Isospin inversion, *n*-*p* interactions, and quartet structures in $N=Z$ nuclei

J. Jänecke,[1,*] T. W. O'Donnell,[1] and V. I. Goldanskii[2,†]
[1]*Department of Physics, University of Michigan, Ann Arbor, Michigan 48109-1120*
[2]*Institute of Physical Chemistry, RU-117334 Moscow, Russia*
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The differences between the excitation energies of isobaric analog states deduced from experimental data have been studied. (i) Isospin inversion is indicated for several nuclei with $N=Z=$odd up to $A=98$ based on experimental information including the systematics of the above energy differences. It is shown that the related approximate equality of the symmetry and pairing energies also extends to this mass region. A recently reported theoretical description of this behavior seems to involve a complex interplay between isoscalar and isovector pairing. (ii) An expression combining three of these excitation energy differences for nuclei with $N\approx Z$ displays residuals up to 8 MeV in the lightest nuclei only when nuclei with $N=Z=$ odd are included. These residuals appear to be related to a combination of isoscalar and isovector p-n interactions. However, other theoretical interpretations have also been reported. (iii) A ratio of excitation energy differences has been introduced. It provides a signature for shell-model or quartet structures in nuclei with $N\approx Z$. Shell-model behavior dominates over the entire range of nuclei except for a region in the fp shell with $A\approx 70$ to 90 where quartet structure is observed. This result is in agreement with the reported theoretical prediction of a coexistence of $T=0$ and $T=1$ nucleon pairs.

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I. INTRODUCTION

Recent theoretical publications [1–7] are concerned with $N\approx Z$ nuclei. Here, neutrons and protons occupy identical shell-model orbits. Questions of $T=1$ and $T=0$ pair interactions, i.e., isovector *and* isoscalar pairing, play an important role. Isospin inversion in odd-odd $N=Z$ nuclei, as discussed earlier [8–11], may be relevant. Pair interactions may also influence the presence of $T=0$ n-p structures (deuteron-like) and/or quartet structures (α-particle-like) in $N=Z$ nuclei. These and other questions have been addressed in the above and other theoretical papers. A mostly phenomenological approach to these questions has been taken recently [10,11]. Extensive experimental work has also concentrated in recent years on $N=Z$ nuclei by using radioactive nuclear beams [12–21]. Nuclei up to ^{100}Sn have now been investigated.

It is the purpose of the present work to again discuss $N=Z$ nuclei from a phenomenological point of view primarily with the use of energy relations. Questions of isospin inversion as well as signatures for n-p interactions and quartet structures will be addressed.

II. PROCEDURES

The experimental basis for this investigation is represented by the differences in the excitation energies $\Delta_{T',T}(A)$ between isobaric analog states with isospins T' and T in the same nucleus. These states are ground states or states analog to ground states in neighboring isobars. A global study of these energy differences has been undertaken over the entire range of atomic nuclei. Results will be reported elsewhere. The emphasis in the present work is on the nuclei with $N\approx Z$.

Experimental energy differences $\Delta_{T',T}(A)$ are directly available from the literature [22]. A much more comprehensive set can be obtained globally from the lightest to the heaviest nuclei from Coulomb-energy-corrected experimental masses [23] of neighboring isobars and the equations

$$\Delta_{T+1,T}(A)=M(A,T_z+1)-M(A,T_z)+\Delta E_C(\overline{Z},A)-M_n+M_H \quad \text{for} \quad T_z\geqslant 0, \tag{1}$$

$$\Delta_{T+1,T}(A)=M(A,T_z-1)-M(A,T_z)-\Delta E_C(\overline{Z},A)+M_n-M_H \quad \text{for} \quad T_z\leqslant 0. \tag{2}$$

These energies are essentially independent of neutron excess and therefore independent of T_z, the z component of isospin T. The two-parameter Eq. (8.98) in Ref. [9] was used for $\Delta E_C(\overline{Z},A)$ with $\overline{Z}=Z+0.5$. The quoted overall uncertainties are $\sim$84 keV but are increased by a factor of $\sim$3 for light nuclei. The simplest case of $\Delta_{T',T}(A)$ and characteristics of sums and ratios of such energies together with the related implications for nuclear structure properties will be discussed in the Secs. III, IV, and V.

III. ISOSPIN INVERSION

Figure 1 displays the energies $\Delta_{1,0}(A)$ obtained in the present work for self-conjugate $T_z=0$ nuclei as function of A. Negative energies indicate isospin inversion between the energetically lowest $T=0$ and $T=1$ states. As noted earlier [8–11] (see also Refs. [24,25]), the energy differences $\Delta_{1,0}(A)$ display strong oscillations as function of A with high values for $A=4n$ and low values for $A=4n+2$. This is due to the fact that symmetry energy contributions and pairing

*Email address: janecke@umich.edu
†Deceased.

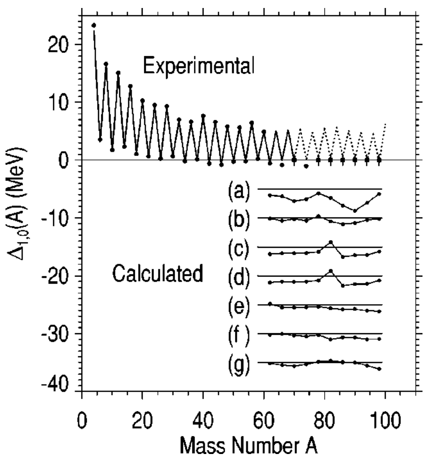

FIG. 1. Experimental [22] (filled circles) and estimated (filled squares; see text) energy differences $\Delta_{1,0}(A)$ for the even-even and odd-odd self-conjugate nuclei. Solid lines are deduced from the 1995 mass tabulation [23] (which includes estimated masses; dotted lines). Also shown are calculated values for $A \geqslant 62$ ($A=4n+2$ only and offset by multiples of 5 MeV) extracted from the following mass equations: (a) Ref. [27], (b) Ref. [28], (c) Ref. [29], (d) Ref. [30], (e) Ref. [31], (f) Ref. [32], (g) Ref. [33].

energies add or subtract [8,9]. In the context of the shell model, symmetry energies account for $T=0$ n-p pairing in identical orbits subject to the Pauli exclusion principle, while pairing energies account for $T=1 N-N$ pairing in identical orbits.

Beginning at $A \approx 26$ the odd-odd self-conjugate nuclei with $A=4n+2$ display almost perfect cancellation of the symmetry and pairing energy contributions resulting in small positive or negative values for the energies $\Delta_{1,0}(A)$. Isospin inversion for the $N=Z=$ odd nucleus ^{34}Cl has been known experimentally for many years. Inversion for heavier odd-odd self-conjugate nuclei up to $A=54$ was predicted [8] and observed. The nucleus ^{58}Cu does not display inversion and/or superallowed β decay. Instead, the excited $0^+ T=1$ state undergoes a γ transition to the $1^+ T=1$ ground state. For heavier nuclei only ^{62}Ga, ^{66}As, and ^{74}Rb appear to have been identified [12,14,15] as nuclei with isospin inversion. The respective energies are included in Fig. 1.

The reasons for the approximate equality of symmetry and pairing energies are not entirely clear and seemingly complex [7]. It is not known whether the approximate equality persists into the region of heavier nuclei or whether an imbalance will results in increased isospin inversion.

Contrary to ^{58}Ni, superallowed β decays have now been observed in all ten heavier odd-odd self-conjugate nuclei up to $A=98$ [19]. Therefore, the $0^+ T=1$ states in these nuclei must be either the ground states, hence isospin inversion, or lie at most ~ 100 keV above the energetically lowest $1^+ T=0$ states. These, however, are not necessarily the ground states because of competition with $T=0$ high-spin states [26]. The unknown experimental energies $\Delta_{1,0}(A)$ for $A>62$ were therefore conservatively estimated as 0.0 ± 1.0 MeV and included in Fig. 1. It follows from these results that the approximate equality of symmetry and pairing energies extends up to at least $A=100$.

Independently, energy differences $\Delta_{1,0}(A)$ have also been deduced using Eq. (1) and the 1995 mass tabulation [23]. They are included in Fig. 1 as solid (and dashed) lines. Isospin inversion is indicated only for a few select values of A. Similarly, given the fact that the energies $\Delta_{1,0}(A)$ can be expressed as sums and differences of symmetry and pairing energies, isospin inversion is subject to the simple condition $a(A,0)/P(A,0)-A<0$ [8] which yields essentially the same values. It is concluded that nuclei with $A=$ 70, 94, and 98 are additional candidates for isospin inversion.

The quantity $a(A,T)$ in the above inequality is the symmetry-energy coefficient in the equation $E_{\text{sym}} = [a(A,T)/A]T(T+1)$, and $P(A,T)$ is the pairing energy. Using a similar inequality, isospin inversion between $T=1$ and $T=2$ states may also occur for $108 \leqslant A \leqslant 124$, but this prediction has not been verified yet.

Predictions for $\Delta_{1,0}(A)$ were also obtained based on several mass equations [27–33]. The results for the heavier odd-odd self-conjugate nuclei with $A \geqslant 62$ are included in Fig. 1. Significant isospin inversion is predicted for most mass equations. Only procedure (g) suggests more limited inversion in agreement with the above results.

Several considerations have shown that the approximate equality in $N=Z$ nuclei between the symmetry energy (reflecting on $T=0$ $n-p$ pairing in identical orbits subject to the Pauli exclusion principle) and the pairing energy (reflecting on $T=1 N-N$ pairing in identical orbits) persists up to at least $A=100$. Only recently [7] has an attempt been made, apparently for the first time, in an extended mean-field model to explain small values for $\Delta_{1,0}(A)$ for nuclei with $N=Z$ $=$odd by invoking both isovector *and* isoscalar pairing. Here, a complex interplay is observed between quasi-particle excitations relevant for the $T=0$ states and isorotations relevant for the $T=1$ states. The calculations were carried out up to $A=74$ and seem to suggest a slow inversion of the sign of $\Delta_{1,0}(A)$ beginning in the $f_{7/2}$ shell near $A=50$ reflecting upon the different mass dependence of the symmetry energy and the $T=0$ pairing correlations. Further experimental and theoretical work seems desirable.

IV. NEUTRON-PROTON INTERACTIONS

It appears that certain mass relations may provide information about the characteristics of n-p interactions including $T=0$ $n-p$ or deuteronlike structures. The transverse Garvey-Kelson mass relation [34–36] represents the difference of effective neutron-proton interactions I_{np} between neighboring isobars [8]. It is valid within uncertainties for all basis nuclei with A and $T_z \geqslant 0$ except for $A=4n+2$ and $T_z=0$.

The transverse Garvey-Kelson mass relation can be separated into nuclear and Coulomb contributions [8,9]. The former becomes

$$\Delta_{T+2,T}(A) - \Delta_{T+3/2,T+1/2}(A-1) - \Delta_{T+3/2,T+1/2}(A+1) \approx 0. \tag{3}$$

Figure 2 displays the residuals from this equation for $T=0$ and $A=4n$, where they are distributed about zero, and for $A=4n+2$, where they are clearly different from zero.

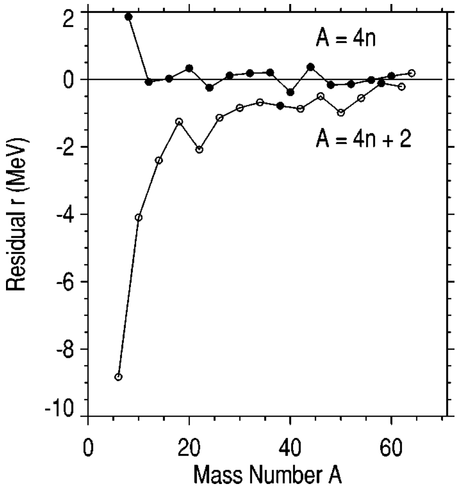

FIG. 2. Residuals $r \equiv \Delta_{2,0}(A) - \Delta_{3/2,1/2}(A-1) - \Delta_{3/2,1/2}(A+1)$ as function of A for nuclei with $A=4n$ and $A=4n+2$, respectively. Data are shown as filled circles (experimental) and open circles [deduced from experimental masses and Eq. (1)].

Unbound nuclei make the residuals less reliable for $A=$ 6, 8, and 10. The residuals for $A=4n+2$ increase with increasing mass numbers of $A=6$ to 60 from about -8 MeV to zero. Since Coulomb energies cancel, the above residuals from the nuclear contributions are almost indistinguishable from those of the transverse Garvey-Kelson mass relation provided that in the case of isospin inversion in $T_z=0$ nuclei the energies of the excited $T=0$ states are used.

As shown schematically in Fig. 3, the transverse Garvey-Kelson mass relation can be represented using simple fourfold degenerate Hartree-Fock or Nilsson-like single particle levels [9,33,35] which are occupied by at most two protons and two neutrons in the lowest energy states. All single-particle energies and contributions from interacting nucleon pairs in the same and different orbits essentially cancel as shown for $T_z=0$ reference nuclei with $A=4n$.

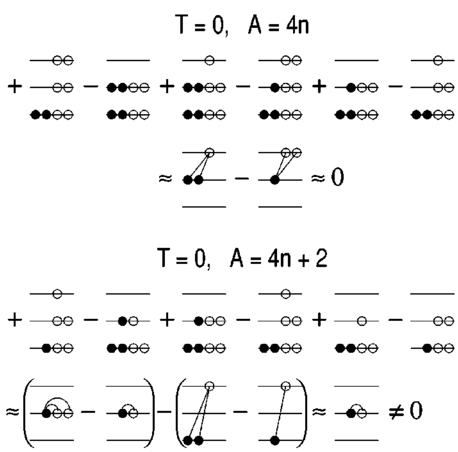

FIG. 3. Schematic representation with fourfold degenerate Hartree-Fock or Nilsson-like single-particle levels of the residuals of the transverse Garvey-Kelson mass relation. The reference nuclei for the two cases are $N=Z=$even and odd, respectively. Filled and open circles describe protons and neutrons, respectively. Lines connecting protons and neutrons represent *n-p* interactions.

However, for $T_z=0$ nuclei with $A=4n+2$ there is indeed an imbalance due to the fact that the *n-p* interaction is stronger between particles in the same level than it is when the particles are in different levels. While this fact has been pointed out earlier [35], the possible connection with $T=0$ *n-p* interactions, which is currently of great interest, has not been established then. It follows from Fig. 3 that the two contributions in the nucleus $(N+1,Z)$, presumably an approximately equal combination of *n-p* interactions with $T=1$ and $T=0$, are not cancelled by the single contribution in the nucleus (N,Z) which must be a $T=0$ *n-p* interaction. The symbolically shown residual *n-p* interaction in Fig. 3 therefore seems to represent approximately equal $T=1$ and $T=0$ contributions. The decrease in the magnitude of the effect with increasing A would be due to reduced *n-p* overlap n heavier nuclei.

In another simple shell-model approach one may apply jj coupling to a single shell. Using the lowest-seniority shell model with isospin [35,37] there is an energy difference between the two odd-odd nuclei in the mass relation. This energy difference results from a difference in pairing energies [the quantity $\kappa-\lambda$ in Eq. (16) of Ref. [37]]. These two terms represent the centroid energies of the degenerate sets of states of seniority $v=2$ with $J=$even $(\neq 0)$ and $J=$odd, respectively. Additional corrections come for $J=$odd from the difference of ground state minus centroid energies. The contributions which are not cancelled are therefore from the nuclei (N,Z) and $(N+2,Z-2)$.

It must be pointed out, though, that the two approaches discussed above seem to be incompatible with each other because the effect is due to different combinations of nuclei. An improved theoretical description of the observed residuals is therefore desirable. It is furthermore not quite clear how the effect is related to the so-called "Wigner energy." Certain mass equations, such as equations based on liquid-drop-type theories [38] or mean field theories, do not contain the experimentally observed linear dependence on isospin T in the symmetry energy, a term which is crucial to describe nuclear masses for $N \approx Z$. A phenomenological Wigner energy has therefore been introduced [38]. This term has its theoretical origin in the Wigner supermultiplet theory [39]. This theory generates two relevant terms [see, e.g., Eq. (8.130) in Ref. [9]], namely, a term linear in isospin T in the expression for the symmetry energy proportional to $T(T+4)$, but also a term which applies only to $N=Z=$odd nuclei. Both of these terms combined represent the Wigner energy [see Eq. (2) in Ref. [40]]. However, only the second term can be related to the residuals observed in the present work since a linear dependence on isospin satisfies the Garvey-Kelson relation. It should be noted that shell-model theories [41,42] also contain a linear dependence on isospin in the symmetry-energy term $T(T+1)$.

V. QUARTET STRUCTURES

Whereas shell-model behavior dominates most light nuclei, recent theoretical work [4] suggests quartet or α-particle-like structures in heavier $N=Z$ nuclei in the region of the fp shell with $A=76-96$. Here, coexistence of

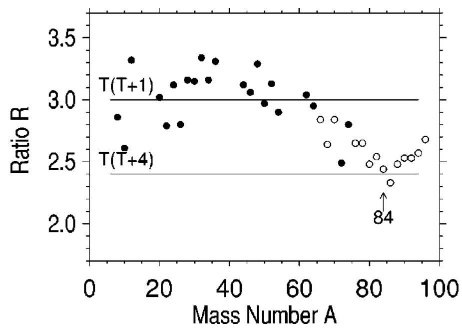

FIG. 4. Ratio R of energy differences $\Delta_{T',T}(A)$ (see text) displayed as function of A. Open circles indicate data points which include estimated mass values. The horizontal lines represent the shell-model and quartet-model predictions, respectively. Averaged values are given in Table I.

$T=0$ and $T=1$ pairs is predicted. The experimental data can be tested with regard to this prediction by establishing whether the curvature of the mass surface near $N=Z$ as determined by the symmetry energy follows a dependence on isospin T given by $T(T+1)$, expected in shell-model approaches [41,42], or by $T(T+4)$, expected in the Wigner supermultiplet formalism [39], suggesting quartet structures.

Ratios of energies $\Delta_{T',T}(A)$ provide information about the quantity x in $E_{\mathrm{sym}}=[a(A,T)/A]T(T+x)$. A ratio which permits the determination of this quantity is

$$R(A)\equiv\frac{\Delta_{2,0}(A)}{\langle\Delta_{1,0}(A)\rangle_{\mathrm{av}}}$$

with

$$\langle\Delta_{1,0}(A)\rangle_{\mathrm{av}}=\frac{1}{4}[\Delta_{1,0}(A-2)+2\Delta_{1,0}(A)+\Delta_{1,0}(A+2)]. \tag{4}$$

This ratio is independent of pairing energy $P(A,T)$ and becomes $R=(4+2x)/(1+x)$. The method is very sensitive to determine the value of x and hence shell-model or quartet-structure behavior in nuclei with $N\approx Z$.

Figure 4 displays this ratio R as function of mass number A up to $A=96$. The calculations for the heavier nuclei include estimated mass values [23]. Only nuclei within major shells are included which explains the gaps at $A=16$, 40, and 58. As expected, the data show some scattering for the lightest nuclei where nuclear structure effects are more pronounced. For nuclei up to $A=60$ the ratio R is compatible with the shell-model predictions. Surprisingly, the ratio R decreases in the region from $A\approx70$ to 90 and approaches near $A=84$ the values expected for quartet structures. The averaged ratios given in Table I show these characteristics even more clearly.

The results shown in Fig. 4 permit a comparison between experiment and theory. The observed compatibility with the shell model in the light nuclei appears to contradict results

TABLE I. Averaged ratios R of energy differences $\Delta_{T',T}(A)$ for nuclei near $N=Z$ as indicators for the dependence of symmetry energy on isospin (see text and Fig. 4).

A range	Shell region	R [a]
8−12	p shell	2.93±0.35
20−36	ds shell	3.09±0.19
44−54	$f_{7/2}$ shell	3.08±0.14
62−96	fp shell	2.64±0.19
80−88	partial fp shell	2.45±0.08

[a] $R=3.00$ for shell structure, $T(T+1)$; $R=2.40$ for quartet structure $T(T+4)$.

described earlier [43,44]. Here, compatibility with the supermultiplet model is reported which is open to question.

However, the compatibility observed in the present work with quartet structures for nuclei with $N\approx Z$ in the upper fp shell in the region $A=70$ to 90 is in very good agreement with recent theoretical predictions [4] (see also Ref. [3]). Here, based on isospin generated BCS and Hartree-Fock-Bogoliubov equations, a coexistence between $T=1$ and $T=0$ Cooper pairs is indicated.

VI. CONCLUSIONS

Primary and secondary experimental data have been used to study energy differences $\Delta_{T',T}(A)$ between isobaric analog states. A brief account of this work has been given earlier [45].

An examination of the energy differences $\Delta_{1,0}(A)$ between the energetically lowest $T=1$ and $T=0$ states in $T_z=0$ nuclei combined with available experimental information on superallowed β decays suggests the approximate equality of the symmetry and the pairing energies up to at least $A=100$. Isospin inversion in some nuclei with $N=Z=$ odd up to $A=98$ is likely, and several candidates for isospin inversion are given. A recent seemingly first attempt to explain these characteristics using an extended mean-field model [7] shows that a complex interplay between isovector and isoscalar pairing excitations has to be invoked.

The study of an expression combining three energy differences $\Delta_{T',T}(A)$ based on the transverse Garvey-Kelson mass relation shows systematic residuals when odd-odd self-conjugate nuclei are included. An interpretation of this effect based on a model using fourfold degenerate Hartree-Fock or Nilsson-like single-particle levels suggests about equal contributions of both $T=1$ and $T=0$ n-p interactions. However, a shell-model approach using the seniority scheme with isospin suggests a connection with the energies of pairs coupled to seniority $v=2$ in the odd-odd members in the relation. These two approaches seem to be incompatible. An improved theoretical understanding of the observed effect related to odd-odd self-conjugate nuclei including the connection with the Wigner energy is desirable.

The investigation of certain ratios of energy differences $\Delta_{T',T}(A)$ provides information about the dependence on isospin of the symmetry energy for nuclei with $N \approx Z$. This makes it possible to establish the dominance of shell-model structures or quartet (or α-particle-like) structures. It was found that the lower shells up to $A=60$ display shell-model behavior. However, in the upper region of the fp shell centered at $A \approx 84$ quartet structures becomes dominant. The latter result is in good agreement with theoretical predictions [4] where coexistence between $T=0$ and $T=1$ is reported.

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