Extended-soft-core *NN* potentials in momentum space. II. Meson-pair exchange potentials

Th. A. Rijken and H. Polinder
Institute for Theoretical Physics Nijmegen, University of Nijmegen, Nijmegen, The Netherlands

J. Nagata*
Venture Business Laboratory, Hiroshima University, Kagamiyama 2-313, Higashi-Hiroshima, Japan
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The partial wave projection of the Nijmegen soft-core potential model for meson-pair-exchange (MPE) for *NN* scattering in momentum space is presented. Here, nucleon-nucleon momentum-space MPE potentials are *NN* interactions where either one or both nucleons contains a meson-pair vertex. Dynamically, the meson-pair vertices can be viewed as describing in an effective way (part of) the effects of heavy-meson exchange and meson-nucleon resonances. From the point of view of "duality," these two kinds of contribution are roughly equivalent. Part of the MPE vertices can be found in the chiral-invariant phenomenological Lagrangians that have a basis in spontaneous broken chiral symmetry. It is shown that the MPE interactions are a very important component of the nuclear force, which indeed enables a very successful description of the low and medium energy *NN* data. Here we present a precise fit to the *NN* data with the extended-soft-core model containing one-boson-exchange, PS-PS-, and MPE potentials. An excellent description of the *NN* data for $T_{Lab} \leqslant 350$ MeV is presented and discussed. Phase shifts are given and a $\chi^2_{pdp}=1.15$ is reached.

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I. INTRODUCTION

In the previous paper (paper I) [1], the techniques for the momentum space treatment of the extended-soft-core (ESC) model are described. This implies first the development of a representation of the ESC model suitable for the projection onto the Pauli-spinor rotational-invariant operators and secondly the partial wave analysis. This partial wave analysis is organized along similar lines as used for the soft-core one-boson-exchange (OBE) models [2]. In [1] the nucleon-nucleon partial wave contributions have been worked out in detail. These are the analogs of the configuration-space two-meson-exchange (TME) potentials given in, e.g., [3]. Here, the TME potentials are defined to contain the planar and crossed-box two-meson-exchange potentials.

In this second paper on soft-core two-meson-exchange potentials in momentum space (paper II), we derive the same representation as in paper I, but now for the contributions to the nucleon-nucleon potentials when either one or both nucleons contains a pair vertex—i.e., the MPE potentials. We give the partial wave potentials in the similar representation as used in paper I. In Ref. [4] the MPE contributions to the configuration-space nucleon-nucleon potentials—i.e., when either one or both nucleons contains a pair vertex—have been derived. The corresponding "seagull" diagrams are referred to as one-pair and two-pair diagrams. This in order to distinct these from the planar and crossed-box diagrams, which were given Ref. [3].

The two types of two-meson-exchange potentials, two-meson exchange (TME) (see I) and meson-pair exchange (MPE) presented here are part of our program to extend the Nijmegen soft-core one-boson-exchange potential [5–7] to arrive at a new extended soft-core nucleon-nucleon model, hereafter referred to as the ESC potential [4,8–10].

In the introduction to Ref. [4] a rather complete description is given of the physical background behind the MPE potentials, and we refer the interested reader to that reference.

We apply the potentials derived in this work to fit the *NN* data. In the TME potentials we restrict ourselves to pseudoscalar-pseudoscalar (PS-PS) exchange. Or, phrased differently, we include only the Goldstone-boson sector. This because it gives the complete long-range contribution of one-plus two-pion-exchange potential (OPEP+TPEP) and the inclusion of η, etc., is necessary for (i) (approximate) chiral symmetry and (ii) for completeness in the sense of $SU_f(3)$, which allows an extension to hyperon-nucleon and hyperon-hyperon potentials [10].

In fact, this fit has been performed in the configuration-space version. However, the results were checked numerically in momentum space, using the formulas of papers I and II.

This paper is organized as follows. In Secs. II and III, we give the essentials of the procedure followed in deriving the new momentum-space representation. In Sec. IV the projection of the MPE on the Pauli-spinor invariants is worked out for the adiabatic contributions. In Sec. V the same is done for the $1/M$ corrections: the nonadiabatic and the pseudovector-vertex terms. In Sec. VI the partial wave analysis is indicated. The procedure for the partial wave projection is completely analogous to that of paper I and can be transcribed immediately comparing the invariant contributions $\Omega_j(\mathbf{k}^2;t,u)$ for MPE to those for TME in I. In Sec. VII the results from a fit to the *NN* data are shown and discussed. Here, phase shifts are given for $T_{lab} \leqslant 350$ MeV and the pair couplings are compared to the values expected from, e.g., chiral Lagrangians.

In Appendix A the pair-interaction Hamiltonians are

*Present address: Kyushu International University, Fukuoka 805-8512, Japan.

listed. In Appendix B the λ representations for the MPE denominators are given. In Appendix C we give the integration dictionary for the Gaussian integrals that occur in MPE but not in TME. In Appendix D a derivation for the potentials due to the "derivative scalar pair" interaction [see the $g'_{(\pi\pi)_0}$ coupling in Eq. (A1a)] is outlined. Thus, for completeness, since although we do not employ this kind of pair interaction, it occurs often in the current literature. In Appendix E the full $SU_f(3)$ content of our pair interactions is shown.

II. MOMENTUM-SPACE REPRESENTATION MPE POTENTIALS

Here, we give an outline the essentials of the procedure to derive our new momentum-space representation for the MPE potentials. These procedures have been described in I, to which we refer for details. Here, we focus on the peculiar features that occur in the application to the MPE potentials.

The starting point is the basic convolutive integral

$$\tilde{V}_{M,N}(\mathbf{k}) = \int\int \frac{d^3k_1 d^3k_2}{(2\pi)^3}\,\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2) \times \tilde{F}_M(\mathbf{k}_1^2,m_1)\tilde{G}_N\big(\mathbf{k}_2^2,m_2\big) = \int \frac{d^3\Delta}{(2\pi)^3}\tilde{F}_M(\mathbf{\Delta}^2,m_1)\tilde{G}_N((\mathbf{k}-\mathbf{\Delta})^2,m_2), \quad (2.1)$$

where $\tilde{F}_M(\mathbf{k}^2)$ and $\tilde{G}_N(\mathbf{k}^2)$ can be of the form

$$M=0:\quad \tilde{F}_0(\mathbf{k}^2) = \exp[-\mathbf{k}^2/\Lambda_1^2],$$

$$M=2:\quad \tilde{F}_2(\mathbf{k}^2) = \frac{\exp[-\mathbf{k}^2/\Lambda_1^2]}{\mathbf{k}^2+m_1^2},$$

$$N=0:\quad \tilde{G}_0(\mathbf{k}^2) = \exp[-\mathbf{k}^2/\Lambda_2^2],$$

$$N=2:\quad \tilde{G}_2(\mathbf{k}^2) = \frac{\exp[-\mathbf{k}^2/\Lambda_2^2]}{\mathbf{k}^2+m_2^2}; \quad (2.2)$$

i.e., $M,N=2$ is the modified Yukawa type and $M,N=0$ is the Gaussian type. Below, we give for the different cases the momentum-space representation, similar to the one that has been developed in paper I.

(i) $M=N=2$: In paper I using twice the identity

$$\frac{\exp[-\mathbf{k}^2/\Lambda^2]}{\mathbf{k}^2+m^2} = e^{m^2/\Lambda^2}\int_1^\infty \frac{dt}{\Lambda^2}\exp\left[-\left(\frac{\mathbf{k}^2+m^2}{\Lambda^2}\right)t\right], \quad (2.3)$$

the Δ integral has been carried out. After a redefinition of the variables $t\to t/\Lambda_1^2$ and $u\to u/\Lambda_2^2$ the result in I is

$$\tilde{V}_{2,2}(\mathbf{k}) = (4\pi)^{-3/2}e^{m_1^2/\Lambda_1^2}e^{m_2^2/\Lambda_2^2} \times \int_{t_0}^\infty dt\int_{u_0}^\infty du\,\frac{\exp[-(m_1^2t+m_2^2u)]}{(t+u)^{3/2}} \times \exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right] \quad (t_0=1/\Lambda_1^2, u_0=1/\Lambda_2^2). \quad (2.4)$$

(ii) $M=2,N=0$: Using the identity (2.3) once and performing similar steps as in paper I, one easily derives that, for this case,

$$\tilde{V}_{2,0}(\mathbf{k}) = (4\pi)^{-3/2}e^{m_1^2/\Lambda_1^2}e^{m_2^2/\Lambda_2^2} \times \int_{t_0}^\infty dt\int_{u_0}^\infty du\,\frac{\exp[-(m_1^2t+m_2^2u)]}{(t+u)^{3/2}} \times \exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right]\delta(u-u_0). \quad (2.5)$$

Here is defined $\delta(u-u_0)\equiv\lim_{\epsilon\downarrow 0}\delta(u-u_{0,\epsilon})$, where $u_{0,\epsilon}=u_0-\epsilon$. This definition implies that in Eq. (2.5) the u integration can simply be performed by the substitution $u\to u_0$ in the integrand.

(iii) $M=0,N=2$: Similarly to the previous case, one has

$$\tilde{V}_{0,2}(\mathbf{k}) = (4\pi)^{-3/2}e^{m_1^2/\Lambda_1^2}e^{m_2^2/\Lambda_2^2} \times \int_{t_0}^\infty dt\int_{u_0}^\infty du\,\frac{\exp[-(m_1^2t+m_2^2u)]}{(t+u)^{3/2}} \times \exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right]\delta(t-t_0); \quad (2.6)$$

the case $\tilde{V}_{0,0}(\mathbf{k})$ does not occur, since double diffractive exchange has not been included. For the integrals $\tilde{V}_{M,N}$ of this section and similar integrals below in this paper, we introduce the following convenient shorthand notation. We write

$$\tilde{V}_{M,N}(\mathbf{k}) = \int_{t_0}^\infty dt\int_{u_0}^\infty du\, w_0(t,u) \times \left\{ v_{M,N}(t,u)\exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right]\right\}, \quad (2.7a)$$

with common weight function $w_0(t,u)$ defined as

$$w_0(t,u) \equiv (4\pi)^{-3/2}e^{m_1^2/\Lambda_1^2}e^{m_2^2/\Lambda_2^2}\frac{\exp[-(m_1^2t+m_2^2u)]}{(t+u)^{3/2}}. \quad (2.7b)$$

The form in which these basic integrals appear in MPE depends on two factors: (i) The denominators $D(\omega_1,\omega_2)$. In the next section we will give a catalog of these. (ii) The operators $\tilde{O}(\mathbf{k}_1,\mathbf{k}_2)$. Also these will be given in the next section.

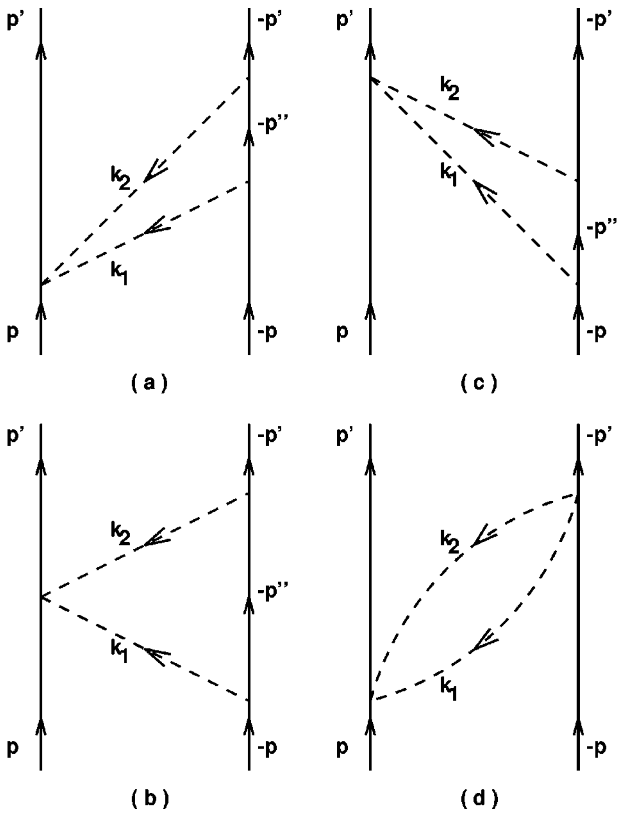

FIG. 1. Time-ordered (a)–(c) one-pair and (d) two-pair diagrams. The dashed line with momentum $\mathbf{k}_1$ refers to the pion and the dashed line with momentum $\mathbf{k}_2$ refers to one of the other (vector, scalar, or pseudoscalar) mesons. To these we have to add the ''mirror'' diagrams, where for the one-pair diagrams the pair vertex occurs on the other nucleon line.

TABLE I. The one-pair isospin factors $C^{(1)}(\alpha\beta)$ and momentum operators $\widetilde{O}^{(1)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)$. The index p labels the type of denominators. Note that $\kappa_1=(f/g)_{(\pi\pi)_1}$.

$(\alpha\beta)$	$C^{(1)}(\alpha\beta)$	$O^{(1)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)$
$(\pi\pi)_0$	6	$-\mathbf{k}_1\cdot\mathbf{k}_2+\frac{i}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{k}_1\times\mathbf{k}_2)$
$(\sigma\sigma)$	2	1
$(\pi\eta)$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$-2\mathbf{k}_1\cdot\mathbf{k}_2$
$(\pi\eta')$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$-2\mathbf{k}_1\cdot\mathbf{k}_2$
$(\pi\pi)_1$	$2i\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$i\Big[\mathbf{k}_1\cdot\mathbf{k}_2-\frac{i}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{k}_1\times\mathbf{k}_2)\Big]$ $+\frac{i}{M}\Big[(1+\kappa_1)\boldsymbol{\sigma}_1\cdot(\mathbf{k}_1\times\mathbf{k}_2)\boldsymbol{\sigma}_2\cdot(\mathbf{k}_1\times\mathbf{k}_2)$ $+\frac{i}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot(\mathbf{k}_1\times\mathbf{k}_2)\,\mathbf{q}\cdot(\mathbf{k}_1-\mathbf{k}_2)\Big]$
$(\pi\rho)_1$	$-2i\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$\frac{i}{M}\Big[\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_1+\frac{1}{2}(1+\kappa_\rho)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2$ $+\boldsymbol{\sigma}_1\cdot\mathbf{k}_2\boldsymbol{\sigma}_2\cdot\mathbf{k}_1-2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\mathbf{k}_1\cdot\mathbf{k}_2)\Big]$
$(\pi\sigma)$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$[\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2+\boldsymbol{\sigma}_1\cdot\mathbf{k}_2\boldsymbol{\sigma}_2\cdot\mathbf{k}_1-2\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_1]$
(πP)	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$[\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2+\boldsymbol{\sigma}_1\cdot\mathbf{k}_2\boldsymbol{\sigma}_2\cdot\mathbf{k}_1-2\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_1]$
$(\pi\rho)_0$	3	$[\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2+\boldsymbol{\sigma}_1\cdot\mathbf{k}_2\boldsymbol{\sigma}_2\cdot\mathbf{k}_1+2\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_1]$
$(\pi\omega)$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$[\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2+\boldsymbol{\sigma}_1\cdot\mathbf{k}_2\boldsymbol{\sigma}_2\cdot\mathbf{k}_1+2\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_1]$

III. MESON-PAIR-EXCHANGE POTENTIALS

In [4] the derivation of the pair-exchange potentials in both momentum and configuration space is given. In this reference the configuration-space potentials are worked out fully. The topic of this paper is to do the same for the momentum-space description. In particular, the partial wave analysis is performed leading to a representation which is very suitable for numerical evaluation.

From [4] and Eq. (3.1) it follows that the momentum-space MPE potential can be represented in general in the form

$$\widetilde{V}^{(n)}_{\alpha\beta}(\mathbf{k})=C^{(n)}(\alpha\beta)g^{(n)}(\alpha\beta)\int\int\frac{d^3k_1d^3k_2}{(2\pi)^3}\,\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)$$
$$\times\widetilde{F}_0(\mathbf{k}_1^2)\widetilde{G}_0(\mathbf{k}_2^2)\sum_p\,\widetilde{O}^{(n)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)D^{(n)}_{\{p\}}(\omega_1,\omega_2), \tag{3.1}$$

where the index n distinguishes one-pair ($n=1$) and two-pair ($n=2$) meson-pair exchange, and $(\alpha\beta)$ refers to the particular meson pair that is being exchanged (see Fig. 1). The subscript $\{p\}=\{ad,na,pv,off\}$ distinguishes, respectively, the adiabatic, the nonadiabatic, pseudovector vertex, and off-shell contributions. Here, the last three are the $1/M$ corrections to the MPE potentials.

The product of the coupling constants in the cases $n=1,2$ is given by

$$g^{(1)}(\alpha\beta)=g_{(\alpha\beta)}g_{NN\alpha}g_{NN\beta},$$
$$g^{(2)}(\alpha\beta)=g^2_{(\alpha\beta)}, \tag{3.2}$$

with appropriate powers of m_π, depending on the definition of the Hamiltonians given in [4], Sec. II.

The momentum-dependent operators $O^{(n)}_{\alpha\beta,p}$ are given in Tables I and II. For completeness, these tables also contain the isospin factors $C^{(n)}(\alpha\beta)$ as derived in Appendix B of [4]. The momentum operators for $(\pi\pi)_0$ and $(\pi\pi)_1$ both contain a term antisymmetric in $\mathbf{k}_1\leftrightarrow\mathbf{k}_2$, which only contributes in the nonadiabatic contribution; see [4], Sec. IV. In the adiabatic potential, as explained in [4], they drop out when we integrate over $\mathbf{k}_1$ and $\mathbf{k}_2$.

The energy denominators $D^{(n)}_p$ are also discussed in detail in [4], Sec. II, in terms of the time-ordered processes involved in one- and two-pair exchange. These denominators depend on the energies of the exchanged mesons, i.e., ω_1 and ω_2. Another source of $\omega_{1,2}$ dependence comes from vertices

TABLE II. The two-pair isospin factors $C^{(2)}(\alpha\beta)$ and momentum operators $\widetilde{O}^{(2)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)$, and denominators $D^{(2)}_p(\omega_1,\omega_2)$.

$(\alpha\beta)$	$C^{(2)}(\alpha\beta)$	$\widetilde{O}^{(2)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)$	$D^{(2)}_p(\omega_1,\omega_2)$
$(\pi\pi)_0$	6	1	$-\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$
$(\sigma\sigma)$	2	1	$-\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$
$(\pi\eta)$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	1	$-\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$
$(\pi\eta')$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	1	$-\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$
$(\pi\pi)_1$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	1	$-\frac{1}{2}\left[\frac{1}{\omega_1}+\frac{1}{\omega_2}-\frac{4}{\omega_1+\omega_2}\right]$
$(\pi\rho)_1$	$2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$-\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$
$(\pi\sigma)$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$\boldsymbol{\sigma}_1\cdot(\mathbf{k}_1-\mathbf{k}_2)\boldsymbol{\sigma}_2\cdot(\mathbf{k}_1-\mathbf{k}_2)$	$-\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$
(πP)	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$\boldsymbol{\sigma}_1\cdot(\mathbf{k}_1-\mathbf{k}_2)\boldsymbol{\sigma}_2\cdot(\mathbf{k}_1-\mathbf{k}_2)$	$-\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$
$(\pi\rho)_0$	3	$\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$-\frac{1}{2}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)$
		$-\boldsymbol{\sigma}_1\cdot(\mathbf{k}_1+\mathbf{k}_2)\boldsymbol{\sigma}_2\cdot(\mathbf{k}_1+\mathbf{k}_2)$	$-\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$
$(\pi\omega)$	$\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$	$\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$	$-\frac{1}{2}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)$
		$-\boldsymbol{\sigma}_1\cdot(\mathbf{k}_1+\mathbf{k}_2)\boldsymbol{\sigma}_2\cdot(\mathbf{k}_1+\mathbf{k}_2)$	$-\frac{1}{2\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$

TABLE III. The one-pair denominators $D^{(1)}_p(\omega_1,\omega_2)$.

$(\alpha\beta)$	$D^{(1)}_{ad}(\omega_1,\omega_2)$	$D^{(1)}_{na}(\omega_1,\omega_2)$	$D^{(1)}_{pv}(\omega_1,\omega_2)$
$(\pi\pi)_0$	$\frac{1}{\omega_1^2\omega_2^2}$	$\frac{1}{\omega_1^2\omega_2^2}\left[\frac{1}{\omega_1}+\frac{1}{\omega_2}-\frac{1}{\omega_1+\omega_2}\right]$	$\frac{1}{\omega_1\omega_2(\omega_1+\omega_2)}$
$(\pi\pi)_1$	$\frac{2}{\omega_1\omega_2(\omega_1+\omega_2)},\ \frac{1}{\omega_1^2\omega_2^2}$	$\frac{2}{\omega_1^2\omega_2^2}$	$\frac{1}{\omega_1^2},\ \frac{1}{\omega_2^2}$
$(\pi\sigma)$	$\frac{1}{\omega_1^2\omega_2^2}$	$\frac{1}{\omega_1^2\omega_2^2}\left[\frac{1}{\omega_1}+\frac{1}{\omega_2}-\frac{1}{\omega_1+\omega_2}\right]$	$\frac{1}{\omega_1\omega_2(\omega_1+\omega_2)}$

TABLE IV. The $d_p(t,u)$ functions corresponding to the denominators $D_p(\omega_1,\omega_2)$, $p=0,1,2,3,4,5$.

$D_{\{p\}}(\omega_1,\omega_2)$			$d_{\{p\}}(t,u))$		
$D_{2,2,0}$	$=$	$\frac{1}{\omega_1^2\omega_2^2}$	$d_{2,2,0}$	$=$	1
$D_{2,0,0}$	$=$	$\frac{1}{\omega_1^2}$	$d_{2,0,0}$	$=$	$\delta(u-u_0)$
$D_{0,2,0}$	$=$	$\frac{1}{\omega_2^2}$	$d_{0,2,0}$	$=$	$\delta(t-t_0)$
$D_{1,0,0}$	$=$	$\frac{1}{\omega_1}$	$d_{1,0,0}$	$=$	$\frac{1}{\sqrt{\pi}}t^{-1/2}\ \delta(u-u_0)$
$D_{0,1,0}$	$=$	$\frac{1}{\omega_2}$	$d_{0,1,0}$	$=$	$\frac{1}{\sqrt{\pi}}u^{-1/2}\ \delta(t-t_0)$
$D_{0,0,1}$	$=$	$\frac{1}{\omega_1+\omega_2}$	$d_{0,0,1}$	$=$	$\frac{1}{2\sqrt{\pi}}(t+u)^{-3/2}$
$D_{1,1,1}$	$=$	$\frac{1}{\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}$	$d_{1,1,1}$	$=$	$\frac{1}{\sqrt{\pi}}(t+u)^{-1/2}$

with derivatives and the nonadiabatic expansion terms. It appears from [4] that in general one can write

$$D^{(n)}_{\{p\}}(\omega_1,\omega_2)=\sum_{p_1,p_2,p_3} c^{(n)}_{p_1,p_2,p_3}D_{\{p_1,p_2,p_3\}}, \tag{3.3}$$

where in terms of the integer powers p_i $(i=1,2,3)$ the denominators can be written

$$D_{\{p_1,p_2,p_3\}}=\frac{1}{\omega_1^{p_1}}\frac{1}{\omega_2^{p_2}}\frac{1}{(\omega_1+\omega_2)^{p_3}}. \tag{3.4}$$

The energy denominators $D_p^{(n)}$ are listed in Tables II and III.

The evaluation of the momentum integrations can now readily be performed using the methods given in [4,11]. There it was shown that the full separation of the $\mathbf{k}_1$ and $\mathbf{k}_2$ dependence can be achieved in all cases using the λ-integral representation, first introduced in [11]. In Appendix B the occurring λ-integrals are listed. From the listing in Appendix B one readily sees that for the derivation of the representation similar to that one in Eqs. (2.4)–(2.6) we need to start out from a generalization of Eq. (2.1):

$$\tilde{V}_{M,N}(\mathbf{k},\lambda)=\frac{2}{\pi}\int_0^\infty d\lambda\, f_{M,N}(\lambda)\int\frac{d^3\Delta}{(2\pi)^3}\tilde{F}_M(\mathbf{\Delta}^2,\sqrt{m_1^2+\lambda^2})\,\tilde{G}_N((\mathbf{k}-\mathbf{\Delta})^2,\sqrt{m_2^2+\lambda^2}). \tag{3.5}$$

In paper I it has been shown that all occurring λ integrals can be performed analytically. The result for all cases can be written as

TABLE V. Coefficients $\Upsilon^{(ad,na,pv)}_{j,k}$ for the $(\pi\pi)_0$ contributions.

	$\Upsilon_0(\Vert)(t,u)$	$\Upsilon_1(\Vert)(t,u)$	$\Upsilon_2(\Vert)(t,u)$
		One-pair exchange	
$\Omega_1^{(1),ad}$	$+\frac{3}{2}\frac{1}{t+u}$	$-\frac{tu}{(t+u)^2}$	—
$\Omega_1^{(1),na}$	$-\frac{1}{\sqrt{\pi}}\frac{15}{4}\frac{\sqrt{t+u}}{(t+u)^2}\frac{1}{M}$	$-\frac{1}{2\sqrt{\pi}}\left(\frac{t^2-8tu+u^2}{t+u}\right)\frac{\sqrt{t+u}}{(t+u)^2}\frac{1}{M}$	$-\frac{1}{\sqrt{\pi}}\left(\frac{t^2u^2}{(t+u)^2}\right)\frac{\sqrt{t+u}}{(t+u)^2}\frac{1}{M}$
$\Omega_4^{(1),na}$	$-\frac{1}{\sqrt{\pi}}\frac{\sqrt{t+u}}{t+u}\frac{1}{M}$	—	—
$\Omega_1^{(1),pv}$	$\frac{3}{2\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}\frac{1}{M}$	$\frac{1}{2\sqrt{\pi}}\left(\frac{t^2+u^2}{t+u}\right)\frac{1}{(t+u)^{3/2}}\frac{1}{M}$	—
$\Omega_4^{(1),pv}$	$+\frac{3}{\sqrt{\pi}}\frac{1}{(t+u)^{1/2}}\frac{1}{M}$	—	—
		Two-pair exchange	
$\Omega_1^{(2),ad}$	$-\frac{1}{2\sqrt{\pi}}\frac{1}{(t+u)^{1/2}}$	—	—

$$\tilde{V}_{p_1,p_2,p_3}(\mathbf{k}) = \int \frac{d^3\Delta}{(2\pi)^3}\tilde{F}_0(\mathbf{\Delta}^2,m_1)\tilde{G}_0((\mathbf{k}-\mathbf{\Delta})^2,m_2)$$

$$D_{\{p_1,p_2,p_3\}}(\omega_1,\omega_2)$$

$$= \int_{t_0}^{\infty} dt \int_{u_0}^{\infty} du\, w_0(t,u)\left\{ d_{\{p_1,p_2,p_3\}}(t,u) \times \exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right]\right\}. \tag{3.6}$$

All functions $d_{\{p_1,p_2,p_3\}}(t,u)$ that occur in this work are given in Table IV. As noted in Sec. II we will use only representations with $M=N=2$, so that no $\delta(t-t_0)$ or $\delta(u-u_0)$ occurs.

IV. PROJECTION MPE ON SPINOR INVARIANTS I: ADIABATIC CONTRIBUTIONS

The MPE contributions from the adiabatic terms—the nonadiabatic and pseudovector-vertex corrections—are the central, spin-spin, tensor, and spin-orbit momentum-space analogs of those given in Ref. [4] in configuration space. From Eq. (3.1), Tables I–III it is readily verified that the projection onto the potentials V_j, similarly to paper I, can be written as

$$\tilde{V}^{(n)}_{\rm pair}(\alpha\beta) = \int_{t_0}^{\infty} dt \int_{u_0}^{\infty} du\, w_0(t,u) \times\left\{ \exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right]\Omega_j^{(n)}(\mathbf{k}^2;t,u)\right\}(\alpha\beta). \tag{4.1}$$

The functions $\Omega_j^{(n)}$ are worked out in the subsections below. Like in I, we also introduce for convenience the expansion in $\mathbf{k}^2$:

$$\Omega_j^{(ad,na,pv)}(\mathbf{k}^2;t,u) = C^{(n)}(\alpha\beta)g^{(n)}(\alpha\beta)\sum_{k=0}^{K} \Upsilon_{j,k}^{(ad,na,pv)}(t,u)(\mathbf{k}^2)^k. \tag{4.2}$$

TABLE VI. Coefficients $\Upsilon_{j,k}^{(ad,na,pv)}$ for the $(\pi\pi)_1$ contributions.

	$\Upsilon_0(\parallel)(t,u)$	$\Upsilon_1(\parallel)(t,u)$	$\Upsilon_2(\parallel)(t,u)$
	One-pair exchange		
$\Omega_1^{(1),ad}$	$+\frac{3}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}$	$-\frac{2}{\sqrt{\pi}}\frac{tu}{(t+u)^{5/2}}$	—
$\Omega_2^{(1),ad}$	—	$-\frac{1}{3\sqrt{\pi}}\frac{(1+\kappa_1)}{M}\frac{1}{t+u}$	—
$\Omega_3^{(1),ad}$	$+\frac{1}{2\sqrt{\pi}}\frac{(1+\kappa_1)}{M}\frac{1}{t+u}$	—	—
$\Omega_4^{(1),ad}$	$-\frac{1}{\sqrt{\pi}}\frac{1}{M}\frac{1}{t+u}$	—	—
$\Omega_1^{(1),na}$	$-\frac{15}{4}\frac{1}{(t+u)^2}\frac{1}{M}$	$-\frac{1}{2}\left(\frac{t^2-8tu+u^2}{(t+u)^3}\right)\frac{1}{M}$	$-\frac{t^2u^2}{(t+u)^4}\frac{1}{M}$
$\Omega_4^{(1),na}$	$-\frac{1}{t+u}\frac{1}{M}$	—	—
$\Omega_1^{(1),pv}$	$+\frac{3}{4}\frac{\delta(t-t_0)+\delta(u-u_0)}{t+u}\frac{1}{M}$	$+\frac{1}{2}\frac{t^2\delta(t-t_0)+u^2\delta(u-u_0)}{(t+u)^2}\frac{1}{M}$	—
$\Omega_1^{(4),pv}$	$+\frac{t\ \delta(t-t_0)+u\ \delta(u-u_0)}{t+u}\frac{1}{M}$	—	—
	Two-pair exchange		
$\Omega_1^{(2),ad}$	$-\frac{1}{2\sqrt{\pi}}\left[\frac{\delta(u-u_0)}{\sqrt{t}}+\frac{\delta(t-t_0)}{\sqrt{u}}-\frac{2}{(t+u)^{3/2}}\right]$		—

TABLE VII. Coefficients $\Upsilon_{j,k}^{(ad,na,pv)}$ for the $(\pi\rho)_1$ contributions.

	$\Upsilon_0(\|)(t,u)$	$\Upsilon_1(\|)(t,u)$	$\Upsilon_2(\|)(t,u)$
	One-pair exchange		
$\Omega_2^{(1),ad}$	$+\frac{1}{M}(\frac{3}{2}+\kappa_\rho)\frac{1}{t+u}$	$\frac{1}{3M}\left(\frac{u^2}{(t+u)^2}-2(1+\kappa_\rho)\frac{tu}{(t+u)^2}\right)$	—
$\Omega_3^{(1),ad}$	$+\frac{1}{M}\frac{u^2+tu(1+\kappa_\rho)}{(t+u)^2}$	—	—
	Two-pair exchange		
$\Omega_2^{(2),ad}$	$-\frac{1}{2\sqrt{\pi}}\frac{1}{(t+u)^{1/2}}$	—	—-

TABLE VIII. Coefficients $\Upsilon_{j,k}^{(ad,na,pv)}$ for the $(\pi\sigma)_1$ contributions.

	$\Upsilon_0(\|)(t,u)$	$\Upsilon_1(\|)(t,u)$	$\Upsilon_2(\|)(t,u)$
	One-pair exchange		
$\Omega_2^{(1),ad}$	$-\frac{2}{t+u}$	$+\frac{2}{3}\frac{tu-u^2}{(t+u)^2}$	—
$\Omega_3^{(1),ad}$	$+2\frac{tu-u^2}{(t+u)^2}$	—	—
$\Omega_2^{(1),na}$	$+\frac{5}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}\frac{1}{M}$	$+\frac{1}{3\sqrt{\pi}}\frac{t^2-13tu+6u^2}{(t+u)^{5/2}}\frac{1}{M}$	$+\frac{2}{3\sqrt{\pi}}\frac{tu^2(t-u)}{(t+u)^{7/2}}\frac{1}{M}$
$\Omega_3^{(1),na}$	$+\frac{1}{\sqrt{\pi}}\frac{t^2-7tu+6u^2}{(t+u)^{5/2}}\frac{1}{M}$	$+\frac{2}{\sqrt{\pi}}\frac{tu^2(t-u)}{(t+u)^{7/2}}\frac{1}{M}$	—
$\Omega_2^{(1),pv}$	$-\frac{1}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}\frac{1}{M}$	$-\frac{1}{3\sqrt{\pi}}\frac{t^2-tu}{(t+u)^{5/2}}\frac{1}{M}$	—
$\Omega_3^{(1),pv}$	$-\frac{1}{\sqrt{\pi}}\frac{t^2-tu}{(t+u)^{5/2}}\frac{1}{M}$	—	—
	Two-pair exchange		
$\Omega_2^{(2),ad}$	$-\frac{1}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}$	$-\frac{1}{6\sqrt{\pi}}\frac{(t-u)^2}{(t+u)^{5/2}}$	—
—	—-		
$\Omega_3^{(2),ad}$	$-\frac{1}{2\sqrt{\pi}}\frac{(t-u)^2}{(t+u)^{5/2}}$	—	—-

TABLE IX. Coefficients $\Upsilon_{j,k}^{(ad,na,pv)}$ for the $(\pi\omega)_1$ contributions.

	$\Upsilon_0(\Vert)(t,u)$	$\Upsilon_1(\Vert)(t,u)$	$\Upsilon_2(\Vert)(t,u)$
		One-pair exchange	
$\Omega_2^{(1),ad}$	—	$+\frac{2}{3}\frac{u}{t+u}$	—
$\Omega_3^{(1),ad}$	$+2\frac{u}{t+u}$	—	—
		Two-pair exchange	
$\Omega_2^{(2),ad}$	$-\frac{1}{2\sqrt{\pi}}\left[\frac{\delta(t-t_0)}{\sqrt{u}}+\frac{\delta(u-u_0)}{\sqrt{t}}\right]$	$+\frac{1}{6\sqrt{\pi}}\frac{1}{\sqrt{t+u}}$	—
$\Omega_3^{(2),ad}$	$+\frac{1}{2\sqrt{\pi}}\frac{1}{\sqrt{t+u}}$	—	—

Below in this section we give the results for the adiabatic contributions. The coefficients $\Upsilon_{j,k}^{ad}$ are tabulated in Tables V–IX.

A. $J^{PC}=0^{++}$: Adiabatic $(\pi\pi)_0$-exchange potentials

The one-pair and two-pair contributions are

$$\Omega_1^{(1)}(\mathbf{k}^2;t,u)=6\left(\frac{g_{(\pi\pi)_0}}{m_\pi}\right)\left(\frac{f_{NN\pi}^2}{m_\pi^2}\right)d_{\{2,2,0\}}(t,u)\times\left\{+\frac{3}{2}-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right\}\frac{1}{t+u}, \tag{4.3a}$$

$$\Omega_1^{(2)}(\mathbf{k}^2;t,u)=-3\left(\frac{g_{(\pi\pi)_0}}{m_\pi^2}\right)^2 d_{\{1,1,1\}}(t,u). \tag{4.3b}$$

B. $J^{PC}=1^{--}$: Adiabatic $(\pi\pi)_1$-exchange potentials

(i) One-pair exchange:

$$\Omega_1^{(1)}(\mathbf{k}^2;t,u)=-4(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\pi)_1}}{m_\pi^2}\right)\left(\frac{f_{NN\pi}^2}{m_\pi^2}\right)\times d_{1,1,1}(t,u)\left\{-\frac{3}{2}+\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right\}\frac{1}{t+u}, \tag{4.4a}$$

$$\Omega_2^{(1)}(\mathbf{k}^2;t,u)=-2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\pi)_1}}{m_\pi^2}\right)\left(\frac{f_{NN\pi}^2}{m_\pi^2}\right)\times\frac{(1+\kappa_1)}{M}d_{2,2,0}(t,u)+\frac{1}{3}\mathbf{k}^2\frac{1}{t+u}, \tag{4.4b}$$

$$\Omega_3^{(1)}(\mathbf{k}^2;t,u)=-2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\pi)_1}}{m_\pi^2}\right)\left(\frac{f_{NN\pi}^2}{m_\pi^2}\right)\frac{(1+\kappa_1)}{M}d_{2,2,0}(t,u)-\frac{1}{2}\frac{1}{t+u}, \tag{4.4c}$$

$$\Omega_4^{(1)}(\mathbf{k}^2;t,u)=-2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\pi)_1}}{m_\pi^2}\right)\left(\frac{f_{NN\pi}^2}{m_\pi^2}\right)\frac{1}{M}d_{2,2,0}(t,u)\frac{1}{t+u}. \tag{4.4d}$$

(ii) Two-pair exchange:

$$\Omega_1^{(2)}(\mathbf{k}^2;t,u)=-\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\pi)_1}}{m_\pi^2}\right)^2[d_{1,0,0}+d_{0,1,0}-4d_{0,0,1}](t,u). \tag{4.4e}$$

C. $J^{PC}=1^{++}$: Adiabatic $(\pi\rho)_1$-exchange potentials

(i) One-pair exchange:

$$\Omega_2^{(1)}(\mathbf{k}^2;t,u)=\frac{2}{M}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\rho)_1}}{m_\pi}\right)\left(\frac{f_{NN\pi}g_{NN\rho}}{m_\pi}\right)d_{2,2,0}(t,u)\left\{\left[\frac{1}{2}+\frac{1}{3}\left(\frac{u^2}{t+u}\right)\mathbf{k}^2\right]+\frac{1}{2}(1+\kappa_\rho)\left[2-\frac{4}{3}\frac{tu}{t+u}\mathbf{k}^2\right]\right\}\frac{1}{t+u}, \tag{4.5a}$$

$$\Omega_3^{(1)}(\mathbf{k}^2;t,u)=\frac{2}{M}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\rho)_1}}{m_\pi}\right)\left(\frac{f_{NN\pi}g_{NN\rho}}{m_\pi}\right)d_{2,2,0}(t,u)\left\{\frac{u^2}{t+u}+\frac{1}{2}(1+\kappa_\rho)\frac{2tu}{t+u}\right\}\frac{1}{t+u}. \tag{4.5b}$$

(ii) Two-pair exchange:

$$\Omega_2^{(2)}(\mathbf{k}^2;t,u)=-(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\rho)_1}}{m_\pi^2}\right)^2 d_{1,1,1}(t,u). \tag{4.5c}$$

D. $J^{PC}=1^{++}$: Adiabatic $(\pi\sigma)$-exchange potentials

(i) One-pair exchange:

$$\Omega_2^{(1)}(\mathbf{k}^2;t,u)=+(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\sigma)}}{m_\pi^2}\right)\left(\frac{f_{NN\pi}g_{NN\sigma}}{m_\pi}\right)d_{2,2,0}(t,u)\left[-2+\frac{2}{3}\left(\frac{tu-u^2}{t+u}\right)\mathbf{k}^2\right]\frac{1}{t+u}, \tag{4.6a}$$

$$\Omega_3^{(1)}(\mathbf{k}^2;t,u)=+2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\sigma)}}{m_\pi^2}\right)\left(\frac{f_{NN\pi}g_{NN\sigma}}{m_\pi}\right)d_{2,2,0}(t,u)\left(\frac{tu-u^2}{t+u}\right)\frac{1}{t+u}. \tag{4.6b}$$

(ii) Two-pair exchange:

$$\Omega_2^{(2)}(\mathbf{k}^2;t,u)=-\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\right)^2 d_{1,1,1}(t,u)\left\{\frac{2}{t+u}+\frac{1}{3}\left(\frac{t-u}{t+u}\right)^2\mathbf{k}^2\right\}, \tag{4.6c}$$

$$\Omega_3^{(2)}(\mathbf{k}^2;t,u)=-\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\right)^2 d_{1,1,1}(t,u)\left(\frac{t-u}{t+u}\right)^2. \tag{4.6d}$$

E. $J^{PC}=1^{+-}$: Adiabatic $(\pi\omega)$-exchange potentials

(i) One-pair exchange:

$$\Omega_2^{(1)}(\mathbf{k}^2;t,u)=(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\omega)}}{m_\pi}\right)\left(\frac{f_{NN\pi}g_{NN\omega}}{m_\pi}\right)d_{2,2,0}(t,u)\left[\frac{2}{3}\left(\frac{tu+u^2}{t+u}\right)\mathbf{k}^2\right]\frac{1}{t+u}, \tag{4.7a}$$

$$\Omega_3^{(1)}(\mathbf{k}^2;t,u)=+2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\omega)}}{m_\pi}\right)\left(\frac{f_{NN\pi}g_{NN\omega}}{m_\pi}\right)d_{2,2,0}(t,u)\left(\frac{tu+u^2}{t+u}\right)\frac{1}{t+u}, \tag{4.7b}$$

(ii) Two-pair exchange:

$$\Omega_2^{(2)}(\mathbf{k}^2;t,u)=-\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\omega)_1}}{m_\pi^2}\right)^2\left\{d_{1,0,0}+d_{0,1,0}-\frac{1}{3}\mathbf{k}^2 d_{1,1,1}\right\}(t,u), \tag{4.7c}$$

$$\Omega_3^{(2)}(\mathbf{k}^2;t,u)=+\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi\omega)_1}}{m_\pi^2}\right)^2 d_{1,1,1}(t,u). \tag{4.7d}$$

F. $J^{PC}=1^{++}$: Adiabatic (πP)-exchange potentials

The treatment of the Pomeron has been explained in [3]. This implies the use of $\widetilde{G}_0/M_N^2$ in Sec. II. Furthermore, with respect to σ exchange there is a $(-)$ sign for P exchange. Therefore, comparing to Eqs. (4.6a)–(4.6d) we obtain the following potentials.

(i) One-pair exchange:

$$\Omega_2^{(1)}(\mathbf{k}^2;t,u)=-(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi P)}}{m_\pi^2}\right)\left(\frac{f_{NN\pi}g_{NNP}}{m_\pi}\right)\frac{1}{M_N^2}d_{2,0,0}(t,u)\left[-2+\frac{2}{3}\left(\frac{tu-u^2}{t+u}\right)\mathbf{k}^2\right]\frac{1}{t+u}\,\delta(u-u_0), \tag{4.8a}$$

$$\Omega_3^{(1)}(\mathbf{k}^2;t,u)=-2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi P)}}{m_\pi^2}\right)\left(\frac{f_{NN\pi}g_{NNP}}{m_\pi}\right)\frac{1}{M_N^2}d_{2,0,0}(t,u)\left(\frac{tu-u^2}{t+u}\right)\frac{1}{t+u}\,\delta(u-u_0). \tag{4.8b}$$

(ii) Two-pair exchange:

$$\Omega_2^{(2)}(\mathbf{k}^2;t,u)=+\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi P)_1}}{m_\pi}\right)^2\frac{1}{M_N^2}d_{1,1,1}(t,u)\left\{\frac{2}{t+u}+\frac{1}{3}\left(\frac{t-u}{t+u}\right)^2\mathbf{k}^2\right\}\delta(u-u_0), \tag{4.8c}$$

$$\Omega_3^{(2)}(\mathbf{k}^2;t,u)=+\frac{1}{2}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g_{(\pi P)_1}}{m_\pi^2}\right)^2\frac{1}{M_N^2}d_{1,1,1}(t,u)\left(\frac{t-u}{t+u}\right)^2\delta(u-u_0). \tag{4.8d}$$

Notice that in Eqs. (4.8a)–(4.8d), $u_0=1/4m_P^2$.

G. $J^{PC}=0^{++}$: Adiabatic "derivative" $(\pi\pi)_0$-exchange potentials

The derivative-pair potentials in coordinate space have been derived in [12] in detail. A summary of this is given in appendix D. A short derivation of the p-space potentials is also can be found there.

(i) One-pair exchange:

$$\Omega_1^{(1)}(\mathbf{k}^2;t,u)=-12\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2 d_{2,2,0}(t,u)\left[\frac{15}{4}+\frac{1}{2}\,\frac{t^2-8tu+u^2}{t+u}\mathbf{k}^2+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right]\frac{1}{(t+u)^2}. \tag{4.9a}$$

(ii) Two-pair exchange:

$$\Omega_1^{(2)}(\mathbf{k}^2;t,u)=-6\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)^2\Bigg\{\left[\frac{15}{4}+\frac{t^2-3tu+u^2}{t+u}\mathbf{k}^2+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right]\frac{d_{1,1,1}(t,u)}{(t+u)^2}$$
$$+\frac{1}{2}\left[\frac{3}{2}(m_1^2+m_2^2)+\frac{m_1^2t+m_2^2u}{t+u}\mathbf{k}^2+m_1^2m_2^2(t+u)\right]\frac{d_{1,1,1}(t,u)}{t+u}+\left[\frac{3}{2}-\frac{tu}{t+u}\mathbf{k}^2\right]\frac{d_{0,0,1}(t,u)}{t+u}\Bigg\}. \tag{4.9b}$$

H. $J^{PC}=0^{++}$: Adiabatic $(\sigma\sigma)$-exchange potentials

(i) One-pair exchange:

$$\Omega_1^{(1)}(\mathbf{k}^2;t,u)=2\left(\frac{g_{(\sigma\sigma)}}{m_\pi}\right)g_{NN\sigma}^2 d_{2,2,0}(t,u). \tag{4.10a}$$

(ii) Two-pair exchange:

$$\Omega_1^{(2)}(\mathbf{k}^2;t,u)=-\left(\frac{g_{(\sigma\sigma)}}{m_\pi}\right)^2 d_{1,1,1}(t,u). \tag{4.10b}$$

V. PROJECTION MPE ON SPINOR INVARIANTS II: 1/M CORRECTIONS

The nonadiabatic and pseudovector-vertex corrections have been given in [4], Sec. IV. Similar to Eq. (4.1) we write these contributions in the form

$$\tilde{V}^{(na,pv)}_{\rm pair}(\alpha\beta)=\int_{t_0}^{\infty}dt\int_{u_0}^{\infty}du\, w_0(t,u)\left\{d^{(n)}_{\{p_1,p_2,p_3\}}(t,u)\exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right]\Omega^{(n)}_j(\mathbf{k}^2;t,u)\right\}. \tag{5.1}$$

A. Nonadiabatic corrections

From Eqs. (4.5)–(4.8) of [4] one readily obtains the momentum-space equivalents using the replacements

$$\int\int\frac{d^3k_1d^3k_2}{(2\pi)^6}e^{i(\mathbf{k}_1+\mathbf{k}_2)}\rightarrow\int\int\frac{d^3k_1d^3k_2}{(2\pi)^3}\,\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2).$$

Then, by comparison one can easily read off the diverse quantities $\tilde{O}^{(na)}_{\alpha\beta,p}$ and $D^{(na)}_p(\omega_1,\omega_2)$ that occur in Eq. (3.1) for the nonadiabatic potentials. The projections onto the $\Omega^{(na)}_j$ in Eq. (5.1) yield the following.

(i) $(\pi\pi)_0$:

$$\Omega^{(na)}_1(\mathbf{k}^2;t,u)=-\frac{g_{(\pi\pi)_0}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{3}{M}d_{\{na\}}(t,u)\left\{\frac{15}{4}++\frac{1}{2}\left(\frac{t^2-8ut+u^2}{t+u}\right)\mathbf{k}^2+\left(\frac{t^2u^2}{(t+u)^2}\right)\mathbf{k}^4\right\}\frac{1}{(t+u)^2}, \tag{5.2a}$$

$$\Omega^{(na)}_4(\mathbf{k}^2;t,u)=-\frac{g_{(\pi\pi)_0}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{3}{M}d_{\{na\}}(t,u)\frac{1}{t+u}. \tag{5.2b}$$

(ii) $(\pi\pi)_1$:

$$\Omega^{(na)}_1(\mathbf{k}^2;t,u)=-2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\pi)_1}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{M}d_{\{2,2,0\}}(t,u)\left\{\frac{15}{4}+\frac{1}{2}\left(\frac{t^2-8ut+u^2}{t+u}\right)\mathbf{k}^2+\left(\frac{t^2u^2}{(t+u)^2}\right)\mathbf{k}^4\right\}\frac{1}{(t+u)^2}, \tag{5.3a}$$

$$\Omega^{(na)}_4(\mathbf{k}^2;t,u)=-2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\pi)_1}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{M}d_{\{2,2,0\}}(t,u)\frac{1}{t+u}. \tag{5.3b}$$

(iii) $(\sigma\sigma)$:

$$\Omega^{(na)}_1(\mathbf{k}^2;t,u)=\frac{g_{(\sigma\sigma)}}{m_\pi}\frac{g^2_{NN\sigma}}{M}d_{\{na\}}(t,u)\left\{-\frac{3}{2}+\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right\}\frac{1}{t+u}. \tag{5.4}$$

(iv) $(\pi\sigma)$:

$$\Omega^{(na)}_2(\mathbf{k}^2;t,u)=+(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\sigma)_1}}{m^2_\pi}\frac{f_{NN\pi}}{m_\pi}\frac{g_{NN\sigma}}{M}d_{\{na\}}(t,u)\left\{\frac{5}{2}+\frac{1}{6}\frac{t^2-13tu+6u^2}{t+u}\mathbf{k}^2+\frac{1}{3}\frac{tu^2(t-u)}{(t+u)^2}\mathbf{k}^4\right\}\frac{1}{(t+u)^2}, \tag{5.5a}$$

$$\Omega^{(na)}_3(\mathbf{k}^2;t,u)=+(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\sigma)_1}}{m^2_\pi}\frac{f_{NN\pi}}{m_\pi}\frac{g_{NN\sigma}}{M}d_{\{na\}}(t,u)\left\{\frac{1}{2}\frac{t^2-7tu+6u^2}{t+u}+\frac{tu^2(t-u)}{(t+u)^2}\mathbf{k}^2\right\}\frac{1}{(t+u)^2}. \tag{5.5b}$$

(v) $(\pi\pi)_0$ (“derivative”):

$$\Omega^{(na)}_1(\mathbf{k}^2;t,u)=-12\left(\frac{g'_{(\pi\pi)_0}}{m^3_\pi}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}\left\{+\left[\frac{15}{4}+\frac{1}{2}\left(\frac{t^2-8tu+u^2}{t+u}\right)\mathbf{k}^2+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right]\frac{d_{1,1,1}(t,u)}{(t+u)^2}\right.$$
$$\left.-\left[\frac{105}{8}+\frac{15}{4}\left(\frac{t^2-5tu+u^2}{t+u}\right)\mathbf{k}^2-\frac{3}{2}tu\left(\frac{t^2-5tu+u^2}{(t+u)^2}\right)\mathbf{k}^4-\frac{t^3u^3}{(t+u)^3}\mathbf{k}^6\right]\frac{d_{na}(t,u)}{(t+u)^3}\right\}, \tag{5.6a}$$

$$\Omega^{(na)}_4(\mathbf{k}^2;t,u)=-12\left(\frac{g'_{(\pi\pi)_0}}{m^3_\pi}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}\left\{\frac{d_{1,1,1}(t,u)}{t+u}+\left[-5+2\frac{tu}{t+u}\mathbf{k}^2\right]\frac{d_{na}(t,u)}{(t+u)^2}\right\}. \tag{5.6b}$$

Here, $d_{\{na\}}(t,u)$ is defined in Eq. (B3).

B. Pseudovector-vertex corrections

From Eqs. (4.9)–(4.11) of [4] likewise as in the case of the nonadiabatic corrections one obtains for the pseudovector-vertex corrections.

(i) $(\pi\pi)_0$:

$$\Omega_1^{(pv)}(\mathbf{k}^2;t,u) = +\frac{g_{(\pi\pi)_0}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{3}{M}d_{\{1,1,1\}}(t,u)\left\{3+\left(\frac{t^2+u^2}{t+u}\right)\mathbf{k}^2\right\}\frac{1}{t+u}, \tag{5.7a}$$

$$\Omega_4^{(pv)}(\mathbf{k}^2;t,u) = +2\frac{g_{(\pi\pi)_0}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{3}{M}d_{\{1,1,1\}}(t,u). \tag{5.7b}$$

(ii) $(\pi\pi)_1$:

$$\Omega_1^{(pv)}(\mathbf{k}^2;t,u) = +(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\pi)_1}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{M}\left[\left(\frac{3}{2}+\frac{u^2}{t+u}\mathbf{k}^2\right)d_{\{2,0,0\}}(t,u)+\left(\frac{3}{2}+\frac{t^2}{t+u}\mathbf{k}^2\right)d_{\{0,2,0\}}(t,u)\right]\frac{1}{t+u}, \tag{5.8a}$$

$$\Omega_4^{(pv)}(\mathbf{k}^2;t,u) = +2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\pi)_1}}{m_\pi}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{M}\left[\frac{u}{t+u}d_{\{2,0,0\}}+\frac{t}{t+u}d_{\{0,2,0\}}\right]. \tag{5.8b}$$

(iii) $(\pi\sigma)$:

$$\Omega_2^{(pv)}(\mathbf{k}^2;t,u) = -(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\frac{f_{NN\pi}}{m_\pi}\frac{g_{NN\sigma}}{M}d_{\{1,1,1\}}(t,u)\left\{1+\frac{1}{3}\frac{t^2-tu}{t+u}\mathbf{k}^2\right\}\frac{1}{t+u}. \tag{5.9a}$$

$$\Omega_3^{(pv)}(\mathbf{k}^2;t,u) = -(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\frac{f_{NN\pi}}{m_\pi}\frac{g_{NN\sigma}}{M}d_{\{1,1,1\}}(t,u)\left(\frac{t^2-tu}{t+u}\right)\frac{1}{t+u}. \tag{5.9b}$$

(iv) $(\pi\pi)_0$ (“derivative”):

$$\Omega_1^{(pv)}(\mathbf{k}^2;t,u) = -6\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}d_{1,1,1}(t,u)\left\{(m_1^2-m_2^2)^2-\left[3(m_1^2+m_2^2)+\frac{m_1^2(3t^2-u^2)+m_2^2(3u^2-t^2)}{t+u}\mathbf{k}^2\right]\frac{1}{t+u}\right.$$
$$\left.-\left[\left(\frac{t^2+2tu+u^2}{t+u}\right)\mathbf{k}^2+2tu\left(\frac{t^2+2tu+u^2}{(t+u)^2}\right)\mathbf{k}^4\right]\frac{1}{(t+u)^2}\right\}, \tag{5.10a}$$

$$\Omega_4^{(pv)}(\mathbf{k}^2;t,u) = -24\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}d_{1,1,1}(t,u)\left\{(m_1^2+m_2^2)+\left[\frac{3}{2}+\left(\frac{t^2+tu+u^2}{t+u}\right)\mathbf{k}^2\right]\frac{1}{t+u}\right\}. \tag{5.10b}$$

The coefficients $\Upsilon_{j,k}^{na,pv}$ defined in Eq. (4.2) are tabulated in Tables V–IX.

For (πP) exchange, the $1/M_N$ nonadiabatic and pseudovector-vertex corrections can be read off from those for $(\pi\sigma)$ and making the same adjustments as given already for the adiabatic contributions. In Table X the $\Omega_i^{(pv,ad)}$ for (πP)-pair exchange are given explicitly.

VI. PARTIAL WAVE ANALYSIS

Like the TME potentials in I, the general form of the MPE potentials in momentum space is

$$\tilde{V}_j^{(n)}(\mathbf{k}) = \int_{t_0}^\infty dt\int_{u_0}^\infty du\left\{w_{\{p_1,p_2,p_3\}}^{(n)}(t,u)\right.$$
$$\left.\times\exp\left[-\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right]\Omega_j^{(n)}(\mathbf{k}^2;t,u)\right\}, \tag{6.1}$$

where

$$w_{\{p_1,p_2,p_3\}}^{(n)}(t,u) = w_0(t,u)d_{\{p_1,p_2,p_3\}}^{(n)}(t,u).$$

TABLE X. Coefficients $\Upsilon_{j,k}^{(ad,na,pv)}$ for the $(\pi P)_1$ contributions. These coefficients have to be multiplied by a factor $-\delta(u-u_0)/M_N^2$.

	$\Upsilon_0(\|)(t,u)$	$\Upsilon_1(\|)(t,u)$	$\Upsilon_2(\|)(t,u)$
	One-pair exchange		
$\Omega_2^{(1),ad}$	$-\frac{2}{t+u}$	$+\frac{2}{3}\frac{tu-u^2}{(t+u)^2}$	—
$\Omega_3^{(1),ad}$	$+2\frac{tu-u^2}{(t+u)^2}$	—	—
$\Omega_2^{(1),na}$	$+\frac{5}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}\frac{1}{M}$	$+\frac{1}{3\sqrt{\pi}}\frac{t^2-13tu+6u^2}{(t+u)^{5/2}}\frac{1}{M}$	$+\frac{2}{3\sqrt{\pi}}\frac{tu^2(t-u)}{(t+u)^{7/2}}\frac{1}{M}$
$\Omega_3^{(1),na}$	$+\frac{1}{\sqrt{\pi}}\frac{t^2-7tu+6u^2}{(t+u)^{5/2}}\frac{1}{M}$	$+\frac{2}{\sqrt{\pi}}\frac{tu^2(t-u)}{(t+u)^{7/2}}\frac{1}{M}$	—
$\Omega_2^{(1),pv}$	$-\frac{1}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}\frac{1}{M}$	$-\frac{1}{3\sqrt{\pi}}\frac{t^2-tu}{(t+u)^{5/2}}\frac{1}{M}$	—
$\Omega_3^{(1),pv}$	$-\frac{1}{\sqrt{\pi}}\frac{t^2-tu}{(t+u)^{5/2}}\frac{1}{M}$	—	—
	Two-pair exchange		
$\Omega_2^{(2),ad}$	$-\frac{1}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}$	$-\frac{1}{6\sqrt{\pi}}\frac{(t-u)^2}{(t+u)^{5/2}}$	—
—	—		
$\Omega_3^{(2),ad}$	$-\frac{1}{2\sqrt{\pi}}\frac{(t-u)^2}{(t+u)^{5/2}}$	—	—

Therefore, the partial wave (PW) analysis runs along the-same lines as described in Secs. VI and VII of paper I for the TME potentials. We refer the reader to this paper for details.

VII. ESC MODEL: RESULTS

The momentum-space formulas for the potentials of this paper and paper I have been checked numerically. This is done by solving the Lippmann-Schwinger equation and comparing the phase shifts with those obtained by solving the Schrödinger equation using the *x*-space equivalent of the potentials. The agreement reached was of the order of one-hundredth of a degree.

After the completion of the *p*-space formalism we performed a χ^2 fit with the ESC model to the 1993 Nijmegen representation of the χ^2 hypersurface of the *NN* scattering data below $T_{lab}=350$ MeV [15].

This fitting was executed in *x* space using the equivalent *x*-space potentials. The reason for this is the much faster evaluation of the ESC model in *x* space. We obtained a $\chi^2/N_{data}=1.15$. The phase shifts are shown in Figs 2–5. In Table XI the results are shown for the ten energy bins, where we compare the results from the updated partial wave analysis with the ESC potentials.

In Table XII we show the OBE coupling constants and the Gaussian cutoffs Λ. The $\alpha\equiv F/(F+D)$ ratios used for the OBE couplings are pseudoscalar mesons $\alpha_{pv}=0.355$, vector mesons $\alpha_V^e=1.0$, $\alpha_V^m=0.275$, and scalar mesons α_S

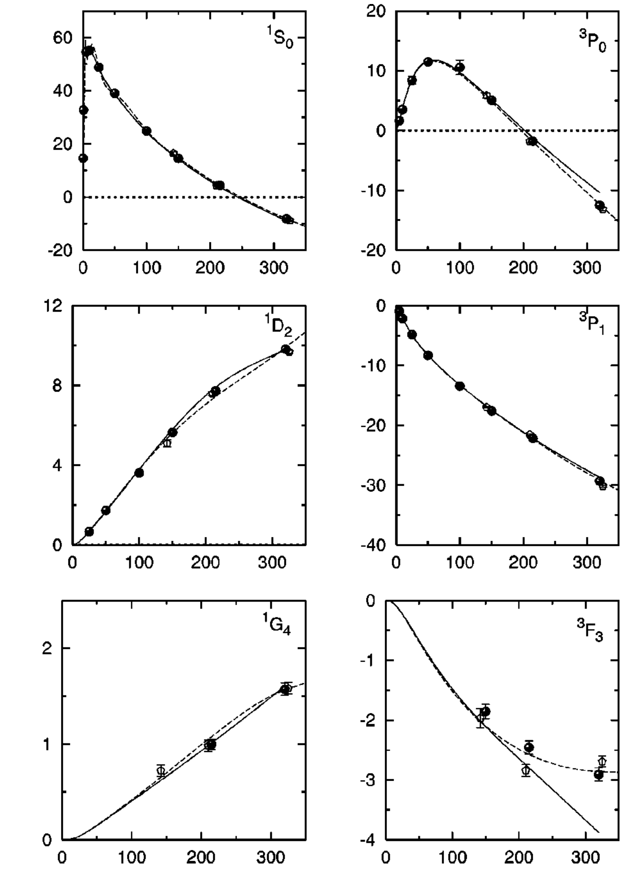

FIG. 2. Solid line: proton-proton $I=1$ phase shifts (degrees), as a function of $T_{\rm lab}$ (MeV), for the ESC model. The dashed line: the multienergy phases of the Nijmegen93 PW analysis [15]. The black dots: the single-energy phases of the Nijmegen93 PW analysis. The diamonds: Bugg single energy [16].

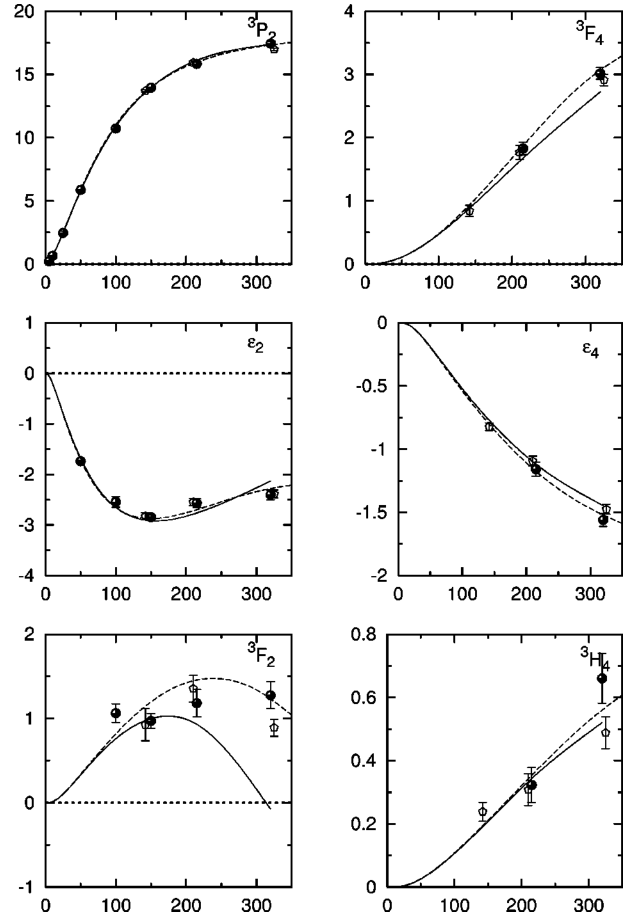

FIG. 3. Solid line: proton-proton $I=1$ phase shifts (degrees), as a function of $T_{\rm lab}$ (MeV), for the ESC model. The dashed line: the multienergy phases of the Nijmegen93 PW analysis [15]. The black dots: the single energy phases of the Nijmegen93 PW analysis. The diamonds: Bugg single energy [16].

$=0.914$, which is computed using the physical $S^*\equiv f_0(993)$ coupling, etc. In Table XIII we show the MPE coupling constants. The $F/(F+D)$ ratios which we used for the MPE couplings are $(\pi\eta)$, etc., and $(\pi\omega)$ pairs $\alpha(\{8_s\})=1.0$, $(\pi\pi)_1$, etc., pairs $\alpha_V^e(\{8\}_a)=0.4$, $\alpha_V^m(\{8\}_a)=0.335$, $(\pi\rho)_1$, etc., pairs $\alpha_A(\{8\}_a)=0.335$.

We emphasize that we use the single-energy (SE) phases and χ^2 surface [17] only as a means to fit the NN data. As stressed in [15] the Nijmegen SE phases have not much significance. The significant phases are the multienergy (ME) ones; see the dashed lines in the figures. One notices that the central values of the SE phases do not correspond to the ME phases in general, illustrating that there has been a certain amount of noise fitting in the SE PW analysis; see, e.g., ϵ_1 and $^1P_1$ at $T_{lab}=100$ MeV. The ME PW analysis reaches $\chi^2/N_{data}=0.99$, using 39 phenomenological parameters plus normalization parameters and the related phenomenological PW potentials NijmI,II and Reid93 [18], with, respectively, 41, 47, and 50 parameters, all with $\chi^2/N_{data}=1.03$. This should be compared to the ESC model, which has $\chi^2/N_{data}=1.15$ using 20 parameters. These are nine meson-nucleon-nucleon couplings, eight meson-pair-nucleon-nucleon couplings, and three Gaussian cutoff parameters. From the figures it is obvious that the ESC model deviates from the ME PW analysis at the highest energy in particular. If we evaluate the χ^2 for the first nine energies only, we obtain $\chi^2/N_{data}=1.10$.

We mentioned that we do not include negative-energy state contributions. It is assumed that a strong pair suppression is operative at low energies in view of the composite nature of the nucleons. This leaves for us the pseudoscalar mesons with two essential equivalent interactions: the direct and derivative ones. In expanding the $NN\pi$, etc., vertex in $1/M_N$ these two interactions differ in the $1/M_N^2$ terms; see [3], Eqs. (3.4) and (3.5). Here, we prefer to cancel these $1/M_N^2$ terms by taking

$$\mathcal{H}_{ps}=\frac{1}{2}[g_{NN\pi}\overline{\psi}i\gamma_5\boldsymbol{\tau}\psi\cdot\boldsymbol{\pi}+(f_{NN\pi}/m_\pi)\gamma_\mu\gamma_5\boldsymbol{\tau}\psi\cdot\partial^\mu\boldsymbol{\pi}], \tag{7.1}$$

where $g_{NN\pi}=(2M_N/m_\pi)f_{NN\pi}$.

As for the OBE couplings, one notices that $G_E=g_{\rho NN}$ is small (see [20]), but $G_M=g_\rho+f_\rho$ is okay. One possible explanation would be that part of the ρ exchange is replaced by the two-pair $(\pi\pi)_1$ exchange, which has identical quantum numbers. This still leaves room for the interpretation of the one-pair $(\pi\pi)_1$ exchange as a form factor correction. An-

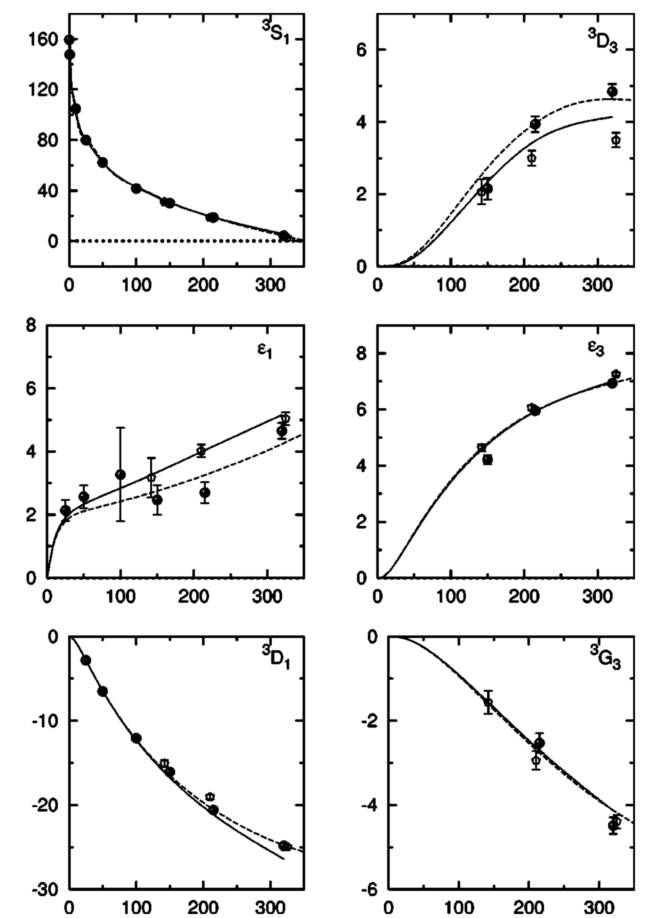

FIG. 4. Solid line: neutron-proton $I=0$ phase shifts (degrees), as a function of $T_{\rm lab}$ (MeV), for the ESC model. The dashed line: the multienergy phases of the Nijmegen93 PW analysis [15]. The black dots: the single-energy phases of the Nijmegen93 PW analysis. The diamonds: Bugg single energy [16].

other interesting possibility is that leaving out the tensor mesons $a_2(1320)$, $f_2(1270)$, and $f_2(1520)$ affects the vector-meson couplings. This can be seen as follows. At high energies and low to moderate momentum transfer there is a strong cancellation between the vector and tensor exchange: $(\rho-a_2)$- and $(\omega-f_2)$ cancellation [19]. This is called exchange degeneracy (EXD). Indeed, by changing $g_\rho/\sqrt{4\pi}=0.3$ to $g_\rho=0.75/\sqrt{4\pi}$ one can cancel the change in the ρ-exchange potential by the inclusion of a_2 exchange rather completely. The inclusion of mesons with a mass $\geqslant 1$ GeV $/c^2$, like the axial and tensor mesons, we leave as a future project.

Unlike in [3,4], we did not fix the pair couplings using a theoretical model based on heavy-meson saturation and chiral symmetry. So in addition to the 14 parameters used in [3,4] we now have eight pair coupling fit parameters. In Table XIII the fitted pair couplings are given. Note that the $(\pi\pi)_0$-pair coupling gets contributions from the $\{1\}$ and the $\{8_s\}$ pairs as well, giving in total $g_{(\pi\pi)}=0.10$, which has the same sign as in [4]. The $f_{(\pi\pi)_1}$-pair coupling has opposite sign as compared to [4]. In a model with a more complex and realistic meson dynamics [9] this coupling is predicted as found in the present ESC-fit. The $(\pi\rho)_1$ coupling agrees

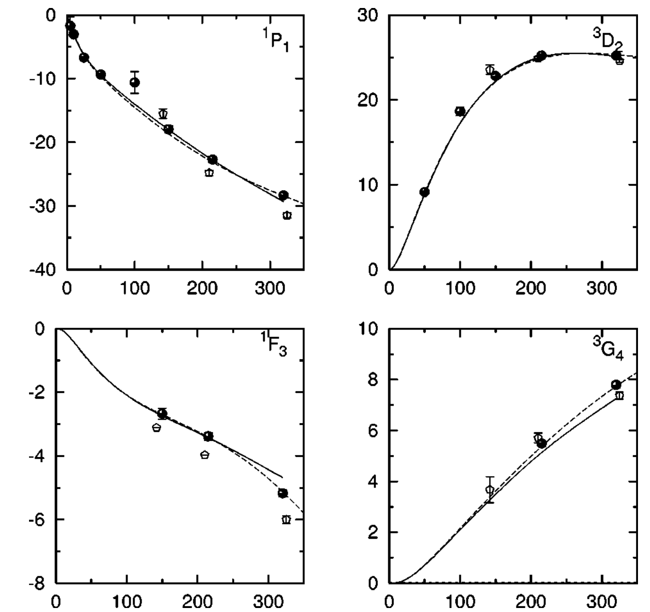

FIG. 5. Solid line: neutron-proton $I=0$ phase shifts (degrees), as a function of $T_{\rm lab}$ (MeV), for the ESC model. The dashed line: the multienergy phases of the Nijmegen93 PW analysis [15]. The black dots: the single-energy phases of the Nijmegen93 PW analysis. The diamonds: Bugg single energy [16].

nicely with the A_1 saturation; see [4]. We conclude that the pair couplings are in general not well understood and deserve more study.

The ESC model described here is fully consistent with SU(3) symmetry. In Appendix E we display the full SU(3) contents of the pair interaction Hamiltonians. For example, $g_{(\pi\rho)_1}=g_{A_8VP}$, and besides $(\pi\rho)$ pairs one sees also that $KK^*(I=1)$ and $KK^*(I=0)$ pairs contribute to the *NN* potentials. All $F/(F+D)$ ratios are taken fixed with heavy-meson saturation in mind. The approximation we have made in this paper is to neglect the baryon mass differences; i.e., we put $m_\Lambda=m_\Sigma=m_N$. This because we have not yet worked out the formulas for the inclusion of these mass differences, which is straightforward in principle.

VIII. CONCLUSIONS AND OUTLOOK

The ESC model presented is very successful and flexible in describing the *NN* data. It can be developed and extended in various ways. First, we plan to extend the OBE potentials in momentum space by including the full OBE propagator, i.e.,

$$\frac{1}{\omega^2}\rightarrow\frac{1}{\omega(\omega+a)},\qquad a=\frac{1}{M}[p_f^2+p_i^2-2p_0^2]. \tag{8.1}$$

This includes retardation at the level of the OBE potentials. Second, one may extend the TME potentials including besides PS-PS also the PS-vector, PS-Scalar, etc., potentials. Third, we may include the axial and tensor mesons, which we discussed in connection with EXD.

TABLE XI. χ^2 and χ^2 per datum at the ten energy bins for the Nijmegen93 partial wave analysis. N_{data} lists the number of data within each energy bin. The bottom line gives the results for the total 0–350 MeV interval. The χ^2 access for the ESC model is denoted by $\Delta\chi^2$ and $\Delta\hat{\chi}^2$, respectively.

$T_{\rm lab}$	No. data	χ_0^2	$\Delta\chi^2$	$\hat{\chi}_0^2$	$\Delta\hat{\chi}^2$
0.383	144	137.5549	21.3	0.960	0.148
1	68	38.0187	55.7	0.560	0.819
5	103	82.2257	13.0	0.800	0.127
10	209	257.9946	78.1	1.234	0.269
25	352	272.1971	44.3	0.773	0.126
50	572	547.6727	137.4	0.957	0.240
100	399	382.4493	27.6	0.959	0.069
150	676	673.0548	82.9	0.996	0.123
215	756	754.5248	108.0	0.998	0.143
320	954	945.3772	305.0	0.991	0.320
Total	4233	4091.122	864.2	0.948	0.201

The momentum-space formulation of the ESC model also suggests a covariant formulation. Consider an effective field theory and suppose that it allows Wick rotation. Then, assuming in Euclidean space a Gaussian cutoff, one can use a representation completely akin to Eq. (2.3), etc. For example, this opens the way to analyze the expansion in loops in the presence of a strong cutoff. Also, one could evaluate the ESC model using the Bethe-Salpeter equation.

The ESC model presented can be applied in various ways: (i) the study of few-body systems in momentum space, (ii) the study of meson-exchange-current (MEC) corrections, (iii) the derivation of three-body forces consistent with the two-body forces, and (iv) G-matrix, etc., description of nuclear matter.

APPENDIX A: PAIR INTERACTION HAMILTONIANS

The pair Hamiltonians are

TABLE XII. Meson parameters of the fitted ESC model. Phases are shown in Figs. 2–5. Coupling constants are at $\mathbf{k}^2=0$. An asterisk denotes that the coupling constant is not searched, but constrained via SU(3) or simply put to some value used in previous work.

Meson	Mass (MeV)	$g/\sqrt{4\pi}$	$f/\sqrt{4\pi}$	Λ (MeV)
π	138.04		0.2663	950.69
η	547.45		0.1461*	950.69
η'	957.75		0.1789*	950.69
ρ	768.10	0.2700	3.6378	688.20
ϕ	1019.41	–1.4717*	0.0149*	688.20
ω	781.95	2.6862	0.3255	688.20
a_0	982.70	0.9851		734.25
f_0	974.10	–0.7998		734.25
ε	760.00	3.7554		734.25
A_2	309.10	–0.4317		
Pomeron	309.10	2.5514		

$$J^{PC}=0^{++}:\mathcal{H}_S=(\overline{\psi'}\psi')\{g_{(\pi\pi)_0}(\boldsymbol{\pi}\cdot\boldsymbol{\pi})+g_{\sigma\sigma}\sigma^2\}/m_\pi + g'_{(\pi\pi)_0}(\overline{\psi'}\psi)(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})/m_\pi^3, \quad \text{(A1a)}$$

$$J^{PC}=1^{--}:\mathcal{H}_V=\left[g_{(\pi\pi)_1}\overline{\psi'}\gamma_\mu\boldsymbol{\tau}\psi'-\frac{f_{(\pi\pi)_1}}{2M}\overline{\psi'}\sigma_{\mu\nu}\boldsymbol{\tau}\psi'\partial^\nu\right]\times(\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi})/m_\pi^2, \quad \text{(A1b)}$$

$$J^{PC}=1^{++}:\mathcal{H}_A=g_{(\pi\rho)_1}\overline{\psi'}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi'(\boldsymbol{\pi}\times\boldsymbol{\rho}^\mu)/m_\pi + g_{(\pi\sigma)}\overline{\psi'}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi'(\sigma\partial^\mu\boldsymbol{\pi}-\boldsymbol{\pi}\partial^\mu\sigma)/m_\pi^2, \quad \text{(A1c)}$$

$$J^{PC}=1^{+-}:\mathcal{H}_B=g_{(\pi\rho)_0}\overline{\psi'}\sigma_{\mu\nu}\gamma_5\psi'\partial^\nu(\boldsymbol{\pi}\cdot\boldsymbol{\rho})/m_\pi^2 + g_{\pi\omega}\overline{\psi'}\sigma_{\mu\nu}\gamma_5\boldsymbol{\tau}\psi'\partial^\nu(\boldsymbol{\pi}\cdot\omega^\mu)/m_\pi^2. \quad \text{(A1d)}$$

APPENDIX B: λ REPRESENTATIONS

The following λ representations [11] are exploited:

$$D_{1,0,0}(\omega_1,\omega_2)=\frac{1}{\omega_1}=\frac{2}{\pi}\int_0^\infty\frac{d\lambda}{\omega_1^2+\lambda^2}, \quad \text{(B1a)}$$

$$D_{0,1,0}(\omega_1,\omega_2)=\frac{1}{\omega_2}=\frac{2}{\pi}\int_0^\infty\frac{d\lambda}{\omega_2^2+\lambda^2}, \quad \text{(B1b)}$$

$$D_{0,0,1}(\omega_1,\omega_2)=\frac{1}{\omega_1+\omega_2}=\frac{2}{\pi}\int_0^\infty\frac{\lambda^2 d\lambda}{(\omega_1^2+\lambda^2)(\omega_2^2+\lambda^2)}, \quad \text{(B1c)}$$

$$D_{1,1,1}(\omega_1,\omega_2)=\frac{1}{\omega_1\omega_2(\omega_1+\omega_2)} =\frac{2}{\pi}\int_0^\infty\frac{d\lambda}{(\omega_1^2+\lambda^2)(\omega_2^2+\lambda^2)}. \quad \text{(B1d)}$$

A special combination occurs in nonadiabatic terms. Here (see Table III) occurs

$$D_{na}^{(1)}(\omega_1,\omega_2)=\frac{1}{\omega_1^2\omega_2^2}\left[\frac{1}{\omega_1}+\frac{1}{\omega_2}-\frac{1}{\omega_1+\omega_2}\right] =\frac{2}{\pi}\int_0^\infty\frac{d\lambda}{\lambda^2}\left[\frac{1}{\omega_1^2\omega_2^2}-\frac{1}{(\omega_1^2+\lambda^2)(\omega_2^2+\lambda^2)}\right]. \quad \text{(B2)}$$

Notice that the denominator $D_{na}^{(1)}=2D_\parallel$; see [3]. The corresponding $d_{na}(t,u)$ is (see paper I, Sec. IV A)

$$d_{na}(t,u)=\frac{2}{\pi}\int_0^\infty\frac{d\lambda}{\lambda^2}[1-e^{-(t+u)\lambda^2}]=\frac{2}{\sqrt{\pi}}\sqrt{t+u}. \quad \text{(B3)}$$

TABLE XIII. Pair-meson coupling constants employed in the MPE potentials. Coupling constants are at $\mathbf{k}^2=0$. An asterisk denotes that the coupling constant is set to zero.

J^{PC}	SU(3) irrep	$(\alpha\beta)$	$g/4\pi$	$f/4\pi$
0^{++}	$\{1\}$	$(\pi\pi)_0$	0.1567	
0^{++}	$\{1\}$	$(\sigma\sigma)$	0*	
0^{++}	$\{8\}_s$	$(\pi\eta)$	−0.2946	
0^{++}		$(\pi\eta')$	0*	
1^{--}	$\{8\}_a$	$(\pi\pi)_1$	0.1093	−0.2050
1^{++}	$\{8\}_a$	$(\pi\rho)_1$	0.6950	
1^{++}	$\{8\}_a$	$(\pi\sigma)$	0.0140	
1^{++}	$\{8\}_a$	(πP)	−0.1604	
1^{+-}	$\{8\}_s$	$(\pi\omega)$	−0.1081	

APPENDIX C: INTEGRATION DICTIONARY

In this appendix we give a dictionary for the evaluation of the momentum integrals that occur in the matrix elements of the TME potentials. The results of the $d^3\Delta$ integration are given apart from a factor $(4\pi a)^{-3/2}$ $(a=t+u)$, common to all integrals. Using the results given in Appendix B of paper I, one obtains the following.

(i) For the operators $\tilde{O}^{(1)}_{\alpha\beta,p}$ and the operators $\tilde{O}^{(2)}_{\alpha\beta,p}$:

(a) $$(\mathbf{k}_1\cdot\mathbf{k}_2)=\mathbf{\Delta}\cdot\mathbf{k}-\mathbf{\Delta}^2\Rightarrow\frac{1}{2}\left\{-3+2\left(\frac{tu}{t+u}\right)\mathbf{k}^2\right\}\frac{1}{t+u}, \tag{C1a}$$

(b) $$[\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\times\mathbf{k}_2][\boldsymbol{\sigma}_2\cdot\mathbf{k}_1\times\mathbf{k}_2]$$
$$=[\boldsymbol{\sigma}_1\cdot\mathbf{\Delta}\times\mathbf{k}][\boldsymbol{\sigma}_2\cdot\mathbf{\Delta}\times\mathbf{k}]$$
$$\Rightarrow\frac{1}{2}\left\{\frac{2}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathbf{k}^2\right.$$
$$\left.-\left[(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})-\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathbf{k}^2\right]\right\}\frac{1}{t+u}, \tag{C1b}$$

(c) $$[(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{k}_1\times\mathbf{k}_2]\mathbf{q}\cdot(\mathbf{k}_1-\mathbf{k}_2)$$
$$=[(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{\Delta}\times\mathbf{k}]\mathbf{q}\cdot(2\Delta-\mathbf{k})$$
$$\Rightarrow[(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\mathbf{q}\times\mathbf{k}]\frac{1}{t+u}, \tag{C1c}$$

(d) $$(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)+(\boldsymbol{\sigma}_1\cdot\mathbf{k}_2\boldsymbol{\sigma}_2\cdot\mathbf{k}_1)$$
$$\Rightarrow-\left\{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2-\frac{2tu}{t+u}(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})\right\}\frac{1}{t+u}, \tag{C1d}$$

(e) $$(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_1)$$
$$=(\boldsymbol{\sigma}_1\cdot\mathbf{\Delta})(\boldsymbol{\sigma}_2\cdot\mathbf{\Delta})$$
$$\Rightarrow\frac{1}{2}\left\{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\frac{2u^2}{t+u}(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})\right\}\frac{1}{t+u}, \tag{C1e}$$

(f) $$\boldsymbol{\sigma}_1\cdot(\mathbf{k}_1-\mathbf{k}_2)\boldsymbol{\sigma}_2\cdot(\mathbf{k}_1-\mathbf{k}_2)$$
$$=\boldsymbol{\sigma}_1\cdot(2\mathbf{\Delta}-\mathbf{k})\boldsymbol{\sigma}_2\cdot(2\mathbf{\Delta}-\mathbf{k})$$
$$\Rightarrow\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\left\{\frac{2}{t+u}+\frac{1}{3}\left(\frac{t-u}{t+u}\right)^2\mathbf{k}^2\right\}$$
$$+\left(\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k}-\frac{1}{3}\mathbf{k}^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)\left(\frac{t-u}{t+u}\right)^2. \tag{C1f}$$

(ii) For the $1/M$ correction operators $\tilde{O}^{(na)}_{\alpha\beta}$, etc., not included in the list (C1a)–(C1f):

(a) $$(\mathbf{k}_1\cdot\mathbf{k}_2)^2=(\mathbf{\Delta}\cdot\mathbf{k}-\mathbf{\Delta}^2)^2$$
$$\Rightarrow\frac{1}{4}\left\{15+2\left(\frac{t^2-8ut+u^2}{t+u}\right)\mathbf{k}^2\right.$$
$$\left.+4\left(\frac{t^2u^2}{(t+u)^2}\right)\mathbf{k}^4\right\}\frac{1}{(t+u)^2}, \tag{C2a}$$

(b) $$(\boldsymbol{\sigma}_1\cdot\mathbf{k}_2)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)$$
$$=\boldsymbol{\sigma}_1\cdot(\mathbf{k}-\mathbf{\Delta})\boldsymbol{\sigma}_2\cdot(\mathbf{k}-\mathbf{\Delta})$$
$$\Rightarrow\frac{1}{2}\left\{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\frac{2t^2}{t+u}(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})\right\}\frac{1}{t+u}, \tag{C2b}$$

(c) $$\mathbf{k}_1^2=\mathbf{\Delta}^2\Rightarrow\left\{\frac{3}{2}+\frac{u^2}{t+u}\mathbf{k}^2\right\}\frac{1}{t+u}, \tag{C2c}$$

(d) $$\mathbf{k}_2^2=(\mathbf{k}-\mathbf{\Delta})^2\Rightarrow\left\{\frac{3}{2}+\frac{t^2}{t+u}\mathbf{k}^2\right\}\frac{1}{t+u}, \tag{C2d}$$

(e) $$\mathbf{k}_1^2\mathbf{k}_2^2=\mathbf{k}^2\mathbf{\Delta}^2-2\mathbf{k}\cdot\mathbf{\Delta}\mathbf{\Delta}^2+\mathbf{\Delta}^4$$
$$\Rightarrow\left\{\frac{15}{4}+\frac{1}{2}\frac{(3t^2-4tu+3u^2)}{t+u}\mathbf{k}^2\right.$$
$$\left.+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right\}\frac{1}{(t+u)^2}, \tag{C2e}$$

(f) $(\mathbf{k}_1\cdot\mathbf{k}_2)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_1)$

$$=\boldsymbol{\Delta}\cdot(\mathbf{k}-\boldsymbol{\Delta})[\boldsymbol{\sigma}_1\cdot\boldsymbol{\Delta}\boldsymbol{\sigma}_2\cdot\boldsymbol{\Delta}]$$

$$\Rightarrow\left\{-\frac{5}{4}+\frac{1}{2}\frac{tu}{t+u}\mathbf{k}^2\right\}\cdot\frac{1}{(t+u)^2}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+(\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k})\left\{\frac{1}{2}\frac{(2t-5u)u}{t+u}+\frac{tu^3}{(t+u)^2}\mathbf{k}^2\right\}\frac{1}{(t+u)^2}, \tag{C2f}$$

(g) $(\mathbf{k}_1\cdot\mathbf{k}_2)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)$

$$=\boldsymbol{\Delta}\cdot(\mathbf{k}-\boldsymbol{\Delta})[\boldsymbol{\sigma}_1\cdot\boldsymbol{\Delta}\boldsymbol{\sigma}_2\cdot(\mathbf{k}-\boldsymbol{\Delta})]$$

$$\Rightarrow\left\{\frac{5}{4}-\frac{1}{2}\frac{tu}{t+u}\mathbf{k}^2\right\}\frac{1}{(t+u)^2}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+(\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot\mathbf{k})\times\left\{\frac{1}{2}-\frac{7}{2}\frac{tu}{(t+u)^2}+\frac{t^2u^2}{(t+u)^3}\mathbf{k}^2\right\}\frac{1}{t+u}. \tag{C2g}$$

APPENDIX D: DERIVATIVE SCALAR-PAIR POTENTIALS

As pointed out by Ko and Rudaz [13], besides the most simple Lagrangian for σ decay, $\mathcal{L}^{(0)}_{\sigma\pi\pi}=g_{\sigma\pi\pi}\sigma\boldsymbol{\pi}\cdot\boldsymbol{\pi}$, also the Lagrangian σ decay $\mathcal{L}^{(1)}_{\sigma\pi\pi}=g'_{\sigma\pi\pi}\sigma\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}$ appears in the linear σ model. The latter is useful in keeping the scalar meson widths within reasonable bounds as the scalar mass increases. Also, derivative couplings to baryons were considered in the context of an $\mathrm{SU}_f(3)$ generalization in [9]. In the $(NN2\pi)$ effective-field-theory Lagrangian [14] the NN-interaction Lagrangian, i.e., the next-to-leading-order (NLO) term, for the pion pairs reads

$$\mathcal{L}^{(1)}=-\bar{\psi}\left[8c_1D^{-1}m_\pi^2\frac{\boldsymbol{\pi}}{F_\pi^2}-4c_3\mathbf{D}_\mu\cdot\mathbf{D}^\mu+2c_4\sigma_{\mu\nu}\boldsymbol{\tau}\cdot\mathbf{D}^\mu\times\mathbf{D}^\nu\right]\psi, \tag{D1}$$

where $D=1+\boldsymbol{\pi}^2/f_\pi^2$ and $\mathbf{D}_\mu=D^{-1}\partial_\mu\boldsymbol{\pi}/F_\pi$, with $F_\pi=2f_\pi=185$ MeV. The correspondence with the pair terms treated in this paper is that $c_1\sim g_{(\pi\pi)_0}$ and $c_3\sim g'_{(\pi\pi)_0}$. The c_4 term has been considered in [9], but not in this paper. The "derivative" Hamiltonian to lowest order in the $\boldsymbol{\pi}$ reads

$$\mathcal{H}_{S'}=g'_{(\pi\pi)_0}(\bar{\psi}'\psi)(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})/m_\pi^3. \tag{D2}$$

1. Adiabatic potentials

For the one-pair graphs in Eq. (3.1),

$$\tilde{O}^{(1)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)\Rightarrow\tilde{O}^{(S')}_{\alpha\beta,p}\tilde{O}^{(2PS)}_{\alpha\beta},$$

$$\tilde{O}^{(2PS)}_{\alpha\beta}=-\left(\frac{f_{NN\pi}}{m_\pi}\right)^2(\mathbf{k}_1\cdot\mathbf{k}_2-i\boldsymbol{\sigma}\cdot\mathbf{k}_1\times\mathbf{k}_2),$$

$$\tilde{O}^{(S')}_{\alpha\beta,p}=2\frac{g'_{(\pi\pi)_0}}{m_\pi^3}(\pm\omega_1\omega_2+\mathbf{k}_1\cdot\mathbf{k}_2). \tag{D3}$$

Here, for $p=a,c$ the $(-)$ sign and for $p=b$ the $(+)$ sign apply. Obviously, $\alpha=\beta=\pi$ in Eqs. (D3). All other quantities in Eq. (3.1) are the same as for a pion pair without derivatives. Here and in the rest of this appendix, we absorb the $g^{(n)}(\alpha,\beta)$ factor in Eq. (3.1) into the definition of the O operators.

Evaluation of the p sum and including the mirror graphs, one gets, collecting all terms and selecting the contributions symmetric in $1\leftrightarrow 2$ the matrix element,

$$\sum_p\tilde{O}^{(1)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)D^{(1)}_p(\omega_1,\omega_2)=-2\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2(\mathbf{k}_1\cdot\mathbf{k}_2)\times\{\mathbf{k}_1\cdot\mathbf{k}_2-i\boldsymbol{\sigma}\cdot\mathbf{k}_1\mathbf{k}_2\}\frac{1}{\omega_1^2\omega_2^2}. \tag{D4}$$

For the two-pair graphs in Eq. (3.1) one has

$$\tilde{O}^{(2)}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)D^{(2)}(\omega_1,\omega_2)=-\frac{1}{2\omega_1\omega_2(\omega_1+\omega_2)}(-\omega_1\omega_2+\mathbf{k}_1\cdot\mathbf{k}_2)^2. \tag{D5}$$

Using the expressions in this appendix we obtain in p space the adiabatic "derivative" $(\pi\pi)_0$-exchange potentials. The one-pair exchange and two-pair graphs give

$$\Omega_1^{(1),ad}(\mathbf{k}^2;t,u)=-12\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2d_{2,2,0}(t,u)\times\left[\frac{15}{4}+\frac{1}{2}\frac{t^2-8tu+u^2}{t+u}\mathbf{k}^2+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right]\frac{1}{(t+u)^2}, \tag{D6a}$$

$$\Omega_1^{(2),ad}(\mathbf{k}^2;t,u)$$

$$=-6\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)^2\left\{\left[\frac{15}{4}+\frac{t^2-3tu+u^2}{t+u}\mathbf{k}^2+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right]\right.$$

$$\times\frac{d_{1,1,1}(t,u)}{(t+u)^2}+\frac{1}{2}\left[\frac{3}{2}(m_1^2+m_2^2)+\frac{m_1^2t+m_2^2u}{t+u}\mathbf{k}^2\right.$$

$$\left.+m_1^2m_2^2(t+u)\right]\frac{d_{1,1,1}(t,u)}{t+u}$$

$$\left.+\left[\frac{3}{2}-\frac{tu}{t+u}\mathbf{k}^2\right]\frac{d_{0,0,1}(t,u)}{t+u}\right\}. \tag{D6b}$$

2. 1/M corrections

The nonadiabatic from the $1/M$ expansion of the energy denominators and the pseudovector-vertex $1/M$ corrections are described in Ref. [11] and used also in Ref. [4], Sec. IV. Below, we give the results for the evaluation of these $1/m$ corrections for the one-pair graphs with the ‘‘derivative’’ pair interaction.

a. Nonadiabatic contributions

For the one-pair graphs in Eq. (3.1) the nonadiabatic operator is

$$\tilde{O}^{(1),na}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)\Rightarrow-2\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}\Big[(\mathbf{k}_1\cdot\mathbf{k}_2)^2$$

$$+\frac{i}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\mathbf{k}_1\times\mathbf{k}_2\mathbf{q}\cdot(\mathbf{k}_1-\mathbf{k}_2)\tilde{\Gamma}^{(S')}_{\pi\pi,p}, \tag{D7}$$

where $\tilde{\Gamma}^{(S')}_{\pi\pi,p}=\pm\omega_1\omega_2+\mathbf{k}_1\cdot\mathbf{k}_2$ and the $\pm$ sign has been explained above. The denominators $D_p^{(na)}(\omega_1,\omega_2)$ have been given in [4]. Again, we select the terms symmetric in $1\leftrightarrow 2$ since the asymmetric terms will not contribute, which is easily seen in x space. The sum over the graph's $p=a,b,c$ yields

$$\sum_p \tilde{\Gamma}^{(S')}_{\pi\pi,p}D_p^{na}(\omega_1,\omega_2)=\frac{1}{\omega_1\omega_2}\frac{1}{\omega_1+\omega_2}+\frac{1}{\omega_1^2\omega_2^2}$$

$$\times\left[\frac{1}{\omega_1}+\frac{1}{\omega_2}-\frac{1}{\omega_1+\omega_2}\right](\mathbf{k}_1\cdot\mathbf{k}_2). \tag{D8}$$

Using the expressions in this appendix we obtain in p space the nonadiabatic ‘‘derivative’’ $(\pi\pi)_0$-exchange potentials

$$\Omega_1^{(na)}(\mathbf{k}^2;t,u)$$

$$=-12\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}$$

$$\left\{+\left[\frac{15}{4}+\frac{1}{2}\left(\frac{t^2-8tu+u^2}{t+u}\right)\mathbf{k}^2+\frac{t^2u^2}{(t+u)^2}\mathbf{k}^4\right]\right.$$

$$\times\frac{d_{1,1,1}(t,u)}{(t+u)^2}-\left[\frac{105}{8}+\frac{15}{4}\left(\frac{t^2-5tu+u^2}{t+u}\right)\mathbf{k}^2\right.$$

$$\left.-\frac{3}{2}tu\left(\frac{t^2-5tu+u^2}{(t+u)^2}\right)\mathbf{k}^4-\frac{t^3u^3}{(t+u)^3}\mathbf{k}^6\frac{d_{na}(t,u)}{(t+u)^3},\right. \tag{D9a}$$

$$\Omega_4^{(na)}(\mathbf{k}^2;t,u)=-12\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}$$

$$\times\left\{\frac{d_{1,1,1}(t,u)}{t+u}\right.$$

$$\left.+\left[-5+2\frac{tu}{t+u}\mathbf{k}^2\right]\frac{d_{na}(t,u)}{(t+u)^2}\right\}. \tag{D9b}$$

Here $d_{\{na\}}(t,u)$ is defined in Eq. (B3).

b. Pseudovector contributions

The pseudovector vertex gives $1/M$ terms as can be seen from

$$\overline{u}(\mathbf{p}')\Gamma_P^{(1)}u(\mathbf{p})=-i\frac{f_{NN\pi}}{m_\pi}\left[\boldsymbol{\sigma}\cdot(\mathbf{p}'-\mathbf{p})\pm\frac{\omega}{2M}\boldsymbol{\sigma}\cdot(\mathbf{p}'+\mathbf{p})\right], \tag{D10}$$

where the upper (lower) sign applies for the creation (absorption) of the pion at the vertex. For graph (a) the operator for the nucleon line on the right is readily seen to be

$$-\left(\frac{f_P}{m_\pi}\right)^2\frac{1}{2M}[(\omega_1\mathbf{k}_2^2-\omega_2\mathbf{k}_1^2)-2\mathbf{q}\cdot(\omega_1\mathbf{k}_2+\omega_2\mathbf{k}_1)$$

$$+2i\boldsymbol{\sigma}_2\cdot\mathbf{q}\times(\omega_1\mathbf{k}_2-\omega_2\mathbf{k}_1)]. \tag{D11}$$

The same expression for graph (b) is obviously obtained from Eq. (D11) by making the the substitution $\omega_1\to-\omega_1$ and for graph (c) the substitution $\omega_{1,2}\to-\omega_{1,2}$. The mirror graphs are included by making the replacement $\boldsymbol{\sigma}_2\to(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)/2$. Combining all this with the adiabatic denominators $D_i^1(\omega_1,\omega_2)$,

$$D_a^1(\omega_1,\omega_2)=\frac{1}{2\omega_1\omega_2^2(\omega_1+\omega_2)},\quad D_b^1(\omega_1,\omega_2)=\frac{1}{2\omega_1^2\omega_2^2}, \tag{D12}$$

TABLE XIV. Coefficients $\Upsilon^{(ad,na,pv)}_{j,k}$ for the $(\pi\pi)_0$ (“derivative”) contributions.

	$\Upsilon_0(\Vert)(t,u)$	$\Upsilon_1(\Vert)(t,u)$	$\Upsilon_2(\Vert)(t,u)$	$\Upsilon_3(\Vert)(t,u)$
$\Omega_1^{(1),ad}$	$-\frac{15}{4}\frac{1}{t+u}$	$-\frac{1}{2}\frac{t^2-8tu+u^2}{(t+u)^3}$	$-\frac{t^2u^2}{(t+u)^4}$	—
$\Omega_1^{(1),na}$	$-\frac{45}{2\sqrt{\pi}}\frac{1}{(t+u)^{5/2}}\frac{1}{M}$	$+\frac{1}{2\sqrt{\pi}}\frac{14t^2-67tu+14u^2}{(t+u)^{7/2}}\frac{1}{M}$	$-\frac{tu}{\sqrt{\pi}}\frac{3t^2-14tu+3u^2}{(t+u)^{9/2}}\frac{1}{M}$	$-\frac{1}{\sqrt{\pi}}\frac{t^3u^3}{(t+u)^{1/2}}\frac{1}{M}$
$\Omega_4^{(1),na}$	$+\frac{9}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}\frac{1}{M}$	$-\frac{4}{\sqrt{\pi}}\frac{tu}{(t+u)^{5/2}}\frac{1}{M}$	—	—
$\Omega_1^{(1),pv}$	$-\frac{1}{2\sqrt{\pi}}\frac{(m_1^2-m_2^2)^2}{(t+u)^{1/2}}\frac{1}{M}$	$\frac{1}{2\sqrt{\pi}}\frac{m_1^2(3t^2-u^2)-m_2^2(t^2-3u^2)}{(t+u)^{5/2}}\frac{1}{M}$	$\frac{1}{\sqrt{\pi}}\frac{tu(t^2+2tu+u^2)}{(t+u)^{9/2}}\frac{1}{M}$	—
$\Omega_4^{(1),pv}$	$+\frac{3}{2\sqrt{\pi}}\frac{m_1^2+m_2^2}{(t+u)^{3/2}}\frac{1}{M}$ $-\frac{2}{\sqrt{\pi}}\frac{(m_1^2+m_2^2)}{(t+u)^{1/2}}\frac{1}{M}$ $-\frac{3}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}\frac{1}{M}$	$+\frac{1}{2\sqrt{\pi}}\frac{t^2+2tu+u^2}{(t+u)^{7/2}}\frac{1}{M}$ $-\frac{2}{\sqrt{\pi}}\frac{(t^2+tu+u^2)}{(t+u)^{5/2}}\frac{1}{M}$	—	—
$\Omega_1^{(2),ad}$	$+\frac{1}{\sqrt{\pi}}\frac{1}{(t+u)^{3/2}}$	$-\frac{1}{2\sqrt{\pi}}\frac{tu}{(t+u)^{5/2}}$	—	—

and $D_c^1(\omega_1,\omega_2)=D_a^1(\omega_2,\omega_1)$. Summing over the one-pair graphs gives

$$\sum_p \tilde{O}^{(1),na}_{\alpha\beta,p}(\mathbf{k}_1,\mathbf{k}_2)D_p^1(\omega_1,\omega_2) = -\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{M}\frac{1}{\omega_1\omega_2(\omega_1+\omega_2)} \times\left[\left\{\frac{1}{2}(m_1^2+m_2^2)(\omega_1^2+\omega_2^2)-2\omega_1^2\omega_2^2\right\}-(\mathbf{k}_1^2+\mathbf{k}_2^2) \times(\mathbf{k}_1\cdot\mathbf{k}_2)-i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{q}\times\mathbf{k}(\omega_1^2+\omega_2^2+\mathbf{k}_1\cdot\mathbf{k}_2)\right]. \tag{D13}$$

Using the expressions in this appendix we obtain in p space the pseudovector-vertex $1/M$ corrections to the “derivative” $(\pi\pi)_0$-exchange potentials

$$\Omega_1^{(1),pv}(\mathbf{k}^2;t,u)=-6\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}d_{1,1,1}(t,u) \times\left\{(m_1^2-m_2^2)^2-\left[3(m_1^2+m_2^2) +\frac{m_1^2(3t^2-u^2)+m_2^2(3u^2-t^2)}{t+u}\mathbf{k}^2\right]\frac{1}{t+u} -\left[\left(\frac{t^2+2tu+u^2}{t+u}\right)\mathbf{k}^2 +2tu\left(\frac{t^2+2tu+u^2}{(t+u)^2}\right)\mathbf{k}^4\right]\frac{1}{(t+u)^2}\right\}, \tag{D14a}$$

$$\Omega_4^{(1),pv}(\mathbf{k}^2;t,u) =-24\left(\frac{g'_{(\pi\pi)_0}}{m_\pi^3}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)^2\frac{1}{2M}d_{1,1,1}(t,u)\left\{(m_1^2+m_2^2) +\left[\frac{3}{2}+\left(\frac{t^2+tu+u^2}{t+u}\right)\mathbf{k}^2\right]\frac{1}{t+u}\right\}. \tag{D14b}$$

The coefficients $\Upsilon^{(ad,na,pr)}_{j,k}$ defined in Eq. (4.2) are tabulated in Table XIV.

APPENDIX E: PAIR COUPLINGS AND $SU_F(3)$ SYMMETRY

Below, $\sigma,\mathbf{a}_0,\mathbf{A}_1,\ldots$ are shorthand for, respectively, the nucleon densities $\bar{\psi}\psi$, $\bar{\psi}\boldsymbol{\tau}\psi$, $\bar{\psi}\gamma_5\gamma_\mu\boldsymbol{\tau}\psi$, ...

The $\mathrm{SU}_f(3)$ octet and singlet mesons, denoted by the subscripts 8 and 1, respectively, are in terms of the physical ones defined as follows.

(i) *Pseudoscalar mesons*:

$$\eta_1 = \cos\theta_{pv}\,\eta' - \sin\theta_{pv}\,\eta,$$

$$\eta_8 = \sin\theta_{pv}\,\eta' + \cos\theta_{pv}\,\eta.$$

Here η' and η are the physical pseudoscalar mesons $\eta(957)$ and $\eta(548)$, respectively.

(ii) *Vector mesons*:

$$\phi_1 = \cos\theta_v\,\omega - \sin\theta_v\,\phi,$$

$$\phi_8 = \sin\theta_v\,\omega + \cos\theta_v\,\phi.$$

Here ϕ and ω are the physical vector mesons $\phi(1019)$ and $\omega(783)$, respectively.

Then, one has the following SU(3)-invariant pair interaction Hamiltonians. (1) SU(3)-singlet couplings $S^\alpha_\beta = \delta^\alpha_\beta \sigma/\sqrt{3}$:

$$\mathcal{H}_{S_1PP} = \frac{g_{S_1PP}}{\sqrt{3}}\{\boldsymbol{\pi}\cdot\boldsymbol{\pi} + 2K^\dagger K + \eta_8\eta_8\}\cdot\sigma.$$

(2) SU(3)-octet symmetric couplings I, $S^\alpha_\beta = (S_8)^\alpha_\beta \Rightarrow (1/4)\mathrm{Tr}\{S[P,P]_+\}$:

$$\mathcal{H}_{S_8PP} = \frac{g_{S_8PP}}{\sqrt{6}}\left\{(\mathbf{a}_0\cdot\boldsymbol{\pi})\eta_8 + \frac{\sqrt{3}}{2}\mathbf{a}_0\cdot(K^\dagger\boldsymbol{\tau}K) + \frac{\sqrt{3}}{2}\{(K_0^\dagger\boldsymbol{\tau}K)\cdot\boldsymbol{\pi} + \mathrm{H.c.}\} - \frac{1}{2}\{(K_0^\dagger K)\eta_8 + \mathrm{H.c.}\} + \frac{1}{2}f_0(\boldsymbol{\pi}\cdot\boldsymbol{\pi} - K^\dagger K - \eta_8\eta_8)\right\}.$$

(3) SU(3)-octet symmetric couplings II, $S^\alpha_\beta = (B_8)^\alpha_\beta \Rightarrow (1/4)\mathrm{Tr}\{B^\mu[V_\mu,P]_+\}$:

$$\mathcal{H}_{B_8VP} = \frac{g_{B_8VP}}{\sqrt{6}}\left\{\frac{1}{2}[(\mathbf{B}_1^\mu\cdot\boldsymbol{\rho}_\mu)\eta_8 + (\mathbf{B}_1^\mu\cdot\boldsymbol{\pi}_\mu)\phi_8] + \frac{\sqrt{3}}{4}[\mathbf{B}_1\cdot(K^{*\dagger}\boldsymbol{\tau}K) + \mathrm{H.c.}] + \frac{\sqrt{3}}{4}[(K_1^\dagger\boldsymbol{\tau}K^*)\cdot\boldsymbol{\pi} + (K_1^\dagger\boldsymbol{\tau}K)\cdot\boldsymbol{\rho} + \mathrm{H.c.}] - \frac{1}{4}[(K_1^\dagger\cdot K^*)\eta_8 + (K_1^\dagger\cdot K)\phi_8 + \mathrm{H.c.}] + \frac{1}{2}H^0\left[\boldsymbol{\rho}\cdot\boldsymbol{\pi} - \frac{1}{2}(K^{*\dagger}\cdot K + K^\dagger\cdot K^*) - \phi_8\eta_8\right]\right\}.$$

(4) SU(3)-octet asymmetric couplings I, $A^\alpha_\beta = (V_8)^\alpha_\beta \Rightarrow (-i/\sqrt{2})\mathrm{Tr}\{V^\mu[P,\partial_\mu P]_-\}$:

$$\mathcal{H}_{V_8PP} = g_{A_8PP}\left\{\frac{1}{2}\boldsymbol{\rho}_\mu\cdot\boldsymbol{\pi}\times\overleftrightarrow{\partial^\mu}\boldsymbol{\pi} + \frac{i}{2}\boldsymbol{\rho}_\mu\cdot(K^\dagger\boldsymbol{\tau}\overleftrightarrow{\partial^\mu}K) + \frac{i}{2}[K_\mu^{*\dagger}\boldsymbol{\tau}(K\overleftrightarrow{\partial^\mu}\boldsymbol{\pi}) - \mathrm{H.c.}] + i\frac{\sqrt{3}}{2}[K_\mu^{*\dagger}\cdot(K\cdot\overleftrightarrow{\partial^\mu}\eta_8) - \mathrm{H.c.}] + \frac{i}{2}\sqrt{3}\phi_\mu(K^\dagger\overleftrightarrow{\partial^\mu}K)\right\}.$$

(5) SU(3)-octet asymmetric couplings II, $A^\alpha_\beta = (A_8)^\alpha_\beta \Rightarrow (-i/\sqrt{2})\mathrm{Tr}\{A^\mu[P,V_\mu]_-\}$:

$$\mathcal{H}_{A_8VP} = g_{A_8VP}\left\{\mathbf{A}_1\cdot\boldsymbol{\pi}\times\boldsymbol{\rho} + \frac{i}{2}\mathbf{A}_1\cdot[(K^\dagger\boldsymbol{\tau}K^*) - (K^{*\dagger}\boldsymbol{\tau}K)] - \frac{i}{2}\{[(K^\dagger\boldsymbol{\tau}K_A)\cdot\boldsymbol{\rho} + (K_A^\dagger\boldsymbol{\tau}K^*)\cdot\boldsymbol{\pi}] - \mathrm{H.c.}\} - i\frac{\sqrt{3}}{2}\{[(K^\dagger\cdot K_A)\phi_8 + (K_A^\dagger\cdot K^*)\eta_8] - \mathrm{H.c.}\} + \frac{i}{2}\sqrt{3}f_1[K^\dagger\cdot K^* - K^{*\dagger}\cdot K]\right\}.$$

The relation with the pair couplings of Appendix A is $g_{S_1PP}/\sqrt{3} = g_{(\pi\pi)_0}/m_\pi$, $g_{A_8VP} = g_{(\pi\rho)_1}/m_\pi$, etc.

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