

**Behavior of charge fluctuations in relativistic nucleus-nucleus collisions**Ben-Hao Sa,<sup>1,2,4,5,\*</sup> Xu Cai,<sup>4,2</sup> Zong-Di Su,<sup>4,1</sup> An Tai,<sup>3</sup> and Dai-Mei Zhou<sup>2</sup><sup>1</sup>*China Institute of Atomic Energy, P.O. Box 275 (18), Beijing, 102413 China*<sup>2</sup>*Institute of Particle Physics, Huazhong Normal University, Wuhan, 430079 China*<sup>3</sup>*Department of Physics and Astronomy, University of California, at Los Angeles, Los Angeles, California 90095*<sup>4</sup>*CCAST (World Lab.), P.O. Box 8730 Beijing, 100080 China*<sup>5</sup>*Institute of Theoretical Physics, Academia Sinica, Beijing, 100080 China*

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Using a hadron and string cascade model JPCIAE we have investigated the dependence of event-by-event charge fluctuations on (pseudo)rapidity window size, final state interaction, resonance decay, centrality, and reaction energy for Pb+Pb collisions at SPS and LHC energies and for Au+Au collisions at RHIC energies. The JPCIAE results of the charge fluctuations as a function of rapidity window size in Pb+Pb collisions at SPS energies are compared with the preliminary NA49 data. Comparisons with PHENIX and STAR data in Au+Au collisions at  $\sqrt{s_{nn}} = 130$  GeV are also given. It is found that the charge fluctuations hardly depend on collision centrality except for very peripheral collisions and are rather insensitive to the change of reaction energy. Our calculations also show that the charge fluctuations are not strongly affected by the final state interaction and the resonance decay.

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In Ref. [1,2], the energy fluctuation (heat capacity) was first related to the liquid-gas phase transition in intermediate energy heavy-ion collisions. It was later proposed to study the phase transition from hadronic matter to a quark-gluon plasma (QGP) through the irregular behavior of the heat capacity, provided that the event-by-event (EE) fluctuation of temperature is observable in relativistic nucleus-nucleus collisions [3,4]. Such irregular behavior also characterizes the order of a phase transition: a jump in a first order phase transition and a singularity in a second order one [3]. The EE fluctuation of an observable might supply important information such as the hadronic matter compressibility [5], the position and property of a critical point in the QCD phase diagram of temperature  $T$  vs chemical potential  $\mu$  [4], etc. In Ref. [4] it was also predicted that the EE fluctuation pattern in average transverse momentum, for instance, would significantly be changed around a critical point.

With the increase of interaction energy a rather high particle multiplicity is accessible and the statistically significant measurements of the EE fluctuations became possible for the first time in Pb+Pb collisions at 158A GeV/c [6–10] and recently in Au+Au collisions at  $\sqrt{s_{nn}} = 130$  GeV [11]. Though it was claimed that nonstatistical contributions to the EE fluctuation of average transverse momentum,  $k/\pi$  ratio, and net charge multiplicity are small [9–11], the calculations of the EE fluctuations based on hadronic transport models [8,12,13] and effective models [14–16] were stimulated.

Since the unit of charge (baryon charge) in the QGP phase is 1/3 while it is 1 in the hadronic phase, the thermal model predicted that the value of the charged particle ratio EE fluctuation,  $D_R$  (defined below), in the hadronic phase would be a factor of  $\sim 2.5$ –4 larger than that in the QGP phase [17–19]. The charged particle ratio EE fluctuation was then proposed as a signal of QGP formation if the initial fluctuations

survive hadronization and their typical relaxation time is longer than the collision time [17,18]. In Ref. [20] URQMD [21] was used to investigate the charged particle ratio fluctuation in Pb+Pb collisions at SPS energies and Au+Au collisions at the full RHIC energy. However, the URQMD predictions for the charged particle ratio fluctuation in Pb+Pb collisions at SPS were around 3 while the preliminary NA49 data [22] were around 4.

The hadron and string cascade model JPCIAE was employed in this paper to further study the charge fluctuations. The model results were compared with the preliminary NA49 data of the charged particle ratio fluctuations as a function of rapidity window size in Pb+Pb collisions at 40, 80, and 158A GeV/c [22] and with PHENIX and STAR data in Au+Au collisions at  $\sqrt{s_{nn}} = 130$  GeV [11,23]. Meanwhile, the dependence of the charge fluctuations on the final state interaction (rescattering), the resonance decay ( $\rho$  and  $\omega$ ), the centrality (impact parameter  $b$ ), and the reaction energy (from SPS up to LHC) was investigated.

The JPCIAE model was developed based on PYTHIA [24], which is a well known event generator for hadron-hadron collisions. In the JPCIAE model the radial position of a nucleon in colliding nucleus  $A$  (indicating the atomic number of this nucleus as well) is sampled randomly according to the Woods-Saxon distribution and the solid angle of the nucleon is sampled uniformly in  $4\pi$ . Each nucleon is given a beam momentum in  $z$  direction and zero initial momenta in  $x$  and  $y$  directions. The collision time of each colliding pair is calculated under the requirement that the least approach distance of the colliding pair along their straight line trajectory (mean field potential is not taken into account in JPCIAE) should be smaller than  $\sqrt{\sigma_{\text{tot}}/\pi}$ . Here  $\sigma_{\text{tot}}$  refers to the total cross section. The nucleon-nucleon collision with the least collision time is then selected from the initial collision list to perform the first collision. Both the particle list and the collision list are then updated such that the new collision list may consist of not only nucleon-nucleon collisions but also collisions

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between nucleons and produced particles and among produced particles themselves. The next collision is selected from the new collision list and the processes above are repeated until the collision list is empty.

For each executing collision pair, if its c.m.s. energy is above a certain threshold (=4 GeV in program), we assume that strings are formed after the collision and PYTHIA is used to deal with particle production. Otherwise, the collision is treated as a two-body collision [25–27]. The threshold above is chosen in such a way that JPCIAE correctly reproduces the charged multiplicity distributions in nucleus-nucleus collisions [28]. It should be noted here that the JPCIAE model is not a simple superposition of nucleon-nucleon collisions since the rescatterings of secondary particles are taken into account. In addition, the particle production from strings in JPCIAE is determined by the Lund fragmentation scheme [29], in which only the lowest excitation state of a resonance is included. We refer to Ref. [28] for more details about the JPCIAE model.

If the deviation (i.e., fluctuation [30]) of a physical variable  $x$  from its average value per event  $\langle x \rangle$  is defined as

$$\delta x = x - \langle x \rangle, \quad (1)$$

the variance of  $x$  reads [30]

$$\langle (\delta x)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2. \quad (2)$$

Suppose  $x \equiv R = N_+ / N_-$  to be the ratio of positively to negatively charged particle multiplicity, the corresponding variance is

$$\langle (\delta R)^2 \rangle = \langle R^2 \rangle - \langle R \rangle^2. \quad (3)$$

Similarly the variance of net charge multiplicity  $Q = N_+ - N_-$  reads

$$\langle (\delta Q)^2 \rangle = \langle Q^2 \rangle - \langle Q \rangle^2. \quad (4)$$

However, what is interesting is neither  $\langle (\delta R)^2 \rangle$  nor  $\langle (\delta Q)^2 \rangle$  but

$$D_R \equiv \langle N_{\text{ch}} \rangle \langle (\delta R)^2 \rangle \quad (5)$$

or

$$D_Q \equiv \frac{\langle (\delta Q)^2 \rangle}{\langle N_{\text{ch}} \rangle}, \quad (6)$$

where  $N_{\text{ch}} = N_+ + N_-$  refers to the total charge multiplicity. When  $\langle N_{\text{ch}} \rangle \gg \langle Q \rangle$ , a relation follows approximately [17]

$$D_R \approx 4D_Q. \quad (7)$$

The thermal (effective) model predictions for  $D_R$  are [17]  $\sim 4$  for a pion gas,  $\sim 3$  for a resonance pion gas (pions from  $\rho$  and  $\omega$  decays), and  $\sim 0.75$  for massless noninteracting quarks and gluons.

As mentioned in Ref. [17], one main assumption made in the thermal model predictions is that the studied system can be described as a grand canonical ensemble. However, in experiments or dynamical simulations, the investigated sub-

system (e.g., within a rapidity interval  $\Delta y$ ) is a finite fraction of the full system (e.g., in the full rapidity region). Therefore, the assumption of a grand canonical ensemble is only valid in the limit of  $\langle N_{\text{ch}} \rangle_{\Delta y} / \langle N_{\text{ch}} \rangle_{\text{total}} \rightarrow 0$ , such that the rest system plays the role of a thermal resource. In order to compare the experiments or dynamical simulations with the thermal model predictions a correction factor is introduced [17],

$$C_y = 1 - \frac{\langle N_{\text{ch}} \rangle_{\Delta y}}{\langle N_{\text{ch}} \rangle_{\text{total}}}. \quad (8)$$

Another assumption adopted in the thermal model predictions is vanishing of the net charge [17]. However, that is actually impossible in experiments or dynamical simulations, the corresponding correction factor [17] reads

$$C_\mu = \frac{\langle N_+ \rangle_{\Delta y}^2}{\langle N_- \rangle_{\Delta y}^2}. \quad (9)$$

The  $D_R$  with corrections above is denoted as

$$\tilde{D}_R = \frac{D_R}{C_y C_\mu}. \quad (10)$$

The fluctuations are usually composed of statistical fluctuations and dynamical fluctuations. There are many sources to be considered as the dynamical fluctuations, such as string fragmentation (or QCD color fluctuations), centrality (impact parameter or participants), rescattering, and resonance decay, etc. On the contrary, the statistical fluctuations are no dynamical origin and could be described in a stochastic scenario by probability distribution functions [11,30]. Only a finite number of events could be generated in experiments or dynamical simulations, for instance, also causes a statistical fluctuation. Though it is necessary to study the influence of reaction energy, centrality, rescattering, and resonance decay individually, an alternative way of investigating the non-statistical contributions is to compare the EE fluctuation distributions extracted from real events with ones from mixed events [10]. The mixed events here are constructed from the real events so that in principle only the statistical fluctuations survive in the mixed events [10].

In Fig. 1 the JPCIAE results of  $\tilde{D}_R$  as a function of  $\Delta y$  in 40, 80, and 158A GeV/c Pb+Pb collisions (full circles) are compared to the NA49 preliminary data (full triangles) [22]. Corresponding to the centrality cut of 7.2% at 40 and 80A GeV/c and 10% at 158A GeV/c in the NA49 experiments the impact parameters in the JPCIAE calculations were set to be  $b \leq 3.57$  fm and  $b \leq 4.20$  fm, respectively.  $\Delta y$  was set around 2.9, 3.2, and 3.6, respectively, for 40, 80, and 158A GeV/c, and the  $p_t$  window was set to be  $0.005 < p_t < 2.5$  GeV/c for all the three beam momenta as in the NA49 experiments. The JPCIAE result of each datum point ( $\tilde{D}_R$ ) and its error bar in this figure were obtained from five independent runs and in each run  $N$  events are generated, where  $N$  is large enough to guarantee the approximate stability of positively and negatively charged multiplicities. The  $N$  is equal to 300 in Pb+Pb collisions at 40A GeV/c, for instance. The datum point in this figure is then the average of the five

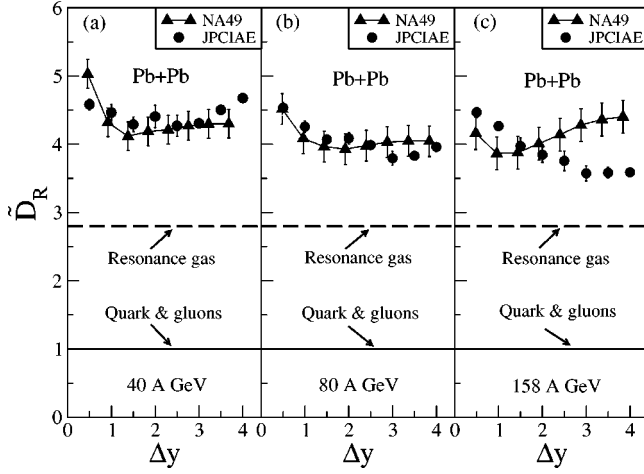


FIG. 1.  $\tilde{D}_R$  as a function of  $\Delta y$  in 40, 80, and 158 A GeV/c Pb+Pb collisions. The preliminary NA49 data were taken from Ref. [22].

corresponding values and the error bar is calculated as the square root of the variance of  $\tilde{D}_R$  divided by five (the same for other JPCIAE results). The dashed and solid lines in this figure are the thermal model predictions for a resonance pion gas and the lattice Monte Carlo result for a quark-gluon gas, respectively [20]. One sees from this figure that the JPCIAE results are generally compatible with the preliminary NA49 data for 40 and 80 A GeV/c Pb+Pb collisions. However, for 158 A GeV/c Pb+Pb collisions there exist discrepancies in the  $\Delta y$  dependence between the preliminary NA49 data and the JPCIAE results. Such differences are not due to statistics and require a further study.

In Fig. 2 the JPCIAE results of  $D_R$  and  $4D_Q$  as a function of  $\langle N_{ch} \rangle$  in peripheral Au+Au collisions at  $\sqrt{s_{nn}} = 130$  GeV ( $-0.35 < \eta < 0.35$ ,  $p_t > 0.2$ ) were compared with PHENIX data [11]. For simplicity only part of the PHENIX  $D_R$  data points were plotted (full squares with error bar in the figure) and compared with the JPCIAE results (open squares with error bar in the figure). The PHENIX data of  $4D_Q$  with error

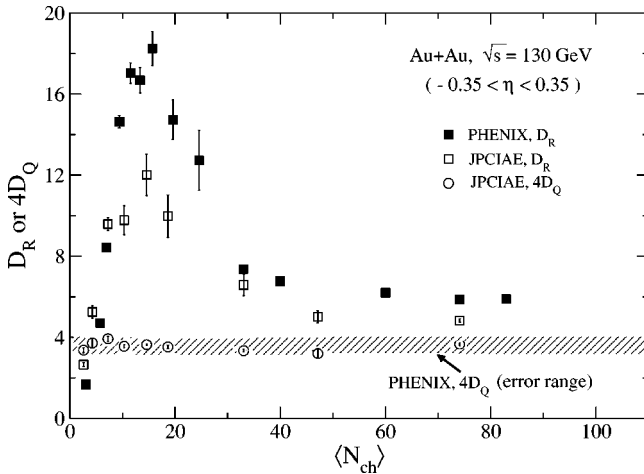


FIG. 2. The charge fluctuations as a function of charged multiplicity  $\langle N_{ch} \rangle$ , in Au+Au peripheral collisions at  $\sqrt{s_{nn}} = 130$  GeV.

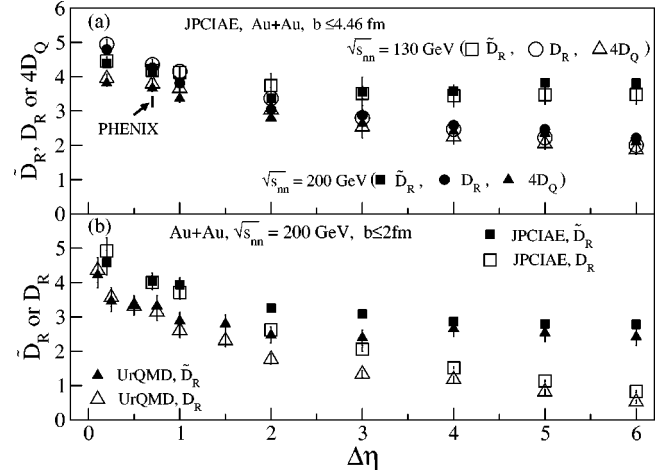


FIG. 3. The  $\tilde{D}_R$ ,  $D_R$ , and  $4D_Q$  as a function of  $\Delta \eta$ : (a) comparison of the JPCIAE results for Au+Au collisions at  $\sqrt{s_{nn}} = 130$  and 200 GeV; (b) comparison for the results from JPCIAE and URQMD (taken from Ref. [20] where  $\Delta y$  was used instead of  $\Delta \eta$ ) for Au+Au collisions at  $\sqrt{s_{nn}} = 200$  GeV.

bars were presented simply by a shaded region in Fig. 2 and compared with corresponding JPCIAE results (open circles). One sees here that the JPCIAE results of  $4D_Q$  are compatible with PHENIX data. A few comments related to the results in Fig. 2 are in order. First, behavior of  $D_R$  is very different from  $4D_Q$  when  $\langle N_{ch} \rangle$  is small, where  $D_R$  is strongly dependent on  $\langle N_{ch} \rangle$  unlike  $4D_Q$ . Second, when  $\langle N_{ch} \rangle$  is large  $D_R$  and  $4D_Q$  become close and show no dependence on  $\langle N_{ch} \rangle$ . Third, most  $D_R$  results from JPCIAE were lower than the PHENIX data, in the peak region especially, which might attribute in part to the fact that the PHENIX spectrometer only covers  $\pi/2$  in azimuthal angle.

Having gained confidence from the results in Figs. 1 and 2, we investigate the dependence of the charge fluctuations on subsystem size, rescattering, resonance decay, centrality, and reaction energy. In Fig. 3(a) the JPCIAE results of fluctuation measures (i.e.,  $\tilde{D}_R$ ,  $D_R$ , and  $4D_Q$ ) as a function of  $\Delta \eta$  in Au+Au collisions at  $\sqrt{s_{nn}} = 130$  (open squares, circles, and triangles, respectively) and at 200 GeV (full squares, circles, and triangles, respectively) are compared with each other. In JPCIAE calculations the charged particles with  $p_t > 0.2$  from 10% the most central collisions were used. The thick stick at  $\Delta \eta = 0.7$  is the PHENIX datum [11] of  $4D_Q$  in Au+Au collisions at  $\sqrt{s_{nn}} = 130$  GeV, which is about 10% lower than the corresponding JPCIAE result (open triangle). It is also interesting to note that the JPCIAE result of  $D_Q \sim 0.9$  at  $\Delta \eta = 1$  in Au+Au collisions at  $\sqrt{s_{nn}} = 130$  GeV is about 10% higher than the corresponding STAR datum of  $\sim 0.8$ , extracted under the assumption of zero net charge [23]. In Fig. 3(a) one sees that globally speaking the dependence of the charge fluctuation measures on  $\Delta \eta$  are not sensitive to the change of energy from 130 to 200 GeV, which is consistent with the conclusions in Refs. [18,20,31]. In Fig. 3(b) the JPCIAE results of  $\tilde{D}_R$  and  $D_R$  (full and open squares) as a function of  $\Delta \eta$  in Au+Au collisions at  $\sqrt{s_{nn}} = 200$  GeV ( $b \leq 2$  fm) were compared

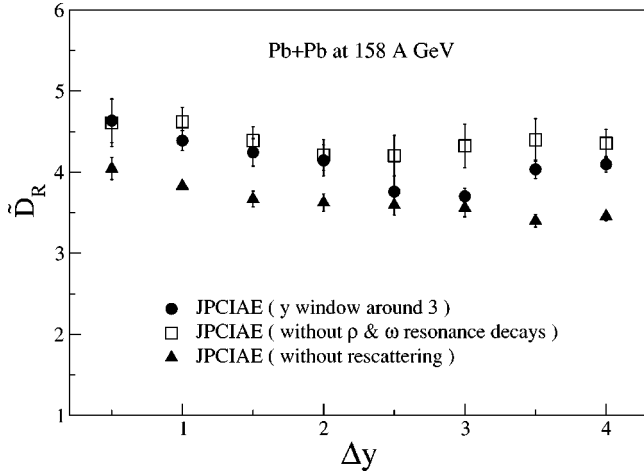


FIG. 4. The effects of rescattering and resonance decay on the distribution of  $\tilde{D}_R$  vs  $\Delta y$  in 158A GeV/c Pb+Pb collisions obtained with JPCIAE.

with the URQMD results (full and open triangles, taken from Ref. [20] where  $\Delta y$  was used instead of  $\Delta \eta$ ). Generally speaking, the results of JPCIAE are systematically higher than those of URQMD, that might attribute to the higher resonance states included in the URQMD model.

The effects of rescattering and resonance decay ( $\rho$  and  $\omega$ ) on the distribution of  $\tilde{D}_R$  vs  $\Delta y$  are shown in Fig. 4 for 158A GeV/c Pb+Pb collisions. In this figure, the circles, the triangles, and the squares are, respectively, the results of default JPCIAE, JPCIAE without rescattering, and JPCIAE without  $\rho$  and  $\omega$  resonance decays. In the JPCIAE calculations the impact parameter was  $b \leq 3.5$  fm,  $\Delta y$  was set around 3 and the  $p_t$  window was  $0 < p_t < 5$  GeV/c. Globally speaking, the rescattering effect on  $\tilde{D}_R$  is not strong, that is consistent with the conclusion from the RQMD model [31]. The effect of resonance decay does not seem strong either and the shift in  $\tilde{D}_R$  due to the resonance decay is smaller than one over all the  $\Delta y$  region unlike what is expected based on the thermal model predictions for a pion gas relative to the resonance pion gas (pions from  $\rho$  and  $\omega$  decays). However, in the default JPCIAE calculations there are still a lot of mesons not from the decays of resonances unlike the case of the resonance pion gas in thermal model predictions, which may explain in part why the shift is smaller than one.

The centrality dependence of  $\tilde{D}_R$  from JPCIAE in Pb+Pb collisions at 158A GeV/c was given in Fig. 5(a) (circles) and compared with URQMD results (squares, taken from Ref. [20]). In both calculations the rapidity window was  $2.5 < y < 4.5$ . The discrepancies between JPCIAE and URQMD results might attribute to the higher resonance states included in the URQMD model. In Fig. 5(b) the centrality dependence of  $\tilde{D}_R$ ,  $D_R$ , and  $4D_Q$  in Au+Au collisions at  $\sqrt{s_{nn}} = 200$  GeV from JPCIAE ( $-0.5 < \eta < 0.5$ ) are given by full circles, open circles, and full squares, respectively. One sees from Fig. 5 that for the impact parameter region considered here the charge fluctuation measures are not so sensitive to centrality within error bars, a conclusion consistent with the corresponding STAR and PHENIX observations [11,23].

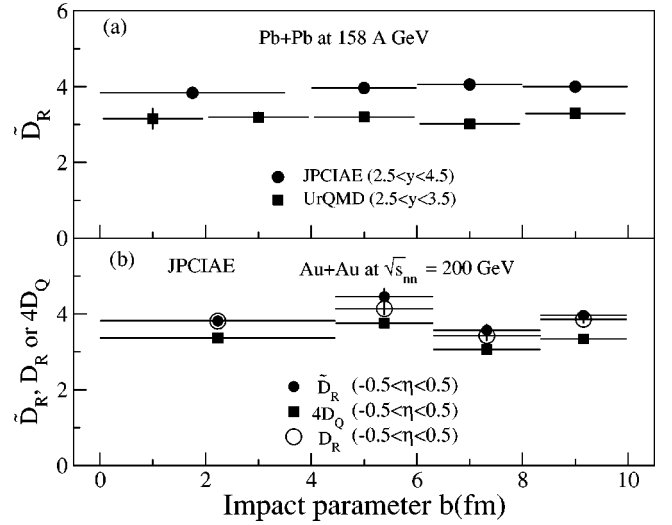


FIG. 5. The centrality dependence of charge fluctuations: (a) comparison of JPCIAE results of  $\tilde{D}_R$  in Pb+Pb collisions at 158A GeV/c with URQMD results (taken from Ref. [20]); (b) JPCIAE results of charge fluctuation measures in Au+Au collisions at  $\sqrt{s_{nn}} = 200$  GeV.

Finally, the JPCIAE results of the energy dependence of  $\tilde{D}_R$  (full circles),  $D_R$  (open circles), and  $4D_Q$  (full triangles) from SPS to RHIC and then to LHC energy were given in Fig. 6. In the JPCIAE calculations the centrality cut was 10% the most central and we set  $2.5 < y < 3.5$  for Pb+Pb at SPS,  $-0.5 < \eta < 0.5$  for Au+Au at RHIC, and Pb+Pb at LHC energy, respectively. One sees from Fig. 6 that while  $D_R$  might decrease slightly with energy,  $\tilde{D}_R$  and  $4D_Q$  show almost no energy dependence within error bar.

In summary, a hadron and string cascade model JPCIAE has been employed in this paper to investigate the dependence of the charge fluctuation measures on (pseudo)rapidity window size, rescattering, resonance decay, centrality, and energy. Within the framework of this model the calculated results seem compatible with the preliminary NA49 data for Pb+Pb collisions at 40 and 80A GeV/c. However, for

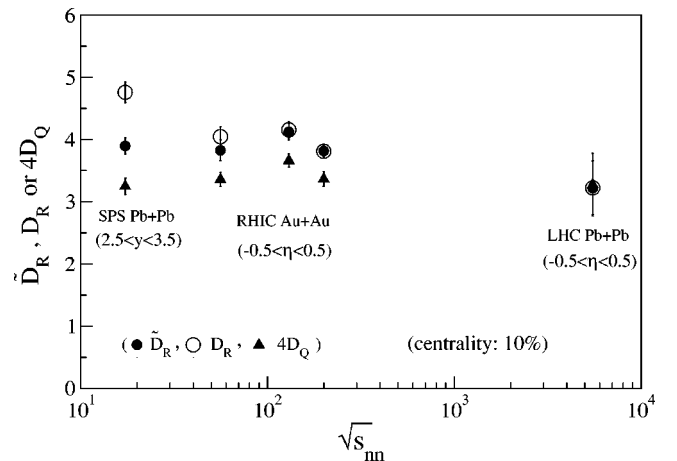


FIG. 6. The energy dependence of  $\tilde{D}_R$ ,  $D_R$ , and  $4D_Q$ .



158A GeV/c Pb+Pb collisions there exist discrepancies in the  $\Delta y$  dependence between the JPCIAE results and the preliminary NA49 data. The JPCIAE results of  $4D_Q$  in Au+Au collisions at  $\sqrt{s_{nn}}=130$  are compatible with PHENIX data, however the JPCIAE results of  $D_R$  are lower than PHENIX data. We give also the JPCIAE results for Au+Au collisions at  $\sqrt{s_{nn}}=56$  and 200 GeV and for Pb+Pb collisions at  $\sqrt{s_{nn}}=5500$  GeV. It seems that the effect of resonance decay ( $\rho$  and  $\omega$ ) on the charge fluctuation measures is not strong,

though the rescattering may affect the charge fluctuation measures somewhat more strongly than the resonance decay. The charge fluctuation measures are almost independent of the collision centrality except for very peripheral collisions and their dependence on the reaction energy is not strong either.

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- [1] Ben-Hao Sa and D. H. E. Gross, Nucl. Phys. **A437**, 643 (1985).
- [2] J. Bondorf, R. Donangelo, I. N. Mishustin, and H. Schulz, Nucl. Phys. **A444**, 460 (1985).
- [3] L. Stodolsky, Phys. Rev. Lett. **75**, 1044 (1995).
- [4] M. Stephanov, K. Rajagopal, and E. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998); M. Stephanov, K. Rajagopal, and E. Shuryak, Phys. Rev. D **60**, 114028 (1999).
- [5] St. Mrówczyński, Phys. Lett. B **430**, 9 (1998); M. Gaździcki and St. Mrówczyński, Z. Phys. C **54**, 127 (1992).
- [6] G. Roland (unpublished).
- [7] S. V. Afanasiev *et al.*, NA49 Collaboration, hep-ex/0009053.
- [8] M. Bleicher, M. Belkacem, C. Ernst, H. Weber, L. Gerland, C. Spieles, S. A. Bass, H. Stöcker, and W. Greiner, Phys. Lett. B **435**, 9 (1998).
- [9] H. Appelshäuser *et al.*, NA49 Collaboration, Phys. Lett. B **459**, 679 (1999).
- [10] S. V. Afanasiev *et al.*, NA49 Collaboration, Phys. Rev. Lett. **86**, 1965 (2001).
- [11] K. Adcox *et al.*, PHENIX Collaboration, nucl-ex/0203014.
- [12] F. Liu, A. Tai, M. Gaździcki, and R. Stock, Eur. Phys. J. C **8**, 649 (1999).
- [13] A. Capella, E. G. Ferreira, and A. B. Kaidalov, hep-ph/9903338.
- [14] S. A. Voloshin, V. Koch, and H. G. Rither, Phys. Rev. C **60**, 024901 (1999).
- [15] G. Baym and H. Heiselberg, Phys. Lett. B **469**, 7 (1999).
- [16] H. Heiselberg and A. D. Jackson, Phys. Rev. C **63**, 064904 (2001).
- [17] S. Jeon and V. Koch, Phys. Rev. Lett. **83**, 5435 (1999); **85**, 2076 (2000).
- [18] M. Asakawa, U. Heinz, and B. Müller, Phys. Rev. Lett. **85**, 2072 (2000).
- [19] Z.-W. Lin and C. M. Ko, Phys. Rev. C **64**, 041901(R) (2001).
- [20] M. Bleicher, S. Jeon, and V. Koch, Phys. Rev. C **62**, 061902 (2000).
- [21] S. A. Bass *et al.*, Prog. Part. Nucl. Phys. **41**, 225 (1998); nucl-th/9803035.
- [22] V. Friese (unpublished); J. Zaranek, hep-ph/0111228.
- [23] S. A. Voloshin, STAR Collaboration, nucl-ex/0109006.
- [24] J. Sjöstrand, Comput. Phys. Commun. **82**, 74 (1994).
- [25] J. Cugnon, T. Mizutani, and J. Vandermeulen, Nucl. Phys. **A352**, 505 (1981).
- [26] G. F. Bertsch and S. Das Gupta, Phys. Rep. **160**, 189 (1988); A. Bonasera, F. Gulminelli, and J. Molitoris, *ibid.* **243**, 1 (1994).
- [27] An Tai and Ben-Hao Sa, Comput. Phys. Commun. **116**, 353 (1999).
- [28] Ben-Hao Sa, An Tai, Hui Wang, and Feng-He Liu, Phys. Rev. C **59**, 2728 (1999); Ben-Hao Sa and An Tai, *ibid.* **62**, 044905 (2000).
- [29] B. Andersson, G. Gustafson, G. Ingelman, and T. Sjöstrand, Phys. Rep. **97**, 31 (1983).
- [30] L. E. Reichl, *A Modern Course in Statistical Physics* (University of Texas Press, Austin, 1980); R. Kubo, *Statistical Mechanics* (North-Holland, Amsterdam, 1965).
- [31] Q. H. Zhang, V. Topor Pop, S. Jeon, and C. Gale, hep-ph/0202057.