⁶Be and ⁸C level widths

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R-matrix formulas are used to calculate the two-proton decay widths of the ⁶Be and ⁸C ground states and of the ⁶Be first excited state. The calculated widths for the ⁶Be states depend strongly on the values taken for the energy and width of the ⁵Li ground state; agreement for the ⁶Be ground-state width can be obtained for a reasonable choice of ⁵Li parameter values, and the same choice gives good agreement for the ⁶Be excited-state width and branching ratio for ²He decay. For ⁸C, contributions from two of the possible decay channels give an appreciable fraction of the experimental width.

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The latest compilation of "Energy Levels of Light Nuclei, A=5-7" [1], gives the width of the 0⁺ ground state of ⁶Be as 92±6 keV. It mentions two published experimental values of 95±28 keV [2] and 89±6 keV [3]. An earlier published value was 140±40 keV [4], while an unpublished value of 126±15 keV [5] was mentioned in an earlier compilation [6]. The compilation [1] also gives the width of the 2⁺ first excited state of ⁶Be as 1.16±0.06 MeV, with the branching ratio for decay of this state via the emission of ²He [*T* = 1, *S*=0] as 0.60±0.15. The width value comes from the average of many measurements (see Ref. [7]). The branching ratio is from a single measurement [8]; however, this paper actually gives the fraction of diproton emission as about 20% [and of this fraction, (60±15)% has *S*=0].

For ⁸C , the latest compilation [9] gives the 0⁺ groundstate width as 230 ± 50 keV. This value was obtained by fitting a Gaussian to an observed peak [10]; the same authors also found a width of 183 ± 56 keV by using a Breit–Wigner peak shape [10]. An earlier published value was 220^{+80}_{-140} keV [11].

We here use previously published *R*-matrix formulas [12,13] to calculate the widths of the ${}^{6}\text{Be}$ and ${}^{8}\text{C}$ ground states and of the first excited state of ${}^{6}\text{Be}$.

The ground state of ⁶Be decays by two-proton emission to the ground state of ⁴He, with an available energy of 1.371 MeV [1]. The decay can be by diproton (²He) emission, or it can occur as sequential emission of the two protons through the low-energy tail of the unstable ⁵Li ground state. *R*-matrix formulas have been given for two-proton sequential decay [12] and for ²He decay [13], where they were used to calculate upper limits on the observed widths for the sequential and ²He decay of the ¹²O ground state. The partial observed widths as defined in Refs. [12,13] are, however, not additive, as each is based on a one-channel approximation; rather, the corresponding formal widths should be added to give the total formal width and the total observed width obtained from this, with the factor containing contributions from both decay channels. Alternatively, with the original [14] multichannel definition of the partial observed widths Γ_c^0 (here c = s or d for sequential or diproton decay), but using the notation of Refs. [12,13], we may write the total observed width as

$$\Gamma_{\rm tot}^0 = \sum_c \ \Gamma_c^0, \tag{1}$$

with

$$\Gamma_{c}^{0} = \frac{\Gamma_{c}}{1 + \sum_{c'} \gamma_{1c'}^{2} \bar{S}_{c'}^{\prime}}, \quad \Gamma_{c} = 2 \gamma_{1c}^{2} \bar{P}_{c}, \quad (2)$$

where

$$\bar{P}_{c} = \int_{0}^{Q_{2p}} P_{1c}(Q_{2p} - U)\rho_{c}(U) \mathrm{d}U, \qquad (3)$$

$$\overline{S}_{c}^{\prime} = \int_{0}^{\infty} \left[\frac{\mathrm{d}S_{1c}(E-U)}{\mathrm{d}E} \right]_{E=\mathcal{Q}_{2p}} \rho_{c}(U) \mathrm{d}U. \tag{4}$$

Here

$$\rho_c(U) = c \frac{\Gamma_{2c}(U)}{[U - Q_{1pc} - \Delta_{2c}(U)]^2 + \frac{1}{4}\Gamma_{2c}^2(U)},$$
 (5)

which can be written for the 2 He channel in the form of Eq. (3) of Ref. [13]. Also

$$\Gamma_{2c}(U) = 2 \gamma_{2c}^2 P_{2c}(U),$$

$$\Delta_{2c}(U) = -\gamma_{2c}^2 [S_{2c}(U) - S_{2c}(Q_{1pc})].$$
(6)

We note that the suffixes 1 and 2 refer to the first and second decays, although Q_{1pc} and Q_{2p} refer to one-proton and twoproton decay energies. We apply these formulas to the ⁶Be case. Because the ⁶Be ground-state width is reasonably small, one would not expect much difference between the observed width and the full width at half maximum (FWHM) as obtained experimentally. One has $Q_{2p} = 1.371$ MeV. Un-

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less otherwise mentioned, we use conventional values of the channel radius $a = \overline{a}(A_1^{1/3} + A_2^{1/3})$, with $\overline{a} = 1.45$ fm [15].

We first consider ²He decay. The wave function for the ⁶Be ground state is taken from the shell-model calculations of Ref. [16]:

$${}^{6}\text{Be}(0^{+}) = 0.934([2]^{31}\text{S}_{0}) - 0.358([11]^{33}\text{P}_{0}).$$
(7)

Then the spectroscopic factor for ⁶Be decay to ⁴He+ ²He is $S_{42} = (0.934)^2 \times 9/8 = 0.981$, where 9/8 is the c.m. correction factor [17]. For this decay, the single-particle dimensionless reduced width, calculated from Eq. (16) of Ref. [12] using Woods-Saxon (WS) parameter values $r_0 = 1.17$ fm, $a_0 = 0.72$ fm, and $r_c = 1.30$ fm as in Ref. [13], is $\theta_{sp}^2 = 1.13$. Then $\theta^2 = 1.11$ and $\gamma_{1d}^2 = 2.03$ MeV. Also $\bar{P}_d = 0.0347$ and $\bar{S}'_d = 0.320$ MeV⁻¹.

The formulas of Ref. [12] are used to calculate the contribution to the total width of the ⁶Be ground state due to sequential decay. These formulas assume a one-level *R*-matrix approximation for the ground state of ⁵Li, which is described in terms of its resonance energy Q_{1ps} above the ⁴He+*p* threshold and reduced width γ_{2s}^2 for decay to ⁴He +*p*. The observed width $\Gamma_{2s}^0(Q_{1ps})$ of the ⁵Li ground state is given by [14]

$$\Gamma_{2s}^{0}(Q_{1ps}) = \Gamma_{2s}(Q_{1ps}) / [1 + \gamma_{2s}^{2}(\mathrm{d}S_{2s}(U)/dU)_{U=Q_{1ps}}],$$
(8)

so that values of γ_{2s}^2 can be obtained from experimental values of either the formal or observed width. The formulas also depend on the reduced width γ_{1s}^2 for the ⁶Be decay to ⁵Li(g.s.)+p, which depends slightly on Q_{1ps} .

Widely varying experimental values have been given for the energy and width of the ⁵Li ground state, in part due to the use of different definitions for the energy and width of an unbound level. In the compilation [1], the recommended prescription based on the extended *R*-matrix method uses the complex energy of a pole of the *S* matrix, giving the energy as 1.69 MeV above the ⁴He+*p* threshold and the width as 1.23 MeV. It is, however, not obvious how Q_{1ps} and γ_{2s}^2 can be obtained from these values. From a conventional *R*-matrix prescription, using definitions as in Lane and Thomas [14], the compilation [1] gives the resonance energy as 2.08 MeV and the observed width as 2.11 MeV, for a channel radius $a_2=2.9$ fm. These values give

$$Q_{1ps} = 2.08 \text{ MeV}, \quad \gamma_{2s}^2 = 11.9 \text{ MeV}.$$
 (9)

The values above all come from a comprehensive multilevel, multichannel *R*-matrix analysis of reactions in the ⁵Li system, including all possible reactions for the two-body channels $d+{}^{3}$ He, $p+{}^{4}$ He, and $p+{}^{4}$ He for c.m. energies corresponding to ⁵Li excitation energies less than 23 MeV. They take no account of reactions in which the ⁵Li ground state is observed as a particle-unstable product nucleus. The compilation [1] lists 20 such reactions, while in only one of the reactions included in the *R*-matrix analysis ($p+{}^{4}$ He elastic scattering) is the ⁵Li ground state expected to contribute significantly.

Values of the ⁵Li ground-state energy and width obtained from some of these 20 reactions are given in the latest Ajzenberg-Selove compilation [18]; they are essentially values of the peak energy and FWHM of the peak, which are not necessarily the same as the resonance energy and either the formal or observed width, though the FWHM is expected to be closer to the observed width. The compilation [18] gives the energy as 1.97 MeV, based on an atomic mass excess of 11.68 MeV, and the width as \approx 1.5 MeV. This value of the mass excess was given originally by Everling et al. [19], where it is derived from seven nuclear-reaction results collected by Van Patter and Whaling [20]. These results give ⁵Li ground-state energy values ranging from 1.53 ± 0.15 MeV from the reaction ${}^{6}\text{Li}(p,d){}^{5}\text{Li}$ to 2.06±0.2 MeV from 3 He (d, γ) ⁵Li, with a weighted mean of 1.74±0.06 MeV, corresponding to a mass excess of 11.45 MeV. It is not clear how Everling et al. [19] obtained a mass excess of 11.68 MeV; thus the compilation [18] energy of 1.97 MeV is suspect. It may be noted that the compilations in 1959 and earlier [21-23] took the energy as 1.80 MeV. The width value of ≈ 1.5 MeV given in the compilation [18] can be traced back to the measurement of Frost and Hanna [24], and appears in all compilations from 1955 [22] to 1988 [18]. Other values of the FWHM have been given [18], ranging from 1.18 ± 0.13 MeV from 4 He(7 Li, 6 He) 5 Li to 2.6 ± 0.4 MeV from ${}^{3}\text{He}(d,\gamma){}^{5}\text{Li}$, while a more recent value of 1 ± 0.2 MeV from ¹H(α, γ)⁵Li is given in Ref. [1]. Values of Q_{1ps} and γ_{2s}^2 are given directly in a two-level *R*-matrix fit to data from reactions in which ⁵Li is formed as an unstable product nucleus [25]:

$$Q_{1ps} = 1.861$$
 MeV,
 $\gamma_{2s}^2 = (0.952 \text{ MeV}^{1/2})^2 = 0.906$ MeV, (10)

for $a_2=5.5$ fm [and $B=S(Q_{1ps})$]. These values give $\Gamma^0_{2s}(Q_{1ps})=1.30$ MeV.

In view of the wide spread of values for the ⁵Li groundstate energy and width, we initially calculate the sequential contribution to the ⁶Be ground-state width using one set of values and consider the sensitivity of the results to changes in these values. From the conventional *R*-matrix prescription [1], we take the values (9). We calculate γ_{1s}^2 using the ⁶Be description (7), giving the spectroscopic factor $S_{51}=1.88 \times 6/5=2.25$. Using conventional WS parameter values r_0 = 1.25 fm and $a_0=0.65$ fm as in Ref. [12], we find $\theta_{sp}^2=0.433$, giving $\theta^2=0.975$ and $\gamma_{1s}^2=3.15$ MeV. The formulas of Ref. [12] also give $\overline{P}_s=0.0404$ and \overline{S}'_s = 0.278 MeV⁻¹. From Eqs. (1) and (2), the total observed width is

$$\Gamma_{\text{tot}}^{0} = \frac{2(2.03 \times 0.0347 + 3.15 \times 0.0404)}{1 + 2.03 \times 0.320 + 3.15 \times 0.278} \text{ MeV} = 157 \text{ keV}.$$
(11)

This is significantly bigger than the experimental FWHM values, which is not surprising as the value $\gamma_{2s}^2 = 11.9 \text{ MeV}$ is very large. It leads to $S_{41}=2.27$, based on $\theta_{sp}^2 = 0.848 \text{ MeV}$ from Eq. (16) of Ref. [12]. Most models for ⁵Li(g.s.) would give $S_{41} \le 1.25$ (as 5/4 is the c.m. correction factor). These

values use $a_2=2.9$ fm as in Ref. [1]. If we fit the observed width 2.11 MeV given in Ref. [1], using the conventional channel radius $a_2=3.75$ fm, we find $\gamma_{2s}^2=3.98$ MeV and $\Gamma_{tot}^0=93$ keV in good agreement with experiment. In this case, the spectroscopic factor is $S_{41}=1.46$.

We consider how sensitive this agreement is to changes in the assumed values of parameters and input data, using the results for $a_2=3.75$ fm as standard. Changing the WS parameter values has little effect on the calculated width—10% changes in r_0 , a_0 , and r_C produce at most a few keV change in the width. Decreasing \bar{a} from 1.45 to 1.35 fm increases Γ_{tot}^0 by 7 keV. If the simplest *LS*-coupled shell-model description is used for the ⁶Be ground state (entirely [2]³¹S₀, so that $S_{51}=1.600$), Γ_{tot}^0 is increased by 4 keV.

For the sequential decay, the calculated values of \overline{P}_s are sensitive to the values assumed for the energy and width of the ⁵Li ground state. If we retain $Q_{1ps}=2.08$ MeV, as above, but reduce the width from 2.11 MeV to 1.91 MeV, corresponding to $S_{41}=1.25$, its expected upper limit, then Γ_{tot}^0 is reduced by 10 keV. From the values (10) (with a_2 = 5.5 fm), which correspond to $S_{41}=0.97$, we find $\Gamma_{tot}^0=69$ keV. If we use $\Gamma_{2s}^0 (Q_{1ps})=1.30$ MeV as derived from Eqs. (10), but then use the conventional channel radius a_2 = 3.75 fm, we find $\gamma_{2s}^2=2.48$ MeV (corresponding to S_{41} = 0.94) and $\Gamma_{tot}^0=79$ keV. Keeping $Q_{1ps}=1.861$ MeV and $a_2=3.75$ fm, and taking $\Gamma_{2s}^0 (Q_{1ps})=1.57$ MeV corresponding to $S_{41}=1.25$, we find $\gamma_{2s}^2=3.32$ MeV and $\Gamma_{tot}^0=95$ keV.

With conventional values of all parameters, including $a_2 = 3.75$ fm, interpolation of the above values shows that the experimental FWHM of 92 keV may be fitted with $S_{41} = 1.25$ and $Q_{1ps} = 1.91$ MeV or, alternatively, with $S_{41} = 1.20$ and $Q_{1ps} = 1.86$ MeV (or with other combinations with smaller S_{41} and smaller Q_{1ps}). As an example, we take the values

$$a_2 = 3.75$$
 fm, $Q_{1ps} = 1.86$ MeV, $\gamma_{2s}^2 = 3.18$ MeV, (12)

which give $S_{41}=1.20$, $\Gamma_{2s}^0 (Q_{1ps})=1.53$ MeV, and $\Gamma_{tot}^0=92$ keV. These values seem to be not unreasonable.

From a three-cluster microscopic model of ⁶Be, Csótó [26] calculated a ground-state width of 160 keV, the large value probably being due to the calculated energy of the state being 150 keV too high (if we use Csótó's energy for the state, we find $\Gamma_{tot}^0 = 145$ keV). It is of interest that his ground-state wave function contains 87.7% S = 0, L = 0, compared with 87.2% from Eq. (7).

The 2^+ excited state of ⁶Be at an excitation energy of 1.67 MeV decays by two-proton emission to the ground state of ⁴He, with an available energy of 3.04 MeV [1]. In this case, the sequential decay through ⁵Li+*p* is energetically allowed, as is the ²He emission. We assume the ⁵Li parameter values (12), which led above to a good fit to the ⁶Be ground-state width. Conventional values are assumed for the other parameters.

From Ref. [16], the wave function of the ⁶Be excited state is

$${}^{6}\text{Be}(2^{+}) = 0.833([2]^{31}\text{D}_{2}) + 0.553([11]^{33}\text{P}_{2}).$$
 (13)

For ²He decay, one has $S_{42} = (0.833)^2 \times 9/8 = 0.780$, and $\theta_{sp}^2 = 0.520$, giving $\gamma_{1d}^2 = 0.742$. Also $\bar{P}_d = 0.272$ and $\bar{S}'_d = 0.260 \text{ MeV}^{-1}$. For sequential decay, $S_{51}(s=1) = 1.693$ and $S_{51}(s=2) = 0.551$ (where *s* is here the channel spin) and $\theta_{sp}^2 = 0.616$, leading to $\gamma_{1s}^2 = 4.46$ MeV. Also $\bar{P}_s = 0.256$ and $\bar{S}'_s = 0.200 \text{ MeV}^{-1}$.

These give $\Gamma_{tot}^0 = 1.29$ MeV, and the branching ratio for ²He emission is 15.0%. An estimate of the corresponding FWHM (using a density-of-states function for sequential decay only) gives a value about 7% less than the observed width—i.e., about 1.20 MeV. There is therefore good agreement with the experimental FWHM of 1.16 ± 0.06 MeV and branching ratio of "about 20%" [of which (60 ± 15) % has S=0]—we have assumed that the ²He emission is entirely S=0.

With the simplest shell-model description of the ⁶Be excited state, one finds $\Gamma_{tot}^0 = 1.16$ MeV and a branching ratio of 26.3%. If we use the parameter values (9), with $a_2 = 2.9$ fm, we obtain $\Gamma_{tot}^0 = 1.66$ MeV with a branching ratio of 12.5%.

Csótó [26] calculated an excited-state width of 0.87 MeV, the small value in this case being associated with a calculated energy that is 230 keV too low (with Csótó's energy, our procedure gives $\Gamma_{tot}^0 = 1.02$ MeV). His wave function contains 59.9% S = 0, L = 2 compared with 69.4% from Eq. (13).

The ⁸C ground state decays by the emission of four protons to the ground state of ⁴He, with an available energy of 3.513 MeV [9]. The decay can proceed in various ways—by direct emission of a four-proton cluster (⁴Be), which seems unlikely, or by sequential decay involving the unstable ground states of ⁵Li, ⁶Be, and ⁷B. We consider only two contributions to the total width of the ⁸C ground state: ²He emission to the ground state of ⁶Be and single-proton emission to the low-energy tail of the ⁷B ground state, which then decays to ⁶Be+p. In both contributions, we neglect the width of the ⁶Be ground state. Then the available energy for decay of ⁸C to ⁶Be+2p is $Q_{2p}=2.142$ MeV [9]. From Ref. [1], we take the energy and width of ⁷B ground state as $Q_{1ps}=2.21$ MeV and $\Gamma_{2s}^0(Q_{1ps})=1.4$ MeV. The procedure is similar to that for ⁶Be above.

From Ref. [16], the relevant shell-model wave functions are

$${}^{3}C(0^{+}) = 0.934([22]^{51}S_{0}) + 0.356([211]^{53}P_{0})$$
 (14)

and

$${}^{7}B(3/2^{-}) = 0.843([21]^{42}P_{3/2}) + 0.510([21]^{42}D_{3/2}) - 0.169([111]^{44}S_{3/2}).$$
(15)

For the ²He decay, $S_{62} = 0.195 \times 8/9 = 0.173$ and $\theta_{sp}^2 = 1.004$, giving $\theta^2 = 0.174$ and $\gamma_{1d}^2 = 0.242$ MeV. Also $\bar{P}_d = 0.0478$ and $\bar{S}'_d = 0.345$ MeV⁻¹.

For the sequential decay through ⁷B, we find $\gamma_{2s}^2 = 2.22$ MeV, $S_{71} = 3.11 \times 8/7 = 3.55$, and $\theta_{sp}^2 = 0.414$, giving γ_{1s}^2 = 3.91 MeV. Also $\overline{P}_s = 0.0412$ and $\overline{S}'_s = 0.322$ MeV⁻¹.

We then obtain an upper limit on the contribution to the total width coming from these two channels of 148 keV, which is about 70% of the experimental FWHM values.

A rough allowance for the nonzero width of the ⁶Be ground state increases the calculated width by less than 0.5 keV. Use of the simplest wave functions for the ⁸C, ⁷B, and ⁶Be ground states reduces the calculated width by about 8 keV.

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The *R*-matrix formulas [12,13] for two-proton decay give calculated widths of the ground and first-excited states of ⁶Be in agreement with experimental values, provided the energy and width of the ⁵Li ground state are suitably and reasonably chosen, and for the excited state the calculated branching ratio for ²He decay agrees with experiment. For the ground state of ⁸C, which decays eventually to ⁴He and four protons, contributions from two of the possible decay channels calculated from the *R*-matrix formulas make up an appreciable fraction of the experimental total width.

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