# Correlating the giant-monopole resonance to the nuclear-matter incompressibility

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Differences in the density dependence of the symmetry energy predicted by nonrelativistic and relativistic models are suggested, at least in part, as the culprit for the discrepancy in the values of the compression modulus of symmetric nuclear matter extracted from the energy of the giant monopole resonance in $^{208}$Pb. "Best-fit" relativistic models, with stiffer symmetry energies than Skyrme interactions, consistently predict higher compression moduli than nonrelativistic approaches. Relativistic models with compression moduli in the physically acceptable range of $K = 200$–$300$ MeV are used to compute the distribution of isoscalar monopole strength in $^{208}$Pb. When the symmetry energy is artificially softened in one of these models, in an attempt to simulate the symmetry energy of Skyrme interactions, a lower value for the compression modulus is indeed obtained. It is concluded that the proposed measurement of the neutron skin in $^{208}$Pb, aimed at constraining the density dependence of the symmetry energy and recently correlated to the structure of neutron stars, will also become instrumental in the determination of the compression modulus of nuclear matter.

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The compression modulus of symmetric nuclear matter is a fundamental property of the equation of state. While some of the existent claims in the literature may be overstated—indeed, there is little evidence in support of a correlation between the compression modulus and the physics of neutron stars [1]—the compression modulus impacts on a diverse set of phenomena ranging from nuclear structure to supernova explosions. In particular, the compression modulus controls the energetics around the nuclear-matter saturation point. This is because the first derivative of the energy per nucleon with respect to the density (i.e., the pressure) vanishes at saturation, so the dynamics of small density fluctuations around the equilibrium position becomes solely determined by the compression modulus.

To date, most efforts devoted to the study of the compression modulus have relied on the excitation of the isoscalar giant-monopole resonance (GMR). While the first set of measurements of the GMR date back to the late 1970s and early 1980s [2,3], a recently improved $\alpha$-scattering experiment finds the position of the giant monopole resonance in $^{208}$Pb at $E_{\rm GMR} = 14.17 \pm 0.28$ MeV [4]. While the experimental story on the GMR in $^{208}$Pb seems to be coming to an end, the theoretical picture remains unclear. On the one hand, nonrelativistic calculations that reproduce the distribution of isoscalar-monopole strength using Hartree-Fock plus random-phase approximation (RPA) approaches with state-of-the-art Skyrme [5,6] and Gogny [7] interactions, predict a nuclear compression modulus in the range of $K = 210$–$220$ MeV. On the other hand, relativistic models that succeed in reproducing a large body of observables, including the excitation energy of the GMR, predict a larger value for the nuclear incompressibility ($K \simeq 275$ MeV) [8,9]. It is the aim of this paper to elucidate the origin of this apparent discrepancy. It is proposed that this discrepancy, at least in part, is due to the density dependence of the symmetry energy; a poorly known quantity that affects physics ranging from the neutron radius of heavy nuclei to the structure of neutron stars [10]. It should be noted that while knowledge of the symmetry energy is at present incomplete, the proposed measurement of the neutron radius of $^{208}$Pb at the Jefferson Laboratory [11] should provide stringent constraints on this fundamental component of the equation of state.

In this paper we follow closely the philosophy of Blaizot and co-workers who advocate a purely microscopic approach for the extraction of the compression modulus of nuclear matter from the energy of the giant-monopole resonance [7,12]. While the merit of macroscopic (semiempirical) formulas for obtaining qualitative information on the compression modulus is unquestionable [13,14], the field has attained a level of maturity that demands stricter standards: it is now expected that microscopic models predict simultaneously the compression modulus of nuclear matter as well as the distribution of isoscalar monopole strength. Moreover, theoretical studies based solely on macroscopic approaches have been proven inadequate [15,16].

The starting point for the calculations is an interacting Lagrangian density of the following form:

$$\mathcal{L}_{\rm int} = \bar{\psi}\left[g_{\rm s}\phi - \left(g_{\rm v}V_{\mu} + \frac{g_{\rho}}{2}\boldsymbol{\tau}\cdot\mathbf{b}_{\mu} + \frac{e}{2}(1+\tau_3)A_{\mu}\right)\gamma^{\mu}\right]\psi - \frac{\kappa}{3!}(g_{\rm s}\phi)^3 - \frac{\lambda}{4!}(g_{\rm s}\phi)^4. \qquad (1)$$

This Lagrangian includes an isodoublet nucleon field ($\psi$) interacting via the exchange of scalar ($\phi$) and vector ($V^{\mu}$, $\mathbf{b}^{\mu}$, and $A^{\mu}$) fields. It also incorporates scalar-meson self-interactions ($\kappa$ and $\lambda$) that are instrumental in reducing the unreasonably large value of the compression modulus predicted in the original (linear) Walecka model [17,18]. Although this effective Lagrangian includes only a subset of "local meson terms" (i.e., scalar cubic and quartic), power counting [19,20] suggests that other terms, such as vector quartic and isoscalar-isovector terms, may be equally important. While predictions for ground-state observables with

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TABLE I. Empirical bulk observables used in the determination of the coupling constants and the scalar mass. The symmetry energy $J$ has been fixed at $k_F=1.15\ \mathrm{fm}^{-1}$, but the quantities in parentheses represent its value at saturation density. The slope of the symmetry energy at saturation density $L$ is an actual prediction of the model. Values for two "best-fit" nonrelativistic Skyrme models, previously used in calculations of the GMR in $^{208}$Pb [5,6], are included for comparison.

| Family | $k_{\rm F}^0$ (fm$^{-1}$) | $\epsilon_0$ (MeV) | $M^*/M$ | $K$ (MeV) | $J$ (MeV) | $L$ (MeV) |
|---|---|---|---|---|---|---|
| A | 1.30 | −16.0 | 0.6 | 200–300 | 26(38) | 120 |
| B | 1.30 | −16.0 | 0.7 | 200–300 | 26(37) | 108 |
| C | 1.30 | −16.0 | 0.7 | 200–300 | 20(28) | 82 |
| SGII | 1.33 | −15.6 | — | 215 | (27) | 38 |
| SKM* | 1.33 | −15.8 | — | 217 | (30) | 46 |

these additional terms are now available [10], RPA calculations of the linear response of the ground state (with these terms) have yet to be done. Thus, in the interest of consistency, all calculations reported here—both for the ground state and for the excited states—are limited to the set of interactions displayed in Eq. (1). Yet incorporating additional local meson terms in the consistent linear response of the mean-field ground is an important area for future investigations. Moreover, data on excited nuclear states may provide new constraints that may determine features of the equation of state that at present are poorly known, such as the density dependence of the symmetry energy.

As it stands, the Lagrangian density of Eq. (1) depends on five unknown coupling constants that may be determined from a fit to ground-state observables. Four of these constants ($g_{\rm s}$, $g_{\rm v}$, $\kappa$, and $\lambda$) are sensitive to isoscalar observables so they are determined from a fit to symmetric nuclear matter. The four nuclear bulk properties selected for the fit are as follows: (i) the saturation density, (ii) the binding energy per nucleon at saturation, (iii) the nucleon effective mass at saturation, and (iv) the compression modulus (see Table I). It is noteworthy, yet little known, that the above four coupling constants can be determined algebraically and uniquely from these four empirical quantities [21–23]. It is also possible for the various meson masses to enter as undetermined parameters. However, here the standard procedure of fixing the masses of the $\omega$ and $\rho$ mesons at their physical value is adopted; that is, $m_{\rm v}=783$ MeV and $m_\rho=763$ MeV. As infinite nuclear matter is only sensitive to the ratio $g_{\rm s}^2/m_{\rm s}^2$, the mass of the $\sigma$ meson must be determined from finite-nuclei properties; the $\sigma$-meson mass has been adjusted to reproduce the experimental root-mean-square (rms) charge radius of $^{208}$Pb ($r_{\rm ch}=5.50\pm0.01$ fm).

The symmetry energy of nuclear matter is a poorly known quantity with an uncontrolled density dependence in nonrelativistic models (for a recent discussion of the symmetry energy in Skyrme models see Refs. [24,25]). In contrast, the symmetry energy displays a weak model dependence in relativistic approaches. It is given by the following simple form:

$$S(k_{\rm F})=\frac{k_F^2}{6E_F^*}+\frac{g_\rho^2}{12\pi^2}\frac{k_F^3}{m_\rho^2}, \tag{2}$$

where $E_F^*=\sqrt{k_{\rm F}^2+M^{*2}}$. The symmetry energy, together with its density dependence, is constrained in relativistic approaches because the only "free" parameter in Eq. (2) is the $NN\rho$ coupling constant. As the effective nucleon mass $M^*$ has been fixed in symmetric nuclear matter (and spin-orbit phenomenology demands a value in the range of $M^*/M=0.6$–$0.7$) reproducing the empirical value of the symmetry energy at saturation ($J\simeq37$ MeV) constrains the $NN\rho$ coupling constant to a relatively small range. Note that relativistically the density dependence of the symmetry energy can also be modified through the inclusion of isoscalar-isovector couplings terms [10], density-dependent coupling constants [26], and isovector-scalar mesons [27]. However, as a consistent RPA formalism that incorporates these additional terms has yet to be developed, none of these contributions will be considered henceforth. Yet work on extending the RPA approach to include these terms is in progress. In reality, the symmetry energy at saturation is not well constrained experimentally. Rather, it is an average of the symmetry energy near saturation density and the surface symmetry energy that is constrained by the binding energy of nuclei. Thus a prescription first outlined in Ref. [10] is adopted here: the value of the $NN\rho$ coupling constant is adjusted, unless otherwise noted, so that the symmetry energy at $k_F=1.15\ \mathrm{fm}^{-1}$ (i.e., $\rho=0.10\ \mathrm{fm}^{-3}$) be equal to 26 MeV (see Table I).

The nuclear observables used as input for the determination of the model parameters are listed in Table I. In all cases the saturation density, binding energy per nucleon, and rms charge radius in $^{208}$Pb have been fixed at their empirical values. Thus the only discriminating factors among the three "families" are the effective nucleon mass and the symmetry energy. While best-fit relativistic models suggest values for the symmetry energy and its slope at saturation density satisfying $J\geqslant35$ MeV and $L\geqslant100$ MeV, respectively [13], family C is defined with an artificially small value for $J$ (and correspondingly for $L$) in a "poor-man's" attempt at simulating nonrelativistic Skyrme forces [25] (see Table I). That nonrelativistic Skyrme models have a softer symmetry energy is revealed by the behavior of one of the most sensitive probes of the density dependence of symmetry energy: the neutron skin of $^{208}$Pb. Indeed, the neutron skin of $^{208}$Pb is predicted to be equal to $R_n-R_p=0.16$ fm for the recent SkX parametrization and falls below 0.22 fm for all eighteen Skyrme parameter sets considered in Ref. [24]. In contrast, best-fit relativistic models consistently predict larger values. For example, the NL3 model of Ref. [8], the TM1 model of

TABLE II. The compression modulus of symmetric nuclear matter, the compression modulus for asymmetric ($I=0.212$) nuclear matter, the neutron skin of $^{208}$Pb, and the energy of the GMR in $^{208}$Pb for the three families discussed in the text.

| Family | $K$ (MeV) | $K_{208}$ (MeV) | $R_n-R_p$ (fm) | $E_{\mathrm{GMR}}$ (MeV) |
|---|---|---|---|---|
| A | 200 | 184 | 0.28 | 12.27 |
| | 225 | 203 | 0.28 | 12.88 |
| | 250 | 224 | 0.28 | 13.58 |
| | 275 | 246 | 0.28 | 14.14 |
| | 300 | 268 | 0.28 | 14.81 |
| B | 200 | 187 | 0.25 | 12.65 |
| | 225 | 208 | 0.25 | 13.35 |
| | 250 | 230 | 0.26 | 14.03 |
| | 275 | 252 | 0.26 | 14.75 |
| | 300 | 276 | 0.26 | 15.36 |
| C | 200 | 190 | 0.19 | 13.13 |
| | 225 | 212 | 0.19 | 13.80 |
| | 250 | 235 | 0.19 | 14.45 |
| | 275 | 258 | 0.19 | 15.09 |
| | 300 | 282 | 0.19 | 15.81 |

Sugahara and Toki [28], and the NLC model of Serot and Walecka [19], predict $R_n-R_p=0.28$, 0.27, and 0.26 fm, respectively (also see Table II).

Within each family defined in Table I, calculations of the isoscalar monopole response have been performed using a compression modulus in the physically acceptable range of $K=200$–300 MeV. To illustrate the similarities and differences between these three families, the equation of state for symmetric nuclear matter (left panel) and the symmetry energy (right panel) are displayed in Fig. 1 at $K=250$ MeV. Clearly, the properties of symmetric nuclear matter at saturation density are identical in all three models. Further, having fixed the value of the effective nucleon mass in symmetric nuclear matter, the full density dependence of the symmetry energy is determined by one sole number: its value at $k_F=1.15\ \mathrm{fm}^{-1}$.

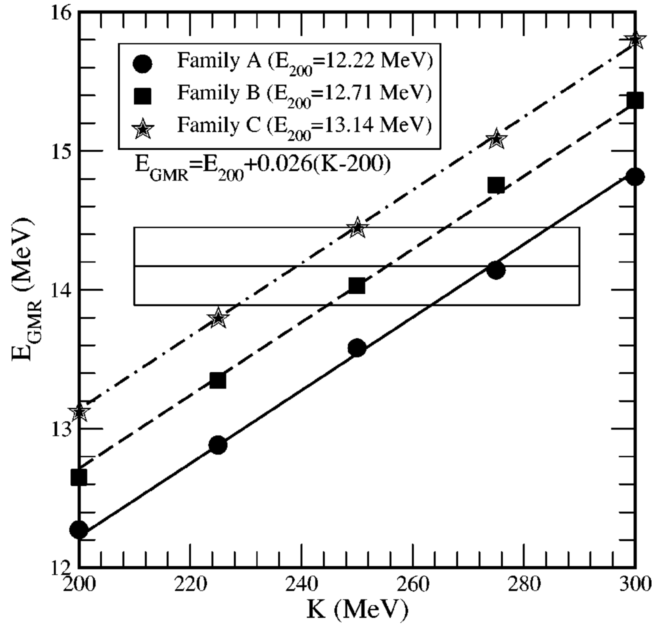


FIG. 2. Energy of the isoscalar giant-monopole resonance as a function of the nuclear matter compression modulus for the three families discussed in the text. The box displays the experimentally allowed range of $E_{\mathrm{GMR}}=14.17\pm0.28$ MeV [4].

Results for the peak energy of the giant-monopole-resonance in $^{208}$Pb as a function of the nuclear incompressibility are listed in Table II and displayed in Fig. 2. All calculations were performed using the nonspectral, relativistic RPA approach of Ref. [29]. Note that while the distribution of isoscalar monopole strength, particularly its spreading width, has been shown to be sensitive to configurations that go beyond the RPA [5], these ("second-RPA") configurations will not be considered here any further as the aim of this paper is limited to understand the discrepancies between equivalent relativistic and nonrelativistic mean-field-plus-RPA models. For each family there is a clear correlation between the compression modulus and the energy of the GMR. Indeed, all of the results are well represented (in this

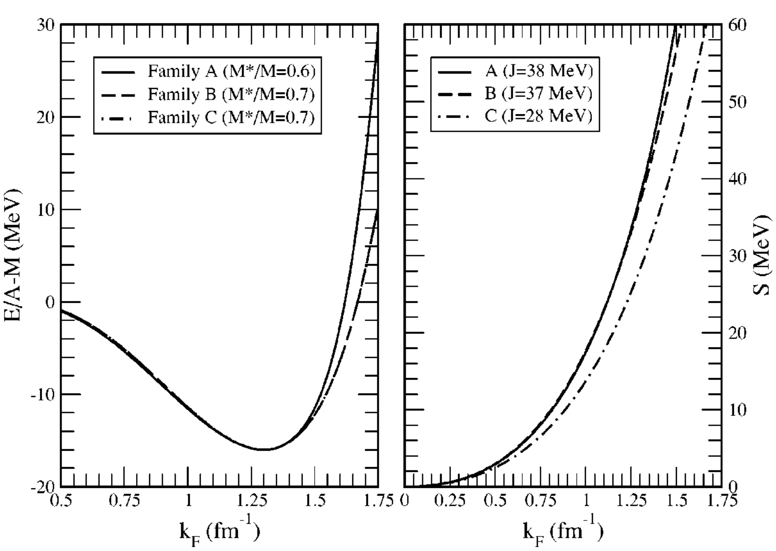


FIG. 1. Equation of state for symmetric nuclear matter (left panel) and the symmetry energy (right panel) as a function of the Fermi momentum for the three families discussed in the text. In all the cases presented here the compression modulus was fixed at $K=250$ MeV.

limited range of $K$) by a linear relation with a ''universal'' slope

$$E_{\rm GMR}=E_{200}+0.026(K-200), \tag{3}$$

where $E_{\rm GMR}$, $E_{200}$, and $K$ are all given in MeV. The intercept is nonuniversal and given by $E_{200}=12.22$ MeV, $E_{200}=12.71$ MeV, and $E_{200}=13.14$ MeV, for families A, B, and C, respectively.

A few comments are now in order. First, the value of the slope (0.026) is obviously small. This suggests that even without theoretical uncertainties, it would not be possible to determine the compression modulus from the $^{208}$Pb measurement alone to better than $\Delta E_{\rm GMR}/0.026$ MeV ($\Delta E_{\rm GMR}$ is the experimental uncertainty). At present, the best determination of the peak position of the GMR is $E_{\rm GMR}=14.17\pm0.28$ MeV [4], thereby resulting in an uncertainty in the compression modulus of about 20 MeV. Second, and more importantly, the journey from the GMR to the compression modulus is plagued by uncertainties unrelated to the physics of symmetric nuclear matter. To illustrate this point we invoke—although never use in any of the calculations—a semiempirical formula based on a leptodermous expansion of the nuclear incompressibility:

$$K(A,I)=K+K_{\rm surf}/A^{1/3}+K_{\rm sym}I^2+K_{\rm Coul}Z^2/A^{4/3}+\cdots, \tag{4}$$

where $K_{\rm surf}$, $K_{\rm sym}$, and $K_{\rm Coul}$ are empirical surface, symmetry, and Coulomb coefficients, and $I=(N-Z)/A$ is the neutron-proton asymmetry. The sizable contribution from the surface term to $K(A,I)$ has been discussed recently by Patra, Viñas, Centelles, and Del Estal [30] in the context of a relativistic Thomas-Fermi theory so we limit ourselves to only a few comments. A surface dependence is modeled here through a change in the value of the effective nucleon mass (surface properties are also sensitive to the $\sigma$-meson mass but this value has been chosen to reproduce the rms charge radius of $^{208}$Pb). As shown in Table I, family A uses an effective nucleon mass of $M^*/M=0.6$ while family B uses $M^*/M=0.7$; all other input observables are identical. A larger $M^*$ generates a slightly compressed single-particle spectrum and a correspondingly smaller spin-orbit splitting. Consequences of this change in $M^*$ result in a larger intercept, as displayed in Fig. 2. Thus compression moduli of approximately $K=275$ MeV (for family A) and $K=250$ MeV (for family B) are required to reproduce the experimental energy of the GMR. Further, if one incorporates the experimental error into this analysis, one concludes that ''best-fit'' relativistic mean-field models are consistent with a compression modulus in the range $K=245-285$ MeV.

We now turn to the central idea behind this work, namely, how our incomplete knowledge of the symmetry energy impacts on the the extraction of the compression modulus. Let us then start by considering two identical models, but with vastly different symmetry energies, that predict a compression modulus of $K=250$ MeV. Further, for simplicity we assume that these two models have identical surface and Coulomb properties so only the first and third term in Eq. (4) are relevant to this discussion. Both models attempt to reproduce the ''experimentally'' accessible quantity

$$K_{208}\equiv\lim_{A\to\infty}K(A,I=0.212)=K+K_{\rm sym}(0.212)^2+\cdots, \tag{5}$$

defined as the compressibility of infinite nuclear matter at a neutron-proton asymmetry identical to that of $^{208}$Pb (see Table II). The first model, having a very stiff symmetry energy (that is, $K_{\rm sym}$ large and negative) reduces $K(A,I)$ from its $I=0$ value of 250 MeV all the way down to, let us say, 200 MeV at $I=0.212$. Comparing this prediction to the assumed experimental value of $K_{208}=225$ MeV, it is concluded that the compression modulus of symmetric nuclear matter must be increased to $K\simeq275$ MeV. The second model predicts a very soft symmetry energy. So unrealistically soft, let us assume, that it generates no shift in going from $I=0$ to $I=0.212$ (i.e., $K_{\rm sym}=0$). In this case, the compression modulus must then be reduced to $K=225$ MeV to reproduce the experimentally determined value. Thus the two models, originally identical as far as symmetric nuclear matter is concerned, disagree in their final values of the compression modulus due to an incomplete knowledge of the symmetry energy. While the situation depicted in Fig. 2 might not be as extreme, it does follow the trends suggested by the above discussion. Indeed, family C, with the softest symmetry energy, generates the largest intercept and consequently predicts the smallest compression modulus of the three families.

In summary, the impact of the poorly known density dependence of the symmetry energy on the extraction of the compression modulus of nuclear matter from the energy of the giant-monopole resonance in $^{208}$Pb was addressed. The nuclear matter equation of state and the distribution of isoscalar monopole strength in $^{208}$Pb were computed using three different families of relativistic models constrained to reproduce a variety of ground-state observables. For each family the compression modulus was allowed to vary within the physically acceptable range of $K=200-300$ MeV. The first family (A) has an effective nucleon mass fixed at $M^*/M=0.6$ and is, at least for $K=275$ MeV, practically indistinguishable from the successful NL3 model of Ref. [8]. The second family (B) differs from the first in that the effective nucleon mass is increased to $M^*/M=0.7$, thereby generating a slightly compressed single-particle spectrum but still a robust phenomenology. Finally, the third family (C) is obtained from the second one by artificially softening the symmetry energy in a ''poor-man's'' attempt at simulating nonrelativistic Skyrme models. When the peak energy of the GMR is plotted against the compression modulus, a linear relation with a universal slope is obtained. In contrast, the intercept is family dependent and it is largest for the model with the softest symmetry energy. Demanding agreement with the experimental value for the peak energy fixes the compression modulus at: $K=275$, 255, and 240 MeV, for families A, B, and, C, respectively. Thus we regard these as our most important conclusions.

(1) The extraction of the compression modulus of symmetric nuclear matter from the energy of the giant-monopole

resonance in $^{208}$Pb is sensitive to the density dependence of the symmetry energy.

(2) Assuming all other things being equal, models with a softer symmetry energy require a lower compression modulus to reproduce the energy of the giant-monopole resonance in $^{208}$Pb.

(3) The discrepancy between accurately calibrated relativistic and nonrelativistic mean-field-plus-RPA models in the prediction of the compression modulus of symmetric nuclear matter is attributed in part to our incomplete knowledge of the symmetry energy.

At present, resolving the density dependence of the symmetry energy is not possible. Yet the proposed Parity Radius Experiment (PREX) at the Jefferson Laboratory should provide a unique constraint on the density dependence of the symmetry energy through a measurement of the neutron skin of $^{208}$Pb. Such a measurement could have far-reaching implications: from the determination of a fundamental parameter of the equation of state ($K$) to the structure of neutron stars [10].

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