Solution to a long-standing problem: $^{12}C(^{14}N,^{13}N)^{13}C_{1/2+}$

N. Keeley,* K. W. Kemper, and D. Robson
Department of Physics, Florida State University, Tallahassee, Florida 32306-4350
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The failure of finite-range distorted-wave Born approximation (DWBA) calculations to describe neutron transfer to $2s_{1/2}$ states in nuclei such as ^{13}C, ^{13}N, ^{17}O, and ^{27}Mg is a long-standing problem. Detailed coupled-channels Born approximation and coupled-reaction-channels (CRC) calculations failed to improve on the much simpler DWBA calculations. This work presents results suggesting that the vital step necessary to obtain agreement between CRC calculations and data in the case of the $2s_{1/2}$ state in ^{13}C is an increase in the radius of the potential binding the neutron to the ^{12}C core, leading to a considerable extension of the neutron radial wave function. The source of this extension is shown to arise from the deformation of the ^{12}C core.

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There has been a long history of the failure of calculations [distorted-wave Born approximation (DWBA), coupled-channels Born approximation (CCBA), and coupled-reaction-channels (CRC)] to describe neutron transfer to $2s_{1/2}$ states in nuclei such as ^{13}C, ^{13}N, ^{17}O, and ^{27}Mg. This problem was first noted by DeVries *et al.* [1] for the $^{12}C(^{14}N,^{13}N)^{13}C_{1/2+}$ transfer at a ^{14}N bombarding energy of 100 MeV, who reported that exact finite-range DWBA calculations were $\approx 5°$ out of phase with the data. Similar problems were found in DWBA analyses of the $^{12}C(^{10}B,^{9}Be)^{13}N_{1/2+}$ transfer at a ^{10}B bombarding energy of 100 MeV [2], the $^{12}C(^{14}N,^{13}C)^{13}N_{1/2+}$ transfer at a ^{14}N bombarding energy of 155 MeV [3], the $^{12}C(^{7}Li,^{6}Li)^{13}C_{1/2+}$ [4], and $^{16}O(^{7}Li,^{6}Li)^{17}O_{1/2+}$ [5] transfers, both at ^{7}Li bombarding energies of 34 MeV, and in CCBA calculations for the $^{26}Mg(^{7}Li,^{6}Li)^{27}Mg_{1/2+}$ transfer at a ^{7}Li bombarding energy of 44 MeV [6]. In the last three cases the measured angular distributions were shifted in phase with respect to the calculations and were relatively structureless while the calculations showed large oscillations.

The original reason for the emphasis on transfers to $s_{1/2}$ states was the realization that these transitions provide a very severe test of heavy-ion reaction models. When a particle is transferred from a $p_{1/2}$ ($p_{3/2}$) orbit in a projectile to any orbit other than an *s* orbit in the target, two (three) *l* transfers occur. These multiple *l* transfers cause the angular distributions to be unstructured so that they are unable to clearly distinguish between theoretical calculations with different underlying assumptions. The transfer to an *s* orbit can only proceed by $l=1$, which results in highly oscillatory angular distributions whose description has proven to be a challenge.

Extensive CCBA calculations by Nagel [7] for the 100 MeV $^{12}C(^{14}N,^{13}N)^{13}C_{1/2+}$ transfer, including couplings between the positive parity states of ^{13}C and transfer via the 2^+ excited state of ^{12}C, showed that these multistep processes were not responsible for the observed phase difference between the data and the calculations. Similarly, CCBA and CRC calculations for the 34-MeV $^{12}C(^{7}Li,^{6}Li)^{13}C_{1/2+}$ transfer [5] that included transfer via the $1/2^-$ first excited state of ^{7}Li were unable to provide a good description of the data. Kubo *et al.* [8] were able to obtain agreement with the data for transfer to many of the ''anomalous'' $1/2^+$ states by increasing the radius of the imaginary part of the exit channel distorting potential. However, this procedure had no effect on the predicted angular distribution for the $^{12}C(^{7}Li,^{6}Li)^{13}C_{1/2+}$ case and the physical meaning of this radius increase remained unclear.

In this work we focus on the 100-MeV $^{12}C(^{14}N,^{13}N)^{13}C_{1/2+}$ transfer data of DeVries *et al.* [1], who first brought this problem to light, and demonstrate that it is possible to obtain good agreement with these data if the radius of the potential binding the transferred neutron to the ^{12}C core is considerably increased compared to that usually used, leading to a significant extension of the neutron radial wave function. We show that this extension is a manifestation of the deformation of the underlying ^{12}C core. As further support for our conclusions, we also present similar calculations for the 34-MeV $^{12}C(^{7}Li,^{6}Li)^{13}C_{1/2+}$ data of Cook *et al.* [4], where a similar increase in the neutron binding potential radius also leads to an improvement in the agreement between the calculations and the data.

In Fig. 1(a) we show the results of two CRC calculations for the $^{12}C(^{14}N,^{13}N)^{13}C_{1/2+}$ transfer. All the calculations that follow were carried out using the code FRESCO [9], version FRXP.14. Both calculations included coupling to the 2^+ excited state of ^{12}C at 4.44 MeV using the conventional rotation-vibration model with a deformation length $\delta = -1.76$ fm, and transfer to the $1/2^+$ state of ^{13}C via the ground and 2^+ states of ^{12}C. No couplings to excited states of ^{14}N were included in the calculations. The optical potential in the entrance channel was that used by DeVries *et al.* [1] with the imaginary strength reduced to 20.0 MeV to take account of the coupling to the ^{12}C 2^+ state. The optical potential in the exit channel was that used by DeVries *et al.*, without alteration. The neutron was considered to be bound in a pure $2s_{1/2}$ state for the $^{12}C_{g.s.}/^{13}C_{1/2+}$ overlap and in a mixture of $1d_{5/2}$ and $1d_{3/2}$ states for the $^{12}C_{2+}/^{13}C_{1/2+}$ overlap. For the $^{13}N/^{14}N$ overlap the neutron was considered to be bound in a mixture of $1p_{1/2}$ and $1p_{3/2}$ states. The spectroscopic amplitudes for the $^{12}C/^{13}C$ overlap (0.938, 0.288, and 0.189 for the $0^+ \otimes 2s_{1/2}$, $2^+ \otimes 1d_{5/2}$, and $2^+ \otimes 1d_{3/2}$ compo-

*Electronic address: keeley@nucmar.physics.fsu.edu

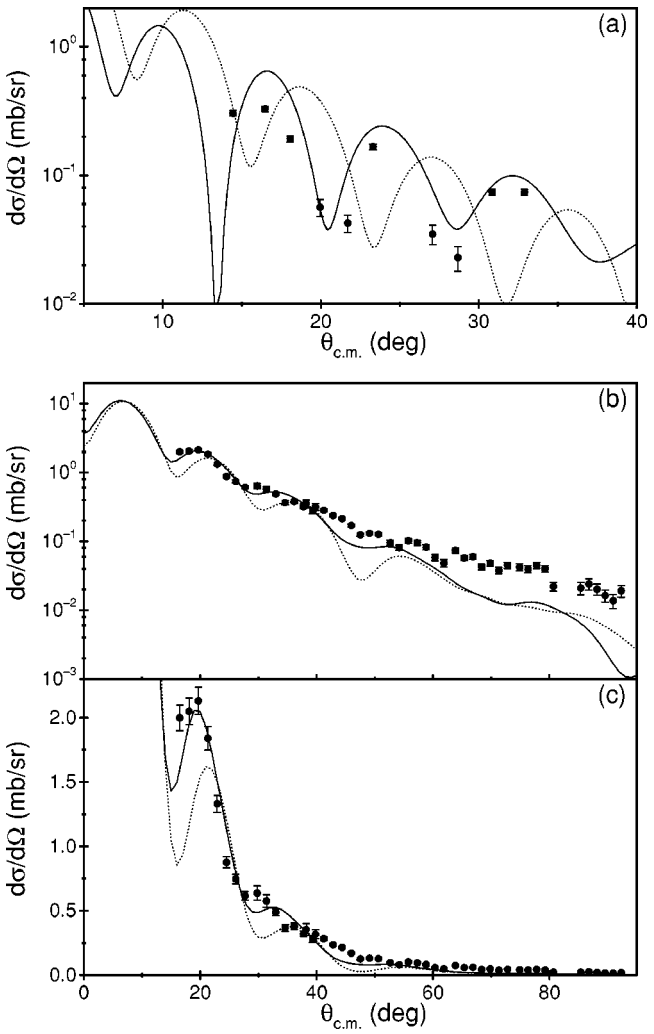

FIG. 1. The $^{12}C(^{14}N,^{13}N)^{13}C_{1/2+}$ transfer for a ^{14}N bombarding energy of 100 MeV (a) and the $^{12}C(^{7}Li,^{6}Li)^{13}C_{1/2+}$ transfer for a ^{7}Li bombarding energy of 34 MeV, on logarithmic (b) and linear (c) scales. The full curves denote the results of CRC calculations where the $^{12}C+n$ binding potential radius was set at $2.3\times12^{1/3}$ fm ($2.0\times12^{1/3}$ fm) for the ^{14}N (^{7}Li) projectile, while the dotted curves denote the result of similar CRC calculations with the neutron binding potential set at the conventional value of $1.25\times12^{1/3}$ fm.

nents, respectively) were obtained from the calculation using a deformed Woods-Saxon $n+^{12}C$ potential with $\beta_2=-0.65$ and $\beta_4=+0.5$ described below. The spectroscopic amplitudes for the $^{13}N/^{14}N$ overlap (0.8293 and 0.0632 for the $1p_{1/2}$ and $1p_{3/2}$ components, respectively) were taken from the calculations of Cohen and Kurath [10].

The dotted curve in Fig. 1(a) denotes the result of a CRC calculation as described above where the radius of the $^{12}C+n$ binding potential was set at the conventional value of $1.25\times12^{1/3}$ fm, while the solid curve denotes the result of an identical CRC calculation where this radius was increased to $2.3\times12^{1/3}$ fm (the elastic scattering given by both calculations is identical). This increase in binding potential radius results in an extended neutron wave function.

It may be seen from Fig. 1(a) that by increasing the radius of the $^{12}C+n$ binding potential, we are able to shift the oscillations in the calculated angular distribution for the $^{12}C(^{14}N,^{13}N)^{13}C_{1/2+}$ transfer to smaller angles, so that they correspond to the data. No such increase in the neutron binding potential radius is required for either the $1/2^-$ ground state or $5/2^+$ excited state of ^{13}C, as shown by the good description of these data by the DWBA calculations of DeVries *et al.* [1] with the conventional radius value of $1.25\times12^{1/3}$ fm.

To further pursue the extended neutron wave function of the ^{13}C 3.09 MeV $1/2^+$ state, CRC calculations were performed for neutron transfer to this state via the $^{12}C(^{7}Li,^{6}Li)^{13}C_{1/2+}$ reaction. Previous studies showed that the data are essentially nonoscillatory, whereas DWBA and CCBA calculations were highly oscillatory [4,5]. In addition, there is an angular shift between the data and calculations as found for the $^{12}C(^{14}N,^{13}N)$ case. The calculations were similar to those carried out for the $^{12}C(^{14}N,^{13}N)^{13}C_{1/2+}$ transfer. The optical potential in the entrance channel was set VII of Vineyard *et al.* [11], with the real and imaginary strengths reduced to 145.0 MeV and 6.8 MeV, respectively, to take account of the coupling to the ^{12}C 2^+ state. The $^{6}Li+^{13}C$ optical potential was taken from Table I of Schumacher *et al.* [12]. No couplings to excited states of ^{7}Li were included in the calculation. Single neutron stripping from the $3/2^-$ ground state of ^{7}Li was included, with the neutron considered to be bound in a mixture of $1p_{1/2}$ and $1p_{3/2}$ states. The spectroscopic amplitudes (-0.5378 and 0.6567 for the $1p_{1/2}$ and $1p_{3/2}$ components, respectively) were taken from Cohen and Kurath [10]. The $^{12}C/^{13}C$ overlap was treated in the same way as for the $^{12}C(^{14}N,^{13}N)^{13}C$ calculations.

Figures 1(b) and 1(c) show the results of two CRC calculations for the 34-MeV $^{7}Li+^{12}C$ system. Figure 1(c) shows the data and calculations on a linear scale to emphasize the agreement at forward angles, where the direct stripping mechanism dominates. The data are taken from Cook *et al.* [4]. The dotted curves show the result of a CRC calculation where the $^{12}C+n$ binding potential radius was set at the conventional value of $1.25\times12^{1/3}$ fm, while the full curves show the result for a similar calculation where this radius was increased to $2.0\times12^{1/3}$ fm.

As can be seen in Figs. 1(b) and 1(c), the CRC calculation (dotted curve) with a $^{12}C+n$ binding potential radius of $1.25\times12^{1/3}$ fm is rather more oscillatory than the data, and shifted in phase with respect to the peak in the data centered at $\theta_{c.m.}\approx18°$. By increasing the radius of the $^{12}C+n$ binding potential (the solid curve) we see that not only does the calculation now match the phase of the data, but also the oscillations are now considerably damped, better matching the data.

In order to provide a more quantitative understanding of this phenomenon, we have calculated neutron wave functions using a deformed Woods-Saxon potential for the $n+^{12}C$ system. The model has the potential given by

$$V(r,\Omega')=\frac{V_0}{1+\exp[(r-R)/a]},$$

wherein R is the radius parametrized by the multipole expansion

$$R(\Omega')=R_0\left\{1+\sum_{L>1}\beta_L\sum_K a_{LK}Y_{LK}(\Omega')\right\},$$

and R_0 and a are the spherical radius and diffuseness, respectively. The magnitude of the deformations for each multipole are given by β_L, and the coefficients of the spherical harmonics Y_{LK} are chosen to yield the shape of the nuclei being considered relative to the body fixed axes. The calculations here consider only the ground state and first 2^+ state of ^{12}C. In all deformed potential calculations we have adopted the conventional values of $R_0=1.25A^{1/3}$ fm and $a=0.65$ fm and assumed a rotational model is appropriate for ^{12}C.

For the $^{12}\mathrm{C}+n$ system the multipoles we need are $L=2$, 3, and 4. The calculations here are weakly sensitive to the $L=3, K=\pm 3$ multipoles and so we show results for only $L=2$ and $L=4$ with $K=0$ harmonics. The latter is consistent with the assumption of an equilateral triangle as used in cluster models [13]. The Nilsson model for the second $1/2^+$ state suggests that the $2s_{1/2}$ state in mass 13 would have a value for β_2 of about -0.6. It should be noted that even for a spherical well with conventional radius and diffuseness parameters the ^{13}C $1/2^+$ state has a radius considerably in excess of that for either the ^{12}C 0^+ or ^{13}C $1/2^-$ ground states. By utilizing a deformed $^{12}\mathrm{C}+n$ potential the existing extended nature of the $2s_{1/2}$ neutron wave function is considerably enhanced. The neutron form factors using a spherical Woods-Saxon potential with $R=2.0A^{1/3}$ fm, which are used in the CRC calculations, are very closely reproduced by the rotational model calculations when we use either $\beta_2=-0.65$ and $\beta_4=+0.5$ or $\beta_2=-0.65$ and $\beta_4=+0.85$. The upper panel of Fig. 2 shows the $2s_{1/2}$ neutron densities, $R^2(r)$, for the CRC calculations for $R=1.25A^{1/3}$ fm and $R=2.0A^{1/3}$ fm. The lower panel shows $R^2(r)$ for the ^{13}C $1/2^-$ ground state for both spherical and deformed potentials. The enhanced radius effect from the $R=2.0A^{1/3}$ fm case is clear and we conclude that the large quadrupole oblate deformation of the carbon system is the major reason for the enhancement of the neutron wave function radius. We also have investigated the effects of deformation for the $1/2^-$ ground state and the first $5/2^+$ state in ^{13}C and find relatively small changes from the conventional spherical case. The effect of deformation in the $2s_{1/2}$ case is large because the quadrupole coupling between the $0^+\otimes 2s_{1/2}$ and $2^+\otimes 1d$ configurations is strong in the outer surface region, whereas the $1p_{1/2}$ and $1d_{5/2}$ states have much weaker coupling.

In summary, we have presented CRC calculations for the $^{12}\mathrm{C}(^{14}\mathrm{N},^{13}\mathrm{N})^{13}\mathrm{C}_{1/2+}$ and $^{12}\mathrm{C}(^{7}\mathrm{Li},^{6}\mathrm{Li})^{13}\mathrm{C}_{1/2+}$ transfers, which are able to provide a good description of the data, in contrast to previous DWBA, CCBA, and CRC calculations. We have found that the essential step necessary to produce agreement with the phase of the data is to greatly increase the radius of the potential binding the neutron to the ^{12}C core. The other essential component needed to dampen the calculated oscillations is to include the transfer of a neutron onto the strongly coupled 2^+ state in ^{12}C as well as the ground state in forming the $1/2^+$ state of ^{13}C. We have shown that a quantitative understanding of the increased binding potential radius required for such states may be obtained by taking account of core deformation, which leads to a considerable enhancement of the neutron wave function radius obtained with a spherical core of conventional geometry. Thus, our conclusion is that for transfers to single particle states where the core is significantly deformed, the well-depth prescription with a spherical potential of conventional geometry is inadequate. Ideally, one should use a deformed binding potential in such cases, a procedure that is unfortunately not available in most standard coupled-channels and DWBA codes. However, we have demonstrated that the core deformation effect may be simulated by utilizing a spherical binding potential with a considerably enlarged radius.

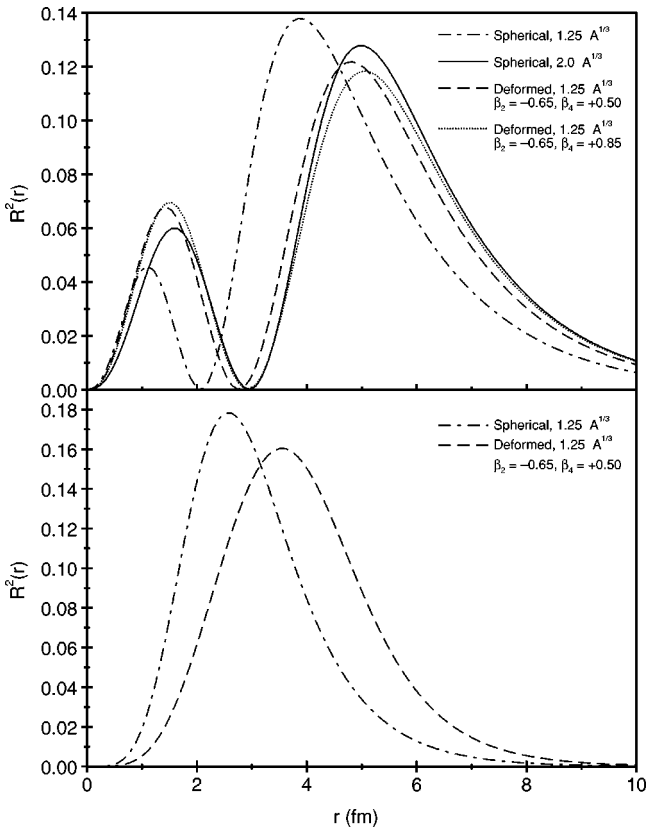

FIG. 2. Upper panel: $R^2(r)$ for the ^{13}C $1/2^+$ state plotted for various spherical and deformed potentials. Note the good agreement between $R^2(r)$ for the spherical potential of radius $2.0A^{1/3}$ and the deformed potentials, and the relative insensitivity to the exact value of β_4. Lower panel: $R^2(r)$ for the ^{13}C $1/2^-$ state plotted for spherical and deformed potentials.

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