass. Our results agree with the experiment very well. In contrast to the case of nucleon electromagnetic form factors, the nucleon axial vector form factor is largely parameter independent. Alternatively, the effect of the difference between the scalar and vector diquarks to the the nucleon axial vector form factor is small. The static properties corresponding to the axial vector current, axial constant g_A, and axial radius $\langle r_A^2\rangle^{1/2}$ are listed in Table I. The axial radius is calculated by

TABLE I. The static electromagnetic properties of nucleons for the three sets of parameters.

	Set I	Set II	Set III	Expt.
g_A	1.253	1.270	1.242	1.2670(35) [14]
$\langle r_A^2\rangle^{1/2}$	0.624	0.703	0.611	0.635(23) [17],0.65(7) [18]

$$r_A^2=-6\frac{1}{G_A(0)}\frac{dG_A(Q^2)}{dQ^2}|_{Q^2=0}. \qquad (12)$$

Our results are very close to the experimental data, and also the effect of the difference between the scalar and vector diquarks to the nucleon axial vector static properties is small.

For the axial constant $g_A=G_A(0)$, the relativistic effects are essential in order to reduce the SU(6) nonrelativistic result $g_A^{NR}=5/3$ to the experimental data. In fact, from Eq. (10) we have

$$g_A=\int \frac{d^2k_\perp dx}{16\pi^3}\left[\frac{1}{6}W_A(x,\vec{k}_\perp)\varphi_V^2(x,\vec{k}_\perp) + \frac{3}{2}W_A(x,\vec{k}_\perp)\varphi_S^2(x,\vec{k}_\perp)\right], \qquad (13)$$

where

$$W_A(x,\vec{k}_\perp)=\frac{(k_q^+ + m_q)^2-\vec{k}_\perp^2}{(k_q^+ + m_q)^2+\vec{k}_\perp^2}, \qquad (14)$$

is the Wigner rotation factor corresponding to the contribution from the relativistic effects due to the quark transversal motion [4,5,15]. In the case of SU(6) symmetry between the vector and scalar diquarks, we get the same results as other light-cone quark models [15,16], $g_A=\langle W_A\rangle g_A^{NR}$. In the nonrelativistic limit, $\langle W_A\rangle=1$ and $g_A=g_A^{NR}$. Therefore, the physical value of the axial constant is reduced by 25% from its nonrelativistic value due to relativistic effects. This is similar to the nucleon spin problem situation [4], where relativistic effects are important for producing the quark spin reduction on the light cone. This is reasonable, since from Eq. (5) it is easy to obtain the following relation between the nucleon axial constant and the quark spin contributions to the nucleon spin:

$$g_A=\Delta u-\Delta d. \qquad (15)$$

Here Δu and Δd are the helicities of the up and down quarks in the nucleon, and $\Delta q=\langle W_A\rangle \Delta q^{NR}$, where Δq^{NR} is the nonrelativistic quark spin contributions to the nucleon spin defined in the quark model.

On the other hand, in our model the anomalous magnetic moment of proton a can be also written as [8]

$$a=2M\int \frac{d^2k_\perp dx}{16\pi^3}\frac{1}{\mathcal{M}}\left[\frac{(1-x)\mathcal{M}(k_q^+ + m_q)-\vec{k}_\perp^2/2}{(k_q^+ + m_q)+\vec{k}_\perp^2}\right] \times\varphi_S^2(x,\vec{k}_\perp)=\langle W_M\rangle a^{NR}, \qquad (16)$$

where $a^{NR}=2M/3m_q$ is the nonrelativistic value of the proton anomalous magnetic moment, and the relativistic effect corresponding to the anomalous magnetic moment

$$W_M=\frac{3m_q}{\mathcal{M}}\left[\frac{(1-x)\mathcal{M}(k_q^+ + m_q)-\vec{k}_\perp^2/2}{(k_q^+ + m_q)+\vec{k}_\perp^2}\right], \qquad (17)$$

which is the same as in Ref. [16]. Thus, the wave-function-independent relations between the nucleon axial coupling g_A and the nucleon magnetic moments of Ref. [16] are still kept in our model.

In conclusion, we extended our studies of nucleon elastic and inelastic scattering processes in a light-cone quark-diquark model to the nucleon axial vector form factor and the Melosh rotations were applied to both the quark and vector diquark. It is shown that the axial vector form factor has very little dependence on the parameters of the model and the relativistic properties of the model are essential for a good agreement with the experimental results. The relation between the nucleon axial constant and the anomalous magnetic moment of nucleons is also discussed.

Therefore the light-cone quark-diquark model gives a very good description of the nucleon. In fact, we have shown that it can reproduce quite accurately both elastic and inelastic electromagnetic and weak nucleon data, as can be shown by our results for the nucleon structure functions and vector and axial vector form factors.

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