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Evidence of X(5) symmetry for $n_\gamma=0,1,2$ bands in ^{104}Mo

P. G. Bizzeti and A. M. Bizzeti-Sona
Dipartimento di Fisica, Università di Firenze, I.N.F.N., Sezione di Firenze, Via G. Sansone 1, I-50019 Sesto Fiorentino (Firenze), Italy
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The dynamical symmetry X(5) characterizes the critical point of phase transition between spherical and axially deformed shape in atomic nuclei. A first example of this new symmetry has been found in ^{152}Sm ($Z=62$) by Casten and Zamfir. We show that the level scheme of the transitional nucleus ^{104}Mo ($N=62$) also presents the characteristic X(5) pattern, not only in the ground-state band but also in its low-lying $n_\gamma=1, K=2$ and $n_\gamma=2, K=4$ bands. An essential point of the model leading to the X(5) symmetry, i.e., the decoupling of the γ vibrations from the other degrees of freedom, is therefore confirmed.

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The possible signature of a phase transition between collective vibrator and axially deformed rotor has received considerable attention, in the more general frame of critical point properties in transitional nuclei [1]. In the frame of IBM, this phase transition would take place when moving continuously from the pure U(5) symmetry to the SU(3) symmetry. It has been shown by Iachello [2] that, under simplifying assumptions, the critical point corresponds to another definite symmetry, called X(5), implying definite relations among the level energies and among the $E2$ transition strengths. In particular, excitation energies are given by the expression

$$E(s,J,n_\gamma,K,M)=E_0+B\ \ (x_{s,J})^2+An_\gamma+CK^2, \qquad (1)$$

where $x_{s,J}$ is the sth zero of the Bessel function $\mathcal{J}_\nu$ of (irrational) order $\nu=[J(J+1)/3+9/4]^{1/2}$, J is the total angular momentum (with projections K on the symmetry axis and M on the quantization axis in the laboratory frame), and $n_\gamma=0,1,2,\ldots$ is the number of γ-vibration quanta, while E_0, A, B, and C are arbitrary parameters. For $n_\gamma=0$, the excitation energies $E(s,J)$ (referred to the ground state) depend, therefore, only on the scaling parameter B and Eq. (1) provides a very stringent condition for the level scheme at the critical point.

Without entering into details, which can be found in the Ref. [2], we remind the reader of the main approximations used in the X(5) model, to derive Eq. (1): (i) The potential surface in β and γ at the critical point is approximated with the sum of two separate potentials, a square-well potential (of width β_W) for the variable β, and a harmonic potential for γ; (ii) the coupled differential equations in β,γ are approximated with two separate equations, one for the variable β and one for γ.

Recently, it was shown that a signature of phase transition is observed in the chain of Sm isotopes [3,2], where ^{152}Sm displays (approximately) the predicted features of the X(5) symmetry and mark therefore—approximately—the critical point.

Here we want to show that a similar pattern can be found in the transitional region around ^{104,106}Mo, where ^{104}Mo ($N=62$) plays the same role as ^{152}Sm ($Z=62$) in its own isotone chain. Actually, already in 1982 Kern *et al.* [4] noticed the similarity between this region and that of Sm isotopes, and in particular between ^{104}Mo and ^{152}Sm (having, in the IBM, the same number of valence bosons).

In Fig. 1, the ratio $E(4^+)/E(2^+)$ is reported, as a function of the proton number Z, for the even isotones with $N=62$ and $N=64$, from $_{38}$Sr to $_{48}$Cd [5]. Immediately below the semimagic $_{50}$Sn, the low-lying levels of $_{48}$Cd and $_{46}$Pd isotopes follow the pattern expected for (anharmonic) vibrations, with the typical two-phonon triplet $0^+, 2^+, 4^+$, while a rotational-like character is clear in the level scheme of the lighter elements of the chain, $_{40}$Zr and $_{38}$Sr. A value of $E(4^+)/E(2^+)$ very close to that expected at the critical point [2.91, for the X(5) symmetry] is found for $N=62$ and $Z=42$, i.e., for ^{104}Mo, whose level scheme [4–6], depicted in Fig. 2, shows in fact a clear similarity to that expected for the X(5) symmetry.

As this region of nuclei is considerably softer with respect to γ vibrations, compared to that of ^{152}Sm, it becomes important to compare the pattern of the low-lying excited γ bands ($n_\gamma=1$, $K=2$ and $n_\gamma=2$, $K=4$) with the predictions of the X(5) model. As the model does not predict the excitation energy of the band heads for the observed γ bands, these energies must be taken from the experimental data and

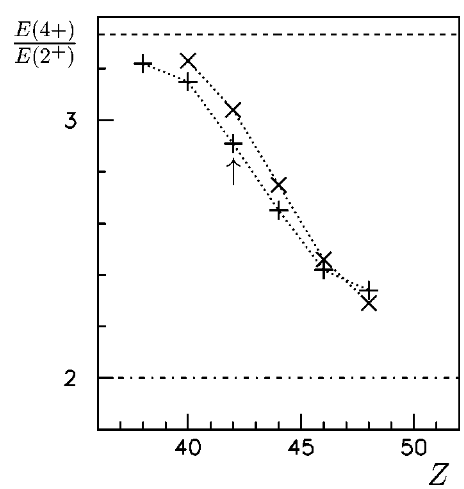

FIG. 1. Values of the ratio $E(4^+)/E(2^+)$ for $N=62$ (+) and $N=64$ (×) isotones, as a function of the proton number Z. The arrow shows the point corresponding to ^{104}Mo. The horizontal dashed and dashed-dotted lines correspond, respectively, to the rigid axial rotor and to the harmonic vibrator.

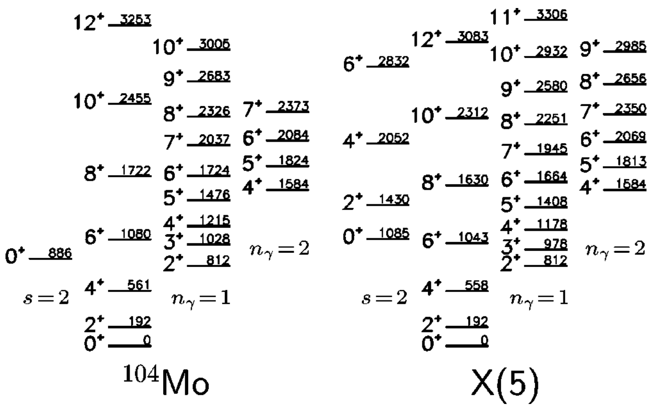

FIG. 2. Partial level scheme of ^{104}Mo compared with the predictions of the X(5) model, with the energy scale (in keV) normalized to the experimental $E(2^+)$ and the band heads of the $n_\gamma=1,2$ bands fixed at the same position of the experimental ones.

fix the values of the parameters A and C of Eq. (1). No other free parameter is required to be included in the comparison with the two γ bands. In doing this, we have assumed a complete decoupling between the β and γ degrees of freedom—i.e., a constant value of the inertial term $\langle\beta^2\rangle$ in the differential equation for the variable γ [see Eq. (4) of Ref. [2]].

The comparison—done here for the first time—of the experimental results with the predictions of Eq. (1) for $n_\gamma>0$, shows that the agreement is as good for the γ bands as for the ground-state band, and this fact gives further support to the validity of the model proposed in Ref. [2], not only for the even-J but also for the odd-J states. In fact, if the assumption of complete decoupling of the β and γ degree of freedom is valid, the vibrational energy remains constant for all states of a given γ-excited band, but rotational energies depend on the zeroes of the Bessel functions, and hence provide an additional test of the X(5) symmetry.

Moreover, we will show that also in ^{152}Sm Eq. (1) not only accounts for the $n_\gamma=0$ but also for the $n_\gamma=1$ level scheme. We emphasize that the observed agreement is not trivial. In fact, the level scheme of the $K^\pi=0^-$ octupole band of ^{152}Sm cannot be reproduced by the $s=1$ energy sequence of X(5) and is very close to a rotational pattern.

A more quantitative comparison of experimental and theoretical results can be found in Table I, where we report—in addition to the X(5) values and ^{104}Mo results—also data concerning the next isotope ^{106}Mo [5,7], and two nuclides of the previously discovered X(5) region [3,5], ^{152}Sm and ^{150}Nd. Values reported in Table I correspond to the excitation energy of the indicated level, divided by that of the first excited state. Excitation energies of the two γ bands are referred to the corresponding band head, 2^+ and 4^+ for the $n_\gamma=1$ and $n_\gamma=2$, respectively. As a rule, the ^{104}Mo data are in better agreement with the X(5) values than those of ^{152}Sm, with one significant exception: the excitation energy of the first $s=2$ level ($J^\pi=0^+$) deviates from the X(5) predictions more for ^{104}Mo than for ^{152}Sm. For this particular level, the agreement with X(5) would be much better for ^{106}Mo, whose ground state band, however, is definitely more displaced from X(5) towards the rotational pattern. A similar situation arises for the pair ^{150}Nd-^{152}Sm: the level scheme of ^{150}Nd is closer to X(5) as for the ground-state band, but the first level of the $s=2$ band deviates from X(5) more than in the case of ^{152}Sm.

TABLE I. Ratios of excitation energy of the level specified in the first three columns to that of the first excited state for the two isotopes ^{104,106}Mo, for the X(5) symmetry and for the two nuclei, ^{152}Sm and ^{150}Nd, discussed in Ref. [3]. For levels of the $K=2$, $n_\gamma=1$ and $K=4$, $n_\gamma=2$ bands, excitation energies are referred to the 2^+ and 4^+ band head.

J^π	s	K	^{104}Mo	^{106}Mo	X(5)	^{152}Sm	^{150}Nd
4^+	1	0	2.917	3.045	2.91	3.009	2.929
6^+	1	0	5.619	6.026	5.43	5.804	5.553
8^+	1	0	8.959	9.840	8.48	9.241	8.676
10^+	1	0	12.776	14.413	12.03	13.214	12.280
12^+	1	0	16.928		16.04	17.642	16.274
0^+	2	0	4.610	5.576	5.65	5.622	5.187
2^+	2	0			7.45	6.655	6.533
4^+	2	0			10.69	8.400	8.738
6^+	2	0			14.75	10.761	11.836
3^+	1	2	1.124	1.017	0.86	1.215	1.065
4^+	1	2	2.094	2.083	1.90	2.347	2.238
5^+	1	2	3.451	3.475	3.10	3.890	
6^+	1	2	4.745	4.973	4.43	5.274	
7^+	1	2	6.370	6.747	5.89	7.062	
8^+	1	2	7.577	8.651	7.48	7.061	
5^+	1	4	1.253	1.299	1.19		
6^+	1	4	2.602	2.667	2.53		
7^+	1	4	4.107	4.458	3.99		

Data concerning the three bands with $s=1$ and (n_γ,K) equal to (0,0), (1,2), and (2,4) can be combined together for comparison with the X(5) predictions, as shown in Fig. 3. Experimental values at high spin are somewhat displaced towards the rotational curve (and more for ^{152}Sm than for ^{104}Mo). To explain this behavior, it has been proposed [3] that^{152}Sm do not correspond exactly to the critical point of phase transition, being somewhat displaced on the side of the deformed region. This is possible, but another possibility exists. It has been noted already that, after all, X(5) is not a symmetry of some algebraic model, but corresponds to a very particular situation in the potential surface of the geometrical model: as a function of the deformation parameter β, the potential energy at $\beta=0$ has a minimum for spherical (vibrational) nuclei, a maximum for deformed nuclei, and a higher-order stationary point in the limiting case of X(5). But the calculations (with phenomenological shell corrections) show that [9] the energy surface moves slowly with increasing angular momentum, due to Coriolis forces, so that more-deformed shapes become more favored at higher values of J. It is therefore possible that the critical point is met at different values of the driving parameter in different regions of angular momenta.

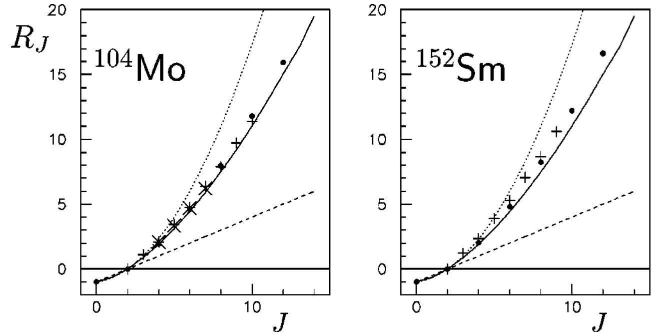

FIG. 3. Experimental and theoretical values of the ratio $R_J(n_\gamma)=[E_J(n_\gamma)-E_2(n_\gamma)]/E_2(0)$ for the $s=1$ ($n_\gamma=0,1$) bands in ^{104}Mo and—for comparison—in ^{152}Sm. Dots: $n_\gamma=0$; crosses: $n_\gamma=1$. For ^{104}Mo, also the $n_\gamma=2,K=4$ band is reported. In this case, the $\times$ symbols give the ratios $[E_J(2)-\Delta E]/E_2(0)$ with $\Delta E=E_4(2)-E_4(1)+E_2(1)$, in order to bring the 4^+ band head at the same position of the corresponding level of the $n_\gamma=1$ band. The full line gives the predictions of the X(5) model; the dotted and the dashed curves correspond, respectively, to the rigid rotor and to the harmonic oscillator.

One could also suspect that the "constants" E_0, A, and C of Eq. (1) vary slowly with the angular momentum, due to their dependence on $\langle\beta^2\rangle$ (see Ref. [2]), if this parameter is assumed to correspond, for each value of s and J, to the expectation value $\langle\beta^2\rangle_{s,J}$ of β^2 over the proper wave function $\xi_{s,J}(\beta)$. This possibility can be explored, at least for constants A and C, by comparing the excitation energy of levels with fixed J, $s=1$ and different n_γ and/or K. The following relations, valid for $n_\gamma=K/2$, have been used: $E(J,K=2)-E(J,K=0)=A+4C$ and $E(J,K=4)-E(J,K=2)=A+12C$. However, the experimental values, reported in Table II, do not show conclusive evidence of a systematic trend with respect to the values of $\beta_W^2/\langle\beta^2\rangle_{s,J}$, shown in the last column.

A further test on the validity of the X(5) symmetry in the case of ^{104}Mo is provided by the comparison of experimental and theoretical values of the reduced strengths of $E2$ transitions. Rules to calculate the values of $B(E2)$ in the X(5) model are given in Ref. [2], where a few numerical values are also given for transitions between states with $n_\gamma=0$.

Within the limits of the model, the $E2$ transition operator

TABLE II. Empirical values (in keV) of the constants A and C—or linear combinations of them—for different values of J. The last column gives the inverse of the expectation value of β^2/β_W^2, calculated with the wave functions $\xi_{s=1,J}$ given in Ref. [2].

J	$A+4C$	$A+12C$	A	C	$\beta_W^2/\langle\beta^2\rangle_{1,J}$
2	620				2.41
4	654	369	796	-35.7	2.15
5		349			2.05
6	644	359	787	-35.7	1.97
7		336			1.90
8	604				1.83
10	549				1.74

[Eq. (12) of Ref. [2]] can be approximated with the expression (valid for small values of γ)

$$T_\mu^{(E2)}=t\beta\left[\mathcal{D}^{(2)}_{\mu,0}+\gamma\frac{1}{\sqrt{2}}(\mathcal{D}^{(2)}_{\mu,2}+\mathcal{D}^{(2)}_{\mu,-2})\right], \qquad (2)$$

where $\mathcal{D}^{(2)}_{\mu,K}(\vartheta_1,\vartheta_2,\vartheta_3)$ are the Wigner $\mathcal{D}$ functions, $\vartheta_1,\vartheta_2,\vartheta_3$ the Euler angles of rotation, and t is a scale factor. In the same approximation, the intrinsic wave function is factorized in a part dependent on β and another dependent on γ:

$$\varphi^{sn_\gamma}_{JK}(\beta,\gamma)=\xi_{s,J}(\beta)\,\eta_{n_\gamma,K}(\gamma)$$

and the complete wave function is

$$\Psi=\frac{1}{\sqrt{2}}[\varphi^{sn_\gamma}_{J,K}\mathcal{D}^{(J)}_{M,K}(\vartheta_i)+(-)^{J+K}\varphi^{sn_\gamma}_{J,-K}\mathcal{D}^{(J)}_{M,-K}(\vartheta_i)].$$

For transitions between states having equal n_γ and K, only the first term of Eq. (2) survives, and all $B(E2)$ values can be deduced from the model without free parameters, apart from the common scale factor t^2. Between states with equal K and different n_γ, the transition is forbidden. For interband transitions with $|\Delta n_\gamma|=1$, $|\Delta K|=2$, the expression for the transition amplitude involves one of the last two terms of Eq. (2) and includes the integral $\langle n_\gamma|\gamma|n'_\gamma\rangle=\int_0^\infty\eta_{n_\gamma}\gamma\eta_{n'_\gamma}\gamma d\gamma$, which contains an additional free parameter. As the γ dependent wave function is that of a two-dimensional oscillator, one gets

$$\langle n_\gamma=2|\gamma|n_\gamma=1\rangle^2=2\langle n_\gamma=1|\gamma|n_\gamma=0\rangle^2.$$

In conclusion, ratios of experimental $B(E2)$ for two intraband or two interband transitions can be directly compared with the model. For the ratio of one interband to one intraband transition, the comparison involves a different scale factor, that must be deduced from the experimental results.

In ^{104}Mo, only the half-lives of the first two excited states are known [8], $T_{1/2}(2^+)=721(41)$ ps and $T_{1/2}(4^+)=26.8(35)$ ps. Taking also into account the electron conversion, we obtain for the ratio of the reduced strengths the value $1.16^{+0.25}_{-0.19}$, in rough agreement with the theoretical value 1.6.

Other data to be compared with the model can be deduced from the branching ratios [5] of pure $E2$ transitions deexciting the same parent state. Only transitions with $\Delta J=2$ have been considered, as no measurement of the multipole mixing ratio has been reported for $\Delta J=0$ or 1. Experimental results are shown in Table III, together with the (properly normalized) model predictions. The "theoretical" value given in italics at the third line of the table has been fixed as the normalization point for transitions with $|\Delta K|=2$, and its coincidence with the experimental value is therefore not significant. Instead, the comparison in the other lines *is* significant, and shows a satisfactory agreement with the predictions of the X(5) model, within the (admittedly large) experimental uncertainties. An apparent exception is the $4_3^+\rightarrow2_1^+$ tran-

TABLE III. Values of the ratio of the reduced strengths $B(E2;K_a,J_a \rightarrow K'_a,J'_a)/B(E2;K_b,J_b \rightarrow K'_b,J'_b)$ for transitions in ^{104}Mo between states with $s=1$ and $n_\gamma = K/2$.

Transition a	Transition b	$B(E2,a)/B(E2,b)$	
$J_a,K_aJ'_a,K'_a$	$J_b,K_bJ'_b,\ K'_b$	Exper.	Theor.
$4_1^+,0 \rightarrow 2_1^+,0$	$2_1^+,0 \rightarrow 0_1^+,0$	1.16(25)	1.60
$4_2^+,2 \rightarrow 2_1^+,0$	$4_2^+,2 \rightarrow 2_2^+,2$	0.029(5)[a]	0.019(2)
$6_2^+,2 \rightarrow 4_1^+,0$	$6_2^+,2 \rightarrow 4_2^+,2$	0.0076(8)	*0.0076(8)*[b]
$8_2^+,2 \rightarrow 6_1^+,0$	$8_2^+,0 \rightarrow 6_2^+,2$	0.0058(13)	0.0057(6)
$4_3^+,4 \rightarrow 2_1^+,0$	$4_3^+,4 \rightarrow 2_2^+,2$	0.012(2)	0.0[c]
$6_3^+,4 \rightarrow 4_2^+,2$	$6_3^+,4 \rightarrow 4_3^+,4$	0.0096(27)	0.0170(17)

[a]Or 0.010(3), according to the branching ratios given in Ref. [6].
[b]Normalization point for $|\Delta K| = 2$ transitions.
[c]Transition a not seen in another experiment [6].

sitions, which should be forbidden, having $\Delta n_\gamma = 2$. However, this branch, although reported [4] as resulting from the decay of ^{104}Nb (4.8 s) and included in the NNDC database, has not been observed in a more recent experiment [6] in which prompt γ rays of ^{104}Mo were observed, following the spontaneous fission of ^{248}Cm. One can add that none of the other $\Delta n_\gamma = 2$ transitions from the $K=4$ γ-band, although energetically favored with respect to the observed decays from the same parent state, has actually been observed.

In conclusion, another example of X(5) symmetry in atomic nuclei, in addition to ^{152}Sm, has been found in the Mo isotopes. In particular, the overall agreement of the ^{104}Mo level scheme with the X(5) prediction is at least as good as for the ^{152}Sm case. The two known γ bands of ^{104}Mo are also accounted for by X(5) and—as we have shown—something similar happens also for ^{152}Sm, where, however, only the lowest γ band is known. Also the comparison of transition strengths, in the few cases where it is possible, confirms the X(5) nature of ^{104}Mo.

Finally, we can notice that, at present, the best indications of X(5) symmetry concern an isotope of Sm ($Z=62$) and ^{104}Mo ($N=62$). Moreover, as already noted by Kern *et al.* [4], in the IBM scheme ^{152}Sm has six proton bosons and four neutron bosons (with respect to the shell closure at $N=82$), while ^{104}Mo has six neutron bosons and four proton-hole bosons (with respect to the shell closure at $Z=50$). Although we cannot exclude that this correspondence be accidental, we think it can be interesting to explore whether the number 62 of identical nucleons has some particular role in making the dependence of the potential energy on the deformation parameter β as flat, near to the origin, as required for the validity of the X(5) symmetry.

After completion of this work, new measurements of $E2$ strengths in ^{150}Nd were published [10]. Results are in good agreement with the predictions of the X(5) model.

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