

Dependence of calculated binding energies and widths of η -mesic nuclei on treatment of subthreshold η -nucleon interaction

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We demonstrate that the binding energies ϵ_η and widths Γ_η of η -mesic nuclei depend strongly on the subthreshold η -nucleon interaction. This strong dependence is made evident from comparing three different η -nucleus optical potentials: (1) a microscopic optical potential taking into account the full effects of the off-shell ηN interaction; (2) a factorization approximation to the microscopic optical potential where a downward energy shift parameter is introduced to approximate the subthreshold ηN interaction; and (3) an optical potential using the on-shell ηN scattering length as the interaction input. Our analysis indicates that the in-medium ηN interaction for bound-state formation is about 30 MeV below the free-space ηN threshold, which causes a substantial reduction of the attractive force between the η and nucleon with respect to that implied by the scattering length. Consequently, the scattering-length approach overpredicts the ϵ_η and caution must be exercised when these latter predictions are used as guide in searching for η -nucleus bound states. We also show that final-state-interaction analysis cannot provide an unequivocal determination of the existence of the η -nucleus bound state. More direct measurements are, therefore, necessary.

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I. INTRODUCTION

The existence of the η -mesic nucleus, a bound state of an η meson in a nucleus, was first predicted by us in 1986 [1]. The formation of such bound systems is caused by the attractive interaction between the η meson and all the nucleons in the nucleus. The attractive nature of the interaction followed naturally from the work of Bhalerao and Liu [2], who found, from a detailed coupled-channel analysis of $\pi N \rightarrow \pi N$, $\pi N \rightarrow \pi \pi N$, and $\pi N \rightarrow \eta N$ reactions, that the near-threshold ηN interaction is attractive. We can easily understand this attraction by observing that the ηN threshold is situated just below the $N^*(1535)$ resonance. However, the attraction given in Ref. [2] is not strong enough to support a bound state in nuclei with mass number $A < 12$. This latter conclusion [1] was confirmed by Li *et al.* [3], who used a standard Green's function technique of many-body theory to study the formation of η -mesic nuclei. Before proceeding further, we would like to mention that in the literature the η -nucleus bound states are also called η -nucleus quasibound states. This is because these bound states have a finite width and they eventually decay. In this work, we shall simply call them bound states.

If the existence of the η -mesic nucleus is experimentally confirmed, many new studies of nuclear and particle physics will become possible. For example, because the binding energies of the η meson depend strongly on the coupling between the ηN and the $N^*(1535)$ channels [2], studies of the η -mesic nucleus can yield additional information on the ηNN^* coupling constant involving bound nucleons. Furthermore, the η -mesic nuclear levels correspond to an excitation energy of ~ 540 MeV, to be compared with an excitation energy of ~ 200 MeV associated with the Λ and Σ hypernuclei. The existence of nuclear bound states with such

high excitation energies provides the possibility of studying nuclear structure far from equilibrium.

Several experiments [4–6] using π^+ beam, as motivated by the theoretical works of Refs. [7,8], were performed to search for η -mesic nuclei. While these experiments could not confirm the existence of the η -mesic nucleus, they did not rule out such a possibility either. More recently, Sokol *et al.* [9,10] have claimed to observe the η -mesic nucleus $^{11}_\eta\text{C}$ through measuring the invariant mass of correlated $\pi^+ n$ pairs in a photo-mesonic reaction. Further confirmation of their experimental result, with improved statistics, is needed before arriving at a definite conclusion.

The existence of these η -mesic nuclear states depends on the value of the ηN scattering length $a_{\eta N}$. (For the scattering length, we use the sign convention of Goldberger and Watson [11].) After 1990, many theoretical models were proposed for the ηN interaction. Fits to various data have yielded very different $a_{\eta N}$ [2,12–23]. The real part of $a_{\eta N}$ ranges from as low as 0.270 fm [2] to as high as 1.000 fm [13], while the imaginary part varies between 0.190 fm [2] and 0.399 fm [23]. This wide range of values of $a_{\eta N}$ are summarized in Table I. These very different values arise because $a_{\eta N}$ is not directly measurable and must be inferred indirectly from other observables, such as the πN phase shifts. In this latter case, the calculated $a_{\eta N}$ depends strongly on the model used to relate the ηN to the πN channels. As pointed out in Ref. [19], the inclusion of both the $N^*(1535)$ and $N^*(1650)$ resonances leads to larger scattering lengths. However, this general rule does not account for all the reasons. For example, in Ref. [16] the inclusion of both these N^* resonances, but with a fit to different reaction data, does not lead to a $\text{Re } a_{\eta N}$ as high as ≈ 0.8 fm, rather only to $\text{Re } a_{\eta N} \approx 0.6$ fm. We caution the readers that it is premature

TABLE I. η -nucleon s -wave scattering lengths $a_{\eta N}$.

$a_{\eta N}$ (fm)	Formalism or reaction	Reference
$0.270+0.220i$	Isobar model	Bhalerao and Liu [2]
$0.280+0.190i$	Isobar model	Bhalerao and Liu [2]
$0.281+0.360i$	Photoproduction of η	Krusche [23]
$0.430+0.394i$		Krusche [23]
$0.579+0.399i$		Krusche [23]
$0.476+0.279i$	Electroproduction of η $pd \rightarrow {}^3\text{He } e \eta$	Tiator <i>et al.</i> [22]
$0.500+0.330i$		Wilkin [24]
$0.510+0.210i$	Isobar model	Sauermann <i>et al.</i> [14]
$0.550+0.300i$		Sauermann <i>et al.</i> [14]
$0.620+0.300i$	Coupled T matrices	Abaev and Nefkens [16]
$0.680+0.240i$	Effective Lagrangian	Kaiser <i>et al.</i> [17]
$0.750+0.270i$	Coupled K matrices	Green and Wycech [12]
$0.870+0.270i$	Coupled K matrices	Green and Wycech [13]
$1.050+0.270i$		Green and Wycech [13]
$0.404+0.343i$	Coupled T matrices	Batinić <i>et al.</i> [18]
$0.876+0.274i$		Batinić and Švarc [19]
$0.886+0.274i$		Batinić and Švarc [19]
$0.968+0.281i$		Batinić <i>et al.</i> [20]
$0.980+0.370i$	Coupled T matrices	Arima <i>et al.</i> [21]

to conclude which published $a_{\eta N}$ is the realistic one. Such a determination can be made only after η -mesic nuclei are experimentally discovered, as only their experimental observation can set a stringent constraint on the in-medium scattering length and enable one to differentiate between different theoretical models.

Using final-state-interaction (FSI) analysis, several theoretical studies have suggested $a_{\eta N} \approx (0.5+0.3i)$ fm and the possibility of the formation of bound η -nucleus states in light nuclei, such as ${}^3,4\text{He}$ [24,25] and deuteron [26–28]. We will revisit the FSI analysis in this work and show the limitation of the method. In spite of the fact that many predicted $a_{\eta N}$ (see Table I) are much larger than the one used in the above-mentioned FSI analysis, to date there has been no direct observation of these bound states in light nuclei. Consequently, we believe it is valuable to understand the current experimental situation by analyzing in detail the dynamics pertinent to the formation of η -mesic nucleus.

We have learned from studying pion-nucleus scattering that, because of the Fermi motion and binding of the target nucleon, the pion-nucleon interaction in nuclei occurs at energies below its free-space value, i.e., below the energy that the pion-nucleon system would have when the nucleon is unbound [29]. As the above causes for the lowering of the interaction energy are also present in η -nucleus scattering, we believe that the η -nucleon interaction energy in η -mesic nucleus formation will occur below its free-space threshold, and this subthreshold interaction can have important effects on the formation of η -nucleus bound states. This observation has motivated us to examine in this work the reliability of using the free-space ηN information, such as the ηN scattering length, to make predictions of the nuclear binding of the η .

We will, therefore, study the dependence of η -nucleus

bound-state formation on off-shell behavior of the ηN interaction in nuclei with $A \geq 3$ through comparing three different theoretical approaches. They are (1) a fully off-shell microscopic optical potential that uses an off-shell ηN model; (2) a factorization approximation (FA) to the preceding microscopic potential, where an energy shift parameter is used to calculate the ηN interaction at subthreshold energies; and (3) an “on-shell” optical potential whose strength depends solely on the scattering length $a_{\eta N}$. The last approach has its roots in the study of K - and π -mesic atoms [30–32] and is also extensively used in predicting η -mesic nuclei. Finally, we mention that as the existence of η -mesic nucleus has not yet been experimentally confirmed with certainty, there are no data to be fitted. Hence, all of our results as well as those of others are purely predictions.

The paper is organized as follows. The three theoretical approaches are described in Sec. II. The important differences in their underlying reaction dynamics are outlined. Results and discussions are presented in Sec. III. As we shall see, for a given ηN interaction model, the “on-shell” approach gives much stronger binding for η than does the microscopic calculation, making evident the important effects of off-shell dynamics. We will further show that these effects are of very general nature and are independent of the specific unitary off-shell model used in the comparative study. The ramifications of off-shell effects on both experimental and theoretical studies of bound states of ηA systems, particularly in systems with $3 \leq A < 12$, are discussed. In view of the interest in the method of FSI, we also reexamine in more detail this method and its applications to the study of η - ${}^3,4\text{He}$ systems. The conclusions of our study are summarized in Sec. IV.

II. THEORY

We calculate the binding energy ϵ_η and width Γ_η of an η -nucleus bound system by solving the momentum-space relativistic three-dimensional integral equation

$$\frac{\mathbf{k}'^2}{2\mu} \psi(\mathbf{k}') + \int d\mathbf{k} \langle \mathbf{k}' | V | \mathbf{k} \rangle \psi(\mathbf{k}) = E \psi(\mathbf{k}'), \quad (1)$$

using the inverse-iteration method [32]. Here $\langle \mathbf{k}' | V | \mathbf{k} \rangle$ are momentum-space matrix elements of the η -nucleus optical potential V , with \mathbf{k} and \mathbf{k}' denoting, respectively, the initial and final η -nucleus relative momenta. The μ is the reduced mass of the η -nucleus system and E is the complex eigenenergy that we will denote as $\epsilon_\eta + i\Gamma_\eta/2 \equiv \kappa^2/2\mu$. For bound states, $\epsilon_\eta < 0$ and $\Gamma_\eta < 0$. As mentioned in Sec. I, three different approaches to V are used by us to calculate E , and they are described below.

A. Covariant η -nucleus optical potential

In spite of its Schrödinger-like form, Eq. (1) is covariant as it can be obtained from applying a specific covariant reduction [33] to the relativistic bound-state equation $\Psi = G_0 \mathcal{V} \Psi$. The three-dimensional relativistic wave function ψ and the covariant potential $\langle \mathbf{k}' | V | \mathbf{k} \rangle$ in Eq. (1) are related to the fully relativistic ones by

$$\psi(\mathbf{k}) = \sqrt{\frac{R(\kappa_r^2)}{R(\mathbf{k}^2)}} \Psi(\mathbf{k}, k^0) \quad (2)$$

and

$$\langle \mathbf{k}' | V | \mathbf{k} \rangle = \sqrt{R(\mathbf{k}'^2)} \langle \mathbf{k}' | \chi(W, k'^0, k^0) | \mathbf{k} \rangle \sqrt{R(\mathbf{k}^2)}. \quad (3)$$

In the above equations,

$$W = \sqrt{M_\eta^2 + \kappa_r^2} + \sqrt{M_A^2 + \kappa_r^2}, \quad \kappa_r^2 \equiv 2\mu\epsilon_\eta, \quad (4)$$

and

$$R(\mathbf{k}^2) = \frac{M_\eta + M_A}{E_\eta(\mathbf{k}) + E_A(\mathbf{k})}. \quad (5)$$

However, as a result of the application of the covariant reduction, the zeroth components of the four-momenta \mathbf{k} and \mathbf{k}' are no longer independent variables but are constrained by

$$k^0 = W - E_A(\mathbf{k}), \quad k'^0 = W - E_A(\mathbf{k}'). \quad (6)$$

The main advantage of working with a covariant theory is that the η -nucleus interaction V can be related to the elementary ηN process by unambiguous kinematical transformations [29].

The first-order microscopic η -nucleus optical potential has the form

$$\langle \mathbf{k}' | V | \mathbf{k} \rangle = \sum_j \int d\mathbf{Q} \langle \mathbf{k}', -(\mathbf{k}' + \mathbf{Q}) | t(\sqrt{s_j})_{\eta N \rightarrow \eta N} | \mathbf{k}, -(\mathbf{k} + \mathbf{Q}) \rangle \phi_j^*(-\mathbf{k}' - \mathbf{Q}) \phi_j(-\mathbf{k} - \mathbf{Q}), \quad (7)$$

where the off-shell ηN interaction $t_{\eta N \rightarrow \eta N}$ is weighted by the product of the nuclear wave functions $\phi_j^* \phi_j$ corresponding to having the nucleon j at the momenta $-(\mathbf{k} + \mathbf{Q})$ and $-(\mathbf{k}' + \mathbf{Q})$ before and after the collision, respectively. The $\sqrt{s_j}$ is the ηN invariant mass and is equal to the total energy in the center-of-mass (c.m.) frame of the η and the nucleon j . It is given by [29]

$$\begin{aligned} s_j &= [\{W - E_{C,j}(\mathbf{Q})\}^2 - \mathbf{Q}^2] \\ &= \left[M_\eta + M_N - |\epsilon_j| - \frac{\mathbf{Q}^2}{2M_{C,j}} \left(\frac{M_\eta + M_A}{M_\eta + M_N} \right) \right]^2 \\ &< (M_\eta + M_N)^2, \end{aligned} \quad (8)$$

where \mathbf{Q} , $E_{C,j}$, and $M_{C,j}$ are, respectively, the momentum, total energy, and mass of the core nucleus arising from removing a nucleon j of momentum $-(\mathbf{k} + \mathbf{Q})$ and binding energy $|\epsilon_j|$ from the target nucleus having the momentum $-\mathbf{k}$. Equations (7) and (8) indicate that the calculation of V involves integration over the Fermi motion variable \mathbf{Q} and requires knowledge of the basic $t_{\eta N \rightarrow \eta N}$ at subthreshold energies.

The matrix element of $t_{\eta N \rightarrow \eta N}$ in the η -nucleus system is related to the ηN scattering amplitude \mathcal{A} in the ηN system by

$$\begin{aligned} &\langle \mathbf{k}', -(\mathbf{k}' + \mathbf{Q}) | t(\sqrt{s_j})_{\eta N \rightarrow \eta N} | \mathbf{k}, -(\mathbf{k} + \mathbf{Q}) \rangle \\ &= \frac{\sqrt{E_\eta(\mathbf{p}') E_N(\mathbf{p}') E_\eta(\mathbf{p}) E_N(\mathbf{p})}}{\sqrt{E_\eta(\mathbf{k}') E_N(\mathbf{k}' + \mathbf{Q}) E_\eta(\mathbf{k}) E_N(\mathbf{k} + \mathbf{Q})}} \mathcal{A}(\sqrt{s_j}, \mathbf{p}', \mathbf{p}), \end{aligned} \quad (9)$$

where \mathbf{p} and \mathbf{p}' are the initial and final relative three-momenta in the ηN c.m. frame. We define the on-shell limit as $p' = p = p_o$ and $\sqrt{s_j} = E_\eta(p_o) + E_N(p_o) \equiv \sqrt{s_o}$, where p_o is the on-shell (asymptotic) momentum. A natural way of parametrizing \mathcal{A} is

$$\begin{aligned} \mathcal{A}(\sqrt{s_j}, \mathbf{p}', \mathbf{p}) &= - \frac{\sqrt{s_j}}{4\pi^2 \sqrt{E_\eta(p') E_N(p') E_\eta(p) E_N(p)}} \\ &\times \mathcal{F}(\sqrt{s_j}, \mathbf{p}', \mathbf{p}), \end{aligned} \quad (10)$$

so that $(d\sigma/d\Omega)_{\eta N \rightarrow \eta N} = |\mathcal{F}|^2$. The \mathcal{F} has the standard partial-wave expansion of a spin-0–spin-1/2 system:

$$\begin{aligned} &\mathcal{F}(\sqrt{s_j}, \mathbf{p}', \mathbf{p}) \\ &= \frac{1}{\sqrt{p'p}} \sum_l \{ [l t_{2T,2j-}^l(\sqrt{s_j}, p', p) + (l+1) t_{2T,2j+}^l] \\ &\quad \times (\sqrt{s_j}, p', p) P_l(z) - i \vec{\sigma} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{p}}') [t_{2T,2j-}^l \\ &\quad \times (\sqrt{s_j}, p', p) - t_{2T,2j+}^l(\sqrt{s_j}, p', p)] P_l'(z) \}, \end{aligned} \quad (11)$$

where $z = \hat{\mathbf{p}} \cdot \hat{\mathbf{p}}'$, $j_\pm = l \pm 1/2$, and T is the isospin of the ηN system and equals to 1/2. In the on-shell limit,

$$\frac{t_{2T,2j_\pm}^\ell(\sqrt{s_j}, p, p)}{\sqrt{p'p}} \rightarrow \frac{1}{2ip} \{ \exp[2i\delta_{2T,2j_\pm}^\ell(\sqrt{s_o})] - 1 \}. \quad (12)$$

The phase shifts δ^ℓ are complex-valued because the thresholds for $\eta N \rightarrow \pi N$ and $\eta N \rightarrow \pi\pi N$ reactions are lower than the threshold for ηN scattering. When $p \rightarrow 0$, $\delta^\ell \rightarrow p^{2\ell+1} a^{(\ell)}$, and

$$\frac{t_{2T,2j_\pm}^\ell(\sqrt{s_j}, p, p)}{\sqrt{p'p}} \rightarrow p^{2\ell} a_{2T,2j_\pm}^{(\ell)}. \quad (13)$$

The $a_{2T,2j}^{(0)}$ and $a_{2T,2j}^{(1)}$ are, respectively, the (complex) ηN scattering length and volume. Near the threshold, only the s -wave term, t_{11}^0 in Eq. (11), is important.

Different off-shell models give different off-shell extensions of \mathcal{A} to kinematic regions where $p \neq p'$ and $\sqrt{s_j} \neq \sqrt{s_o}$. In the separable model of Ref. [2], the off-shell amplitude is given by

$$t_\alpha(\sqrt{s_j}, p', p) = K(\sqrt{s_j}, p', p) \sqrt{p'p} \left(\frac{N_\alpha(\sqrt{s_j}, p', p)}{D_\alpha(\sqrt{s_j})} \right), \quad (14)$$

with

$$K = -\frac{\pi}{\sqrt{s_j}} \sqrt{E_\eta(p') E_N(p') E_\eta(p) E_N(p)}, \quad (15)$$

$$N_\alpha = h_\alpha(\sqrt{s_j}, p') h_\alpha(\sqrt{s_j}, p) \propto \frac{g_{\eta N \alpha}^2}{2\sqrt{s_j}} (p' p)^\ell v_\ell(p') v_\ell(p), \quad (16)$$

and

$$D_\alpha = \sqrt{s_j} - M_\alpha - \Sigma_\eta^\alpha(\sqrt{s_j}) - \Sigma_\pi^\alpha(\sqrt{s_j}) - \Sigma_{\pi\pi}^\alpha(\sqrt{s_j}). \quad (17)$$

Here α is a shorthand notation for the quantum numbers $(\ell, 2T, 2j)$ of the isobar resonance α . M_α is the bare mass of isobar α and Σ_η^α , Σ_π^α , and $\Sigma_{\pi\pi}^\alpha$ in Eq. (17) are the self-energies of the isobar α associated, respectively, with its coupling to the ηN , πN , and $\pi\pi N$ channels [2]. The coupling constants and form factors are denoted by g and v . At the ηN threshold, only the s -wave ηN interaction is important, which limits the isobar α to $N^*(1535)$ or $\alpha = (\ell, 2T, 2j) = (0, 1, 1)$. Clearly, different models will have different off-shell extensions in energy and momenta for $t_\alpha(\sqrt{s_j}, p', p)$. However, they should all satisfy Eq. (12) in the on-shell limit.

B. Factorization approximation

We define the factorization approximation (FA) by taking the ηN scattering amplitude in Eq. (7) out of the \mathbf{Q} integration at an *ad hoc* fixed momentum $\langle \mathbf{Q} \rangle$:

$$\langle \mathbf{k}' | V_{\text{FA}} | \mathbf{k} \rangle = \langle \mathbf{k}', -(\mathbf{k}' + \langle \mathbf{Q} \rangle) | t(\sqrt{s})_{\eta N \rightarrow \eta N} | \mathbf{k}, -(\mathbf{k} + \langle \mathbf{Q} \rangle) \rangle f(\mathbf{k}' - \mathbf{k}), \quad (18)$$

where

$$f(\mathbf{k}' - \mathbf{k}) = \sum_j \int d\mathbf{Q} \phi_j^*(-\mathbf{k}' - \mathbf{Q}) \phi_j(-\mathbf{k} - \mathbf{Q}), \quad (19)$$

is the nuclear form factor having the normalization $f(0) = A$. In Eq. (18), $t_{\eta N \rightarrow \eta N}$ is still defined by the same functional dependences on various momenta and energies as given by Eq. (14), except that p' and p are now determined from an ad-hoc momentum $\langle \mathbf{Q} \rangle$ in the η -nucleus system and that the interaction is given by an ad-hoc energy \sqrt{s} . The choice of $\langle \mathbf{Q} \rangle$ is not unique. One option is to take an average of two geometries corresponding, respectively, to having a motionless target nucleon fixed before and after the ηN interaction. This leads to

$$\langle \mathbf{Q} \rangle = -\left(\frac{A-1}{2A}\right)(\mathbf{k}' - \mathbf{k}). \quad (20)$$

This choice has the virtue of preserving the symmetry of the t matrix with respect to the interchange of \mathbf{k} and \mathbf{k}' . (There are other possible schemes; see, for example, Ref. [34].) An inspection of Eq. (8) suggests that it is reasonable to assume $\sqrt{s} = M_\eta + M_N - \Delta \equiv \sqrt{s_{\text{th}}} - \Delta$, with Δ being an energy shift

TABLE II. Binding energies and half-widths (both in MeV) of η -mesic nuclei given by the full off-shell calculation. The solutions were obtained with the ηN interaction parameters determined from the πN phase shifts of Arndt *et al.* (Ref. [38]). No bound state solutions of Eq. (1) were found for $A < 12$.

Nucleus	Orbital ($n\ell$)	$\epsilon_\eta + i\Gamma_\eta/2$
^{12}C	1s	-(1.19+3.67i)
^{16}O	1s	-(3.45+5.38i)
^{26}Mg	1s	-(6.39+6.60i)
^{40}Ca	1s	-(8.91+6.80i)
^{90}Zr	1s	-(14.80+8.87i)
^{208}Pb	1p	-(4.75+6.70i)
	1s	-(18.46+10.11i)
	2s	-(2.37+5.82i)
	1p	-(12.28+9.28i)
	1d	-(3.99+6.90i)

parameter. In πN scattering, the downward shift Δ that fit the data was determined to be ~ 30 MeV [29,35].

C. “On-shell” optical potential

We will use the term “on-shell” optical potential for the optical potential where the on-shell hadron-nucleon (hN) scattering length is used to generate the hN interaction. In the literature, the first-order low-energy hadron-nucleus “on-shell” optical potential is often given as [36,37]

$$\begin{aligned} \langle \mathbf{k}' | U | \mathbf{k} \rangle = & -\frac{1}{4\pi^2\mu} \left(1 + \frac{M_h}{M_N}\right) f(\mathbf{k}' - \mathbf{k}) \\ & \times \sum_{\ell=0,1} \frac{|\mathbf{k}'|^\ell |\mathbf{k}|^\ell}{(1 + M_h/M_N)^{2\ell}} a_{hN}^{(\ell)} P_\ell(\hat{\mathbf{k}}' \cdot \hat{\mathbf{k}}), \end{aligned} \quad (21)$$

where M_h is the hadron mass, μ the hadron-nucleus reduced mass, and f the nuclear form factor normalized as $f(0) = A$. For η -mesic nuclei, $M_h = M_\eta$ and $a_{\eta N}^{(0)}$ is the s -wave ηN scattering length. As has been shown in Ref. [1], U corresponds to V_{FA} with no energy shift ($\Delta=0$) and in a static limit of the target nucleon. In this latter limit, the ηN relative momenta are $\mathbf{p} = \mathbf{k}/(1 + M_\eta/M_N)$ and $\mathbf{p}' = \mathbf{k}'/(1 + M_\eta/M_N)$. In addition, $\hat{\mathbf{k}}' \cdot \mathbf{k} = \hat{\mathbf{p}}' \cdot \hat{\mathbf{p}}$.

III. RESULTS AND DISCUSSION

The binding energies and half-widths of η -mesic nuclei given by the off-shell microscopic calculation are presented in Table II. The covariant optical potential [Eq. (7)] used in the calculation is based on the model of Ref. [2]. We use this model to demonstrate the effect of the subthreshold ηN interaction on the formation of η -mesic nucleus. The solutions are obtained with the ηN interaction parameters $g_{\eta N \alpha}$, M_α , and $\Lambda_{\eta N \alpha}$ determined from the πN phase shifts of Arndt *et al.* [38]. The p -wave and d -wave interactions are also attractive at the threshold but their magnitudes are very small

TABLE III. Binding energies and half-widths (both in MeV) of η -mesic nuclei obtained with the factorization approach for different values of the energy shift parameter Δ (in MeV).

Nucleus	Orbital ($n\ell$)	$\Delta=0$	$\Delta=10$	$\Delta=20$	$\Delta=30$
^{12}C	$1s$	$-(2.18+9.96i)$	$-(1.80+6.80i)$	$-(1.42+5.19i)$	$-(1.10+4.10i)$
^{16}O	$1s$	$-(4.61+11.57i)$	$-(3.92+8.13i)$	$-(3.33+6.37i)$	$-(2.84+5.17i)$
^{26}Mg	$1s$	$-(10.21+15.41i)$	$-(8.95+11.17i)$	$-(7.94+8.97i)$	$-(7.11+7.46i)$
^{40}Ca	$1s$	$-(14.34+17.06i)$	$-(12.75+12.55i)$	$-(11.53+10.21i)$	$-(10.51+8.59i)$
^{90}Zr	$1s$	$-(21.32+18.59i)$	$-(19.15+13.97i)$	$-(17.58+11.54i)$	$-(16.29+9.84i)$
	$1p$	$-(8.27+16.01i)$	$-(7.19+11.47i)$	$-(6.23+9.48i)$	$-(5.40+7.94i)$
^{208}Pb	$1s$	$-(24.06+19.18i)$	$-(21.88+14.44i)$	$-(20.28+11.96)$	$-(18.96+10.22i)$
	$2s$	$-(4.89+11.04i)$	$-(3.67+8.28i)$	$-(2.81+6.79i)$	$-(2.12+5.72i)$
	$1p$	$-(18.33+18.97i)$	$-(16.31+14.27i)$	$-(14.81+11.79i)$	$-(13.56+10.06i)$
	$1d$	$-(8.27+14.07i)$	$-(6.17+10.56i)$	$-(5.58+8.71i)$	$-(4.66+7.41i)$

and have negligible effect on ϵ_η and Γ_η . The nuclear wave functions in Eq. (7) are derived from the experimental form factors with the proton finite size corrected for [39]. Details of the calculation can be found in Refs. [1,7]. As can be seen from the table, the binding energy increases as the nucleus becomes heavier. In addition, the number of nuclear orbital in which the η is bound ($1s$, $2s$, $1p$, etc.) increases with increasing mass number A . The reason for this trend is discussed in Ref. [1]. We would like to point out that our microscopic calculation does not use the on-shell scattering length as an input.

We emphasize that the reason we use the off-shell model of Ref. [2] in our comparative analysis is because we have at our disposal the detailed unitary off-shell momenta and energy dependences of that model, which allow us to study the off-shell effects in the formation of the η -nucleus bound state. Such off-shell information is not readily obtainable from other ηN models, either because the models cannot be extended to the off-shell domain or because of the elaborate computation required to generate the needed off-shell p, p' , and \sqrt{s} dependences of the ηN interaction. Fortunately, as we shall see later, the effects of off-shell dynamics on any

reasonable off-shell model will be qualitatively similar.

In Table III, we present the bound-state solutions obtained from using the factorized covariant potential V_{FA} [Eq. (18)] with $\Delta=0, 10, 20, 30$ MeV. The same interaction parameters were used. The nuclear form factors [40,41] used in the calculations are summarized in Table IV. A comparison between Tables II and III indicates that the FA results with $\Delta=30$ MeV are very close to the off-shell results. This value of Δ is similar to the one found in pion-nucleus elastic scattering [35] and can be understood by noting that the average nuclear binding and Fermi motion amount to about a 30-MeV downward shift [29] of the hadron-nucleon interaction energy \sqrt{s} . Our full off-shell dynamical calculations indicate, therefore, that the ηN interaction in η bound-state formation takes place at energies about 30 MeV below the (free-space) threshold.

The subthreshold nature of the hadron-nucleon interaction in a nucleus is also evident in K -mesic and π -mesic atoms. For example, although the free-space KN scattering length is repulsive, the effective KN scattering length needed to fit the K -mesic atom data is attractive [30]. The sign change of the scattering length can be easily understood. Indeed, one may

TABLE IV. Nuclear form factors used in the factorization approach and scattering-length approach.

Nucleus	Form factor	Parameters ^a
^3He	Hollow exponential	$a = 1.82$ fm
	Gaussian	$a = 1.77$ fm
^4He	Three-parameter Fermi	$c = 1.01$ fm, $z = 0.327$ fm, $w = 0.445$ fm
	Frosch model	$a = 0.316$ fm, $b = 0.680$ fm
^6Li	Modified harmonic well	$a_1 = 1.71$ fm, $a_2 = 2.08$ fm
^9Be	Harmonic well	$\alpha = 2/3$, $a = 2.42$ fm
^{10}B	Harmonic well	$\alpha = 1$, $a = 2.45$ fm
^{11}B	Harmonic well	$\alpha = 1$, $a = 2.42$ fm
^{12}C	Harmonic well	$\alpha = 4/3$, $a = 2.53$ fm
^{16}O	Harmonic well	$\alpha = 1.6$, $a = 2.75$ fm
^{26}Mg	Two-parameter Fermi	$c = 3.050$ fm, $z = 0.524$ fm
^{40}Ca	Two-parameter Fermi	$c = 3.510$ fm, $z = 0.563$ fm
^{90}Zr	Three-parameter Gaussian	$c = 4.500$ fm, $z = 2.530$ fm, $w = 0.20$ fm
^{208}Pb	Two-parameter Fermi	$c = 6.624$ fm, $z = 0.549$ fm

^aReference [40] for $A=3-16$ and Ref. [41] for the rest of the nuclei.

TABLE V. Binding energies and half-widths (both in MeV) of η -mesic nuclei given by the scattering-length approach for two different values of the scattering length $a_{\eta N}$. A blank entry indicates the absence of bound state. No bound state exists in ${}^3\text{He}$.

Nucleus	Orbital	$a_{\eta N} = (0.28 + 0.19i)$ fm	
	$(n\ell)$	$a_{\eta N} = (0.28 + 0.19i)$ fm	$a_{\eta N} = (0.51 + 0.21i)$ fm
${}^4\text{He}^a$	1s		− (6.30 + 11.47i)
${}^6\text{Li}$	1s		− (3.47 + 6.79i)
${}^9\text{Be}$	1s		− (13.78 + 12.45i)
${}^{10}\text{B}$	1s	− (0.93 + 8.70)	− (15.85 + 13.05i)
${}^{11}\text{B}$	1s	− (2.71 + 10.91i)	− (20.78 + 15.42i)
${}^{12}\text{C}$	1s	− (2.91 + 10.22i)	− (19.61 + 14.20i)
${}^{16}\text{O}$	1s	− (5.42 + 11.43i)	− (23.26 + 14.86i)
	1p		− (0.95 + 7.72i)
${}^{26}\text{Mg}$	1s	− (11.24 + 14.76i)	− (33.11 + 17.73i)
	1p		− (13.41 + 12.33i)
${}^{40}\text{Ca}$	1s	− (15.46 + 16.66i)	− (38.85 + 19.16i)
	2s		− (5.59 + 6.14i)
	1p	− (1.22 + 10.58i)	− (22.84 + 14.32i)
	1d		− (4.28 + 9.52i)
${}^{90}\text{Zr}$	1s	− (22.41 + 19.97i)	− (48.40 + 22.60i)
	2s		− (26.07 + 10.07i)
	1p	− (10.18 + 14.33i)	− (31.53 + 15.93i)
	2p		− (18.51 + 8.57i)
${}^{208}\text{Pb}$	1s	− (24.55 + 19.57i)	− (50.27 + 21.42i)
	2s	− (10.56 + 13.32i)	− (22.27 + 11.50i)
	1p	− (20.19 + 19.05i)	− (34.03 + 10.03i)
	2p		− (1.89 + 3.75i)
	1d	− (12.22 + 16.07i)	− (27.89 + 12.17i)

^aForm factor used is the three-parameter Fermi.

note that the $\Lambda(1405)$ resonance is situated about 26 MeV below the KN threshold [30]. Using a downward shift of 30 MeV found in this work, we can see that the in-medium KN interaction occurs actually below the resonance. Hence, an attraction arises (see Sec. I). In the π -mesic atom studies, a strong s -wave repulsion of the effective πN interaction is indicated by the data. This is in sharp contrast to the free-space πN s -wave interaction where there is a near-perfect cancellation between the S_{31} and S_{11} scattering lengths. Bhalerao and Shakin [42] showed that the strong s -wave repulsion is due to different energy dependences of the S_{31} and S_{11} interactions, which make the cancellation no longer near complete at subthreshold energies. The mesic-atom experiments, therefore, indicate unambiguously that in bound-state formation the hadron-nucleon interaction inside a nucleus occurs at subthreshold energies.

The results of the “on-shell” optical potential [Eq. (21)] are given in Table V for two different values of $a_{\eta N}$. The first one is the same one used for Tables II and III, and the other is chosen from Table I, such that it has an imaginary part very similar to the former one. The nuclear form factors are the same as those used in the FA calculations. For these two scattering lengths, no bound state can exist in ${}^3\text{He}$. Upon comparing the third column of Table V with the off-shell calculation (Table II), we can see that the “on-shell”

approximation predicts more strongly bound η -mesic nuclei. Also, as expected, the “on-shell” results for $a_{\eta N} = (0.28 + 0.19i)$ fm are similar to those of FA with $\Delta = 0$ MeV. We further note from Table V that the effect on decreasing the calculated nuclear binding energies as caused by the decrease in $\text{Re } a_{\eta N}$ is not a linear function of the nuclear mass number A . For light nuclei ($A \leq 12$) a decrease of $\text{Re } a_{\eta N}$ by a factor of ~ 2 causes ϵ_{η} to decrease by a factor greater than 10.

While the in-medium ηN interaction strength is a function of the off-shell variables p', p, \sqrt{s} , one can appreciate the main feature of the off-shell effects by considering only the \sqrt{s} , or the energy, dependence. Our study of the three different optical potentials indicates that this strength decreases with energy. However, the specific subthreshold energy dependence is model-dependent. We may define a phenomenological reduction factor R by

$$t_{\eta N \rightarrow \eta N}(\sqrt{s_{\text{th}}} - \Delta, p', p) = R(\Delta) t_{\eta N \rightarrow \eta N}(\sqrt{s_{\text{th}}}, p', p), \quad (22)$$

and introduce an effective in-medium scattering length by

$$a_{\eta N}^{\text{eff}} = R(\Delta) a_{\eta N}. \quad (23)$$

In the model of Ref. [2], $R(\Delta) = \sqrt{s_{\text{th}}} D(\sqrt{s_{\text{th}}}) / [(\sqrt{s_{\text{th}}} - \Delta) D(\sqrt{s_{\text{th}}} - \Delta)]$ [Eqs.(10) and (14)–(17)]. Our calculation indicates that $a_{\eta N}^{\text{eff}} = (0.26 + 0.13i)$, $(0.25 + 0.11i)$, $(0.23 + 0.09i)$ fm for $\Delta = 10, 20, 30$ MeV, respectively, while at the threshold $a_{\eta N} = (0.28 + 0.19i)$ fm. The $R(\Delta)$ is about 0.89 at $\Delta = 20$ MeV and 0.82 at $\Delta = 30$ MeV. This reduction is the origin of the Δ dependence of the ϵ_{η} and Γ_{η} in Table III. For the scattering length of the model of Ref. [12], it was mentioned that at 20 and 30 MeV below the threshold, $a_{\eta N}^{\text{eff}} = (0.49 + 0.10i)$ fm and $(0.45 + 0.08i)$ fm, respectively. Upon comparing these values with $a_{\eta N} = (0.75 + 0.27i)$ fm given by that model at the threshold, we see a reduction of more than 1/3 of the real part of $a_{\eta N}$ (i.e., $R = 0.6$). It seems that the higher the $\text{Re } a_{\eta N}$ at the threshold the greater subthreshold reduction. We note that at 20 to 30 MeV below the threshold, both models give an imaginary part of $a_{\eta N}^{\text{eff}}$ about 0.09i fm. This is because the $\mathcal{I}m a_{\eta N}^{\text{eff}}$ is related to the reaction channel. At the subthreshold region, the only reaction channels that the ηN channel can couple to are the πN and $\pi\pi N$ channels, which were taken into account by both the models [2,12]. On the other hand, the real part of the effective scattering length is still very model dependent, though in lesser degree than at the threshold. Because of this model dependence, it is not possible to guess the subthreshold reduction for the other models listed in Table I for which the off-shell dependence cannot be easily reconstructed from the corresponding publications. We conclude from the two models analyzed above that a substantial reduction of attractive strength, $\text{Re } a_{\eta N}$, must occur at subthreshold energies.

This reduction in attractive strength is of very general nature as it is a direct consequence of the $N^*(1535)$ resonance. We recall that the attraction between the η and nucleon at low energies arises because the ηN threshold is situated below this resonance. However, as the energy shift Δ becomes larger, the ηN interaction energy moves farther

TABLE VI. Values of $a_{\eta N}^{\min}$ for nuclei with mass number $A < 10$. The $\text{Im } a_{\eta N}^{\min}$ is fixed at 0.09 fm (see the text).

Nucleus	Nuclear form factor	$a_{\eta N}^{\min}$ (fm)
${}^3\text{He}$	Hollow exponential	$0.49 + 0.09i$
${}^4\text{He}$	Three-parameter Fermi	$0.35 + 0.09i$
${}^6\text{Li}$	Modified harmonic well	$0.35 + 0.09i$
${}^9\text{Be}$	Harmonic well	$0.24 + 0.09i$

downward away from the resonance. Any reasonable off-shell model leads, therefore, to a reduced attraction. Consequently, calculations making use of free-space scattering length (corresponding to $\Delta=0$) necessarily overestimate ϵ_η . Because of the large sensitivity of the binding energies to the η -nucleon interaction in light nuclei and because $\text{Re } a_{\eta N}^{\text{eff}} \ll \text{Re } a_{\eta N}$, many η -nucleus bound states in very light nuclei, as predicted by using some of the $a_{\eta N}$ in Table I, may not exist in real situations. We caution, therefore, against using the ‘‘on-shell’’ (or scattering-length) prediction as a guide for searching η -mesic nuclei.

In view of the existing interest in searching for η -nucleus bound states in light nuclear systems, we believe that it is informative to determine values of the ‘‘minimal’’ scattering length, $a_{\eta N}^{\min}$, which represents the smallest value of an $a_{\eta N}^{\text{eff}}$ that can bind the η into an $1s$ nuclear orbital. Clearly, the real and imaginary parts of this scattering length are not independent of each other. We fixed $\text{Im } a_{\eta N}^{\min}$ to 0.09 fm, as suggested by the two off-shell models discussed above, and searched for $\text{Re } a_{\eta N}^{\min}$. The results for several light nuclei are given in Table VI.

It is interesting to revisit the FSI analysis and to examine the results of Refs. [24,43] in light of our findings. The FSI analysis was first applied by Wilkin [24] to the η - ${}^3\text{He}$ system, which is a final state of the $pd \rightarrow \eta^3\text{He}$ reaction. The analysis raised a great deal of theoretical interest [25] and was later extended to the study of the η - ${}^4\text{He}$ system [42], which is a final state of the $dd \rightarrow \eta^4\text{He}$ reaction. Let us denote the η -nucleus scattering length as $A_0 \equiv A_R + iA_I$. According to the Goldberger-Watson FSI theory [11] for a weak transition and a strong FSI, one can approximate the total reaction amplitude, f , at low energies by

$$|f|^2 = \frac{|f_B|^2}{|1 - iA_0 p_\eta|^2} = \frac{|f_B|^2}{(1 + A_I p_\eta)^2 + (A_R p_\eta)^2}, \quad (24)$$

where p_η is the c.m. momentum of the η meson. The f_B is the transition amplitude and was treated as a constant in Ref. [24]. The unitarity condition requires $A_I > 0$. Hence, $|f|^2$ increases monotonically when p_η decreases. A good fit to the p_η dependence of $|f|^2$ was obtained by Wilkin, who used $a_{\eta N} = (0.55 \pm 0.20 + 0.30i)$ fm, which led to $A_0(\eta^3\text{He}) = (-2.31 + 2.57i)$ fm [24]. In a later work, Wilkin and his collaborators [43] used $a_{\eta N} \approx (0.52 + 0.25i)$ fm, which gave $A_0(\eta^3\text{He}) \approx (-2.3 + 3.2i)$ fm and $A_0(\eta^4\text{He}) \approx (-2.2 + 1.1i)$ fm, respectively. Upon introducing these A_0 into Eq. (24), they obtained a good representation of the experimental p_η dependence of $|f|^2$ for both ${}^3\text{He}$ and ${}^4\text{He}$. Because in the

case of a real potential, a negative scattering length has the possibility of corresponding either to a repulsive interaction or to an attractive interaction that supports a s -wave bound state [44], the findings of Ref. [43] have raised the hope that η -nucleus bound states might exist in ${}^3\text{He}$ and ${}^4\text{He}$.

A closer look at the value of the η -helium scattering lengths can provide partial answer to the question as to whether η can be bound in ${}^3\text{He}$ and ${}^4\text{He}$ nuclei. This is because for a complex potential the existence of a bound state imposes a constraint on the complex scattering length. Equation (24) indicates that, if a weakly bound state exists, the amplitude f will have a pole, p_{pol} , in the complex p_η plane at

$$p_{\text{pol}} = \frac{-i}{A_0} = \frac{-A_I - iA_R}{d}, \quad (25)$$

where $d = A_R^2 + A_I^2$ and A_I is positive. The condition for a bound state is $\text{Re } p_{\text{pol}} < 0$. This requires that $|A_I| < |A_R|$. It is easy to see that $A_0(\eta^3\text{He})$ of Ref. [43] does not satisfy this inequality, while $A_0(\eta^4\text{He})$ does. Our optical-potential calculations have verified this, namely, with $a_{\eta N}$ used in Ref. [43] we have found that there is no bound state in ${}^3\text{He}$ but there is one in ${}^4\text{He}$. In other words, $A_R < 0$ is not a sufficient condition for having a bound state. Consequently, in spite of the much greater slope of $|f|^2$ with respect to p_η in the η - ${}^3\text{He}$ system [43], we conclude that there is no η -nuclear bound state in ${}^3\text{He}$. For the same reason, η cannot be bound onto the deuteron because none of the various calculated $A_0(\eta d)$ of Ref. [26] can satisfy the condition $|A_I| < |A_R|$. On the other hand, the formation of a bound state in ${}^4\text{He}$ remains a possibility.

This possibility is, however, hampered by the fact that $|f|^2$ is insensitive to the sign of A_R , as can be seen from Eq. (24). Indeed, we have found an A_0 that has a positive real part and can equally describe the data. For example, with $a_{\eta N} = (0.30 + 0.09i)$ fm, we have obtained from our optical potential, Eq. (21), $A_0(\eta^3\text{He}) = 2.10 + 2.88i$ fm, which can well describe the ${}^3\text{He}$ data. Similarly, a good representation of the ${}^4\text{He}$ data can be obtained with $A_0(\eta^4\text{He}) = (0.80 + 1.75i)$ fm by using $a_{\eta N} = (0.16 + 0.09i)$ fm. The quality of representation is shown in Fig. 1. In both cases $A_R > 0$, which corresponds to the situation where the interaction is attractive but not strong enough to support a bound state. Hence, we have shown that FSI can also admit solutions corresponding to having no η bound state. We emphasize that our analysis above does not imply our preference for weaker $a_{\eta N}$ but rather exemplifies numerically that FSI cannot provide a unique answer concerning the formation of bound states.

At this point, several comments are in order. First, although we need different $a_{\eta N}$ for ${}^3\text{He}$ and ${}^4\text{He}$ while in Ref. [43] only one $a_{\eta N}$ was needed, our result is reasonable because it reflects the fact that the transition amplitudes f_B in the two reactions may not be treated as p_η -independent and thus do not scale each other by a multiplicative constant. Indeed, by using the two-step model of Ref. [45], Willis *et al.* [43] cannot fit the data of $|f(p_\eta)|^2$ for the η - ${}^3\text{He}$ and η - ${}^4\text{He}$ systems with the same $a_{\eta N}$. In addition, one may

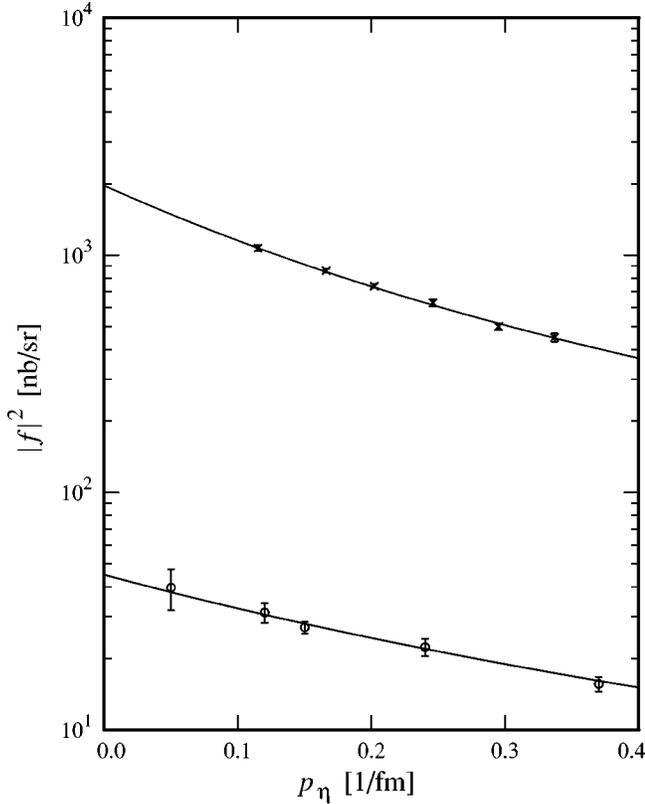


FIG. 1. Dependences of $|f|^2$ on p_η . Data for the η - ${}^3\text{He}$ (crosses) and η - ${}^4\text{He}$ (open circles) systems are from Refs. [25] and [43], respectively. Solid curves are the results of using the scattering lengths having $A_R > 0$.

note that second-order η -nucleus optical potential is important in light nuclei. This potential is sensitive to the nucleon-nucleon (NN) correlation, which is very different in ${}^3\text{He}$ and ${}^4\text{He}$, as evidenced by the fact that the matter densities derived from the measured charge densities of these two nuclei have spatial dependences that do not scale each other. Inclusion of second-order optical potential will lead to different modifications of $A_0(\eta^{3,4}\text{He})$. Consequently, when only first-order optical potential is used to generate the A_0 in fitting the data, the effective $a_{\eta N}$ will be different. In a similar manner, the different NN correlations in the two helium nuclei also contribute to the nonscaling of the f_B . Indeed, many reaction processes contribute to the transition matrix element. Important subsets of the processes are those that cannot be approximated by the $pp \rightarrow \pi^+ d$ and $pd \rightarrow \pi^+ {}^3\text{He}$ doorway mechanisms [45]. This is because in these processes the intermediate pion interacts with two different nucleons that do not belong to the same deuteron cluster in the nucleus. Hence, these processes depend strongly on NN correlations having a momentum dependence that cannot be obtained from using the deuteron wave function. The nonsimple nature of f_B is further exemplified by the work of Santra and Jain [46], who can describe the η - ${}^3\text{He}$ data even without FSI when the exchanges of ρ and other mesons are included in the calculation of f_B . While we strongly believe that FSI must be taken into account, Ref. [46] does indicate that there are many more aspects of the dynamics left to be thoroughly

TABLE VII. Dependence of binding energies and half-widths (both in MeV) of the η - ${}^3\text{He}$ bound state on two different form factors (hollow exponential and Gaussian) for a few values of the scattering length $a_{\eta N}$. All bound states are in the $1s$ orbital.

$a_{\eta N}$ fm	Hollow exponential	Gaussian
$0.680 + 0.240i$	$-(3.74 + 7.89i)$	$-(4.16 + 7.93i)$
$0.750 + 0.270i$	$-(6.24 + 9.94i)$	$-(6.79 + 9.94i)$
$0.876 + 0.274i$	$-(11.86 + 11.26i)$	$-(12.59 + 11.10i)$

investigated before a definitive conclusion could be drawn from the η - ${}^3,4\text{He}$ data.

Second, our use of $\text{Im } a_{\eta N}^{\text{eff}} = 0.09$ fm is merely suggested by the 30-MeV downward shift of the effective ηN interaction energy, as has been discussed after Eq. (23). With $\text{Im } a_{\eta N}^{\text{eff}} = 0.09$ fm, the helium data constrain the real part of the effective scattering length to be between 0.16 and 0.30 fm. However, we emphasize that the value of 0.09 fm is model-dependent and it is important to see if different ηN off-shell models would give a similar subthreshold value. Consequently, our finding of the repulsive scattering length for the helium systems should not be interpreted as if $\text{Im } a_{\eta N}^{\text{eff}} = 0.09$ fm were the final answer, but rather as an example showing the nonuniqueness of the solution to the problem. In view of the many needed improvements in the modeling of the transition amplitude, searching a same repulsive effective $a_{\eta N}$ for both data sets is beyond the scope of this work.

One would certainly like to be able to use the η - ${}^3,4\text{He}$ data to obtain a more stringent constraint on the effective subthreshold ηN interaction. However, we conclude from the above discussion that the currently used FSI analysis cannot provide an unequivocal answer not only because there is no simple correspondence between the sign of A_R and the existence of an η -nucleus bound state, but also because the assumption of a p_η -independent f_B is uncertain. Consequently, our main message is that one should not rely on using the on-shell scattering length and that direct detection of bound states is necessary.

Finally, we have also examined effects of nuclear form factors on the binding of η . The results for ${}^3\text{He}$ are presented in Table VII. An inspection of this last table indicates that the binding of η is not very sensitive to the finer aspect of a realistic nuclear form factor. A similar situation has also been noted for ${}^4\text{He}$. The binding energies and widths of the η as given by using the three-parameter Fermi or the Frosch form factor are close to each other. The most important effect on the formation of η -mesic nucleus is, therefore, from the subthreshold dynamics of the ηN interaction.

IV. CONCLUSIONS

Our study shows that calculated binding energies and widths of η -nucleus bound states strongly depend on the subthreshold dynamics of the ηN interaction. The results of mesic atoms and the present analysis indicate that the aver-

age ηN interaction energy in mesic-nucleus formation is below the threshold. What matters for the bound-state formation is not the ηN interaction at the threshold but the effective in-medium interaction. Because the subthreshold behavior of the ηN interaction is very model dependent, we believe that it is useful for theorists to publish not only the η -nucleon scattering length $a_{\eta N}$, but also the corresponding subthreshold values as a function of Δ . Before the availability of this information, we suggest to look for bound states in nuclear systems much heavier than those indicated by the on-shell scattering length.

The FSI analysis represents an interesting approach. Since the analysis by itself cannot provide a definitive answer as to

whether there is an attractive η -nucleus interaction strong enough to bind the η , more direct measurements such as the one used in Ref. [9] are necessary.

The downward shift in the effective interaction energy can lead to a substantial reduction of the attraction of the in-medium η -nucleon interaction with respect to its free-space value. Consequently, predictions based upon using the free-space ηN scattering length inevitably overestimates the binding of η . This overestimation of the binding, as revealed by this study, has never been taken into account in discussing η -nuclear bound states. One must bear this in mind when using the predictions given by such calculations as a guide in searching for η -nucleus bound states.

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