Methods for the study of particle production fluctuations

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We discuss various measures of net charge (conserved quantities) fluctuations proposed for the identification of critical phenomena in heavy ion collisions. We show the dynamical component of fluctuations of the net charge can be expressed simply in terms of integrals of two- and single-particle densities. We discuss the dependence of the fluctuation observables on detector acceptance, detection efficiency and colliding system size, and collision centrality. Finally, we present a toy model of particle production including charge conservation and resonance production to gauge the effects of such resonances and finite acceptance on the net-charge fluctuations.

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I. INTRODUCTION

The numbers of particles produced in relativistic nuclear collisions differ dramatically from collision to collision due to the variation of impact parameter, energy deposition, baryon stopping, and other dynamical effects [1–3]. Such fluctuations can also be influenced by novel phenomena such as disoriented chiral condensate [4,5] or the appearance of multiple event classes [6]. Even globally conserved quantities such as net charge, baryon number, and strangeness can fluctuate when measured, e.g., in a limited rapidity interval. The rapid hadronization of a quark-gluon plasma (QGP) can reduce net-charge fluctuations compared to hadronic expectations [7,8], while phase separation can increase net-baryon fluctuations [9]. Fluctuations of conserved quantities are possibly the best probes of such dynamics, because conservation laws limit the degree to which final-state scattering can dissipate them.

Many statistical measures have been suggested for analyzing particle number fluctuations in experiments [6,7,10,11]. Although these measures superficially appear to be different in nature and unrelated, closer examination reveals they are infact connected. On the other hand, each measure exhibits different dependence on collision centrality, detector acceptance (rapidity and p_t region used to calculate the observable), particle detection efficiency, and susceptibility to experimental biases. The utility of each measure depends on the particle species measured and the physical phenomena one wishes to extract. For example, ''robust'' efficiency-independent measures are best for observing the correlations between neutral and charged-particles produced by disoriented chiral condensate [12,13].

Experimental efforts to measure event-by-event fluctuations have followed two approaches. Many advocate a statistical approach in which fluctuations of particle numbers are characterized by variances, covariances or other moments [6,7,10,11,14–16]. These moments can be compared to expectations based on thermal equilibrium or other statistical models; any difference can be attributed to novel dynamics. Others emphasize the importance of the momentum-dependent correlation functions, such as the balance function [17]. The correlation-function approach has yielded great success in the case of identical pion Hanbury Brown–Twiss (HBT) correlations.

In this paper, we discuss relations between correlation functions and moment measures of net-charge fluctuations to study the dependence of these measures on collision centrality, experimental efficiency and acceptance. We focus initially on the variance ν_{dyn} suggested in Ref. [18], which is derived from integrals of the single- and two-particle distribution functions. Next, we compare these measures to alternatives suggested in Refs. [7,10]. Our correlation-function based analysis complements a study by Mrowczynski using a statistical point of view [10]. Specifically, we begin in Sec. II by defining the fluctuation measure ν_{dyn} in relation to the microscopic correlation functions. In the following section, we determine the scaling properties of ν_{dyn} with system size and, equivalently, collision centrality. We then introduce alternative fluctuation measures, and discuss their relationship with ν_{dyn} in Sec. IV. A relation between the net-charge fluctuations, and the balance function introduced by Bass *et al.* [17] is presented in Secs. VII and VIII is devoted to a discussion on the robustness of fluctuation observables, i.e., whether and how fluctuation measures introduced in Sec. IV depend on detection efficiency. Finally, we consider and compare, in Sec. IX, the various fluctuation measures in the context of simple particle production models.

II. NET-CHARGE FLUCTUATIONS AS A MEASURE OF TWO-PARTICLE CORRELATIONS

In this section, we show that multiplicity fluctuations are driven by intrinsic two-particle correlations. Statistical quantities that we discuss are constructed from the one-body and two-body densities:

$$\rho_1(\eta)=\frac{dN}{d\eta},$$

$$\rho_2(\eta_1,\eta_2)=\frac{d^2N}{d\eta_1 d\eta_2}. \tag{1}$$

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For simplicity, we focus on pseudorapidity dependence, although results can be generalized to address transverse-momentum and azimuthal-angular dependence. Our approach and notation in this section follows Refs. [19,20].

To extract statistical information from these microscopic densities, we use Eq. (1) to write the multiplicity in the rapidity range $\Delta\eta$ as

$$\langle N\rangle=\int_{\Delta\eta}\rho_1(\eta)d\eta. \tag{2}$$

Here $\langle N\rangle$ represents an average of the observable N over an event ensemble. Fluctuations of the particle number in this rapidity range are determined by integrating the two-particle density,

$$\langle N(N-1)\rangle=\int_{\Delta\eta}\rho_2(\eta_1,\eta_2)d\eta_1 d\eta_2. \tag{3}$$

The "-1" appears on the left side because the integral over $\rho_2(\eta_1,\eta_2)$ counts the average number of particle pairs in the rapidity interval. Note that $\langle N\rangle$ and $\langle N(N-1)\rangle$ are the first and second order factorial moments of the multiplicity distribution.

A familiar statistical measure of particle number fluctuations is the variance

$$V=\langle (N-\langle N\rangle)^2\rangle. \tag{4}$$

We can obtain the variance from Eqs. (2) and (3), since $V=\langle N(N-1)\rangle-\langle N\rangle(\langle N\rangle-1)$. In the absence of particle-particle correlations, the two-body density factorizes into a product of two one-body densities. In that case, we find

$$\langle N(N-1)\rangle_{\mathrm{uncorr}}=\int_{\Delta\eta}\rho_1(\eta_1)\rho_1(\eta_2)d\eta_1 d\eta_2=\langle N\rangle^2. \tag{5}$$

The variance is then $V=\langle N\rangle$, as expected since the number of particles produced in a sequence of independent events follows Poisson statistics [21]. Note that the relative uncertainty in the mean number $\langle N\rangle$ is $\sqrt{V}/\langle N\rangle=1/\sqrt{\langle N\rangle}$ for this case. Observe that the particle number in a grand canonical ensemble in thermal equilibrium follows Poisson statistics.

Information on net-charge fluctuations is contained in the two-body density for distinct particles with opposite charges. We determine these fluctuations from

$$\langle N_\alpha N_\beta\rangle=\int_{\Delta\eta}\rho_2(\eta_\alpha,\eta_\beta)d\eta_\alpha d\eta_\beta, \tag{6}$$

where α and β label the particle species. In a statistical framework, this average is related to the two-particle covariance

$$V_{\alpha\beta}=\langle N_\alpha N_\beta\rangle-\langle N_\alpha\rangle\langle N_\beta\rangle. \tag{7}$$

The covariance vanishes if there are no correlations between the species α and β, since $\rho_2(\eta_\alpha,\eta_\beta)=\rho_1(\eta_\alpha)\rho_1(\eta_\beta)$.

Following Refs. [19,20] we define the robust variance

$$R_{\alpha\alpha}=\frac{V-\langle N\rangle}{\langle N\rangle^2}, \tag{8}$$

and the robust covariance

$$R_{\alpha\beta}=\frac{V_{\alpha\beta}}{\langle N_\alpha\rangle\langle N_\beta\rangle} \tag{9}$$

for particle species α and β. These quantities have the same sensitivity to fluctuations as the variance (4) and covariance (7) but have three significant advantages. First, these quantities vanish for $V=\langle N\rangle$ and $V_{\alpha\beta}=0$, so that they measure the deviation from Poisson-statistical behavior. Second—and of greater practical importance—the ratios (8) and (9) are "robust" in that they are independent of experimental efficiency, To see why Eq. (8) is robust, let the probability of detecting each charged particle be ϵ and the probability of missing it be $1-\epsilon$. For a binomial distribution the average number of measured particles is $\langle N\rangle_{\mathrm{exp}}=\epsilon\langle N\rangle$ while the average square is $\langle N^2\rangle_{\mathrm{exp}}=\epsilon^2\langle N^2\rangle+\epsilon(1-\epsilon)\langle N\rangle$. The variance $V_{\mathrm{exp}}=\langle N^2\rangle_{\mathrm{exp}}-\langle N\rangle^2_{\mathrm{exp}}=\epsilon^2(\langle N^2\rangle-\langle N\rangle^2)+\epsilon(1-\epsilon)\langle N\rangle$, so that $V_{\mathrm{exp}}-\langle N\rangle_{\mathrm{exp}}=\epsilon^2(V-\langle N\rangle)$. We then find

$$R^{\mathrm{exp}}_{\alpha\alpha}=R_{\alpha\alpha}, \tag{10}$$

independent of ϵ; the proof that Eq. (9) is robust is similar. The ratios (8) and (9) are strictly robust only if the efficiency ϵ is independent of multiplicity. We discuss this point in more detail in Sec. VIII.

Third, $R_{\alpha\beta}$ are directly related to the particle correlations. For $\alpha\neq\beta$, we combine Eqs. (2), (6), (7), and (9) to obtain

$$R_{\alpha\beta}=\frac{\int_{\Delta\eta}\rho_2(\eta_\alpha,\eta_\beta)d\eta_\alpha d\eta_\beta}{\int_{\Delta\eta}\rho_1(\eta_\alpha)d\eta_\alpha\int_{\Delta\eta}\rho_1(\eta_\beta)d\eta_\beta}-1; \tag{11}$$

one can check that Eq. (11) also holds for $\alpha=\beta$. As in an HBT analysis, we define a correlation function C by

$$\rho_2(\eta_1,\eta_2)=\rho_1(\eta_1)\rho_1(\eta_2)[1+C(\eta_1,\eta_2)], \tag{12}$$

so that Eq. (11) yields

$$R_{\alpha\beta}=\frac{\int_{\Delta\eta}\rho_1(\eta_\alpha)\rho_1(\eta_\beta)C_{\alpha\beta}(\eta_\alpha,\eta_\beta)d\eta_\alpha d\eta_\beta}{\langle N_\alpha\rangle\langle N_\beta\rangle}. \tag{13}$$

We use this result to illustrate how to extract microscopic information on the rapidity range of correlations from the $\Delta\eta$ dependence of $R_{\alpha\beta}$ in Sec. VII.

To study net-charge fluctuations, one can measure the robust covariance for charged hadrons R_{+-}. On the other hand, it would be better to isolate the potentially interesting net-charge fluctuations from factors that cause the numbers of positive and negative hadrons to fluctuate together, such as variations in energy deposition or collision volume. Toward that end, we consider dynamic charge observable defined as the linear combination

$$\nu_{dyn}=R_{++}+R_{--}-2R_{+-}. \tag{14}$$

Ratio fluctuations considered by Jeon and Koch [7] are an alternative, see, Sec. IV. This combination vanishes when the positive and negative hadrons fluctuate simultaneously, since all the $R_{\alpha\beta}$ are then the same. We also see that ν_{dyn} is both robust (see, Sec. VIII) and straightforwardly related to the microscopic correlators (1), as are the $R_{\alpha\beta}$. We find an alternative expression for ν_{dyn} in terms of

$$\nu_{+-}=\left\langle \left(\frac{N_+}{\langle N_+\rangle}-\frac{N_-}{\langle N_-\rangle}\right)^2\right\rangle, \tag{15}$$

where N_+ and N_- are respectively the multiplicities of positive and negative hadrons. In the limit of independent particle production, ν becomes

$$\nu_{stat}=\frac{1}{\langle N_+\rangle}+\frac{1}{\langle N_-\rangle}. \tag{16}$$

The dynamic charge observable is the difference,

$$\nu_{dyn}=\nu-\nu_{stat}, \tag{17}$$

as we see by expanding the square in Eq. (15). Observe that ν_{dyn} is nonzero when net-charge fluctuations are correlated (non-Poissonian). Furthermore, Eqs. (15)–(17) are more useful than Eq. (14) for extracting correlations from numerical data since the net-charge fluctuations are typically smaller than the fluctuations of the total number of hadrons.

We examine the scaling properties of the ν_{dyn} variance with collision system size in the following section.

III. SCALING OF ν_{dyn} WITH SYSTEM SIZE AND COLLISION CENTRALITY IN $A+A$ COLLISIONS

We now study the scaling of the observables $C_{\alpha\beta}$, $R_{\alpha\beta}$, and ν_{dyn}, with collision centrality, target and projectile mass. For concreteness, we assume that nuclear collisions are a superposition of independent nucleon-nucleon (NN) subcollisions and neglect the rescattering of the hadrons. These assumptions imply that charged-particle pairs can be correlated only if produced in the same subcollision. We expect the contribution to the two-body density from these related pairs to grow linearly with the number of subcollisions M. Related pairs will be diluted by random pairs. The AA densities are

$$\rho_1^{AA}(\eta)=M\rho_1^{NN}(\eta), \tag{18}$$

$$\rho_2^{AA}(\eta_1,\eta_2)=M\rho_2^{NN}(\eta_1,\eta_2)+M(M-1)\rho_1^{NN}(\eta_1)\rho_1^{NN}(\eta_2). \tag{19}$$

The first term of Eq. (19) describes the related pairs while the second accounts for the $M(M-1)$ random pairs. These expressions apply generally to particle production from M sources; we focus on the independent-collision model for simplicity. We apply these considerations to more realistic models at the end of this section.

To compute the correlation function, we substitute the AA densities Eqs. (18)–(19) in Eq. (12) to find

$$C_{\alpha\beta}^{AA}(\eta_1,\eta_2)=\frac{C_{\alpha\beta}^{NN}(\eta_1,\eta_2)}{M}. \tag{20}$$

For independent subcollisions and in the absence of rescattering, we therefore expect the AA correlation function to have the same rapidity dependence as in pp collisions, with an overall scale that is reduced by a factor M^{-1}.

Before turning to realistic experiments, we consider for the moment a collision with a fixed number of subcollisions. The statistical observables then satisfy

$$R_{\alpha\beta}^{AA}=\frac{R_{\alpha\beta}^{NN}}{M} \tag{21}$$

and

$$\nu_{dyn}(AA)=\frac{\nu_{dyn}(pp)}{M}. \tag{22}$$

We see that all quantities scale as M^{-1}.

More realistically, suppose that one specifies a centralility range by measuring the total charge multiplicity, the zero degree energy, or some analogous global observable. The number of subcollisions will then fluctuate, adding to the variance and covariance of particle numbers and changing Eq. (21). Specifically, the fluctuations of M contribute a term $\langle N_\alpha\rangle\langle N_\beta\rangle(\langle M^2\rangle-\langle M\rangle^2)$ to the variance and covariance, V_n and $V_{\alpha\beta}$, so that Eqs. (8) and (9) give

$$R_{\alpha\beta}^{AA}=\frac{R_{\alpha\beta}^{NN}}{\langle M\rangle}+\frac{\langle M^2\rangle-\langle M\rangle^2}{\langle M\rangle^2}. \tag{23}$$

See, the appendix for a full derivation. We remark that these M fluctuations are essentially equivalent to the "volume fluctuations" discussed in a local equilibrium framework [7,8].

On the other hand, random changes in the number of independent subcollisions can change the total number of particles but not the net charge, so that Eq. (22) is effectively unchanged. We find

$$\nu_{dyn}(AA)=R_{++}+R_{--}-2R_{+-}=\frac{\nu_{dyn}(pp)}{\langle M\rangle}. \tag{24}$$

The contributions from subcollision or volume fluctuations are the same for all α and β, so that Eq. (14) implies that this contribution does not affect v_{dyn}. The second term in Eq. (21) is of order $1/\langle M\rangle$ and comparable to the first, since ISR and FNAL experiments suggest that $R_{++}^{NN}\sim R_{--}^{NN}\sim R_{+-}^{NN}/2$, each of order unity in $\Delta\eta=1$–2 at RHIC [19,20].

We now extend these considerations to the wounded-nucleon model, which sucessfully describes many global features in SPS and AGS experiments. There, one assumes that only the first subcollision of each nucleon drives particle production and neglects all subsequent interactions [22]. Since Eqs. (18) and (19) formally describe particle production from M independent sources, we can adapt Eqs. (18) and

(19) to the wounded-nucleon scenario by replacing the number of subcollisions M with the number of participant nucleons $\mathcal{M}$. We must also replace the densities ρ_1^{NN} and ρ_2^{NN} in Eqs. (18) and (19) with coefficients ρ_1^0 and ρ_2^0 that describe the production per participant. Observe that nucleons are counted as particpants if they interact at least once and that there are two participants per NN collision.

Results of the form (23) and (24) then follow from the wounded-nucleon model if we replace M with one half the number of participants $\mathcal{M}$. The average number of participants at impact parameter b for a symmetric AA collision is $\langle\mathcal{M}(b)\rangle = 2\int ds T(s)\{1-e^{-\sigma_{NN}T(b-s)}\}$, where $T(b) = \int\rho(z,b)dz$ is the familiar nuclear thickness function and ρ is the nuclear density. By comparison, the number of subcollisions is $\langle M(b)\rangle = \sigma_{NN}\int ds T(s)T(b-s)$. We remark that both wounded-nucleon and independent-collision approximations imply that the total multiplicity of pions N_π scales as the respective number (participants or subcollisions). Therefore, both models imply $\nu_{dyn}\propto N_\pi^{-1}$, albeit with different coefficients.

We point out that particle production at RHIC energy has contributions from soft interactions, which scale as the number of participants, and hard processes, which scale as the number of subcollisions [23,24]. In this case the scaling of ν_{dyn} with N_π can be more complex. Furthermore, final-state scattering effects can certainly modify this scaling.

IV. ALTERNATIVE MEASURES OF FLUCTUATIONS

In this section we consider the connection between the variance ν_{dyn} and other fluctuations measures. We discuss some of the merits and problems associated with each observable.

A. Φ measure

The Φ measure of the net-charge fluctuation was introduced by Mrowczynski [10] and is based on statistical considerations. It consists of the difference between the mean of particle production variances calculated event-by-event and the variance calculated over the entire dataset. Consider x an observable of interest, e.g., the net charge of produced particles. The inclusive mean of x (i.e., average over all particles an events) is noted $\overline{x}$. Deviation from the inclusive mean are noted $\Delta x = x-\overline{x}$. By construction, one has $\overline{\Delta x}=0$. The root mean square (RMS) deviation is $\overline{\Delta x^2} = \overline{(x-\overline{x})^2}$. To investigate the dynamics, one determines how the event-wise net value of "x," defined as $X=\Sigma_i x_i$, changes event-by-event. One defines $\Delta X = X - N\overline{x}$ as the event deviation from the inclusive mean (with N being the number of particles in the given event). By construction, its event average $\langle\Delta X\rangle$ vanishes, whereas $\langle\Delta X^2\rangle$ does not. The Φ measure is defined as [10]

$$\Phi = \sqrt{\frac{\langle\Delta X^2\rangle}{\langle N\rangle}} - \sqrt{\overline{\Delta x^2}}. \tag{25}$$

For a system with particles of charge q_+ and q_-, the inclusive standard deviation is

$$\overline{\Delta x^2} = (q_+ - q_-)^2 \frac{\langle N_+\rangle\langle N_-\rangle}{\langle N\rangle^2}. \tag{26}$$

The magnitude of $\langle\Delta X^2\rangle$ is determined by both statistical and dynamic fluctuations. Defining Q as the net charge of an event, one has $\Delta X = Q - N\langle Q\rangle/\langle N\rangle$, from which one finds indeed $\langle\Delta X\rangle=0$. The average $\langle\Delta X^2\rangle$ is however nonzero. One finds

$$\langle\Delta X^2\rangle = (q_+-q_-)^2\frac{\langle N_+\rangle^2\langle N_-\rangle^2}{\langle N\rangle^3}\left(\frac{\langle N_+^2\rangle - \langle N_+\rangle^2}{\langle N_+\rangle^2} + \frac{\langle N_-^2\rangle - \langle N_-\rangle^2}{\langle N_-\rangle^2} - 2\frac{\langle N_+N_-\rangle - \langle N_+\rangle\langle N_-\rangle}{\langle N_+\rangle\langle N_-\rangle}\right), \tag{27}$$

so that

$$\Phi = (q_+-q_-)\left\{\frac{\langle N_+\rangle\langle N_-\rangle}{\langle N\rangle^{3/2}}\left(\frac{\langle N_+^2\rangle - \langle N_+\rangle^2}{\langle N_+\rangle^2} + \frac{\langle N_-^2\rangle - \langle N_-\rangle^2}{\langle N_-\rangle^2} - 2\frac{\langle N_+N_-\rangle - \langle N_+\rangle\langle N_-\rangle}{\langle N_+\rangle\langle N_-\rangle}\right)^{1/2} - \left(\frac{\langle N_+\rangle\langle N_-\rangle}{\langle N\rangle^2}\right)^{1/2}\right\}, \tag{28}$$

Examination of Eqs. (26), (27), and (28) reveals that they can, in fact, be expressed as the ν and ν_{stat} variances as follows [as also reported by Mrowczynski [10]]:

$$\langle\Delta X^2\rangle = (q_+-q_-)^2\frac{\langle N_+\rangle^2\langle N_-\rangle^2}{\langle N\rangle^3}\nu, \tag{29}$$

$$\overline{\Delta x^2} = (q_+-q_-)^2\frac{\langle N_+\rangle^2\langle N_-\rangle^2}{\langle N\rangle^3}\nu_{stat}, \tag{30}$$

so one can express Φ as

$$\Phi = \frac{2\langle N_+\rangle\langle N_-\rangle}{\langle N\rangle}\left(\sqrt{\frac{\nu}{\langle N\rangle}} - \sqrt{\frac{\nu_{stat}}{\langle N\rangle}}\right). \tag{31}$$

In general, the dynamic component of the fluctuations is much smaller than the statistical component, $\nu_{dyn}\ll\nu_{stat}$ implying $\sqrt{\nu/\langle N\rangle} - \sqrt{\nu_{stat}/\langle N\rangle} = \sqrt{\nu_{stat}/\langle N\rangle}(\sqrt{1+\nu_{dyn}/\nu_{stat}} - 1) \approx \nu_{dyn}(2\sqrt{\nu_{stat}\langle N\rangle})^{-1}$. Substituting the value of ν_{stat} given by Eq. (16), the above expression can thus be approximated by

$$\Phi \approx \frac{\langle N_+\rangle^{3/2}\langle N_-\rangle^{3/2}}{\langle N\rangle^2}\nu_{dyn}. \tag{32}$$

One thus finds that indeed the Φ measure is determined (mostly) by the dynamical fluctuations of the system, i.e., by the particle correlations implicit in the sum $R_{++}+R_{--}-2R_{+-}$.

Equation (32) further simplifies, as follows, for cases where $\langle N_+\rangle = \langle N_-\rangle$:

$$\Phi \approx \frac{\langle N\rangle}{8}\nu_{dyn}. \tag{33}$$

Given, as we discussed in Sec. III, that the variance ν_{dyn} should vary inversely to the multiplicity of charge particles in the limit of independent particle collisions and absence of rescattering of the secondaries, one should expect that $\Phi \approx \nu_{dyn,pp}/8$ in that limit, and independent of the collision centrality if the collision dynamic do not vary with collision centrality. Note, however, one must exercise caution while comparing Φ measured by experiments with different acceptances (see, Sec. VI for details). Note finally that unlike ν_{dyn}, the Φ measure is a nonrobust observable given it explicitly depends on the detection efficiency of positive and negative particles through the factor $\langle N_+\rangle$ and $\langle N_-\rangle$ as we shall discuss in more detail in Sec. VIII.

B. Particle ratios

Another approach advocated in Ref. [8] focuses on the variance of the ratio of positive and negative particle multiplicities, $R=\langle N_+\rangle/\langle N_-\rangle$. As shown in Ref. [8], the fluctuations of the ratio offer the advantage that "volume" fluctuation effects cancel to first order. This is also true for ν_{dyn} (see, Sec. III) and Φ [10].

For small fluctuations, the variance of the ratio can be related to the charge variance ν (15). A small fluctuation of $R=\langle N_+\rangle/\langle N_-\rangle$ satisfies

$$\frac{\Delta R}{R} = \frac{\Delta N_+}{N_+} - \frac{\Delta N_-}{N_-}, \tag{34}$$

so that

$$\frac{\langle \Delta R^2\rangle}{\langle R\rangle^2} = \frac{\langle \Delta N_+^2\rangle}{\langle N_+\rangle^2} + \frac{\langle \Delta N_-^2\rangle}{\langle N_-\rangle^2} - 2\frac{\langle \Delta N_+\Delta N_-\rangle}{\langle N_+\rangle\langle N_-\rangle}. \tag{35}$$

Expanding the square in Eq. (15), we see that

$$\langle \Delta R^2\rangle = \langle R\rangle^2 \nu. \tag{36}$$

Observe that neither ν nor $\langle \Delta R^2\rangle$ are robust. Also, note that this equivalence holds only when $\langle \Delta N_\pm^2\rangle^{1/2} \ll \langle N_\pm\rangle$; an approximation, which breaks down at small multiplicities. Problems with these quantities for small multiplicities are discussed in Ref. [15]. The D measure used by Koch, Bleicher, and Joen [8]

$$D \equiv \langle N\rangle\langle \Delta R^2\rangle = \langle N_+ + N_-\rangle\langle R\rangle^2\nu \tag{37}$$

is also efficiency dependent.

C. Reduced variance

Last, we consider the reduced variance ω_Q used by authors [6,8,11,15,25]. If we write $N=N_+ + N_-$ and $Q=N_+ - N_-$, then the reduced variance is

$$\omega_Q = \frac{\langle \Delta Q^2\rangle}{\langle N\rangle}. \tag{38}$$

As before, we expand the square to find

$$\omega_Q = \frac{\langle \Delta N_+^2\rangle + \langle \Delta N_-^2\rangle - 2\langle \Delta N_+\Delta N_-\rangle}{\langle N_+\rangle + \langle N_-\rangle}. \tag{39}$$

This ratio is unity for Poissonian fluctuations or for a thermal ensemble in chemical equilibrium; any measured multiplicity dependence would be interesting. In terms of robust ratios, we obtain

$$\omega_Q = 1 + \frac{\langle N_+\rangle^2}{\langle N\rangle}R_{++} + \frac{\langle N_-\rangle^2}{\langle N\rangle}R_{--} - 2\frac{\langle N_+\rangle\langle N_-}{\langle N\rangle}R_{+-}. \tag{40}$$

Generally, this quantity has a complicated dependence on the correlators $R_{\alpha\beta}$. However, for $\langle N_+\rangle \approx \langle N_-\rangle$, the above expression reduces to

$$\omega_Q \approx 1 + \frac{\langle N_+ + N_-\rangle}{4}\nu_{dyn}, \tag{41}$$

indicating that this quantity has the same efficiency dependence as the total number of charged particles.

We note, in closing this section, that the reduced variance ω_Q unlike ν_{dyn}, and Φ, has an explicit dependence on collision volume fluctuations, as given by the following expression:

$$\omega_Q = \omega_{Q,V} + \frac{(\langle N_+\rangle - \langle N_-\rangle)^2}{\langle N_+\rangle + \langle N_-\rangle}\frac{\langle \Delta V^2\rangle}{\langle V\rangle^2}, \tag{42}$$

where $\omega_{Q,V}$ corresponds to the reduced variance at fixed volume, while $\langle V\rangle$, and $\langle \Delta V\rangle^2$ are respectively the mean and variance of the collision volume. The importance of volume fluctuations was pointed out by Jeon and Koch [26]. Following their work, it is straightforward to show that ν_{dyn}, and Φ are independent of volume fluctuations.

V. CHARGE CONSERVATION EFFECTS

The total charge of the system is fixed due to the charge conservation. It implies some "trivial" correlation in particle production regardless of other dynamical effects. As such, it only affects the two-particle density $\rho_{+-}(\eta_+,\eta_-)$. We proceed to study the effect of charge conservation on the net charge fluctuation by calculating the correlation function $C_{+-}(\eta_+,\eta_-)$ as a function of single- and two-particle density expressed in terms of probability distributions of positive and negative particle in order to emphasize the role of charge conservation. One writes, for fixed number $N_\pm$ of positive and negative particles:

$$\rho_\pm(\eta_\pm) = N_\pm P_\pm(\eta_\pm), \tag{43}$$

$$\rho_{+-}(\eta_+,\eta_-) = N_- P_{+-}(\eta_+,\eta_-) + (N_- N_+ - N_-) P_+(\eta_+) P_-(\eta_-). \tag{44}$$

$P_\pm(\eta_\pm)$ are probabilities to find one $+$ or $-$ particle at rapidity $\eta_\pm$. P_{+-} is the probability to find one positive particle and one negative particles at rapidities η_+ and η_-, respectively. N_- and N_+ are, respectively, the total number of negative and positive particles produced (over 4π solid angle) by a collision. By virtue of charge conservation, and given the total charge $Q \geqslant 0$, one has $Q = N_+ - N_-$, and $N_+ \geqslant N_-$. The first term of Eq. (44) accounts for correlations between positive and negative particles. As there are N_- +- pairs created, one has a contribution $N_- P_{+-}$. The second term arises because there are $N_+ N_- - N_-$ ways to pair the uncorrelated +- particles. In general, at a fixed impact parameter (or number of nn collisions), the multiplicities N_- and N_+ shall fluctuate event-by-event. One must then average over such fluctuations and rewrite the above expression as

$$\rho_\pm(\eta_\pm) = \langle N_\pm \rangle_{4\pi} P_\pm(\eta_\pm), \tag{45}$$

$$\rho_{+-}(\eta_+,\eta_-) = \langle N_- \rangle_{4\pi} P_{+-}(\eta_+,\eta_-) + (\langle N_- N_+ \rangle_{4\pi} + \langle N_- \rangle_{4\pi}) P_+(\eta_+) P_-(\eta_-), \tag{46}$$

where the notation $\langle O \rangle_{4\pi}$ represents an average taken over 4π acceptance. In the absence of dynamical correlations, and by virtue of charge conservation, one has

$$\begin{aligned} \langle N_-^2 \rangle_{4\pi} - \langle N_- \rangle_{4\pi}^2 &= \langle N_- \rangle_{4\pi}, \\ \langle N_- N_+ \rangle_{4\pi} &= \langle N_- \rangle_{4\pi}^2 + \langle N_- \rangle_{4\pi} - \langle N_- \rangle_{4\pi} Q. \end{aligned} \tag{47}$$

The correlation function $C_{+-}(\eta_+,\eta_-)$ can then be calculated and written as

$$\begin{aligned} C_{+-}(\eta_+,\eta_-) &= \frac{\rho_{+-}(\eta_+,\eta_-)}{\rho_+(\eta_+)\rho_-(\eta_-)} - 1 \\ &= \frac{1}{\langle N_+ \rangle_{4\pi}} \frac{P_{+-}(\eta_+,\eta_-)}{P_+(\eta_+) P_-(\eta_-)}. \end{aligned} \tag{48}$$

This result is fairly generic and includes the possibility of dynamical spatial (or rapidity) correlations between the particles of a created pair. Neglecting such a correlation however, and for the purpose of evaluating the role of charge conservation alone, one sets $P_{+-} = P_+ P_-$. One then finds that charge conservation implies

$$C_{+-}(\eta_+,\eta_-) = -\frac{1}{\langle N_+ \rangle_{4\pi}} \approx -\frac{2}{\langle N \rangle_{4\pi}}, \tag{49}$$

where $\langle N \rangle_{4\pi}$ stands for the mean *total* number of charged-particles produced in the event. Obviously, at large multiplicities one can neglect the difference between N_+ and $N/2$.

The correlator R_{+-} is obtained by integration [see, Eq. (11)] of $C_{+-}(\eta_+,\eta_-)$ over the experimental acceptance. Given that $C_{+-}(\eta_+,\eta_-)$ is actually independent of $\eta_\pm$, R_{+-} is independent of the experimental acceptance. One thus finds that the charge conservation contribution to ν_{dyn} amounts to

$$\Delta \nu_{dyn} = -\frac{4}{\langle N \rangle_{4\pi}}. \tag{50}$$

It is independent of the experimental acceptance, and only determined by the total charge-particle multiplicity at a given impact parameter.

We emphasize that $\nu_{dyn} \neq 0$ for a 4π acceptance because charge conservation imposes a correlation on the system. The total ν_{+-} given by (15) is strictly zero when all particles are detected. However, Eq. (16) implies that $\nu_{stat} \neq 0$ in this case, since the Poisson distributions used to calculate ν_{stat} do not incorporate a global charge conservation constraint. It follows that $\nu_{dyn} = \nu - \nu_{stat} \to -\nu_{stat}$ for a 4π acceptance, as seen in Eq. (50). This estimate of the effect of charge conservation is in agreement with a correction reported in Ref. [27]. Note, however, that the correction is additive not multiplicative as stated in Ref. [27].

VI. RAPIDITY DEPENDENCE OF FLUCTUATIONS AND DETECTOR ACCEPTANCE

Measuring the dependence of $R_{\alpha\beta}$ and ν_{dyn} on the rapidity window $\Delta\eta$ can yield information on the rapidity range of correlations, as well as their magnitude. Information on the rapidity dependence of $R_{\alpha\beta}$ is also needed to compare data from experiments with different geometric acceptance. The microscopic correlations themselves can and indeed must be determined from balance function and similar measurements [17]; such experiments have different practical issues. We relate ν_{dyn} and balance function measurements in the following section.

To exhibit the rapidity dependence of $R_{\alpha\beta}$, we assume that ρ_1 are η independent and that $C = C_{\alpha\beta}(0)\exp\{-(\eta_1 - \eta_2)^2/2\sigma^2\}$. ISR and FNAL data [19,20] show that charged-particle correlation are functions of the relative rapidity $\eta_1 - \eta_2$ with only a weak dependence on the average rapidity of the pair. Data can be roughly characterized as Gaussian near midrapidity. Using Eq. (13) we find

$$R_{\alpha\beta} \approx \frac{C_{\alpha\beta}(0)}{x^2}\{\sqrt{\pi}x\,\mathrm{erf}(x) - (1 - e^{-x^2})\}, \tag{51}$$

where $x = \sqrt{2}\Delta\eta/\sigma$. The function $R_{\alpha\beta}$ is shown as a function of $\Delta\eta$ in Fig. 1. ISR and FNAL data suggest that the rapidity range of correlations is roughly from one to two rapidity units.

Both R and the microscopic correlator C depend on the value $C_{\alpha\beta}(0)$ at $\eta_1 = \eta_2$ and the rapidity range of correlations, σ. Equation (51) carries the same value—and caveats—as does the Gaussian parametrization of HBT correlations. The range σ depends on the dynamics and may vary with centrality, as well as target and projectile mass.

One must account for this rapidity dependence when comparing experiments of different geometrical acceptance. We estimate, for instance, that the difference between of the fluc-

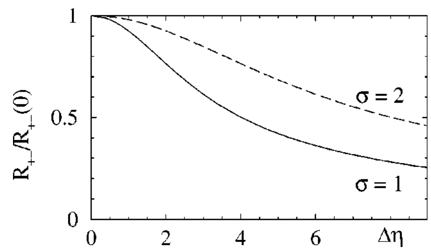

FIG. 1. Rapidity dependence of the robust covariance R_{+-} assuming a Gaussian correlation function of width σ.

tuations measured by the STAR ($|\eta|\leqslant 1.5,\Delta\phi=2\pi$) and PHENIX ($|\eta|\leqslant 0.35,\Delta\phi=\pi/2$) experiments to be roughly $\sim 10\%$ for $\sigma\sim 1-2$. While this is a rather small correction, we emphasize that the experiments should measure the rapidity dependence. In general σ can differ from pp to AA collisions and, moreover, is expected to depend on centrality.

VII. RELATION BETWEEN THE BALANCE FUNCTION AND ν_{dyn}

The balance function was proposed by Bass *et al.* [17] as a technique to study the dynamics of hadronization in relativistic heavy ion collisions. The idea is that the rapidity range of correlations is changed when a collisions forms quark-gluon plasma. Specifically, charged hadrons form late in the reaction, after hadronization, resulting in shorter-ranged correlations in rapidity space for charge/anticharge pairs than expected in the absence of plasma.

The balance function as defined by Bass *et al.* [17] is written (here again focusing, without loss of generality on the rapidity dependence)

$$\begin{aligned} B(\Delta\eta_2|\Delta\eta_1) &= \frac{1}{2}\{D(-,\Delta\eta_2|+,\Delta\eta_1) \\ &\quad -D(+,\Delta\eta_2|+,\Delta\eta_1)+D(+,\Delta\eta_2|-,\Delta\eta_1) \\ &\quad -D(-,\Delta\eta_2|-,\Delta\eta_1)\}, \end{aligned} \tag{52}$$

where

$$D(b,\Delta\eta_2|a,\Delta\eta_1) = \frac{\int_{\eta_1-\Delta\eta_1/2}^{\eta_1+\Delta\eta_1/2}\int_{\eta_2-\Delta\eta_2/2}^{\eta_2+\Delta\eta_2/2} d\eta_a d\eta_b \rho_2(\eta_b,\eta_a)}{\int_{\eta_2-\Delta\eta_1/2}^{\eta_2+\Delta\eta_1/2} d\eta_a \rho_1(\eta_a)}. \tag{53}$$

The ratio $D(b,\Delta\eta_2|a,\Delta\eta_1)$ is essentially a conditional probability for finding a number of particles of type b in the phase space bin $\Delta\eta_2$ of centroid η_2 given the presence of particles of type a in the phase space bin $\Delta\eta_1$ of centroid η_1, i.e.,

$$D(b,\Delta\eta_2|a,\Delta\eta_1) = \frac{N(b,\Delta\eta_2;a,\Delta\eta_1)}{N(a,\Delta\eta_1)}. \tag{54}$$

The bins need not overlap. Experimentally, evaluations of the balance function can be restricted to a determination of the correlation of particle a and b as a function of their relative rapidity $\Delta\eta$. In this case, particle a can be anywhere within the full detector acceptance Y, and particle b is at a rapidity $\Delta\eta$ relative to a. This leads to a one-dimensional balance function $B(\Delta\eta|Y)$ defined as

$$\begin{aligned} B(\Delta\eta|Y) &= \frac{1}{2}\{D(-,\Delta\eta|+,Y)-D(+,\Delta\eta|+,Y) \\ &\quad +D(+,\Delta\eta|-,Y)-D(-,\Delta\eta|-,Y)\}. \end{aligned} \tag{55}$$

To understand this expression better, observe that for a sufficiently narrow bin $\Delta\eta$ we can write

$$D(b,\Delta\eta|a,Y) \approx \frac{\Delta\eta}{\langle N_a\rangle}\int_{-Y/2}^{Y/2} d\eta_a \rho_2(\eta_a,\eta), \tag{56}$$

where $\langle N_a\rangle$ is the number in the full domain $-Y/2\leqslant\eta\leqslant Y/2$. For a boost invariant system the pair correlation function C is a function only of the rapidity difference, so that this integral is essentially C averaged over the system volume, plus a constant term that cancels in Eq. (55).

The integral of this function over the entire acceptance Y is noted $B(Y|Y)$. By virtue of Eq. (54), it amounts to

$$\begin{aligned} B(Y|Y) &= \int_0^Y d\Delta\eta B(\Delta\eta|Y) \\ &= \frac{1}{2}\left\{\frac{\langle N_+N_-\rangle_Y}{\langle N_+\rangle_Y}\frac{\langle N_+N_-\rangle_Y}{\langle N_-\rangle_Y}\frac{\langle N_+(N_+-1)\rangle_Y}{\langle N_+\rangle_Y}\right. \\ &\quad \left.\times\frac{\langle N_-(N_--1)\rangle_Y}{\langle N_-\rangle_Y}\right\}. \end{aligned} \tag{57}$$

The four terms of this equation are part of the expression of the correlators R_{ab} given in Eqs. (8) and (11). The integral $B(Y|Y)$ can thus be rewritten as

$$\begin{aligned} B(Y|Y) &= \frac{1}{2}\{R_{+-}\langle N_-\rangle+R_{+-}\langle N_+\rangle \\ &\quad -R_{++}\langle N_+\rangle-R_{--}\langle N_-\rangle\}, \end{aligned} \tag{58}$$

which establishes a relationships between the integral $B(Y|Y)$ of the balance function, and the correlators R_{++}, R_{--}, and R_{+-}.

At RHIC, one observes that $\langle N_-\rangle\approx\langle N_+\rangle=\langle N\rangle/2$ near central rapidities in Au+Au collisions [28]. The above expression simplifies

$$B(Y|Y) = \frac{\langle N\rangle}{4}\{2R_{+-}-R_{++}-R_{--}\} = -\frac{\langle N\rangle}{4}\nu_{dyn}. \tag{59}$$

The integral $B(Y|Y)$ of the balance function $B(\Delta y|Y)$ is thus indeed proportional to the variance ν_{dyn} and the total multiplicity $\langle N\rangle$ when $\langle N_-\rangle \approx \langle N_+\rangle$.

VIII. FINITE RECONSTRUCTION EFFICIENCY EFFECTS

We consider the effect of finite reconstruction efficiency on measurements of fluctuations studied as a function of collision centrality. We assume the centrality is experimentally determined based on the total multiplicity of charged-particles detected in a reference acceptance, Ω_M whereas the multiplicity fluctuations of interest are measured in a fiducial acceptance Ω_N. We account for the finite detection efficiency, in a given acceptance, Ω_α, by introducing a detector response function $P_D(n_\alpha|N_\alpha)$ expressing the probability of detecting a multiplicity n_α given a produced multiplicity N_α. In general, $P_D(n_\alpha|N_\alpha)$ shall account for finite efficiency effects as well as measurements of ghost tracks. We shall calculate, quite generally, moments $M_{k,\alpha}$ and factorial moments $F_{k\alpha}$, of the particle multiplicity distribution defined, respectively, as

$$M_{k,\alpha} = \langle N_\alpha^k\rangle = \frac{1}{N_{ev}} \sum N_\alpha^k,$$

$$F_{k,\alpha} = \langle N_\alpha(N_\alpha - 1)(N_\alpha - k)\rangle = \frac{1}{N_{ev}} \sum N_\alpha(N_\alpha - 1)\cdots(N_\alpha - k), \tag{60}$$

where N_{ev} is the number of events studied. The mean is $\mu_\alpha = M_{1,\alpha}$ and the variance, $V = \langle \delta N_\alpha^2\rangle = M_{2,\alpha} - M_{1,\alpha}^2$. Here we will restrict our calculation to these lowest moments, but the calculation can easily be generalized to higher moments.

We shall use lower case letter (e.g., $m_{k,\alpha}$) to distinguish measured moments from the intrinsic or actual moment of the produced particles (i.e., that one wishes to infer) represented with capital letters (e.g., $M_{k,\alpha}$).

We assume that moments of the multiplicity distributions are measured as a function of the collision centrality estimated based on the total multiplicity m measured in the reference acceptance. The moments can then be expressed (neglecting for simplicity the particle type label α) as

$$m_k = \sum_{n=0}^{\infty} n^k P(n|m), \tag{61}$$

where the sum is taken over all relevant multiplicities, and $P(n|m)$ is the probability to measure "n" given the centrality estimator "m." We emphasize that both "n" and "m" are influenced by the finite efficiency of the detector. We in fact seek to extract the intrinsic moments of the particle production

$$M_k = \sum_{N=0}^{\infty} N^k P(N|M), \tag{62}$$

where $P(N|M)$ is the probability "N" particles are produced at a given centrality "M." The measured distribution $P(n|m)$ can be expressed as a function of the intrinsic distribution as follows:

$$P(n|m) = \sum_{N,M} P_D(n|N)P(N|M)P_D(M|m), \tag{63}$$

with the sum extending over all relevant produced multiplicities N and M. The factor $P_D(M|m)$ corresponds to the probability of having a produced multiplicity M given the measured value m. It is evaluated using Bayes rule

$$P_D(M|m) = \frac{P_D(m|M)P(M)}{P(m)}, \tag{64}$$

where $P(M)$ and $P(m)$ are, respectively, the probability of the produced M and measured m multiplicities. The measured probability distribution is thus

$$P(n|m) = \frac{1}{P(m)} \sum P(n|N)P(N|M)P(m|M)P(M). \tag{65}$$

Measured moments can be calculated as function of the intrinsic (produced) moment by inserting the above expression in Eq. (62). Introducing for convenience the functions $h_s(N)$ and $g_s(M)$ defined as follows:

$$h_s(N) = \sum_n n^s P(n|N), \tag{66}$$

$$g_s(M) = \sum_N h_s(N)P(N|M). \tag{67}$$

one finds a general expression for the moments as follows:

$$\langle m_k\rangle = \frac{1}{P(m)} \sum_M P(m|M)P(M)g_k(M). \tag{68}$$

Assuming $P(n|N)$ can be appropriately approximated by a binomial distribution, the above expressions can be readily simplified. The moments $h_s(N)$ yield

$$h_1(N) = \varepsilon N, \qquad h_2(N) = \varepsilon^2 N^2 + \varepsilon(1-\varepsilon)N, \tag{69}$$

where ε_n is the detection efficiency achieved in the measurement of "n." Substituting these quantities in Eq. (67) leads to

$$g_1(M) = \varepsilon_n \langle N\rangle, \qquad g_2(M) = \varepsilon_n^2\langle N^2\rangle + \varepsilon_n(1-\varepsilon_n)\langle N\rangle. \tag{70}$$

The first and second moments, are thus in general

$$\langle n\rangle = \frac{1}{P(m)} \sum_M P_D(m|M)P(M)\varepsilon_n\langle N\rangle,$$

$$\langle n^2\rangle = \frac{1}{P(m)} \sum_M P_D(m|M) P(M) (\varepsilon_n^2 \langle N^2\rangle + \varepsilon_n (1-\varepsilon_n)\langle N\rangle), \tag{71}$$

with the moments $\langle N^s\rangle$ evaluated at a fixed value of M. Clearly, the measured moments are determined by the intrinsic moments smeared over the response function of the multiplicity, M. Assuming the efficiency of the total multiplicity detection process is near unity, one can approximate the response function $P_D(m|M)$ with a delta function $\delta_{m,M}$, and the above expressions simplifies as follows:

$$\begin{aligned}\langle n\rangle &= \varepsilon_n \langle N\rangle,\\ \langle n^2\rangle &= \varepsilon_n^2 \langle N^2\rangle + \varepsilon_n(1-\varepsilon_n)\langle N\rangle.\end{aligned} \tag{72}$$

We show in Appendix the above results holds for finite efficiency, as long as "n" has a linear dependence on the total multiplicity "m" over the range of the response function $P_D(m|M)$.

We now proceed to use these for the calculation of the various fluctuation measures introduced in Sec. IV. We use subindices "$+$," "$-$," "Q," and "CH" to denote positively and negatively charged particles, net charge, and total charge particle multiplicity, respectively. We use overlined symbols to represent the intrinsic measures. We find using Eqs. (72), (32), and (41)

$$\begin{aligned}\omega_\pm &= 1-\varepsilon_\pm + \varepsilon_\pm \overline{\omega}_\pm,\\ \omega_Q &= 1-\varepsilon_\pm + \varepsilon_\pm \overline{\omega}_Q,\\ \omega_{CH} &= 1-\varepsilon_\pm + \varepsilon_\pm \overline{\omega}_{CH},\\ \Phi &= \frac{\epsilon_+^{3/2}\epsilon_-^{3/2}}{\epsilon^2}\overline{\Phi}.\end{aligned} \tag{73}$$

The above fluctuation measures display an explicit dependence on the charged-particle detection efficiencies $\varepsilon_\pm$ or the total efficiency ε. The Φ observable, in particular, has a nontrivial dependence on the detection efficiencies of positively and negatively charged particles. This dependence, however, simplifies to a single factor ε if the positive, negative, and global efficiencies are equal (i.e., $\varepsilon_+ = \varepsilon_- = \varepsilon$). By contrast, one finds that the dynamic variance $\nu_{dyn} = \overline{\nu}_{dyn}$, i.e., it is independent of the detection efficiencies, and is thus, in that sense, a robust observable. Note that this conclusion remains strictly correct as long as the Gaussian approximation is valid. See, the Appendix for a discussion of the Gaussian approximation.

IX. SIMPLE PRODUCTION MODELS

A. Poissonian particle production

We first consider a multiparticle production model where no correlation are involved. Specifically, we assume that on average, particle species i are produced in fixed fractions f_i of the total particle production. We consider cases where the fluctuation measures are evaluated over kinematic ranges that might be identical (case A) or smaller (case B) than the kinematic range used to calculate the total (charge) particle production.

The probability to produce species i with multiplicities N_i is evaluated with a multinomial distribution. In general, one has

$$P(N_1, N_2, \ldots, N_k|M) = \frac{1}{M!}\prod_{\alpha=1}^{k} \frac{f_\alpha^{N_\alpha}}{N_\alpha!}. \tag{74}$$

In case A, one shall have $M = \Sigma N_\alpha$ and $\Sigma f_\alpha = 1$, whereas in case B, $M \geqslant \Sigma N_\alpha$, and $\Sigma f_\alpha < 1$.

The multiplicity moments, and variance are calculated at fixed total multiplicity M assumed to be representative of the collision impact parameter

$$\begin{aligned}\langle N_\alpha\rangle_m &= f_\alpha M,\\ \langle N_\alpha^2\rangle_m &= f_\alpha M + f_\alpha^2 M(M-1),\\ \langle N_\alpha N_\beta\rangle_m &= M(M-1) f_\alpha f_\beta,\\ \langle N_\alpha(N_\alpha - 1)\rangle_m &= f_\alpha^2 M(M-1),\\ V_\alpha &= M f_\alpha (1-f_\alpha).\end{aligned} \tag{75}$$

Consider now the specific case of net-charge fluctuations with the index α taking values $+$ and $-$. One has in case "B"

$$\begin{aligned}V_Q &= M[f_+ + f_- - (f_+ - f_-)^2],\\ \omega_Q &= 1 - \frac{(f_+ - f_-)^2}{f_+ + f_-},\\ \omega_{ch} &= 1-(f_+ - f_-),\\ \nu_{+-} &= \frac{f_+ - f_-}{M f_+ f_-},\\ \nu_{dyn} &= 0,\\ \Phi &= 0.\end{aligned} \tag{76}$$

Case A is easily calculated from the above by setting $f_+ - f_- = 1$.

The coefficients $f_\pm$ can be experimentally determined. It is thus straightforward to determine the normalized variances expected for particle independent production and compare with measured values to seek for the presence of sub- or super-Poissonian fluctuations. Note additionally that both the ν_{dyn} and Φ variables have null expectation values irrespective of the fraction of the fractions $f_\pm$. They thus constitute a more reliable measure of the dynamic fluctuations.

B. Simple resonance production model

Two-particle correlations are determined by a host of phenomena such as collective (flow) effects, production of resonances, jet production, Fermi/Bose statistics, as well as intrinsic phenomena related to the underlying collision dynamics. Here we examine the role of resonance decays (e.g., ρ^o, Δ^o) on measurements of the net-charge fluctuations. We show that the production of neutral resonances that decay into pairs of positively and negatively charged-particles produce an effective dynamical correlation.

We formulate a simple toy model, where we include only three types of particles: π^+, π^-, and ρ^o. The ρ^o shall be viewed as a generic neutral resonance, which decays into π^+ and π^-. Obviously, this is an oversimplification of the problem and a fuller treatment shall account for other species, all relevant resonances, and the finite acceptance of the detection apparatus.

We consider the π^+, π^-, and ρ^o to be produced independently (neglecting Bose effects) at freeze out in relative fractions f_1, f_2, and f_3 respectively, and model the multiplicity production according to a multinomial distribution (as in the previous section). The probability of producing n_1 π^+, n_2 π^-, and n_3 ρ^o is expressed

$$P(n_1,n_2,n_3;N)=\frac{N!}{n_1!n_2!n_3!}f_1^{n_1}f_2^{n_2}f_3^{n_3}. \tag{77}$$

Given our assumption that all ρ^o decay into a pair π^+ and π^-, the probability of measuring n_+ positive n_- negative particles respectively can be written as

$$P(n_+,n_-;N)$$

$$=\sum_{n_1,n_2,n_3} P(n_1,n_2,n_3;N)\,\delta_{n_+,n_1+n_3}\delta_{n_-,n_2+n_3}. \tag{78}$$

One then writes the moment generating function of the probability $P(n_+,n_-;N)$ as

$$G(t_+,t_-;N)=(p_1e^{t_+}+p_2e^{t_-}+p_3e^{t_++t_-})^N, \tag{79}$$

which one uses to computes the moments of the pion multiplicity distributions. One finds

$$\langle N_+\rangle=N(f_1+f_3),$$

$$\langle N_-\rangle=N(f_2+f_3),$$

$$\langle N_+(N_+-1)\rangle=N(N-1)(f_1+f_3)^2,$$

$$\langle N_-(N_--1)\rangle=N(N-1)(f_2+f_3)^2,$$

$$\langle N_+N\rangle=N(N-1)(f_1+f_3)(f_2+f_3)+Nf_3. \tag{80}$$

The variance ν_{dyn}, in the presence of resonances, is thus simply

$$\nu_{dyn}=\frac{-2p_3}{N(p_1+p_3)(p_2+p_3)}. \tag{81}$$

One finds that the variance ν_{dyn} increases with the fraction of resonances, p_3 produced in the final state. One also finds it to scale inversely to the number of particles produced in the initial state. Note that in the limit $p_3=0$, ν_{dyn} vanishes by our assumption of independent production. The simple treatment done here does not account for finite acceptance effects on the decay of resonances. Obviously, if too small a rapidity region is integrated, one of the decay partners may on average be missed, and $|\nu_{dyn}|$ shall be increased accordingly.

In AA collisions, one does not expect resonance production to be the sole cause of correlation, i.e., $\nu_{dyn}<0$, but it is yet to be determined what fraction of the observed fluctuations may be attributed to resonance production or to truly dynamic correlations. In that respect, it shall be interesting to consider fluctuations of specific particle species such $p/\overline{p}$ in contrast to $\pi^\pm$ or $K^\pm$ given no known resonance decay into $p+\overline{p}$ whereas many resonances exist that decay into $\pi^+ + \pi^-$ or $K^+ + K^-$.

X. SUMMARY AND CONCLUSIONS

We introduced the net-charge fluctuation measure ν_{dyn} on the basis of two-particle correlation functions. We showed that for heavy ion collisions involving independent-nucleon collision and negligible rescattering of secondaries, ν_{dyn} scales as the multiplicative inverse of the produced charged-particle multiplicity. We also showed that ν_{dyn} is simply related to other observables used or proposed for fluctuation measurement by various authors. We found, however, that the different fluctuation measures have different dependence on the experimental acceptance, detector efficiency, and collision centrality. We showed that ν_{dyn} has a weak dependence on the rapidity range used experimentally to measure the fluctuations provided the rapidity range is of the order or smaller than the two-particle correlator width, whereas observables such as Φ have basically a linear dependence on the size of the acceptance. We found also that ν_{dyn} is, by construction, independent, to first order, of the detection efficiency whereas measures such as Φ, ω_Q have a explicit dependence on the detection efficiency. We also found that charge conservation has a finite, and actually sizable effect on the charge fluctuation measure ν_{dyn} determined by the total charge-particle multiplicity (over 4π and independent of the detector acceptance used to measure the net-charge fluctuations. We further showed, as also pointed out by Mrowczynski [10], that the Φ measure shall be independent of the collision centrality provided the collision dynamic is also independent of the collision centrality. Note, however, that because the detection efficiency may be a subtle function of the detector occupancy, and hence the collision centrality, caution has to be exercised when interpreting uncorrected measurement of Φ vs collision centrality. Finally, we presented, as an example, a simple particle production model that can be used to account for the production of resonances, as well as charge conservation.

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APPENDIX: FINITE EFFICIENCY EFFECTS ON THE MEASUREMENT OF ν_{dyn}

A range of collision impact parameters is selected in experiments using a measured multiplicity m (or a similar observable). This introduces additional fluctuations because a single m corresponds to a range of impact parameters. In this appendix we estimate the effect of centrality selection. We use these results in Secs. III and VIII.

We assume, in the Gaussian approximation, that the moments scale with the true multiplicity M as

$$\langle N_a\rangle=\mu_a M,$$
$$\langle N_a^2\rangle=\mu_a^2M^2+\sigma_a^2M,$$
$$\langle N_aN_b\rangle=\mu_a\mu_bM^2+\xi_{ab}M, \qquad (A1)$$

where μ_a and μ_b are average branching fractions for the production of species "a" and "b," respectively, while σ_a^2 and ξ_{ab} are their variance and covariance. These relations are strictly true in the independent-collision model or the wounded-nucleon model, where both M and N_a are, respectively, proportional to the number of subcollisions or the number or strings. The first moment (71) is then

$$\langle n_a\rangle=\frac{1}{P(m)}\sum_M P_D(m|M)P(M)\varepsilon_a\mu_aM$$
$$=\varepsilon_a\mu_a\frac{1}{P(m)}\sum_M MP_D(m|M)P(M)=\varepsilon_a\mu_a\langle M\rangle_m, \qquad (A2)$$

where we have introduced the expectation value of M at fixed m defined as

$$\langle M\rangle_m=\frac{1}{P(m)}\sum_M MP_D(m|M)P(M). \qquad (A3)$$

The factor ϵ_a is the probability that a particle of type "a" is detected. One gets similarly for the second moment and cross term:

$$\langle n_a^2\rangle=\frac{1}{P(m)}\sum_M P_D(m|M)P(M)[\varepsilon_a^2(\mu_a^2M^2+\sigma_a^2M)+\varepsilon_a(1-\varepsilon_a)\mu_aM],$$
$$\langle n_a^2\rangle=[\varepsilon_a^2(\sigma_a^2-1)+\varepsilon_a]\langle M\rangle_m+\varepsilon_a^2\mu_a^2\langle M^2\rangle_m, \qquad (A4)$$

and

$$\langle n_an_b\rangle=\varepsilon_a\varepsilon_b\mu_a\mu_b(\langle M^2\rangle_m-\langle M\rangle_m^2)+\varepsilon_a\varepsilon_b\xi_{ab}\langle M\rangle_m. \qquad (A5)$$

The correlators R_{aa} and R_{ab} are therefore,

$$R_{aa}=\frac{\sigma_a^2-\mu_a}{\mu_a^2}\frac{1}{\langle M\rangle_m}+\frac{\langle M^2\rangle_m-\langle M\rangle_m^2}{\langle M\rangle_m^2} \qquad (A6)$$

and

$$R_{ab}=\frac{\xi_{ab}}{\mu_a^2}\frac{1}{\langle M\rangle_m}+\frac{\langle M^2\rangle_m-\langle M\rangle_m^2}{\langle M\rangle_m^2}. \qquad (A7)$$

The variance $\nu_{dyn}=R_{aa}+R_{bb}-2R_{ab}$ measured at a given m is then

$$\nu_{dyn}(m)=\frac{\nu_0}{\langle M\rangle_m}, \qquad (A8)$$

where

$$\nu_0=\frac{\sigma_a^2-\mu_a}{\mu_a^2}+\frac{\sigma_b^2-\mu_b}{\mu_b^2}-2\frac{\xi_{ab}}{\mu_a\mu_b}. \qquad (A9)$$

This expression amounts to the value of ν_{dyn} evaluated at $M=\langle M\rangle_m$. One finds that the correlators R_{ab} exhibit a contribution from the variance $\langle M^2\rangle_m-\langle M\rangle_m^2$ whose magnitude depends on the detector response function width. The variance ν_{dyn}, however, does not have such a contribution and as such is also independent of the detection efficiency for measuring M.

Note that the above result implies that ν_{dyn} is robust, i.e., independent of detection efficiencies, in the Gaussian approximation (A1). An explicit dependence on efficiencies would arise if the Gaussian approximation is not valid, e.g., if the detector response functions differ markedly from Binomial or Gaussian functions, or if the efficiencies exhibit very large variations with detector occupency.

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