# Fusion from an excited state

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We discuss the effects of intrinsic degrees of freedom on the barrier transmission probability of a macroscopic variable especially in the case when the intrinsic motion is initially in an excited state based on coupled-channels calculations. We analyze in detail the dependence of channel-coupling effects on the degree of adiabaticity, the properties of the intrinsic motion such as the number of coupled intrinsic states and the nature of either vibrational or rotational couplings, and the location of the coupling form factor. We show that significant transitions from excited states to the ground state take place in the low-energy region even in the cases where one would expect almost no nonadiabatic transitions because of the large excitation energy of the intrinsic motion. This suggests the fusion barrier to be renormalized by a tunneling-assisted intrinsic transition. We also analytically show that the potential renormalization by a fast environment crucially depends on whether it is initially in the ground or in the excited states and on the number of coupled intrinsic states.

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## I. INTRODUCTION

It is now well established that the coupling of the relative motion between the colliding nuclei to nuclear intrinsic degrees of freedom significantly enhances the cross section of heavy-ion fusion reactions at energies below the Coulomb barrier [1,2]. On the other hand, one of the current topics in nuclear physics is to clarify the screening effects by bound electrons in the target nucleus in low-energy nuclear reactions in laboratories [3].

A question that has not yet been fully explored is whether some peculiar effects arise if the intrinsic degrees of freedom are initially in an excited state. This corresponds, e.g., to the effects of transfer reactions with a positive $Q$ value on heavy-ion fusion reactions and to the screening effects in the case, where the electronic state is initially an excited state in the sense of the united system. The distribution of the intrinsic states after tunneling, i.e., the transition probability among the intrinsic states in the tunneling process, has also not been analyzed in detail, because so far almost all studies dealt with only the inclusive transmission probability. The transition properties, however, play an important role in determining the effective potential for the tunneling process such as the potential renormalization due to the screening effects.

The aim of this paper is to shed light on these questions based on the direct numerical solution of schematic coupled-channels equations. A simple schematic model is advantageous to analyze in detail the dependence of the effects of intrinsic degrees of freedom on various conditions such as the degree of adiabaticity, the properties of the intrinsic motion, and the location of the coupling form factor. These analyses will provide useful informations on realistic screening problems, and also on the fusion reactions induced by unstable nuclei [4–8].

The paper is organized as follows. In Sec. II, we briefly review the coupled-channels formalism. In Sec. III we discuss the numerical results of the effects of intrinsic degrees of freedom on the barrier transmission probability under various circumstances. In Sec. IV we discuss the transition properties of the intrinsic motion during the barrier transmission process. We show that the intrinsic motion makes a strong transition to the ground state to facilitate the quantum tunneling. We summarize the paper in Sec. V. We add an appendix to elucidate the properties of the potential renormalization by a fast environment, i.e., in the slow or adiabatic tunneling. To that end we use the influence functional method of the path integral formalism. This will help one to understand the numerical results in Sec. III that a fast environment hinders the barrier transmission probability for wide range of energies including the tunneling region if it starts initially from the excited state in the two-channels model, while it enhances in the three level problem irrespective of whether it starts initially from the ground state or the first excited state.

## II. COUPLED-CHANNELS FORMALISM

We consider a system consisting of a macroscopic variable $x$, which undergoes a quantum tunneling, and the other degrees of freedom $\xi$, which we call the intrinsic degrees of freedom. We assume that the total Hamiltonian is given by

$$H=\frac{\hat{p}^2}{2\mu}+U(x)+H_{in}(\xi)+V_c(x,\xi), \tag{1}$$

where $\hat{p}$ and $\mu$ are the conjugate momentum to $x$ and the mass, respectively, and $U(x)$ the bare potential barrier. The $H_{in}(\xi)$ is the Hamiltonian of the intrinsic degrees of freedom, and $V_c(x,\xi)$ the coupling Hamiltonian between $x$ and $\xi$. The $x$ corresponds to the coordinate of the relative motion between the target and projectile nuclei in both heavy-ion collisions and screening problem, while $\xi$ nuclear intrinsic degrees of freedom in heavy-ion collisions and the coordinates of electrons in the screening problem.

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We assume that the eigenfunctions and the corresponding eigenvalues of $H_{in}$ are known,

$$H_{in}|\phi_m\rangle = \epsilon_m|\phi_m\rangle, \tag{2}$$

and expand the total wave function on the basis of those eigenfunctions,

$$\psi(x,\xi) = \sum_m \chi_m(x)\phi_m(\xi). \tag{3}$$

We consider the case, where the coupling Hamiltonian is given by a product of two factors,

$$V_c(x,\xi) = f(x)\hat{M}(\xi), \tag{4}$$

where $f(x)$ is called the coupling form factor, and $\hat{M}$ is an operator in the space of intrinsic motions. The following coupled-channels equations then follow:

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dx^2} + U(x)\right]\chi_m(x) + \epsilon_m\chi_m(x) + f(x)\sum_{m'}\langle m|\hat{M}|m'\rangle\chi_{m'}(x) = E\chi_m(x). \tag{5}$$

We assume that both $U(x)$ and $f(x)$ are nonzero only for small values of $x$, and that we are interested in the barrier transmission probability when the $x$ variable impinges upon the potential barrier from $x=\infty$ towards $x=-\infty$. The coupled equations are then solved with the following boundary conditions:

$$\psi(x,\xi) \to \begin{cases} \displaystyle\sum_n t_n \frac{1}{\sqrt{k_n}} e^{-ik_n x}\phi_n(\xi) \quad (x \ll 0) & (6) \\ \displaystyle\sum_m \frac{1}{\sqrt{k_m}}[I_m e^{-ik_m x} + r_m e^{ik_m x}]\phi_m(\xi) \quad (x \gg 0), & (7) \end{cases}$$

where $k_m = \sqrt{2\mu(E-\epsilon_m)}/\hbar$. The amplitude of the incident wave $I_m$ is taken to be $\delta_{mm_0}$ if the intrinsic motion is initially in the state $\phi_{m_0}$. Once the transmission amplitudes $t_n$ are determined, the barrier transmission probability is given by

$$P = \sum_n |t_n|^2. \tag{8}$$

We assume throughout the paper that the bare potential and the coupling form factor are given by

$$U(x) = \frac{U_0}{\cosh^2(x/a)}, \quad f(x) = \frac{f_0}{\cosh[(x-x_f)/a_f]}. \tag{9}$$

Furthermore, we fix the values of $U_0$, $a$, $a_f$, and $\mu$ to be 10 MeV, 15 fm, 15 fm, and 2000 MeV/$c^2$, $c$ being the light velocity, respectively.

The effects of intrinsic motion can significantly differ in two opposite situations concerning adiabaticity [9–13]. In this connection, we denote the curvature of the bare potential barrier and the energy quanta of the intrinsic excitation, $\epsilon_2 - \epsilon_1$, as $\hbar\Omega$ and $\hbar\omega$, respectively, and call $\lambda = \omega/\Omega$ the adiabaticity parameter. The larger value of $\lambda$ corresponds to slower or more adiabatic tunneling or equivalently to faster intrinsic motion. Note, however, that the adiabaticity is influenced also by other parameters such as the strength of coupling [12].

## III. EFFECTS OF INTRINSIC MOTION ON THE BARRIER TRANSMISSION PROBABILITY

### A. Two-level model with channel coupling in the tunneling region

We first consider a two-level model that is given by the following coupling matrix

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{10}$$

The coupling matrix given by Eq. (10) appears to be the two-channel truncation of the vibrational coupling. However, as we show shortly, this model can lead to a very different effect from that of the genuine vibrational coupling when the initial state is the excited state. We further assume $x_f = 0$ to represent the channel coupling in the tunneling region.

Figure 1 shows the barrier transmission probability $P$ as a function of the incident kinetic energy $E_K$ for four representative values of adiabaticity. The dashed line is the transmission probability in the absence of the channel coupling, i.e., when $f_0$ is set to be zero. As it should be, the transmission probability is about 0.5 when the incident energy coincides with the height of the potential barrier, i.e., at $E_K = 10$ MeV.

The dot-dashed and solid lines are the barrier transmission probability when the coupling to the intrinsic motion is set on. The former and the latter correspond to the cases, where the intrinsic state is initially the ground state and the excited state, respectively.

The behavior of the dot-dashed line for the case when the intrinsic motion is initially in the ground state is rather familiar in the study of heavy-ion fusion reactions at energies near and below the Coulomb barrier in the past decades. The coupling to a fast intrinsic motion enhances the barrier transmission probability over the whole energy region as is shown for the case of $\lambda = \omega/\Omega = 2.0$. On the other hand, the coupling to slow intrinsic motion enhances the barrier transmission probability at low energies and hinders it at high energies. This trend can be seen for the cases of $\lambda = 0.25$, 0.5, and 1.0. As is also well known, this can be understood by considering the limit of degenerate spectrum, i.e., when $\omega = 0$, where the coupled equations can be decoupled into two one-dimensional problems with lower and higher potential barriers than the bare potential barrier [9]. This fact shows up in the barrier transmission probability even for finite values of $\omega$ in the form of the step-function-like increase at two energies. Note that the decoupling of the coupled-channels

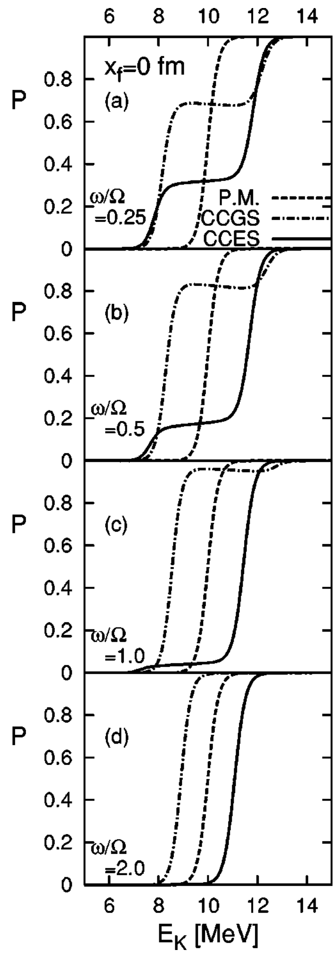


FIG. 1. The barrier transmission probability as a function of the incident kinetic energy in the case where $x_f = 0$.

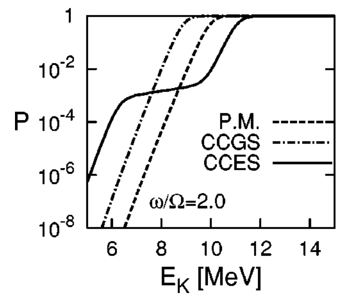


FIG. 2. The barrier transmission probability as a function of the incident kinetic energy in the case where $x_f = 0$ in the semilogarithmic plot.

equations in the limit of degenerate spectrum physically means that one can fix the intrinsic coordinate during the barrier transmission process in this limit. There exists a barrier for each of the two fixed values of the intrinsic coordinate [14].

Figure 1 shows that the coupling to intrinsic motion influences the barrier transmission probability qualitatively in the same manner in the sense that it enhances or hinders the transmission probability at low or high energies, respectively, when the intrinsic state is initially the excited state as far as the adiabaticity parameter is small. One should, however, notice also the difference in the dot-dashed and solid lines. They significantly differ in the amount of change of the barrier transmission probability $P$ at two energies where $P$ shows a step-function-like increase. This means that the fusion barrier distribution appears to be quite different depending on whether the intrinsic state is initially in the ground or in the excited states. In the degenerate spectrum limit, this is a natural consequence of the difference of the weight of the averaging procedure in the so-called zero-point motion formulas in these two cases [13].

A surprise occurs when the adiabaticity parameter $\lambda = \omega/\Omega$ is 2. The figure shows that the fast intrinsic motion hinders the barrier transmission probability for a wide energy region if the intrinsic motion is initially in the excited state. This contrasts with the enhancement in the case, where the intrinsic state is initially the ground state. The semilogarithmic plot, Fig. 2, shows that the hindrance changes to enhancement only at very low energies, where the barrier transmission probability is as small as $10^{-3}$. A more accurate expression of the results presented in Fig. 1 is to state that the barrier transmission probability gets more and more enhanced for wider energy region with increasing value of λ if the intrinsic motion is initially in the ground state. To the contrary, it gets more and more hindered for wider energy region with increasing λ if the intrinsic state is initially the excited state.

It would be worth comparing our results with those in Refs. [15,16], where the effects of channel coupling on the barrier transmission probability are discussed using a similar two-level model. We are especially interested in their results concerning the $Q$ value dependence of the channel-coupling effects. References [15,16] have pointed out that the barrier transmission probability as a function of the energy for negative $Q$ value coupling appears to be smoother than that for positive $Q$ value coupling. In other words, a larger enhancement of the barrier transmission probability is induced by a negative $Q$ value coupling than by a positive $Q$ value coupling at energies below the bare potential barrier. Since the cases of positive and negative $Q$ values in Refs. [15,16] correspond to the cases of the intrinsic motion being initially in the ground and in the excited states, respectively, in our study, this accords with our results shown in Figs. 1 and 2. Assuming a constant coupling form factor, Ref. [16] attributes this dependence of the channel coupling effect on the sign of the $Q$ value to the fact that the weights of the lower and higher effective potential barriers, which become relevant terms for the constant coupling model, interchange the dominance with the sign of the $Q$ value.

Returning to Figs. 1 and 2, it is important to notice that the barrier transmission probability initiated from the excited state eventually overwhelms that initiated from the ground state at very low energies when one decreases the energy from the barrier region. This implies that the effects of channel coupling with positive $Q$ values dominate at extremely low energies, and provides an explanation for an extra large enhancement of heavy-ion fusion cross section at very low energies in systems, where there exist transfer reaction channels with positive $Q$ values [17–21].

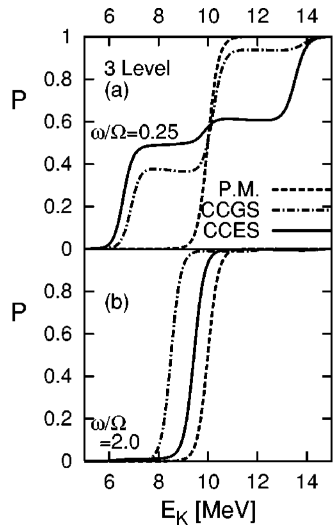


FIG. 3. The barrier transmission probability as a function of the incident kinetic energy in the case where the intrinsic motion has three levels.

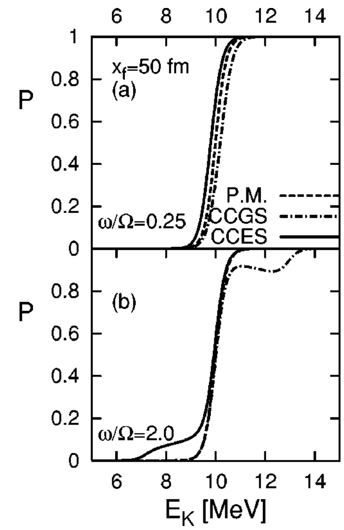


FIG. 4. The barrier transmission probability as a function of the incident kinetic energy in the case where $x_f = 50$ fm. In the lower panel, the dashed line is indistinguishable from the solid and the dot-dashed lines at high and low energies, respectively.

### B. Dependence on the dimension of the intrinsic states: Three-level vibrational model with channel coupling in the tunneling region

The result for the case of slow or adiabatic tunneling, i.e., for $\omega/\Omega = 2.0$, presented in Fig. 1(d) can be understood by a perturbation theory, which predicts a negative and a positive potential renormalization for the cases where the intrinsic state is initially the ground and excited states, respectively. In this connection, the two-level property of the model in the preceding section plays a crucial role in leading to the hindrance of the barrier transmission probability for the case starting from the excited intrinsic state. One can anticipate a quite different behavior if there are more levels and the excited state is allowed to make transitions not only to the ground state, but also to higher excited states. We show analytically in the Appendix that this is indeed the case.

In order to demonstrate this through a concrete example, we discuss in this section the results for the case, where the intrinsic motion has three levels. We assume the following coupling matrix:

$$M = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix}. \qquad (11)$$

The other parameters including $x_f$ are kept to be the same as those for Fig. 1. Though the levels are truncated at the third level, this coupling will mimic the vibrational coupling as long as the coupling is weak and in the case, where the intrinsic state is initially either in the ground state or in the first excited state.

The results are shown in Fig. 3 for two different values of $\omega/\Omega$. The meaning of the dashed and dot-dashed lines is the same as in Fig. 1. The solid line has been obtained by setting the intrinsic motion to be initially in the first excited state. Figure 3(b) shows that the coupling to the fast intrinsic motion now leads to enhancement over whole energy region irrespective of whether the intrinsic motion is initially in the ground or the excited states, and confirms our anticipation. The semilogarithmic plot shows that the barrier transmission probability starting from the excited state overwhelms that starting from the ground state at very low energies, though it is not clearly seen in the present linear scale plot. As for the coupling to slow intrinsic motion, the change from Fig. 1(a) to Fig. 3(a) is what one would expect. There now appear three energies where the transmission probability makes step-function-like increase reflecting three effective potential barriers in the limit of degenerate spectrum, or fast tunneling.

### C. Dependence on the coupling form factor: Two-level model with channel coupling outside the tunneling region

Besides the degree of adiabaticity and the dimension of the intrinsic states which we studied in the preceding sections, the location of the coupling form factor also governs the effects of the coupling to intrinsic motion on the barrier transmission probability. In order to demonstrate this aspect, we show in Fig. 4 the barrier transmission probability for two values of $\omega/\Omega$ for the case, where the coupling to intrinsic state is located outside the tunneling region. As an example, we assume $x_f = 50$ fm. The other parameters are the same as those for Fig. 1.

We observe in the figure that the barrier transmission probability is hindered over the whole energy region by the coupling to intrinsic motion if the intrinsic motion is initially in the ground state, while it is enhanced if the intrinsic state is initially excited state. This holds irrespective of the values of $\lambda = \omega/\Omega$ shown in Fig. 4. We have confirmed that the same conclusion holds for the other values of $\lambda$.

This result can be easily understood by considering the energy transfer from the intrinsic motion to the macroscopic motion. Since the coupling form factor in this section has been assumed to be situated in the region outside the potential barrier, the positive (negative) energy transfer leads di-

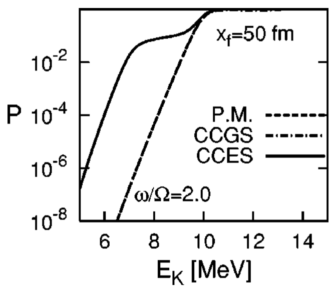


FIG. 5. The barrier transmission probability as a function of the incident kinetic energy in the case where $x_f = 50$ fm in the semilogarithmic plot.

rectly to the enhancement (hindrance) of the barrier transmission probability. This contrasts to the case for Fig. 1, where the coupling to the intrinsic motion takes place in the barrier region, thus leading to the renormalization of the potential barrier as well as the energy transfer between the intrinsic and the macroscopic spaces. The recognition of the simultaneous existence of the potential renormalization and the energy transfer is very important to properly assess the role of channel coupling in the barrier transmission probability, and is one of the crucial issues to settle down the debates concerning the role of breakup reactions in heavy-ion fusion reactions induced by unstable nuclei.

In passing, we remark that the solid and the dashed lines for the case of $\omega/\Omega = 2.0$ are almost parallel at low energies in the semilogarithmic plot suggesting the validity of the interpretation of the channel-coupling effects in terms of the energy transfer from the intrinsic to macroscopic spaces (see Fig. 5).

### D. Dependence on the property of the intrinsic motion: Two-level rotational model with channel coupling in the tunneling region

Another interesting situation is the case where the coupling matrix has a finite diagonal component,

$$M = \begin{pmatrix} 0 & 1 \\ 1 & d \end{pmatrix}. \tag{12}$$

Such a coupling matrix with $d = \frac{2}{7}\sqrt{5}$ is encountered when one discusses the effects of the rotational excitation of the deformed target on its fusion cross section with a spherical projectile at low energies [22]. In this case, the diagonal component describes the so-called reorientation effect and the sign of $f_0$ is also important, positive representing the prolate and negative the oblate deformations of the target nucleus, respectively [1,2]. In this section, we study the properties of the effects of intrinsic motion given by the coupling matrix Eq. (12) by assuming $d = \frac{2}{7}\sqrt{5}$.

Figure 6 shows the results for the case where $f_0 = 2$ MeV. All the other parameters are the same as those for Fig. 1. We see close similarities between the vibrational and rotational couplings shown in Figs. 1 and 6. However, when one compares the two cases carefully, one finds clear differences between them.

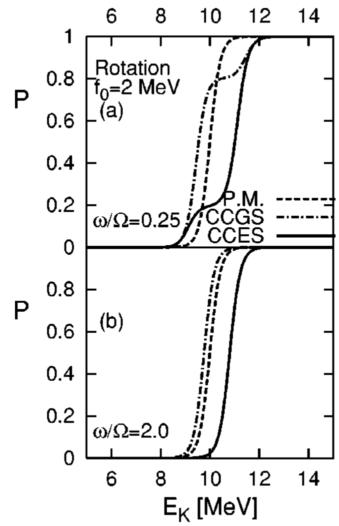


FIG. 6. The barrier transmission probability as a function of the incident kinetic energy in the case of rotational model with positive $f_0$.

Let us first compare the dot-dashed lines in Figs. 1(a) and 6(a). Both of them show step-function-like increase at two energies. A difference is that these two energies are located nearly symmetrically from the bare barrier position, i.e., 10 MeV, in Fig. 1(a), while fairly asymmetrically in Fig. 6(a). Also, the amount of increase of $P$ is more asymmetric in Fig. 6(a). These differences are associated with the reorientation term, i.e., the finite value of $d$ in Eq. (12), and have been used in the past decade in the so-called fusion barrier distribution analysis of heavy-ion fusion reactions at energies near and below the Coulomb barrier in order to identify the important nuclear intrinsic degrees of freedom. It is interesting to notice that these differences remain for the case, when the intrinsic state is initially in the excited state. Another important observation is that the unexpected hindrance of the barrier transmission probability in the case, where the initial state starts from the excited state, for $\omega/\Omega = 2$ coupling appears also for the rotational coupling. A difference of the rotational coupling from the vibrational coupling is that the deviation of the dot-dashed and the solid lines from the dashed line becomes visibly asymmetric.

Figure 7 shows the results for $f_0 = -2$ MeV. The change of the dot-dashed line from Fig. 6(a) to Fig. 7(a) is familiar in the barrier distribution analysis of heavy-ion fusion reactions. Considering the limit of extremely slow rotation, it can be understood in terms of the changes of the position and the weight of two effective barriers caused by the change of deformation from the prolate to oblate shapes. A novel phenomenon happens for the case of $\omega/\Omega = 2$. Contrary to the cases of vibrational coupling and the rotational coupling for the prolate shape shown in Figs. 1(d) and 6(b), respectively, the barrier transmission probability is enhanced over the whole energy region also for the case, where the intrinsic motion is initially in the excited state.

## IV. TRANSITION PROPERTIES OF THE INTRINSIC MOTION

Another issue that we wish to discuss in this paper is the transition properties of the intrinsic motion. To this end, we

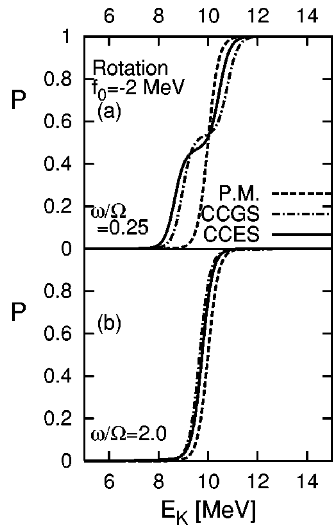


FIG. 7. The barrier transmission probability as a function of the incident kinetic energy in the case of rotational model with negative $f_0$.

introduce the notations $T(g\rightarrow e)$ and $T(g\rightarrow g)$ to represent the squared barrier transmission coefficients $|t_2|^2$ and $|t_1|^2$, respectively, in the case when the intrinsic state is initially the ground state, and $T(e\rightarrow e)$ and $T(e\rightarrow g)$, respectively, to represent them in the case when the intrinsic state is initially the excited state.

As an example, here we consider the case studied in Sec. III A. The results are shown in Fig. 8, which shows the ratios of the transmission probabilities in different channels $R_{TR} \equiv T(g\rightarrow e)/T(g\rightarrow g)$ and $T(e\rightarrow g)/T(e\rightarrow e)$ for two different cases of adiabaticity by the dashed and solid lines, respectively. In the case of $\omega/\Omega=2.0$, there is almost no transition over the whole energy region if the initial intrinsic state is the ground state. At high energies, there is almost no transition also in the case when the intrinsic state is initially the excited state. These can be attributed to the large excitation energy of the intrinsic motion. To the contrary, there occurs a very strong transition to the ground state at low energies, where the barrier transmission is dominated by the quantum tunneling, if the initial intrinsic state is the excited state. This is surprising, because the large value of the adiabaticity parameter $\lambda=\omega/\Omega$ would predict almost no nonadiabatic transition to take place in the tunneling energy region. This large transition is, however, natural, to enhance the tunneling probability. We name this phenomenon as the tunneling-assisted intrinsic transition.

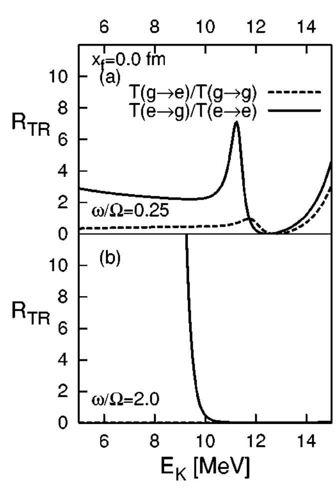


FIG. 8. Ratios of the transmission probabilities in different channels $T(g\rightarrow e)/T(g\rightarrow g)$ and $T(e\rightarrow g)/T(e\rightarrow e)$.

The situation for the case of $\omega/\Omega=0.25$ is less dramatic. However, one can still observe a large transition from the excited state to the ground state in the tunneling energy region when the intrinsic state is initially the excited state. The transition from the ground to excited states exists in this case also in the tunneling energy region. A large transition is predicted to occur at high energies irrespective of whether the intrinsic motion is initially in the ground state or excited state.

## V. SUMMARY

We have discussed the effects of intrinsic degrees of freedom on the barrier transmission probability of a macroscopic variable. We have compared their features in the cases where the intrinsic state is initially in an excited state and in the ground state under various circumstances. The most striking difference has been observed for an adiabatic, i.e., a slow, barrier transmission when the intrinsic motion has only two states and when the coupling form factor is located in the tunneling region. In this case, the barrier transmission probability is hindered for a wide range of energy if the intrinsic state is initially in the excited state contrary to the enhancement in the case where the initial intrinsic state is the ground state. We have shown also that this effect is intimately related to the two-dimensional property of the model, and disappears indeed when the two-dimensional model is replaced by a three-dimensional model.

We have then discussed the transition properties of the intrinsic state by taking a two-channel model with the coupling form factor in the tunneling region as an example. We have shown that the intrinsic state makes a very strong transition from the excited state to the ground state in the tunneling energy region even for the case where the energy scale of the intrinsic motion is as large as twice the barrier curvature, so that one would normally expect almost no nonadiabatic transition to occur. The transition properties for the other cases studied in Sec. IV are qualitatively similar. We will discuss the implications of these intrinsic transitions for the potential renormalization in a separate paper [23] based on the semiclassical mean-field theory of quantum tunneling [24,25].

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## APPENDIX: PROPERTIES OF THE POTENTIAL RENORMALIZATION BY A FAST ENVIRONMENT

In this appendix, we discuss the properties of the potential renormalization by a fast environment, i.e., in the slow or adiabatic tunneling of a macroscopic variable, which we call a translational motion.

### 1. Path integral representation of the inclusive barrier transmission probability

We start from the path integral representation of the inclusive barrier transmission probability [9]

$$P(E)=\lim_{\substack{x_i\to\infty\\ x_f\to-\infty}}\left(\frac{p_i p_f}{\mu^2}\right)\int_0^\infty dT e^{(i/\hbar)ET}\int_0^\infty d\tilde{T}\ \ e^{-(i/\hbar)E\tilde{T}}$$
$$\times\int\mathcal{D}[x(t)]\int\mathcal{D}[\tilde{x}(\tilde{t})]e^{(i/\hbar)[S_t(x,T)-S_t(\tilde{x},\tilde{T})]}$$
$$\times\rho(\tilde{x}(\tilde{t}),\tilde{T};x(t),T),\tag{A1}$$

where $S_t(x,T)$ is the bare action of the translational motion along a path $x(t)$,

$$S_t(x,T)=\int_0^T dt\left(\frac{1}{2}\mu\dot{x}(t)^2-U(x(t))\right).\tag{A2}$$

The effects of environment are described by the two-time influence functional defined by

$$\rho(\tilde{x}(\tilde{t}),\tilde{T};x(t),T)=\sum_{n_f}\ \langle n_i|\hat{u}^\dagger(\tilde{x}(\tilde{t}),\tilde{T})|n_f\rangle$$
$$\times\langle n_f|\hat{u}(x(t),T)|n_i\rangle,\tag{A3}$$

where $|n_i\rangle$ is the initial state of the environment, which evolves with time according to the time evolution operator satisfying

$$i\hbar\frac{\partial}{\partial t}\hat{u}(x,t)=[H_{in}(\xi)+V_c(x(t),\xi)]\hat{u}(x,t),\tag{A4}$$

along each path $x(t)$ with the initial condition

$$\hat{u}(x(t),t=t_i)=1.\tag{A5}$$

### 2. Influence functional and potential renormalization by a fast environment: Two-level model

What one needs is to determine the influence functional. To that end, we study the time evolution of the intrinsic state $\phi(t)=\hat{u}(x,t)\phi(t_i)$ instead of $\hat{u}(x,t)$. In the two-channel model that we considered in Sec. II, we put

$$\phi(\xi,t)=b_1(t)e^{-i\epsilon_1 t/\hbar}\begin{pmatrix}1\\0\end{pmatrix}+b_2(t)e^{-i\epsilon_2 t/\hbar}\begin{pmatrix}0\\1\end{pmatrix}.\tag{A6}$$

Assuming that the intrinsic motion is initially in the state $\phi_1$, we obtain up to the second order of the coupling Hamiltonian,

$$b_1(t)=1-\frac{1}{\hbar^2}\int_{t_i}^t dt_1 f(x(t_1))e^{-i\epsilon t_1/\hbar}$$
$$\times\int_{t_i}^{t_1}dt_2 f(x(t_2))e^{i\epsilon t_2/\hbar},\tag{A7}$$

$$b_2(t)=\frac{1}{i\hbar}\int_{t_i}^t dt_1 f(x(t_1))e^{i\epsilon t_1/\hbar},\tag{A8}$$

with $\epsilon=\epsilon_2-\epsilon_1$.

We perform the partial integration to the second integral in Eq. (A7) and the integral in Eq. (A8) in order to discuss the effects of a fast environment, and ignore all the terms including the ratio of $df/dt$ to $\omega=\epsilon/\hbar$ [9,10]. The results read

$$b_1(t)\sim 1+\frac{i}{\epsilon\hbar^2}\int_{t_i}^t dt_1[f(x(t_1))]^2,\tag{A9}$$

$$b_2(t)\sim-\frac{1}{\epsilon}f(x(t)).\tag{A10}$$

We have used that $f(x(t_i))=0$.

It is now straightforward to obtain the following expression of the influence functional:

$$\rho\sim e^{i\epsilon(\tilde{T}-T)/\hbar}\left(1-\frac{i}{\hbar\epsilon}\int_{t_i}^{\tilde{T}}d\tilde{t}_1[f(\tilde{x}(\tilde{t}_1))]^2+\cdots\right)$$
$$\times\left(1+\frac{i}{\hbar\epsilon}\int_{t_i}^{T}dt_1[f(x(t_1))]^2+\cdots\right).\tag{A11}$$

We have used the fact that the coupling is absent at $T$ and $\tilde{T}$. By exponentiating, we obtain

$$\rho\sim e^{i\epsilon_1(\tilde{T}-T)/\hbar}\exp\left(i\int_{t_i}^T dt\frac{1}{\epsilon}[f(x(t))]^2/\hbar\right)$$
$$\times\exp\left(-i\int_{t_i}^{\tilde{T}}d\tilde{t}\frac{1}{\epsilon}[f(\tilde{x}(\tilde{t}))]^2/\hbar\right).\tag{A12}$$

Inserting this result into Eq. (A1), we see that the effects of a fast environment can be represented by a static potential renormalization given by

$$\Delta V=-\frac{1}{\epsilon}[f(x)]^2.\tag{A13}$$

We can identify the initial state $\phi_1$ with the ground state. In that case, $\epsilon$ is positive, so that Eq. (A13) implies that the fast environment lowers the tunneling barrier. Alternatively, one can identify the initial state $\phi_1$ with the excited state. In that case, $\epsilon$ is negative, so that Eq. (A13) means that the fast environment makes the tunneling barrier higher. This explains why the barrier transmission probability has been hindered for a wide range of energy when the fast environment is initially in the excited state in the two-channel problem, as has been demonstrated in Fig. 1(d).

### 3. Three-level model

One can do the same study for a three-level model. We consider a general case, where the coupling matrix $M$ is given by

$$M=\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & \kappa \\ 0 & \kappa & 0 \end{pmatrix} \tag{A14}$$

and assume that $\epsilon_2-\epsilon_1=\epsilon_3-\epsilon_2=\epsilon$. If the intrinsic motion is initially in the second state, i.e., in the $\phi_2$ state, then the influence functional representing the effects of a fast environment is given by

$$\rho \sim e^{i\epsilon_2(\tilde{T}-T)/\hbar}\exp\left(-i(\kappa^2-1)\int_{t_i}^{T}dt\frac{[f(x(t))]^2}{-\epsilon}\Big/\hbar\right)$$
$$\times\exp\left(i(\kappa^2-1)\int_{t_i}^{\tilde{T}}d\tilde{t}\frac{[f(\tilde{x}(\tilde{t}))]^2}{-\epsilon}\Big/\hbar\right). \tag{A15}$$

The potential renormalization is now given by

$$\Delta V=(\kappa^2-1)\frac{[f(x)]^2}{-\epsilon}. \tag{A16}$$

Equation (A16) agrees with Eq. (A13) for $\kappa=\sqrt{2}$. This explains why the fast environment has a similar enhancement effect when it is initially in the first excited state in the three-level model as that when it starts from the ground state in the two-level model.


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