Cluster interpretation of enhanced electric dipole transitions in nuclei with strong collective multipole correlations

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Experimental data on strong $E1$ transitions from the ground state in collective nuclei are analyzed. A model based on the idea of cluster-type correlations is suggested to interpret these experimental data. The calculated results show that a cluster mode is responsible for strong $E1$ transitions in spherical and near deformed nuclei.

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I. INTRODUCTION

In systematic investigations of the $E1$ transitions [1–5] low-lying 1^- states, which are characterized by strong $B(E1;0^+_{\rm g.s.}\to 1^-)$, have been observed in spherical nuclei. It was demonstrated on the basis of experimental data that these low-lying 1^- states arise by coupling the collective quadrupole 2^+_1 and collective octupole 3^-_1 states: $|2^+_1 \otimes 3^-_1;1^-_1 M\rangle$. The energy of these 1^-_1 states is very close to the summed energy $E(2^+_1)+E(3^-_1)$ [4,6,7]. Similar 1^-_1 states have been observed in Cd, Sn, Ba, Ce, Nd, and Sm isotopes. Their two–phonon character has been proved by the observed strong $E2$ and $E3$ transitions to the corresponding one-phonon states [6,8,9].

These two-phonon states exhibit relatively strong $B(E1;0^+_{\rm g.s.}\to 1^-)$ transitions of the order of several units $\times 10^{-3} e^2\,{\rm fm}^2$. However, the nature of these strong $E1$ transitions is not finally clarified. The following explanations can be found in literature:

(1) The two-phonon nature of the states and the one-body character of the standard shell model $E1$ transition operator suggest a two-body form of the effective $E1$ transition operator. The fact that the observed $1^-_1 \to 0^+_1$ transitions are of the same order of magnitude as the $B(E1;3^-_1\to 2^+_1)$, strongly supports the two-body structure of the effective $E1$ operator. Such an operator has been constructed in IBA [10–12] and in the shell model [13].

(2) The possible important or even decisive role of the $1p$-$1h$ admixture to the two-phonon 1^-_1 states in semimagic nuclei has been stressed in Ref. [13] for the explanation of the strength of the $0^+_1 \to 1^-_1$.

(3) In microscopical calculations based on the quasiparticle-phonon model [14–17] two important sources of the strong $E1$ transition matrix elements have been noted: $1p$-$1h$ admixture to the two-phonon 1^-_1 state and the ground state correlations of the RPA type, i.e., the presence of the $2p$-$2h$ components in the ground state wave function. The first one was mentioned in the preceding paragraph. The second one indicates the important role of the collective effects since the ground state correlations increase with increasing collectivity.

(4) In Ref. [18] it was suggested that a clusterization of nuclei can be responsible for the large magnitude of the $E1$ transitions. (See also Ref. [19], where the molecular $E1$ sum rule was obtained.)

The recent compilation of the experimental data [5] show that by moving away from the semimagic nuclei, the $B(E1;0^+_1\to 1^-_1)$ value decreases first with a minimum at $N=86$ and 78 and then increases again approaching to deformed nuclei. This minimum probably indicates the presence of two sources of strong $E1$ transition matrix elements; the contribution of one of them decreases and of the other one increases when moving away from closed shell. Strong correlations between the values of $B(E1;0^+_1\to 1^-_1)$ and the product of the average squares of the quadrupole $\langle \beta_2^2\rangle$ and octupole $\langle \beta_3^2\rangle$ deformation parameters in nonmagic nuclei [20] definitely show that away from the closed shells the large $B(E1)$ value has a collective nature connected with the motion of the nuclear shape. The ratio $B(E1)/(\langle \beta_2^2\rangle\langle \beta_3^2\rangle)$ is amazingly constant, although the $B(E1)$ strength varies by one order of magnitude in the considered nuclei. For semimagic nuclei this ratio is typically about a factor of 10 higher, indicating that in this case the mechanism producing strong $B(E1)$ values is not related to the collective quadrupole and octupole vibrations. It is possible that in semimagic nuclei this mechanism is connected with the $1p$-$1h$ admixture as it is suggested in Ref. [13], where it is shown that a $1p$-$1h$ admixture to the two-phonon $|2^+_1\otimes 3^-_1;1^-_1 M\rangle$ state can account for $B(E1)$ values of a magnitude of (0.5–1.0) $\times 10^{-2} e^2\,{\rm fm}^2$. The aim of the present paper is to show that the possible mechanism of strong E1 transitions from the ground state to the two-phonon state in nuclei away from closed shells can be connected to clusterization.

II. MODEL

When a nucleus clusterizes into two fragments a lighter fragment has a larger charge-to-mass ratio than a heavier one. Our calculations performed for heavy nuclei have shown that the probability of the formation of an α cluster can be quite significant already near the ground state [21]. The α-cluster charge-to-mass ratio is equal to 0.5, i.e., larger than the one of a mononucleus.

In the cluster model the $E1$ transition operator has the form

$$Q^{(e)}_{1\mu}=e_{\text{eff}}\frac{A_1A_2}{A}\left(\frac{Z_2}{A_2}-\frac{Z_1}{A_1}\right)R_m\sqrt{\frac{3}{4\pi}}D^{1*}_{\mu 0}, \tag{1}$$

where A_1 and A_2 are the mass numbers of the heavy and light cluster, respectively, Z_i/A_i is the charge to mass ratio in the cluster i, and R_m is the intercluster distance which can be well approximated by $R_m=r_0(A_1^{1/3}+A_2^{1/3})+0.5$ fm. Taking a coupling of the cluster mode to the giant dipole vibrations into account, we set $e_{\text{eff}}=e(1+\chi)$, where $\chi\approx-0.7$ [22]. In order to have a contribution of the clusterization effects into $E1$ transitions it is not needed that a geometrical cluster is connected by a thin neck to the rest of the nucleus. It can be an α particle formed with some probability due to an enhanced α particle correlation in the low density [23] surface region which fluctuates with multipole collective surface vibrations.

To be able to calculate the matrix elements of the operator (1) between the ground state and the quadrupole-octupole two-phonon state we have to express this operator, i.e., the mass number of the light cluster A_2, in terms of the quadrupole and octupole collective variables. The idea is the following. Due to multipole shape vibrations a part of the nucleons spend some time outside of the sphere of the equivalent radius $R_0=r_0A^{1/3}$. The octupole mode introduces a mirror asymmetry in the nucleon distribution, showing a tendency to a formation of a dinuclear-type shape. Also, the higher multipole vibrations being connected to the quadrupole and octupole vibrational modes may contribute to the dinuclear shape formation. Because of the α-particle-type correlations, which are enhanced in the surface region, where the density is lower, the numbers of protons and neutrons coincide in this small part of the nucleus volume. We estimate the size of the nuclear volume, where clusterization is possible , in the following way: we consider only those quadrupole and octupole vibrational amplitudes in the intrinsic frame which conserve axial symmetry, i.e., a_{20} and a_{30}. Then we calculate the part of the nuclear volume located outside of the plane orthogonal to the axial symmetry axis and touching the sphere with the equivalent radius R_0. We take a mirror asymmetric part of it since axially symmetrically located matter will not contribute to the dipole moment compensating each other. In the lowest order the corresponding number of nucleons is quadratic in the vibrational amplitudes and we obtain the following expression for A_2:

$$A_2=A\frac{15}{8\pi}\sqrt{\frac{7}{5}}a_{20}a_{30}. \tag{2}$$

Depending on the values of a_{20} and a_{30}, A_2 can be smaller or larger than 4. When A_2 is equal or larger than 4 we assume that $Z_2/A_2=0.5$ because the formation of an α cluster is energetically more favorable than a fragmentation in a cluster with any other four nucleons. For smaller values of A_2 we note that $Z_2/A_2=Z_1/A_1$. Thus, only a part of the collective wave function distributed in the a_{20},a_{30} plane contributes to the $E1$ transition. With this assumption we have calculated the intrinsic dipole transition moment related to the $B(E1;0^+_1\to1^-_1)$ value through the relation

$$D_1=\sqrt{\frac{4\pi}{3}B(E1;0^+_1\to1^-_1)}. \tag{3}$$

For the wave function of the ground state and the two-phonon 1^-_1 state we have taken the harmonic oscillator expressions. Of course, such an approximation is unsatisfactory if we approach a region of a permanent deformation.

The results of such calculations show that in the case of uncorrelated quadrupole and octupole vibrations the produced transitional dipole moment is too small. However, it is natural to assume that if an α clusterization exists, it does not only produce a contribution to the $E1$ transition operator but also introduces some correlation terms in the Hamiltonian. Such correlations between quadrupole and octupole vibrations, are preferably responsible for the formation of a cluster structure and increase the value of the transitional dipole moment. However, they can simultaneously destroy the harmonic picture of the multipole shape vibrations, which is reflected in relations between the energies of the 2^+_1, 3^-_1, and 1^-_1 states and in relations between the electric multipole transition probabilities.

To keep the picture of harmonic vibrations we assume the following collective ground state wave function:

$$\tilde{\Psi}(0^+_1)=U\Psi(0^+_1). \tag{4}$$

Here

$$\Psi(0^+_1)\sim\exp\left(-\frac{\sqrt{B_2C_2}}{2\hbar}\sum_\mu(-1)^\mu\alpha_{2\mu}\alpha_{2-\mu}-\frac{\sqrt{B_3C_3}}{2\hbar}\sum_\mu(-1)^\mu\alpha_{3\mu}\alpha_{3-\mu}\right) \tag{5}$$

is the harmonic oscillator ground state wave function of the system with quadrupole and octupole modes. In Eq. (5) the coordinates $\alpha_{\lambda\mu}$ ($\lambda=2,3$) are the collective multipole variables, and B_λ and C_λ are the inertia and stiffness parameters of the corresponding modes. The unitary operator U

$$U=\exp\left(-g\sqrt{\frac{5}{3}}\sum_\mu\frac{1}{R}(\alpha_2R)_{3\mu}\frac{\partial}{\partial\alpha_{3\mu}}\right) \tag{6}$$

introduces correlations between the quadrupole and octupole vibrations corresponding to a formation of a small cluster. The operator U can also be taken in a more general form

$$U=\exp\left(-g\sqrt{\frac{5}{3}}\sum_\mu\frac{1}{R}(\alpha_2R)_{3\mu}\frac{\partial}{\partial\alpha_{3\mu}}-\tilde{g}\sqrt{\frac{7}{3}}\sum_\mu\frac{1}{R}(\alpha_3R)_{2\mu}\frac{\partial}{\partial\alpha_{2\mu}}\right) \tag{7}$$

with two parameters. But we use the simpler form (6) below since the amplitude of the quadrupole vibrations is usually larger than those of the octupole ones. The fixed vector $\vec{R}$ represents the direction of preferable correlations of quadrupole and octupole vibrations, i.e., the direction from the center of mass to the small cluster. This vector can be considered as a static limit of the dipole boson operator in the extended version of the interacting boson model [24,25].

The new ground state wave function takes the following form with U of Eq. (6):

$$\tilde{\Psi}(0_1^+)\sim\exp\Bigg(-\frac{\sqrt{B_2C_2}}{2\hbar}\sum_\mu(-1)^\mu\alpha_{2\mu}\alpha_{2-\mu} - \frac{\sqrt{B_3C_3}}{2\hbar}\sum_\mu(-1)^\mu\alpha_{3\mu}\alpha_{3-\mu} + g\frac{\sqrt{B_3C_3}}{\hbar}\sqrt{\frac{5}{3}}\frac{1}{R}\sum_\mu(-1)^\mu(\alpha_2R)_{3\mu}\alpha_{3-\mu} - \frac{\sqrt{B_3C_3}}{2\hbar}\frac{5}{3}g^2\frac{1}{R^2}\times\sum_\mu(-1)^\mu(\alpha_2R)_{3\mu}(\alpha_2R)_{3-\mu}\Bigg). \tag{8}$$

At the same time the unitary operator U conserves the harmonic picture of the shape vibrations. As in the case of the harmonic oscillator, the ground state wave function $\Psi(0_1^+)$ represents the vacuum with respect to the phonon annihilation operators

$$b_{2\mu}=\left(\frac{\sqrt{B_2C_2}}{2\hbar}\right)^{1/2}(-1)^\mu\alpha_{2-\mu}+\left(\frac{\hbar}{2\sqrt{B_2C_2}}\right)^{1/2}\frac{\partial}{\partial\alpha_{2\mu}}, \tag{9}$$

$$b_{3\mu}=\left(\frac{\sqrt{B_3C_3}}{2\hbar}\right)^{1/2}(-1)^\mu\alpha_{3-\mu}+\left(\frac{\hbar}{2\sqrt{B_3C_3}}\right)^{1/2}\frac{\partial}{\partial\alpha_{3\mu}}, \tag{10}$$

the new ground state wave function $\tilde{\Psi}(0_1^+)$ is the vacuum of the new phonon operators

$$\tilde{b}_{2\mu}=Ub_{2\mu}U^+=\left(\frac{\sqrt{B_2C_2}}{2\hbar}\right)^{1/2}(-1)^\mu\alpha_{2-\mu}+\left(\frac{\hbar}{2\sqrt{B_2C_2}}\right)^{1/2}\frac{\partial}{\partial\alpha_{2\mu}}-\left(\frac{\hbar}{2\sqrt{B_2C_2}}\right)^{1/2}g\sqrt{\frac{7}{3}}\frac{1}{R}\left(\frac{\partial}{\partial\alpha_3}R^+\right)_{2\mu}, \tag{11}$$

$$\tilde{b}_{3\mu}=Ub_{3\mu}U^+=\left(\frac{\sqrt{B_3C_3}}{2\hbar}\right)^{1/2}(-1)^\mu\alpha_{3-\mu}+\left(\frac{\hbar}{2\sqrt{B_3C_3}}\right)^{1/2}\frac{\partial}{\partial\alpha_{3\mu}}-\left(\frac{\sqrt{B_3C_3}}{2\hbar}\right)^{1/2}\times g\sqrt{\frac{5}{3}}\frac{1}{R}(-1)^\mu(\alpha_2R)_{3\mu}. \tag{12}$$

With the new phonon creation operators $\tilde{b}_{2\mu}^+=Ub_{2\mu}^+U^+$ and $\tilde{b}_{3\mu}^+=Ub_{3\mu}^+U^+$ we construct the wave functions of the excited states

$$\tilde{\Psi}_M(2_1^+)=\tilde{b}_{2\mu}^+\tilde{\Psi}(0_1^+), \tag{13}$$

$$\tilde{\Psi}_M(3_1^-)=\tilde{b}_{3\mu}^+\tilde{\Psi}(0_1^+), \tag{14}$$

$$\tilde{\Psi}_M(1_1^-)=(\tilde{b}_2^+\tilde{b}_3^+)_{1M}\tilde{\Psi}(0_1^+). \tag{15}$$

Using expression (1) with Eq. (2) for the electric dipole transition operator and the wave functions given by Eqs. (13)–(15) we have calculated the values of the electric dipole transition moment. We found it convenient to present the parameter g as

$$g=\frac{16\pi}{3}\sqrt{\frac{5}{7}}\frac{1}{A\langle\beta_2^2\rangle}g', \tag{16}$$

where the value of the parameter g' is found to be approximately equal to 0.2 from the fit of the experimental data for all nuclei considered. The quantity $\langle\beta_2^2\rangle$ is expressed through $B(E2;0_1^+\to2_1^+)$. The experimental values of $B(E2;0_1^+\to2_1^+)$ and $B(E3;0_1^+\to3_1^-)$, which are needed to determine $(\hbar/\sqrt{B_2C_2})^{1/2}$ and $(\hbar/\sqrt{B_3C_3})^{1/2}$, have been taken from Refs. [26] and [27], respectively.

The results of our calculations are presented in Table I together with the experimental data. The indicated errors of the calculated values are connected with the experimental uncertainties in the values of $B(E2;0_1^+\to2_1^+)$ and $B(E3;0_1^+\to3_1^-)$. We notice in Table I for semimagic nuclei (Sn isotopes and $N=82$ nuclei) and also in near lying nuclei with $N=80$ and 84 that the calculated values of D are small compared to the experimental data, whereas the agreement with the experimental data is good in other nuclei. The results presented in Table I show that the mechanism responsible for the strong $E1$ transitions in nuclei away from the closed shells can be related to the shape oscillations leading to the formation of a cluster state in the lower density surface region.

Let us consider well deformed nuclei. In this case a picture of harmonic quadrupole vibrations cannot be applied. The intrinsic ground state wave function has its maximum at a nonzero value of a_{20}, corresponding to the equilibrium deformation. For the electric dipole operator we should use the same expression (1) with Eq. (2) as for nondeformed nuclei, substituting, however, the expression

$$a_{20}=\beta_2+a'_{20}, \tag{17}$$

instead of a_{20}, where a'_{20} describes oscillations around the equilibrium value β_2. The amplitude of these oscillations is taken to be equal to $\langle(a'_{20})^2\rangle^{1/2}=0.12\beta_2$ [22]. We should consider this number as an average value. At the same time our calculations have shown that the results are sensitive to the accepted value of the amplitude of vibrations. So, we obtain only an averaged description of deformed nuclei.

TABLE I. Calculated and experimental values of the electric dipole transition moment for collective spherical nuclei (left part of the table) and semimagic and near lying nuclei (right part of the table). The experimental data are taken from Ref. [5], where they are accumulated for most of the considered nuclei, Refs. [29,30] for Cd isotopes, and Ref. [31] for Sn isotopes. The values of dipole moment are given in e fm.

Nucleus	$D_{1,\mathrm{expt}}$	$D_{1,\mathrm{calc}}$	Nucleus	$D_{1,\mathrm{expt}}$	$D_{1,\mathrm{calc}}$
${}^{108}_{48}\mathrm{Cd}_{60}$	0.10 ± 0.002	0.08 ± 0.01	${}^{116}_{50}\mathrm{Sn}_{66}$	0.17 ± 0.01	0.06 ± 0.01
${}^{110}_{48}\mathrm{Cd}_{62}$	0.10 ± 0.01	0.08 ± 0.01	${}^{118}_{50}\mathrm{Sn}_{68}$	0.17 ± 0.01	0.06 ± 0.01
${}^{112}_{48}\mathrm{Cd}_{64}$	0.08 ± 0.002	0.09 ± 0.01	${}^{120}_{50}\mathrm{Sn}_{70}$	0.18 ± 0.01	0.06 ± 0.01
${}^{114}_{48}\mathrm{Cd}_{66}$	0.09 ± 0.004	0.12 ± 0.02	${}^{122}_{50}\mathrm{Sn}_{72}$	0.17 ± 0.01	0.06 ± 0.01
${}^{116}_{48}\mathrm{Cd}_{68}$	0.07 ± 0.01	0.11 ± 0.02	${}^{124}_{50}\mathrm{Sn}_{74}$	0.16 ± 0.01	0.05 ± 0.01
${}^{134}_{56}\mathrm{Ba}_{78}$	0.10 ± 0.005	0.095 ± 0.015	${}^{136}_{56}\mathrm{Ba}_{80}$	0.14 ± 0.02	0.08 ± 0.02
${}^{146}_{60}\mathrm{Nd}_{86}$	0.145 ± 0.02	0.125 ± 0.015	${}^{138}_{56}\mathrm{Ba}_{82}$	0.23 ± 0.03	0.06 ± 0.01
${}^{148}_{60}\mathrm{Nd}_{88}$	0.24 ± 0.06	0.21 ± 0.01	${}^{140}_{58}\mathrm{Ce}_{82}$	0.26 ± 0.01	0.07 ± 0.01
${}^{150}_{60}\mathrm{Nd}_{90}$	0.26 ± 0.06	0.23 ± 0.02	${}^{142}_{58}\mathrm{Ce}_{84}$	0.22 ± 0.04	0.09 ± 0.01
${}^{148}_{62}\mathrm{Sm}_{86}$	0.11 ± 0.01	0.12 ± 0.02	${}^{142}_{60}\mathrm{Nd}_{82}$	0.26 ± 0.02	0.06 ± 0.01
${}^{150}_{62}\mathrm{Sm}_{88}$	0.20 ± 0.01	0.17 ± 0.02	${}^{144}_{60}\mathrm{Nd}_{84}$	0.20 ± 0.01	0.10 ± 0.01
			${}^{144}_{62}\mathrm{Sm}_{82}$	0.29 ± 0.02	0.06 ± 0.01

We consider below only transitions from the ground to the 1^-, $K=0$ states. If we want to calculate the probability of the $0_1^+\to1^-$, $K=1$ transition, we have to extend our model by introducing angular oscillations of the position of the α cluster with respect to the symmetry axis of the axially symmetric quadrupole deformation. In other words, we have to consider dynamics of the vector $\vec{R}$, introduced above [see Eq. (6)], which was treated in a static limit. This dynamics means the introduction of additional parameters into the Hamiltonian and, therefore, leads to additional uncertainties which we prefer to avoid here.

The results of calculations for deformed nuclei of the $B(E1;0_1^+\to1_1^-,K=0)$ are sensitive to the value of $B(E3;0_1^+\to3_1^-,K=0)$. However, in deformed nuclei the Coriolis interaction, which is not included in our consideration, is very important for the description of the octupole states [22,28] and leads to a concentration of the octupole strength into the octupole transitions with lowest energy [22]. The K quantum number is in many cases not even approximately a good quantum number of the states [28]. Among the nuclei with known $B(E1;0_1^+\to1_1^-,K=0)$ we find only the ^{152,154}Sm and ^{160}Gd nuclei which have $K=0$ octupole bands as the lowest ones. However, the energy interval between the 3^- $K=0$ state and the next excited 3^- state in ^{160}Gd is rather small (170 keV) and the Coriolis mixing can be important. Only the $K=0$ octupole band in ^{152,154}Sm is well separated from the octupole states with $K\neq0$. The corresponding energy interval is equal to 538 keV in ^{152}Sm and 573 keV in ^{154}Sm. So, we can expect an approximate K purity of the low angular momentum negative parity states in these two isotopes. Using relation (16) with $g'=0.2$ we obtain $g=0.20$ for ^{152}Sm and $g=0.16$ for ^{154}Sm. Then the calculated values of the dipole transition moment are $D_{\mathrm{calc}}=0.309$ e fm ($D_{\mathrm{expt}}=0.311\pm0.015$ e fm) for ^{152}Sm and $D_{\mathrm{calc}}=0.320$ e fm ($D_{\mathrm{expt}}=0.334\pm0.024$ e fm) for ^{154}Sm.

In conclusion we suggest an interpretation of the strong $E1$ transitions between the ground and first excited 1^- states in collective spherical nuclei which is based on the idea of α clusterization. The model contains the parameter g' only whose value was taken to be the same for all considered nuclei in Table I: $g'=0.2$. The results of the model calculations, especially the variation of the dipole transition moment from nucleus to nucleus, agree with the experimental data for collective spherical and deformed nuclei.

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