Four-body cluster structure of $A=7-10$ double-Λ hypernuclei

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Energy levels of the double-Λ hypernuclei ${}_{\Lambda\Lambda}^{7}$He, ${}_{\Lambda\Lambda}^{7}$Li, ${}_{\Lambda\Lambda}^{8}$Li, ${}_{\Lambda\Lambda}^{9}$Li, ${}_{\Lambda\Lambda}^{9}$Be, and ${}_{\Lambda\Lambda}^{10}$Be are predicted on the basis of an $\alpha+x+\Lambda+\Lambda$ four-body model, where $x=n,p,d,t,{}^3$He, and α, respectively. Interactions between the constituent particles are determined so as to reproduce reasonably the observed low-energy properties of the $\alpha+x$ nuclei (^{5}He, ^{5}Li, ^{6}Li, ^{7}Li, ^{7}Be, ^{8}Be) and the existing data for Λ-binding energies of the $x+\Lambda$ and $\alpha+x+\Lambda$ systems (${}_{\Lambda}^{3}$H, ${}_{\Lambda}^{4}$H, ${}_{\Lambda}^{5}$He, ${}_{\Lambda}^{6}$He, ${}_{\Lambda}^{6}$Li, ${}_{\Lambda}^{7}$Li, ${}_{\Lambda}^{8}$Li, ${}_{\Lambda}^{8}$Be, ${}_{\Lambda}^{9}$Be). An effective $\Lambda\Lambda$ interaction is constructed so as to reproduce, within the $\alpha+\Lambda+\Lambda$ model, the $B_{\Lambda\Lambda}$ of ${}_{\Lambda\Lambda}^{6}$He, which was extracted from the emulsion experiment, the NAGARA event. With no adjustable parameters for the $\alpha+x+\Lambda+\Lambda$ system, $B_{\Lambda\Lambda}$ of ground and bound excited states of the double-Λ hypernuclei with $A=7-10$ are calculated within the Gaussian-basis coupled-rearrangement-channel method. The *Demachi-Yanagi* event, identified recently as ${}_{\Lambda\Lambda}^{10}$Be, is interpreted as an observation of its 2^+ excited state on the basis of the present calculation. Structural changes in the $\alpha+x$ core nuclei, due to the interaction of the Λ particles, are found to be substantial, and these play an important role in estimating the $\Lambda\Lambda$ bond energies of those hypernuclei.

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I. INTRODUCTION

A recent observation of the double-Λ hypernucleus ${}_{\Lambda\Lambda}^{6}$He, which is called the NAGARA event in the KEK-E373 experiment [1], should have a great impact on both the study of baryon-baryon interactions in the strangeness $S=-2$ sector and on the study of dynamics of many-body systems with multistrangeness. The importance of this event derives from complete identification of all elements of the decay process and the precise experimental value of the $\Lambda\Lambda$-binding energy $B_{\Lambda\Lambda}=7.25\pm0.19\pm^{0.18}_{0.11}$ MeV [1], which leads to a smaller $\Lambda\Lambda$ binding, $\Delta B_{\Lambda\Lambda}=1.01\pm0.20\pm^{0.18}_{0.11}$ MeV, than the previous value of $\Delta B_{\Lambda\Lambda}=4.6\pm0.5$ MeV [3]. Sometimes emulsion events involve ambiguities related to the difficulty of identifying the emission of neutral particles such as neutrons and γ rays. In the NAGARA event, however, the production of ${}_{\Lambda\Lambda}^{6}$He has been uniquely identified, free from such an ambiguity, on the basis of the observed sequential weak decays.

Historically, in the 1960s, there appeared two reports on the observation of double-Λ hypernuclei, ${}_{\Lambda\Lambda}^{10}$Be [2], and ${}_{\Lambda\Lambda}^{6}$He [3], but the validity of the latter case was considered doubtful [4]. Two decades later the modern emulsion-counter hybrid technique has been applied in the KEK-E176 experiment [5], in which a new double-Λ hypernucleus event was found but no unique identification has been made so far: One explanation has that ${}_{\Lambda\Lambda}^{10}$Be leads to a repulsive $\Lambda\Lambda$ interaction ($\Delta B_{\Lambda\Lambda}<0$), while the other possibility involving ${}_{\Lambda\Lambda}^{13}$B leads to an attractive $\Lambda\Lambda$ interaction [6,7]. In the latter case, the extracted strength of the $\Lambda\Lambda$ interaction is attractive with $\Delta B_{\Lambda\Lambda}\simeq4$ MeV. Although the latter option seems consistent with the old data of ${}_{\Lambda\Lambda}^{10}$Be [2], the substantially attractive $\Lambda\Lambda$ interaction has not been convincing.

In strangeness nuclear physics, the most fundamental problem is to understand the different facets of interactions among the octet of baryons (N, Λ, Σ, Ξ) in a unified way. Our detailed knowledge of the $S=0$ NN sector is based on the rich database of NN scattering. Recent studies for $S=-1$ many-body systems such as Λ hypernuclei have clarified interesting features of the ΛN and ΣN interactions in spite of scarce data for the free-space scattering. On the other hand, for the baryon-baryon interactions with the $S=-2$ sector, there is presently no experimental information from two-body scattering experiments. Therefore, the observed $\Lambda\Lambda$ bond energies of double-Λ hypernuclei should be able to provide the only reliable source of information on the S

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$=-2$ interaction, and such data play a pivotal role in determining the strength of the underlying $\Lambda\Lambda$ interactions.

In view of this, the NAGARA event is an epoch-making one, which provides us with a new basis for modeling the $\Lambda\Lambda$ interaction and understanding all other double-Λ hypernuclei. In recent years several experiments have been performed to produce $S=-2$ systems (E176 and E373 at KEK, E885 and E906 at BNL), and some of the data analyses are still in progress. Their goal is to obtain novel information on the $S=-2$ interactions.

Stimulated by this exciting experimental situation, we have performed careful theoretical calculations of double-Λ hypernuclei from a new viewpoint. We think it is timely to make the NAGARA data for the ${}_{\Lambda\Lambda}^{\ 6}$He binding energy a new basis for a systematic study of double-Λ species. In order to utilize the extracted $\Lambda\Lambda$ interaction, we emphasize that hypernuclear calculations should be complete and sufficiently realistic to make structural ambiguity as negligible as possible. All the dynamic changes due to Λ interaction should be fully taken into account. To satisfy these requirements we explore light p-shell double-Λ hypernuclei ($A=6$–10) comprehensively using microscopic three- and four-body models. As a result of these systematic, realistic calculations, we will give reliable predictions of both the ground-state binding energies and possible excited-state energies, which should encourage double-Λ hypernuclear spectroscopic studies in the near future.

So far several cluster models have appeared to estimate the ground-state binding energies of double-Λ species: Based on the earlier data of ${}_{\Lambda\Lambda}^{\ 6}$He and ${}_{\Lambda\Lambda}^{10}$Be, Takaki *et al.* [8] applied a simplified version of the $\alpha+x+\Lambda+\Lambda$ cluster model to $A=6$–10 systems in which they imposed several angular-momentum restrictions and neglected rearrangement channels. Bodmer and co-workers [9,10] performed variational Monte Carlo calculations for $\alpha+\Lambda+\Lambda$ and $\alpha+\alpha+\Lambda+\Lambda$ to investigate consistency between the $\Lambda\Lambda$-binding energies, $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{\ 6}\mathrm{He})$ and $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{10}\mathrm{Be})$, although using the earlier data. In the latter stage of this work, we encountered the Faddeev-Yakubovsky calculations for ${}_{\Lambda\Lambda}^{\ 6}$He and ${}_{\Lambda\Lambda}^{10}$Be by Filikhin and Gal [11] who restricted the equations to the s wave. They compared their results with our previous cluster-model calculation [13] which was performed with a wider model space but a stronger $\Lambda\Lambda$ interaction strength. In our previous work [13], $\Lambda\Lambda$ binding energies were calculated for ${}_{\Lambda\Lambda}^{\ 6}$He and ${}_{\Lambda\Lambda}^{10}$Be within the framework of the $\alpha+\Lambda+\Lambda$ three-body model and the $\alpha+\alpha+\Lambda+\Lambda$ four-body model, respectively, where the adopted $\Lambda\Lambda$ interaction is taken to be considerably attractive on the basis of the traditional interpretation for the earlier double-Λ events.

In the present work, using a $\Lambda\Lambda$ interaction based upon the NAGARA datum, we extend this four-body model to more general cases consisting of $\alpha+x+\Lambda+\Lambda$ systems with $x=n,p,d,t,{}^3$He, and α (${}_{\Lambda\Lambda}^{\ 7}$He, ${}_{\Lambda\Lambda}^{\ 7}$Li, ${}_{\Lambda\Lambda}^{\ 8}$Li, ${}_{\Lambda\Lambda}^{\ 9}$Li, ${}_{\Lambda\Lambda}^{\ 9}$Be, and ${}_{\Lambda\Lambda}^{10}$Be), where nuclear core parts are quite well represented by $\alpha+x$ cluster models (see, for example, Ref. [14]). We emphasize that extensive calculations are presented for the first time for $A=7$–9 double-Λ species and that the old predictions for ${}_{\Lambda\Lambda}^{\ 6}$He and ${}_{\Lambda\Lambda}^{10}$Be have been updated using the same model assumptions.

The four-body calculations are accurately performed by using the Jacobian-coordinate Gaussian-basis method of Refs. [15–21] with all the rearrangement channels taken into account (see Ref. [21] for a test of the high accuracy of the method in the case of a four-nucleon bound state with a realistic NN force). In our model, structure changes of nuclear cores caused by adding one and two Λ particles are treated precisely. Namely, we take into account the rearrangement effects on $\Lambda\Lambda$ bond energies induced by changes in nuclear cores. It is worthwhile to point out that the important effects of core excitations and core rearrangement are lacking in the frozen-core approximation used often for calculations of double-Λ hypernuclei.

In our model, it is possible to determine the αx and Λx interactions so as to reproduce all the existing binding energies of subsystems ($\alpha+x$, $x+\Lambda$, $\alpha+x+\Lambda$, and $\alpha+\Lambda+\Lambda$) in an $\alpha+x+\Lambda+\Lambda$ system, where that of $\alpha+\Lambda+\Lambda$ is determined by the NAGARA event. This feature is important in an analysis of the energy levels of double-Λ hypernuclei and in any prediction of the $\Lambda\Lambda$ bond energies, because the ambiguities of NN and ΛN effective interactions are renormalized by phenomenologically fitting the observed binding energies of subsystems. Our analysis is performed systematically for ground and bound excited states of the series of $\alpha+x+\Lambda+\Lambda$ systems with no more adjustable parameters in this stage, so that these predictions offer important guidance for the interpretation of the upcoming double-Λ experiments and the level structure which should be observed.

In Sec. II, the microscopic $\alpha+x+\Lambda+\Lambda$ four-body model calculational method is described. In Sec. III, the interactions are introduced. Calculated results are presented and discussed in Sec. IV. A summary is given in Sec. V.

II. MODEL AND METHOD

In Ref. [13], the present authors studied ${}_{\Lambda\Lambda}^{\ 6}$He and ${}_{\Lambda\Lambda}^{10}$Be using an $\alpha+\Lambda+\Lambda$ three-body model and an $\alpha+\alpha+\Lambda+\Lambda$ four-body model, respectively. In the same manner, we study in this work the double-Λ hypernuclei ${}_{\Lambda\Lambda}^{\ 7}$He, ${}_{\Lambda\Lambda}^{\ 7}$Li, ${}_{\Lambda\Lambda}^{\ 8}$Li, ${}_{\Lambda\Lambda}^{\ 9}$Li, ${}_{\Lambda\Lambda}^{\ 9}$Be, and ${}_{\Lambda\Lambda}^{10}$Be using as a basis the $\alpha+x+\Lambda+\Lambda$ four-body model with $x=n,p,d,t,{}^3$He, and α, respectively. The $d,t({}^3$He) and α clusters are assumed to be inert having the $(0s)^2$, $(0s)^3$, and $(0s)^4$ shell-model configurations, respectively, and are denoted by $\Phi_s(x)$ with spin s $(=1,\frac{1}{2}$ or 0, respectively).

All nine sets of the Jacobian coordinates of the four-body systems are illustrated in Fig. 1 in which we further take into account the antisymmetrization between the two Λ particles and the symmetrization between two α clusters when $x=\alpha$. The total Hamiltonian and the Schrödinger equation are given by

$$H=T+\sum_{(a,b)} V_{ab}+V_{\mathrm{Pauli}}, \tag{2.1}$$

$$(H-E)\Psi_{JM}({}_{\Lambda\Lambda}^{A}\mathrm{Z})=0, \tag{2.2}$$

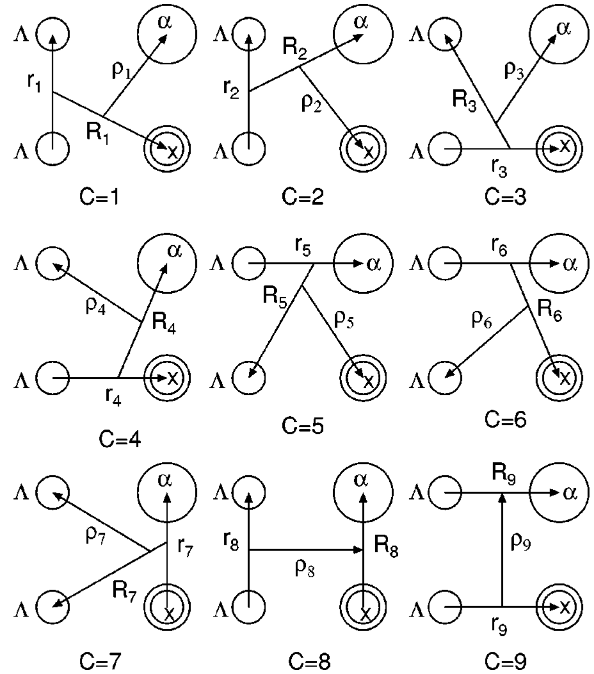

FIG. 1. Jacobian coordinates for all the rearrangement channels ($c=1$–9) of the $\alpha+x+\Lambda+\Lambda$ four-body system. Two Λ particles are to be antisymmetrized, and α and x are to be symmetrized when $x=\alpha$.

where T is the kinetic-energy operator and V_{ab} is the interaction between the constituent particle pair a and b. The Pauli principle between the nucleons belonging to α and x clusters is taken into account by the Pauli projection operator $V_{\rm Pauli}$, which is explained in the next section, as well as V_{ab}. The total wave function is described as a sum of amplitudes of the rearrangement channels ($c=1$–9) of Fig. 1 in the LS coupling scheme:

$$\Psi_{JM}({}^{\,A}_{\Lambda\Lambda}\mathrm{Z})=\sum_{c=1}^{9}\sum_{n,N,\nu}\sum_{l,L,\lambda}\sum_{S,\Sigma,I,K}C^{(c)}_{nlNL\nu\lambda S\Sigma IK}$$
$$\times\mathcal{A}_\Lambda\mathcal{S}_\alpha[\Phi(\alpha)\{\Phi_s(x)\ \ [\chi_{1/2}(\Lambda_1)\chi_{1/2}(\Lambda_2)]_S\}_\Sigma$$
$$\times\{[\phi^{(c)}_{nl}(\mathbf{r}_c)\psi^{(c)}_{NL}(\mathbf{R}_c)]_I\xi^{(c)}_{\nu\lambda}(\boldsymbol{\rho}_c)\}_K]_{JM}. \qquad (2.3)$$

Here the operator $\mathcal{A}_\Lambda$ stands for antisymmetrization between the two Λ particles, and $\mathcal{S}_\alpha$ is the symmetrization operator for exchange of α clusters when $x=\alpha$. $\chi_{1/2}(\Lambda_i)$ is the spin function of the ith Λ particle. Following the Jacobian-coordinate coupled-rearrangement-channel Gaussian-basis variational method of Refs. [15–21], we take the functional form of $\phi_{nlm}(\mathbf{r})$, $\psi_{NLM}(\mathbf{R})$, and $\xi^{(c)}_{\nu\lambda\mu}(\boldsymbol{\rho}_c)$ as

$$\phi_{nlm}(\mathbf{r})=r^l\,e^{-(r/r_n)^2}Y_{lm}(\hat{\mathbf{r}}),$$
$$\psi_{NLM}(\mathbf{R})=R^L\,e^{-(R/R_N)^2}Y_{LM}(\hat{\mathbf{R}}),$$
$$\xi_{\nu\lambda\mu}(\boldsymbol{\rho})=\rho^\lambda e^{-(\rho/\rho_\nu)^2}Y_{\lambda\mu}(\hat{\boldsymbol{\rho}}), \qquad (2.4)$$

where the Gaussian range parameters are chosen to lie in geometrical progressions:

$$r_n=r_1a^{n-1}\quad(n=1-n_{\max}),$$
$$R_N-R_1A^{N-1}\quad(N=1-N_{\max}),$$
$$\rho_\nu=\rho_1\alpha^{\nu-1}\quad(\nu=1-\nu_{\max}). \qquad (2.5)$$

These basis functions have been verified to be suitable for describing both short-range correlations and the long-range tail behavior of few-body systems [15–21]. The eigenenergy E in Eq. (2.2) and the coefficients C in Eq. (2.3) are determined by the Rayleigh-Ritz variational method.

For the angular-momentum component of the wave function, the approximation with $l,L,\lambda\leqslant 2$ was found to be sufficient to obtain in getting satisfactory convergence of the binding energies for the states concerned. Note that no truncation of the *interactions* is made in the angular-momentum space. As for the numbers of the Gaussian basis, $n_{\max}$,$N_{\max}$, and $\nu_{\max}$, 4–10 are enough.

In so far as the single-Λ hypernuclei ${}^{6}_{\Lambda}$He, ${}^{6}_{\Lambda}$Li, ${}^{7}_{\Lambda}$Li, ${}^{8}_{\Lambda}$Li, ${}^{8}_{\Lambda}$Be, and ${}^{9}_{\Lambda}$Be are concerned, the wave functions are described by Eq. (2.3) but with one Λ particle omitted. As for the core nucleus itself, $\alpha+x$, the wave function is given by

$$\Psi_{JM}(\alpha+x)=\sum_{n,l}C_{nl}\mathcal{S}_\alpha\Phi(\alpha)[\Phi_s(x)\phi_{nl}(\mathbf{r})]_{JM}. \qquad (2.6)$$

III. INTERACTIONS

In the study of double-Λ hypernuclei based on the $\alpha+x+\Lambda+\Lambda$ four-body model, it is absolutely necessary to examine, before the four-body calculation, whether the model with the interactions adopted is able to reproduce reasonably well the following observed quantities: (i) energies of the low-lying states and scattering phase shifts of the $\alpha+x$ nuclear systems, (ii) B_Λ of hypernuclei composed of $x+\Lambda$, x being $d,t,{}^3$He, α, (iii) B_Λ of hypernuclei composed of $\alpha+x+\Lambda$, x being $n,p,d,t,{}^3$He, α, and (iv) $B_{\Lambda\Lambda}$ of ${}^{\;\,6}_{\Lambda\Lambda}$He $=\alpha+\Lambda+\Lambda$. We emphasize that these severe constraints were successfully met in the present model as mentioned below. This encourages us to perform the four-body calculations, with no adjustable parameters at this stage, expecting high reliability of the results.

A. Pauli principle between α and x clusters

The Pauli principle between nucleons belonging to α and x clusters is taken into account by the orthogonality condition model (OCM) [22]. The OCM projection operator $V_{\rm Pauli}$ is represented by

$$V_{\rm Pauli}=\lim_{\lambda\to\infty}\lambda\sum_f|\phi_f(\mathbf{r}_{\alpha x})\rangle\langle\phi_f(\mathbf{r}'_{\alpha x})|, \qquad (3.1)$$

which rules out the amplitude of the Pauli-forbidden $\alpha-x$ relative states $\phi_f(\mathbf{r}_{\alpha x})$ from the four-body total wave function [23]. The forbidden states are $f=0S$ for $x=n(p)$, $f=\{0S,0P\}$ for $x=d$, $f=\{0S,1S,0P,0D\}$ for $x=t({}^3$He), and $f=\{0S,1S,0D\}$ for $x=\alpha$. The Gaussian range parameter b of the single-particle $0s$ orbit in the α particle is taken to be $b=1.358$ fm so as to reproduce the size of the α particle. The same size is assumed for clusters $x=d,t$, and ^{3}He, to manage the Pauli principle while avoiding calculational difficulty. In the actual calculations, the strength λ for $V_{\rm Pauli}$ is taken to be 10^5 MeV, which is large enough to push the unphysical forbidden states into the very high energy region while keeping the physical states unchanged. Usefulness of this Pauli operator method of OCM has been verified in many cluster-model calculations.

In some calculations [9–12,24] of three-body systems including two or three α clusters, use is made of an $\alpha\alpha$ potential with a strong repulsive core [25] so as to describe the Pauli exclusion role which prevents the two α clusters from overlapping. But, it is well known [27] that this approximate prescription of the Pauli principle is not suited for the case where the presence of the third particle makes the two α clusters come closer to each other; in other words, the off-energy-shell behavior of the repulsive potential is not appropriate in the three-body system. As for $\alpha-N$ interaction, the same thing was pointed out by Ref. [26] in the study of the binding energies of ^{6}He and ^{6}Li. Moreover, there is no available potential reported for the αx systems ($x=d,t$, and ^{3}He) of this type. Therefore, we do not employ this prescription in the present systematic study of the structure change of the αx systems due to the addition of Λ particles. We take the orthogonality condition model instead, which is suited even for the case of heavy overlapping between the two clusters.

B. αx interactions

Regarding the potentials $V_{\alpha x}$ between the clusters α and x, we employ those which have been often used in the OCM-based cluster-model study of light nuclei. Namely, they are the $V_{\alpha N}$ potential introduced in Ref. [28], the $V_{\alpha d}$ and $V_{\alpha t}$ potentials given in Ref. [14], and the $V_{\alpha\alpha}$ potential used in Ref. [29], which reproduce reasonably well the low-lying states and low-energy scattering phase shifts of the αx systems. The potentials are described in the following parity-dependent form with the central and spin-orbit terms:

$$V_{\alpha x}(r)=\sum_{i=1}^{i_{\max}} V_i e^{-\beta_i r^2}+\sum_{i=1}^{i'_{\max}}(-)^l V_i^{\rm p} e^{-\beta_i^{\rm p} r^2} +\left[\sum_{i=1}^{i''_{\max}} V_i^{ls} e^{-\gamma_i r^2}+\sum_{i=1}^{i'''_{\max}}(-)^l V_i^{ls,{\rm p}} e^{-\gamma_i^{\rm p} r^2}\right]\boldsymbol{l}\cdot\boldsymbol{s}_x, \tag{3.2}$$

where $\boldsymbol{l}$ is the relative angular momentum between α and x, and $\boldsymbol{s}_x$ is the spin of x. In the $\alpha\alpha$ system the spin-orbit term is missing and the odd wave is forbidden by the Pauli principle. The additional Coulomb potentials are constructed by folding the pp Coulomb force into the proton densities of the α and x clusters. The parameters in Eq. (3.2) are listed in Table I (we slightly modified the strength of the central force in $V_{\alpha d}$ and that of the spin-orbit force in $V_{\alpha t}$ to obtain better agreement with the energy levels of ^{6}Li and ^{7}Li, respectively).

TABLE I. Parameters of (a) the $\alpha\alpha$ interaction, (b) the αt (Λ^3He) interaction, (c) the αd interaction, and (d) the αN interaction defined in Eq. (3.2). Size parameters are in fm^{-2} and strengths are in MeV. The $^1S_0$ scattering length is -0.575 fm and the effective range is 6.45 fm.

	(a) $\alpha\ \alpha$ interaction		
i	1	2	3
β_i	0.1111	0.2777	0.3309
V_i	−1.742	−395.9	299.4
$V_i^{\rm p}$	0.0	0.0	0.0
	(b) $\alpha t(\alpha^3$He) interaction		
i	1	2	3
β_i	0.0913	0.1644	0.2009
V_i	6.9	−43.35	−51.7
$\beta_i^{\rm p}$	0.0913	0.1644	0.2009
$V_i^{\rm p}$	6.9	43.35	−51.7
γ_i	0.28		
V_i^{ls}	−1.2		
$\gamma_i^{\rm p}$	0.28		
$V_i^{ls,{\rm p}}$	1.2		
	(c) $\alpha\ d$ interaction		
i	1		
β_i	0.2		
V_i	−64.21		
$\beta_i^{\rm p}$	0.2		
$V_i^{\rm p}$	−10.21		
γ_i	0.3		
V_i^{ls}	−4.0		
$\gamma_i^{\rm p}$	0.3		
$V_i^{ls,{\rm p}}$	−4.0		
	(d) $\alpha\ N$ interaction		
i	1	2	3
β_i	0.36	0.9	
V_i	−96.3	77.0	
$\beta_i^{\rm p}$	0.2	0.53	2.5
$V_i^{\rm p}$	34.0	−85.0	51.0
γ_i	0.396	0.52	2.2
V_i^{ls}	−20.0	−16.8	20.0
$\gamma_i^{\rm p}$	0.396	2.2	
$V_i^{ls,{\rm p}}$	6.0	−6.0	

C. Λx interactions

We derive the interaction between the Λ particle and the x cluster by folding the G-matrix-type hyperon-nucleon (YN) interaction (the YNG interaction) into the density of the x cluster in the same manner as our previous work on double-Λ hypernuclei [13]. The YNG interactions between Λ and N are derived from the YN one-boson exchange (OBE) models as follows: First the G-matrix equation is solved in nuclear matter at each $k_{\rm F}$, where the so-called QTQ prescription [30] is adopted for simplicity. Next the resulting G matrix is simulated by a three-range Gaussian form with the strengths as a function of $k_{\rm F}$. The obtained *YNG* interactions are given in Ref. [30] as

$$v_{\Lambda N}(r;k_F)=\sum_{i=1}^{3}\left[(v_{0,\mathrm{even}}^{(i)}+v_{\sigma\sigma,\mathrm{even}}^{(i)}\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_N)\frac{1+P_r}{2}+(v_{0,\mathrm{odd}}^{(i)}+v_{\sigma\sigma,\mathrm{odd}}^{(i)}\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_N)\frac{1-P_r}{2}\right]e^{-\mu_i r^2}, \tag{3.3}$$

where P_r is the space exchange (Majorana) operator. The strengths $v^{(i)}$ are represented as quadratic functions of $k_{\rm F}$; see Eq. (2.7) of Ref. [30] and Table V of Ref. [13] for various original *YN* interactions. In the present work, we employ the Nijmegen model D interaction (ND).

The Λx interaction is derived by folding the above $v_{\Lambda N}(r;k_F)$ interaction into the x-cluster wave function. The $k_{\rm F}$ depends on the mass number of the cluster x. Because of the operator P_r in Eq. (3.3), the resultant Λx potential becomes nonlocal, the explicit form of which is given in the Appendix of Ref. [13]. We summarize the functional form of the local and nonlocal parts of the Λx potentials as

$$V_{\Lambda x}(\mathbf{r},\mathbf{r}')=\sum_{i=1}^{3}(V_i+V_i^{\rm s}\mathbf{s}_\Lambda\cdot\mathbf{s}_x)e^{-\beta_i r^2}\delta(\mathbf{r}-\mathbf{r}')+\sum_{i=1}^{3}(U_i+U_i^{\rm s}\mathbf{s}_\Lambda\cdot\mathbf{s}_x)e^{-\gamma_i(\mathbf{r}+\mathbf{r}')^2-\delta_i(\mathbf{r}-\mathbf{r}')^2}, \tag{3.4}$$

where $\mathbf{s}_\Lambda=\boldsymbol{\sigma}_\Lambda/2$. Table II lists the parameters in Eq. (3.4) for the (a) $\Lambda\alpha$ interaction, (b) $\Lambda t(\Lambda^3{\rm He})$ interaction, and (c) Λd interaction. They were determined in the following manner:

(i) $\Lambda\alpha$ *interaction*. The ΛN spin-spin part vanishes when folded into the α particle. The odd-force contribution is negligible in the Λ-binding energy of ${}^5_\Lambda$He. We determined the $k_{\rm F}$ parameter as $k_{\rm F}=0.925$ fm^{-1} in order to reproduce this binding energy (3.12 MeV) within the $\alpha+\Lambda$ two-body model. The ΛN odd force having the same $k_{\rm F}$ was determined by tuning the magnitude of $v_{0,\rm odd}^{(3)}$ so as to reproduce, within the $\alpha+\alpha+\Lambda$ model, the Λ-binding energy of the $1/2^+$ ground state of ${}^9_\Lambda$Be.

(ii) Λd *interaction*. We determined the value of $k_{\rm F}=0.84$ fm^{-1} by fitting the experimental Λ-binding energy of the $1/2^+$ ground state of ${}^3_\Lambda$H within the $d+\Lambda$ model where the ΛN odd force plays a negligible role. The odd force was determined, retaining the same $k_{\rm F}$, by reproducing the Λ-binding energies of the $1/2_1^+$ and $3/2_1^+$ states of ${}^7_\Lambda$Li within the $\alpha+d+\Lambda$ model; we tuned $v_{0,\rm odd}^{(2)}$ and $v_{\sigma\sigma,\rm odd}^{(2)}$.

(iii) Λt *interaction*. The experimental Λ-binding energies of the 0^+ and 1^+ states of ${}^4_\Lambda$H were used to determine the even force of the ΛN interaction. The magnitudes of $k_{\rm F}$ and $v_{\sigma\sigma,\text{ even}}^{(2)}$ were adjusted to reproduce the energies, $k_{\rm F}$ being 0.84 fm^{-1}. This value of $k_{\rm F}$ was substituted into the $k_{\rm F}$ used in the odd force of the ΛN interaction of the Λd interaction with no other change. The resulting Λt interaction reproduces, by chance, the Λ-binding energy of the 1^+ ground state of ${}^8_\Lambda$Li within the $\alpha+t+\Lambda$ model; the calculated energy is 6.80 MeV while the observed one is 6.80 ± 0.03 MeV.

TABLE II. Parameters of (a) the $\Lambda\alpha$ interaction, (b) the $\Lambda t(\Lambda^3{\rm He})$ interaction, and (c) the Λd interaction defined in Eq. (3.4). Size parameters are in fm^{-2} and strengths are in MeV.

	(a) $\Lambda\alpha$ interaction		
i	1	2	3
β_i	0.2752	0.4559	0.6123
V_i	−17.49	−127.0	497.8
$V_i^{\rm s}$	0.0	0.0	0.0
γ_i	0.1808	0.1808	0.1808
δ_i	0.4013	0.9633	2.930
U_i	−0.3706	−12.94	−331.2
$U_i^{\rm s}$	0.0	0.0	0.0
	(b) $\Lambda t(\Lambda^3{\rm He})$ interaction		
i	1	2	3
β_i	0.2874	0.4903	0.6759
V_i	−14.16	−108.0	425.9
$V_i^{\rm s}$	2.379	10.91	−126.9
γ_i	0.2033	0.2033	0.2033
δ_i	0.3383	0.8234	2.521
U_i	−0.2701	−9.553	−231.6
$U_i^{\rm s}$	−0.2615	1.433	97.05
	(c) Λd interaction		
i	1	2	3
β_i	0.3153	0.5773	0.8532
V_i	−10.84	−88.36	167.2
$V_i^{\rm s}$	2.734	14.35	−179.9
γ_i	0.2710	0.2710	0.2710
δ_i	0.2470	0.4870	1.924
U_i	−0.1862	−5.844	−3.065
$U_i^{\rm s}$	−0.2705	1.566	100.4

D. ΛN interaction in ${}^{\ 7}_{\Lambda\Lambda}$He (${}^{\ 7}_{\Lambda\Lambda}$Li)

In the study of ${}^{\ 7}_{\Lambda\Lambda}$He (${}^{\ 7}_{\Lambda\Lambda}$Li) within the $\alpha+N+\Lambda+\Lambda$ model and of the subsystem ${}^6_\Lambda$He (${}^6_\Lambda$Li) within the $\alpha+N+\Lambda$ model, it is inappropriate to use the G-matrix-type ΛN interaction because ΛN correlations are fully taken into account in our model space. Here, we employ a simple free-space ΛN interaction with a three-range Gaussian form, which simulates the Nijmegen model F (NF) ΛN interaction. Here, the ΣN channel coupling contribution is renormalized into the ΛN single channel using the closure approximation. The even- and odd-state parts of our ΛN interaction are represented as follows:

$$V_{\Lambda N}(r)=\sum_{i=1}^{3}\left[(v_i^{\rm even}+v_i^{{\rm even},\sigma}\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_N)\frac{1+P_r}{2}+(v_i^{\rm odd}+v_i^{{\rm odd},\sigma}\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_N)\frac{1-P_r}{2}\right]e^{-\mu_i r^2}. \tag{3.5}$$

TABLE III. Parameters of the ΛN interaction defined in Eq. (3.5), which is used only in the $\alpha+N+\Lambda$ and $\alpha+N+\Lambda+\Lambda$ systems ($x=N$). Size parameters are in fm^{-2} and strengths are in MeV.

ΛN interaction when $x=N$			
i	1	2	3
μ_i	0.5487	1.384	6.250
$v_i^{\rm even}$	-10.40	-87.05	1031
$v_i^{{\rm even},\sigma}$	0.2574	17.09	-256.3
$v_i^{\rm odd}$	-5.816	-18.29	4029
$v_i^{{\rm odd},\sigma}$	-0.959	-9.184	-573.8

First, the parameters are determined so as to simulate the ΛN scattering phase shifts calculated with NF. Next, the second-range strengths $v_2^{\rm even}$ and $v_2^{{\rm even},\sigma}$ are adjusted so as to reproduce the Λ-binding energies of the 0^+ and 1^+ states of ${}^4_\Lambda$H with the use of the $N+N+N+\Lambda$ four-body model. Furthermore, strengths $v_2^{\rm odd}$ and $v_2^{{\rm odd},\sigma}$ are adjusted within the framework of the $\alpha+\Lambda+n+p$ four-body model so as to reproduce the observed binding energies of the ground-state spin doublet, $1/2^+$ and $3/2^+$ of ${}^7_\Lambda$Li . Our resulting parameters in Eq. (3.5) are listed in Table III. We further found that the energy of the ground state of ${}^6_\Lambda$He (${}^6_\Lambda$Li) measured from the ${}^5_\Lambda$He-N threshold can be well reproduced with our ΛN interaction in the $\alpha+N+\Lambda$ three-body calculation; for ${}^6_\Lambda$He (${}^6_\Lambda$Li), the calculated energy is -0.17 MeV (0.57 MeV), while the observed one is -0.17 MeV (0.59 MeV).

E. $\Lambda\Lambda$ interactions

In the present model, since the $\Lambda\Lambda$ relative motion is solved rigorously including the short-range correlations, it is not adequate to use the $\Lambda\Lambda$ G-matrix interaction given in Ref. [30]. However, our $\Lambda\Lambda$ interaction to be used in the present calculation should be still considered as an effective interaction, since the couplings to ΞN and $\Sigma\Sigma$ channels are not treated explicitly. Thus we employ the $\Lambda\Lambda$ interaction represented in the following three-range Gaussian form:

$$v_{\Lambda\Lambda}(r)=\sum_{i=1}^{3}(v_i+v_i^{\sigma}\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_\Lambda)e^{-\mu_i r^2}. \tag{3.6}$$

It is enlightening here to keep some linkage to the OBE models in determining the interaction parameters μ_i, v_i, and $v_i^\sigma (i=1-3)$. In our previous work on ${}^{\ 6}_{\Lambda\Lambda}$He and ${}^{\ 10}_{\Lambda\Lambda}$Be [13], the interaction parameters were chosen so as to simulate the $\Lambda\Lambda$ sector of the ND interaction, which is a reasonable model for the strong attraction suggested by the interpretation of earlier double-Λ hypernuclei data. The characteristic feature of ND is that there is only a scalar singlet instead of a scalar nonet, which gives a strongly attractive contribution in $\Lambda\Lambda$ as well as NN.

The other versions of the Nijmegen models [31–33] with a scalar nonet lead to much weaker $\Lambda\Lambda$ attractions, which seems to be appropriate for the weak $\Lambda\Lambda$ binding indicated by the NAGARA event. The NF is the simplest among these versions, which is adopted here as a guide to construct our $\Lambda\Lambda$ interaction: The outer two components of the above Gaussian potential ($i=1,2$) are determined so as to simulate the $\Lambda\Lambda$ sector of NF, and then the strength of the core part ($i=3$) is adjusted so as to reproduce the experimental value of $B_{\Lambda\Lambda}({}^{\ 6}_{\Lambda\Lambda}\mathrm{He})$. The values of the parameters obtained are given in Table IV. It is interesting that the resulting $\Lambda\Lambda$ interaction is almost equivalent to the interaction obtained by multiplying the above ND-simulated interaction employed in Ref. [13] by a factor of 0.5.

TABLE IV. Parameters of the $\Lambda\Lambda$ interaction defined in Eq. (3.6). Size parameters are in fm^{-2} and strengths are in MeV. The ${}^1S_0$ scattering length is -0.575 fm and the effective range is 6.45 fm.

$\Lambda\Lambda$ interaction			
i	1	2	3
μ_i	0.555	1.656	8.163
v_i	-10.67	-93.51	4884
v_i^σ	0.0966	16.08	915.8

IV. RESULTS AND DISCUSSION

Let us examine the calculated results for a series of double-Λ hypernuclei with the $\alpha+x+\Lambda+\Lambda$ structure ($x=0,n,p,d,t,{}^3\mathrm{He},\alpha$) studied in the microscopic four-body cluster model. In order to understand the role of two Λ particles attached to the core nuclei, it is useful to compare the level structures of the $\alpha+x+\Lambda+\Lambda$ double-Λ hypernuclei with those of the corresponding $\alpha+x$ nuclei and the $\alpha+x+\Lambda$ single-Λ hypernuclei. Then, we can see clearly how the ground and excited states of $\alpha+x$ nuclei are changed due to the addition of the Λ particles. It should be noted again here that in the model description of $\alpha+x+\Lambda+\Lambda$, the observed low-energy properties of the $\alpha+x$ nuclei and the existing Λ-binding energies of the $x+\Lambda$ and $\alpha+x+\Lambda$ hypernuclei have been reproduced accurately enough to provide reliable predictions for double-Λ hypernuclei experiments, with no adjustable parameters of the interactions in the four-body calculations. That the threshold energies for every partition into subcluster systems are reproduced is a necessary condition for a reliable cluster-model calculation.

A. Energy spectra

In Figs. 2–7, the calculated level structure of $\alpha+x$ core nuclei, $\alpha+x+\Lambda$ hypernuclei, and $\alpha+x+\Lambda+\Lambda$ hypernuclei are illustrated side by side. All the ground and bound excited states of double-Λ hypernuclei predicted in the present model are exhibited. In these figures, one sees clearly that injection into an $\alpha+x$ core nucleus of one and two Λ particles leads to stronger binding of the whole system and a prediction of more bound states in most systems. But, there is no bound "p orbit" of a Λ particle in single- or double-Λ hypernuclei with $A\leqslant 10$. In the bound states of double-Λ hypernuclei, two Λ particles are coupled to $S=0$, and therefore the spins and parities are the same as those of its nuclear core.

Table V summarizes the calculated ground-state energies for the double-Λ hypernuclei including the 2^+ excited state of ${}^{\;10}_{\Lambda\Lambda}$Be. The results are expressed in terms of two quantities: One is the total energy measured from the breakup threshold of $\alpha+x+\Lambda+\Lambda$, which is denoted as $E_{\Lambda\Lambda}$. The other is $B_{\Lambda\Lambda}$, which is the binding energy of two Λ particles with respect to the ground-state nuclear core $\alpha+x$.

The calculated values of $B_{\Lambda\Lambda}$ can be compared with some experimental data, though the data are quite limited at present. The most recent and precise data of the NAGARA event are used as a basic input of our model, so that our $\Lambda\Lambda$ interaction is adjusted to reproduce the experimental value $B^{\rm exp}_{\Lambda\Lambda}({}^{\;6}_{\Lambda\Lambda}{\rm He})=7.25\pm0.19\pm^{0.18}_{0.11}$ MeV [1]. It is of particular interest to compare the present result with another datum which is not used in the fitting procedure. There is an event

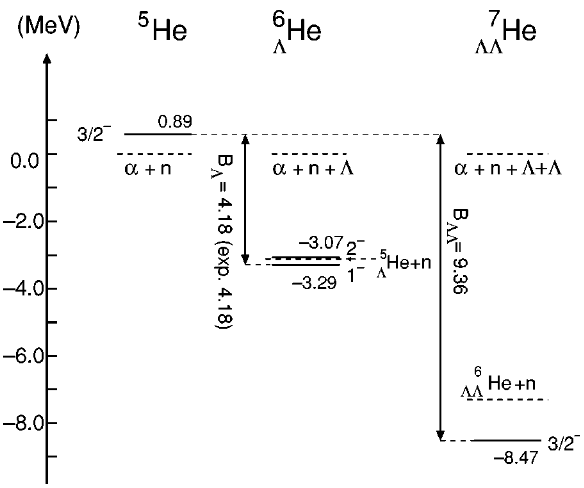

FIG. 2. Calculated energy levels of ^{5}He, ${}^6_\Lambda$He, and ${}^{\;7}_{\Lambda\Lambda}$He on the basis of the $\alpha+n$, $\alpha+n+\Lambda$, and $\alpha+n+\Lambda+\Lambda$ models, respectively. The level energies are measured from the particle breakup thresholds.

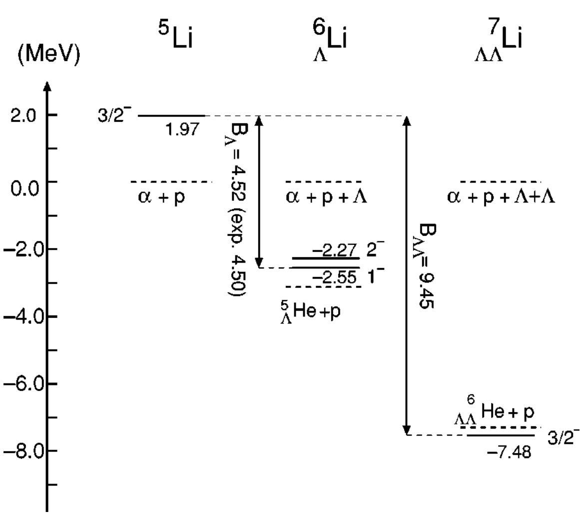

FIG. 3. Calculated energy levels of ^{5}Li, ${}^6_\Lambda$Li, and ${}^{\;7}_{\Lambda\Lambda}$Li on the basis of the $\alpha+p$, $\alpha+p+\Lambda$, and $\alpha+p+\Lambda+\Lambda$ models, respectively. The level energies are measured from the particle breakup thresholds.

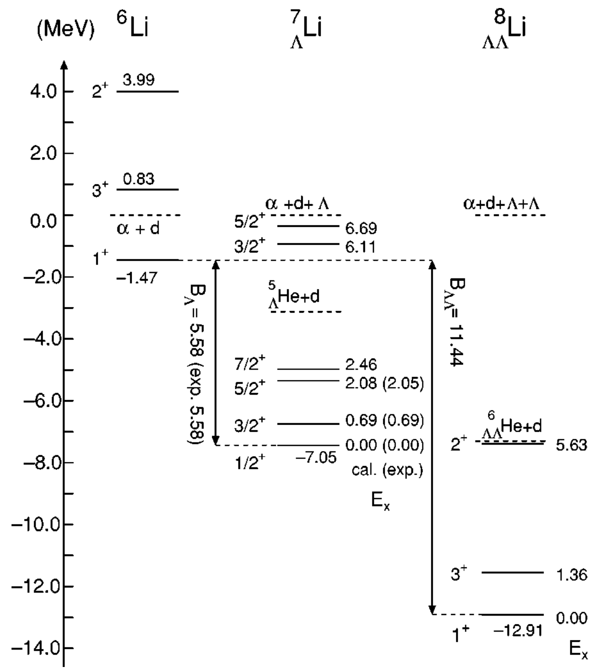

FIG. 4. Calculated energy levels of ^{6}Li, ${}^7_\Lambda$Li, and ${}^{\;8}_{\Lambda\Lambda}$Li on the basis of the $\alpha+d$, $\alpha+d+\Lambda$, and $\alpha+d+\Lambda+\Lambda$ models, respectively. The level energies are measured from the particle breakup thresholds or are given by excitation energies E_x.

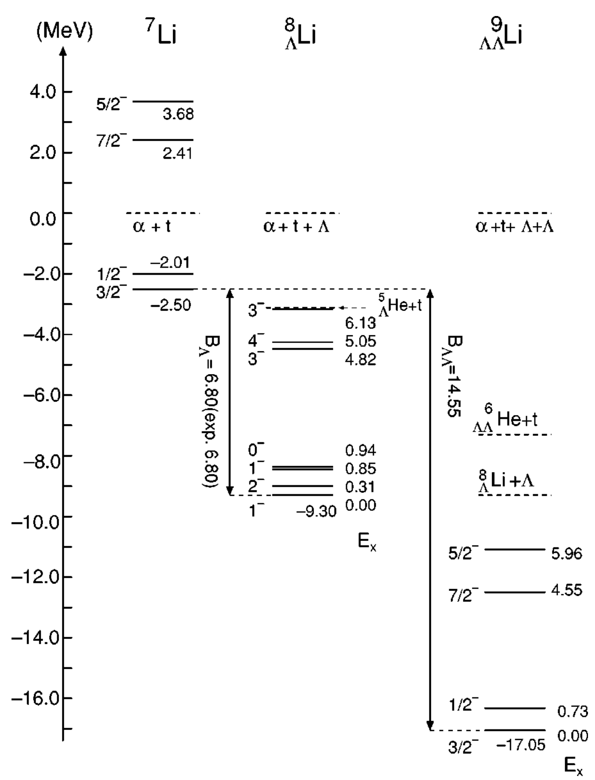

FIG. 5. Calculated energy levels of ^{7}Li, ${}^8_\Lambda$Li, and ${}^{\;9}_{\Lambda\Lambda}$Li on the basis of the $\alpha+t$, $\alpha+t+\Lambda$, and $\alpha+t+\Lambda+\Lambda$ models, respectively. The level energies are measured from the particle breakup thresholds or are given by excitation energies E_x.

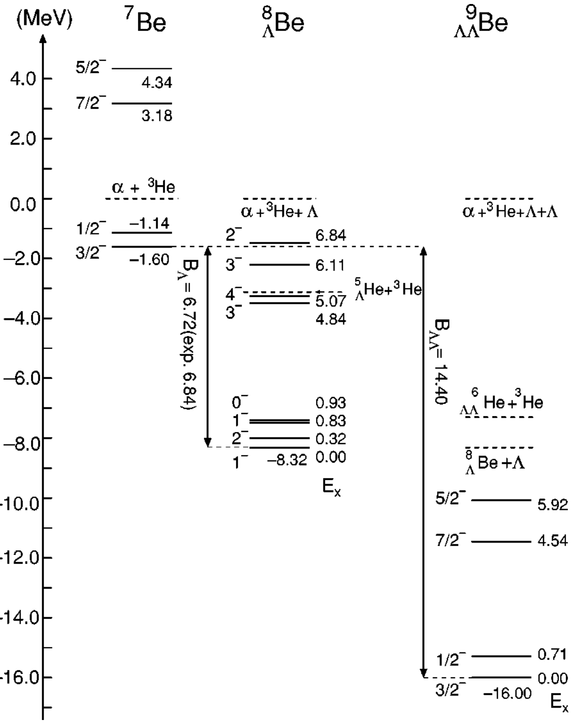

FIG. 6. Calculated energy levels of ^{7}Be, $^8_\Lambda$Be, and $^9_{\Lambda\Lambda}$Be on the basis of the $\alpha+{}^3$He, $\alpha+{}^3\mathrm{He}+\Lambda$, and $\alpha+{}^3\mathrm{He}+\Lambda+\Lambda$ models, respectively. The level energies are measured from the particle breakup thresholds or are given by excitation energies E_x.

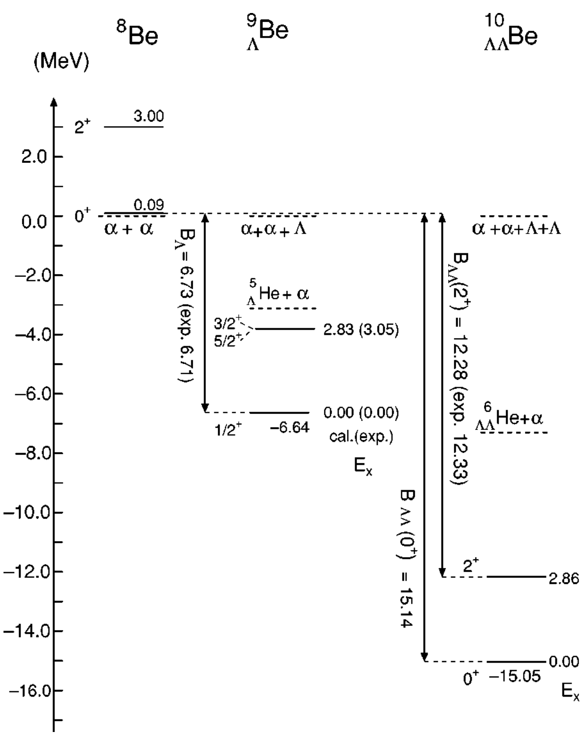

FIG. 7. Calculated energy levels of ^{8}Be, $^9_\Lambda$Be, and $^{10}_{\Lambda\Lambda}$Be on the basis of the $\alpha+\alpha$, $\alpha+\alpha+\Lambda$, and $\alpha+\alpha+\Lambda+\Lambda$ models, respectively. The level energies are measured from the particle breakup thresholds or are given by excitation energies E_x.

TABLE V. Calculated energies of the ground states of $A=6-10$ double-Λ hypernuclei based on the $\alpha+x+\Lambda+\Lambda$ four-body model ($x=0,n,p,d,t,{}^3$He, and α). $E_{\Lambda\Lambda}$ are measured from the $\alpha+x+\Lambda+\Lambda$ threshold. The $\Lambda\Lambda$ bond energy $\mathcal{V}^{\rm bond}_{\Lambda\Lambda}$ is defined by Eq. (4.1). Information on the 2^+ excited state of $^{10}_{\Lambda\Lambda}$Be is specially added so as to demonstrate the agreement with the experimental result.

	J^π	$E_{\Lambda\Lambda}$ (MeV)	$B_{\Lambda\Lambda}$ (MeV)	$B^{\rm exp}_{\Lambda\Lambda}$ (MeV)	$\mathcal{V}^{\rm bond}_{\Lambda\Lambda}$ (MeV)
$^6_{\Lambda\Lambda}$He	0^+	−7.25	7.25	7.25 ± 0.19 [a]	0.88
$^7_{\Lambda\Lambda}$He	$\frac{3}{2}^-$	−8.47	9.36		0.96
$^7_{\Lambda\Lambda}$Li	$\frac{3}{2}^-$	−7.48	9.45		0.95
$^8_{\Lambda\Lambda}$Li	1^+	−12.10	11.44		0.98
$^9_{\Lambda\Lambda}$Li	$\frac{3}{2}^-$	−17.05	14.55		0.98
$^9_{\Lambda\Lambda}$Be	$\frac{3}{2}^-$	−16.00	14.40		0.97
$^{10}_{\Lambda\Lambda}$Be	0^+	−15.05	15.14	17.7 ± 0.4 [b]	0.93
				14.6 ± 0.4 [b]	
$^{10}_{\Lambda\Lambda}$Be	2^+	−12.19	12.28	$12.33\pm^{0.35}_{0.21}$ [c]	0.93

[a] Reference [1].
[b] Reference [2]. Also see text for the second value.
[c] References [34,35].

found in the E373 experiment named the *Demachi-Yanagi* event [34,35]; the most probable interpretation of this event is a bound state of $^{10}_{\Lambda\Lambda}$Be having $B^{\rm exp}_{\Lambda\Lambda}=12.33\pm^{0.35}_{0.21}$ MeV, which is obtained by assuming $B^{\rm exp}_{\Xi^-}=0.15\pm^{0.3}_{0.1}$ MeV. In the emulsion analysis there is no direct evidence for the production of $^{10}_{\Lambda\Lambda}$Be in an excited state. However, if the produced $^{10}_{\Lambda\Lambda}$Be is interpreted to be in the ground state, the resultant $\Lambda\Lambda$ bond energy becomes repulsive, contradicting the NAGARA event. From the viewpoint of the present study, the *Demachi-Yanagi* event can be interpreted most probably as the observation of the 2^+ excited state in $^{10}_{\Lambda\Lambda}$Be; our calculated value of $B_{\Lambda\Lambda}(^{10}_{\Lambda\Lambda}\mathrm{Be}(2^+))$ is 12.28 MeV, which agrees with the above experimental value. This good agreement suggests that our systematically calculated level structures are predictive and useful for interpreting upcoming events expected to be found in the further analysis of the E373 data. Now it should be stressed that the above experimental datum on $^{10}_{\Lambda\Lambda}\mathrm{Be}(2^+)$ leads to no information about the ground-state value of $B_{\Lambda\Lambda}$ unless the theoretical value (2.86 MeV in our case) of the excitation energy of $^{10}_{\Lambda\Lambda}\mathrm{Be}(2^+)$ is utilized.

On the other hand, the earlier experiment by Danysz *et al.* [2] on the pionic decay of $^{10}_{\Lambda\Lambda}\mathrm{Be}(0^+)\to{}^9_\Lambda\mathrm{Be}(1/2^+)+p+\pi^-$ gave $B^{\rm exp}_{\Lambda\Lambda}(^{10}_{\Lambda\Lambda}\mathrm{Be}(0^+))=17.7\pm0.4$ MeV. This value has been used for a long time, which implies a strongly attractive $\Lambda\Lambda$ interaction. However, it should be noted that the authors also suggested the possibility of another decay $^{10}_{\Lambda\Lambda}\mathrm{Be}(0^+)\to{}^9_\Lambda\mathrm{Be}(3/2^+,5/2^+)+p+\pi^-$ (Table 5 of Ref. [2]); the same was pointed out in Ref. [12]. In this case, the value of $B^{\rm exp}_{\Lambda\Lambda}(^{10}_{\Lambda\Lambda}\mathrm{Be}(0^+))$ is modified to 14.6 ± 0.4 MeV, which is obtained by using 3.05 MeV [36] as the excitation energy of $^9_\Lambda\mathrm{Be}(3/2^+,5/2^+)$. This modified value turns out to not contradict our calculated value, 15.14 MeV. A similar reinterpretation, with the hypernuclear excited states taken into account, may be needed also for the E176 event which was identified as ${}_{\Lambda\Lambda}^{13}\mathrm{B}$ (${}_{\Lambda\Lambda}^{10}\mathrm{Be}$) with a strongly attractive (repulsive) $\Lambda\Lambda$ interaction.

Thus, we have understood the consistency between the experimental data and our theoretical results for ${}_{\Lambda\Lambda}^{10}\mathrm{Be}$. We, therefore, discuss the level structures of double-Λ hypernuclei in more detail. As seen in Figs. 2–7 and Table V, the Λ particle plays a gluelike role, so that a whole system becomes more strongly bound. This effect in a double-Λ nucleus is more enhanced than in the corresponding single-Λ nucleus. One can see a typical example in the case of ${}_{\Lambda\Lambda}^{7}\mathrm{Li}$ in Fig. 3. For the unbound nuclear system ${}^{5}\mathrm{Li}$, a single Λ cannot make a bound system of ${}_{\Lambda}^{6}\mathrm{Li}$, but the addition of one more Λ particle leads to a bound system of ${}_{\Lambda\Lambda}^{7}\mathrm{Li}$ whose ground state is weakly bound with respect to the ${}_{\Lambda\Lambda}^{6}\mathrm{He}+p$ threshold.

The bound excited states of double-Λ hypernuclei predicted in the present cluster model are summarized as follows: In ${}_{\Lambda\Lambda}^{7}\mathrm{He}$ and ${}_{\Lambda\Lambda}^{7}\mathrm{Li}$, the ground states are both bound but no excited states are predicted. Needless to say, there are no bound excited states in double-Λ hypernuclei with $A\leqslant 6$ since there is no bound excited state in their core nuclei. The lightest double-Λ hypernucleus that has at least one excited state is ${}_{\Lambda\Lambda}^{8}\mathrm{Li}$. In ${}_{\Lambda\Lambda}^{8}\mathrm{Li}$ we predict two $T=0$ excited states in the bound-state region. One might expect to see a $T=1,0^{+}$ bound excited state in ${}_{\Lambda\Lambda}^{8}\mathrm{Li}$, which would correspond to the $T=1,0^{+}$ state in ${}^{6}\mathrm{Li}$ at $E_{\mathrm{x}}=3.56$ MeV, but the state is not shown in Fig. 4 because the $T=1$ state may have a five-body structure and is outside the scope of the present cluster model. We predict three bound excited states in ${}_{\Lambda\Lambda}^{9}\mathrm{Li}$ (${}_{\Lambda\Lambda}^{9}\mathrm{Be}$). There is one bound excited state in ${}_{\Lambda\Lambda}^{10}\mathrm{Be}$ as mentioned before. It will be challenging to discover these excited states one by one as well as the ground states.

B. Dynamical change of the core nucleus

It is interesting to look at the dynamical change of the $\alpha+x$ nuclear cores, which occurs due to the injection of two Λ particles. The possibility that a nuclear core shrinks when a Λ particle is added was pointed out using the $\alpha+x+\Lambda$ cluster model for light p-shell Λ hypernuclei [37]. An updated calculation [38] specifically predicted a 21% shrinkage in size in ${}_{\Lambda}^{7}\mathrm{Li}$. The recent measurement of the γ-ray transition rate in ${}_{\Lambda}^{7}\mathrm{Li}$ [39] has confirmed quantitatively the shrinkage effect predicted in both the old calculation and the updated one. It is quite reasonable, therefore, that in a double-Λ hypernucleus the participation of one more Λ particle can induce further shrinkage of the nuclear core. Such an effect has been also investigated systematically using the molecular orbital model for ${}_{n\Lambda}^{8+n}\mathrm{Be}(n=1-4)=\alpha+\alpha+n\Lambda$ [40].

In order to see such a shrinkage effect, we show three physical quantities: First, in Table VI we list the rms distance between α and x, $\overline{r}_{\alpha x}$. As the number of the Λ particles increases, $\overline{r}_{\alpha x}$ turns out to shrink significantly due to the gluelike role of the bound Λ particles. For example, one sees $\overline{r}_{\alpha x}$ changing as $4.10\rightarrow 3.44\rightarrow 3.16$ fm for ${}^{6}\mathrm{Li}\rightarrow {}_{\Lambda}^{7}\mathrm{Li}\rightarrow {}_{\Lambda\Lambda}^{8}\mathrm{Li}$. Participation of the second Λ gives rise to about an 8% reduction of $\overline{r}_{\alpha x}$ except for $x=n$. Second, in more detail, we demonstrate in Fig. 8 the change of the $\alpha-n$ two-body density (correlation function) $\rho(r_{\alpha n})$ in ${}^{5}\mathrm{He}$, ${}_{\Lambda}^{6}\mathrm{He}$, and ${}_{\Lambda\Lambda}^{7}\mathrm{He}$ when Λ particles are added, which again manifests the shrinkage effect. Third, this shrinkage effect is seen in the large change of the expectation value of the relative kinetic energy, $\langle T_{\alpha x}\rangle$, and that of the potential energy, $\langle V_{\alpha x}\rangle$, in the $\alpha-x$ subsystems. When the α and x clusters approach each other, the increase of $\langle T_{\alpha x}\rangle$ overcomes the gain in $\langle V_{\alpha x}\rangle$, and the sum $\langle T_{\alpha x}+V_{\alpha x}\rangle$ increases appreciably. In spite of this energy loss in the $\alpha-x$ core system, the core shrinkage is realized because of the larger energy gain of the $\Lambda-\alpha$ and $\Lambda-x$ parts.

TABLE VI. Calculated rms distances between α and x, $\overline{r}_{\alpha x}$, in core nuclei, single-Λ hypernuclei, and double-Λ hypernuclei ($x=n,d,t,\alpha$). The expectation values of the kinetic energy and the potential energy between α and x, $\langle T_{\alpha x}\rangle$, $\langle V_{\alpha x}\rangle$, and $\langle T_{\alpha x}+V_{\alpha x}\rangle$ are also listed. For ${}^{5}\mathrm{He}$ and ${}^{8}\mathrm{Be}$, $\overline{r}_{\alpha - x}$ are not calculated since they are resonant states.

	$\overline{r}_{\alpha x}$	$\langle T_{\alpha x}\rangle$	$\langle V_{\alpha x}\rangle$	$\langle T_{\alpha x}+V_{\alpha x}\rangle$
${}^{5}\mathrm{He}$		7.86	−6.97	0.89
${}_{\Lambda}^{6}\mathrm{He}$	5.79	11.38	−9.92	1.46
${}_{\Lambda\Lambda}^{7}\mathrm{He}$	3.92	15.19	−11.95	2.24
${}^{6}\mathrm{Li}$	4.10	11.59	−13.06	−1.47
${}_{\Lambda}^{7}\mathrm{Li}$	3.44	15.59	−16.70	−1.11
${}_{\Lambda\Lambda}^{8}\mathrm{Li}$	3.16	18.86	−19.54	−0.68
${}^{7}\mathrm{Li}$	3.69	17.45	−19.95	−2.50
${}_{\Lambda}^{8}\mathrm{Li}$	3.30	21.85	−24.00	−2.15
${}_{\Lambda\Lambda}^{9}\mathrm{Li}$	3.05	26.74	−28.33	−1.59
${}^{8}\mathrm{Be}$		7.21	−7.12	0.09
${}_{\Lambda}^{9}\mathrm{Be}$	3.78	14.90	−14.14	0.76
${}_{\Lambda\Lambda}^{10}\mathrm{Be}$	3.44	19.49	−17.96	1.53

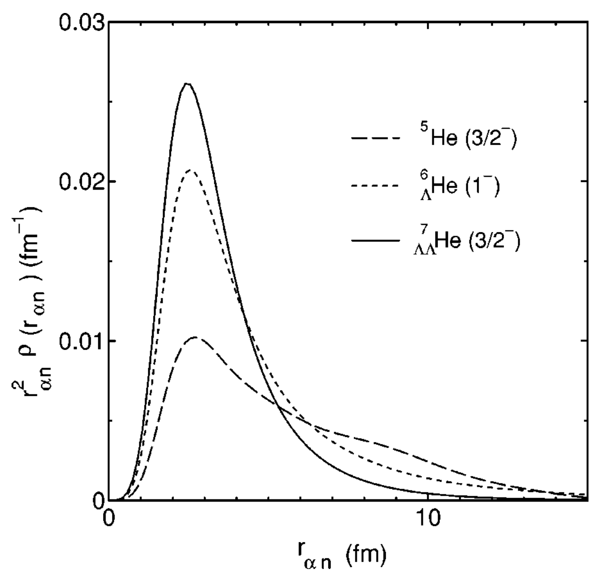

FIG. 8. The $\alpha-n$ two-body densities (correlation functions) $\rho(r_{\alpha n})$ of ${}^{5}\mathrm{He}(3/2^{-})$, ${}_{\Lambda}^{6}\mathrm{He}(1^{-})$, and ${}_{\Lambda\Lambda}^{7}\mathrm{He}(3/2^{-})$. Here, $\rho(r_{\alpha n})$ is multiplied by $r_{\alpha n}^{2}$.

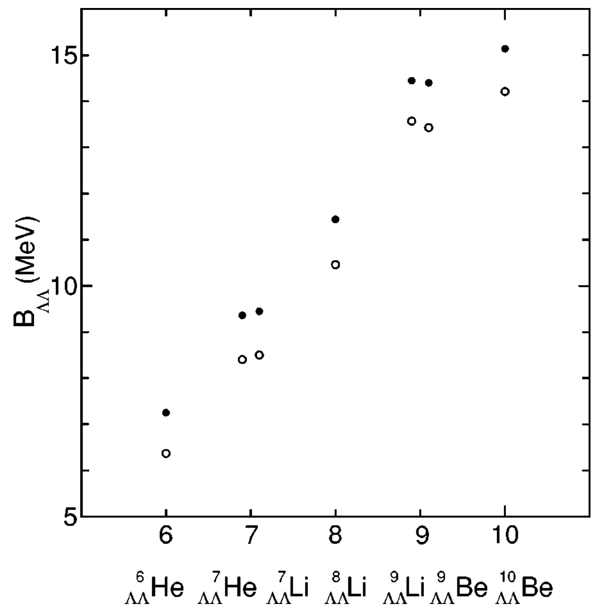

FIG. 9. Calculated values of $B_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z)$ in the ground states given by closed circles. The same quantities but calculated by putting $V_{\Lambda\Lambda}=0$, namely, $B_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z;\, V_{\Lambda\Lambda}=0)$, are shown by open circles.

C. ΛΛ bond energy

In Fig. 9 we display the contributions of the ΛΛ interaction to the total binding energies of double-Λ hypernuclei $^{A}_{\Lambda\Lambda}Z$. Here the calculated values of $B_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z)$ in the ground states are shown by closed circles. In order to extract the contribution of the ΛΛ interaction, we perform the same calculations by putting $V_{\Lambda\Lambda}=0$. The obtained values are denoted as $B_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z;\, V_{\Lambda\Lambda}=0)$ and shown by open circles in the figure. It should be noted that the effect of the dynamical change of the $\alpha+x$ core due to the ΛN interactions is included in the four-body estimate of $B_{\Lambda\Lambda}$ and $B_{\Lambda\Lambda}(V_{\Lambda\Lambda}=0)$. Since the ΛΛ interaction is not so strong compared with the ΛN interaction, the core-rearrangement effects included in $B_{\Lambda\Lambda}$ and $B_{\Lambda\Lambda}(V_{\Lambda\Lambda}=0)$ are similar to each other. Then, naturally the pure effect of the ΛΛ interaction is given by the difference

$$\mathcal{V}^{\rm bond}_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z)\equiv B_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z)-B_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z;V_{\Lambda\Lambda}=0). \quad (4.1)$$

We consider $\mathcal{V}^{\rm bond}_{\Lambda\Lambda}$ as the ΛΛ bond energy which should be determined essentially by the strength of the ΛΛ interaction. Now in Fig. 9, we find that the magnitude of $\mathcal{V}^{\rm bond}_{\Lambda\Lambda}$, the energy difference between the closed and open circles, is almost constant at ~ 1 MeV for all the double-Λ hypernuclei with $A=6-10$. The detailed values of $\mathcal{V}^{\rm bond}_{\Lambda\Lambda}$ are listed in Table V.

So far the following intuitive formula has been often used to estimate the ΛΛ interaction strength:

$$\Delta B_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z)\equiv B_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z)-2B_{\Lambda}(^{A-1}_{\Lambda}Z). \quad (4.2)$$

It is worthwhile to point out the problems underlying in this formula: This expression includes three problems which come from (i) the mass-polarization term of the three-body kinetic-energy operator, (ii) the ΛN spin-spin interaction, and (iii) the dynamical change of the core-nuclear structure.

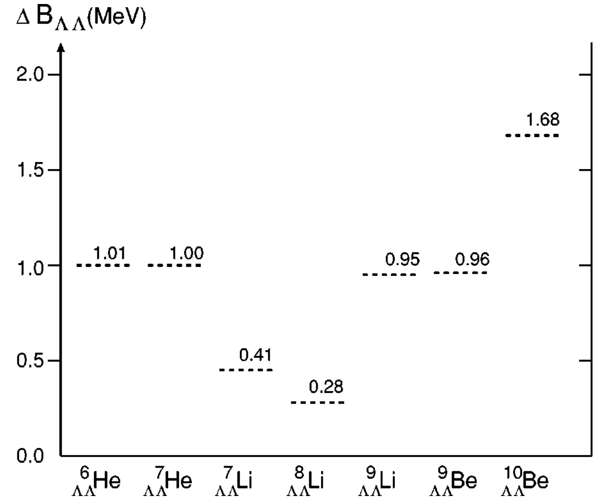

FIG. 10. Calculated values of $\Delta B_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z)$ defined in Eq. (4.2).

Problem (i) is understood as follows: In the $\alpha+\Lambda+\Lambda$ three-body model for $^{6}_{\Lambda\Lambda}$He (generally, α may be replaced by a spinless frozen-core nucleus), if one takes the *non-Jacobian* coordinate set $\mathbf{r}_{\alpha\Lambda_1}$ and $\mathbf{r}_{\alpha\Lambda_2}$, the Schrödinger equation may be written, in a self-explanatory notation, as

$$\left[-\frac{\hbar^2}{2\mu_{\alpha\Lambda_1}}\nabla^2_{\alpha\Lambda_1}-\frac{\hbar^2}{2\mu_{\alpha\Lambda_2}}\nabla^2_{\alpha\Lambda_2}-\frac{\hbar^2}{m_\alpha}\boldsymbol{\nabla}_{\alpha\Lambda_1}\cdot\boldsymbol{\nabla}_{\alpha\Lambda_1}+V_{\alpha\Lambda_1}\right.$$
$$\left.+V_{\alpha\Lambda_2}+V_{\Lambda_1\Lambda_2}-E\right]\Psi_{JM}(^{6}_{\Lambda\Lambda}\mathrm{He})=0. \quad (4.3)$$

If the third term of the kinetic energy, the so-called mass-polarization term, and $V_{\Lambda_1\Lambda_2}$ are neglected, we have the trivial solution $-E(=B_{\Lambda\Lambda})=2B_{\Lambda}$. Therefore, the quantity $\Delta B_{\Lambda\Lambda}=B_{\Lambda\Lambda}-2B_{\Lambda}$ stands for the contribution from the two neglected terms. In $^{6}_{\Lambda\Lambda}$He, the contribution to $B_{\Lambda\Lambda}$ from the mass-polarization term is $+0.13$ MeV, which explains the difference between $\Delta B_{\Lambda\Lambda}=1.01$ MeV and the ΛΛ bond energy $\mathcal{V}^{\rm bond}_{\Lambda\Lambda}=0.88$ MeV in Table V. This contribution decreases rapidly as the core-nuclear mass increases ($+0.01$ MeV in $^{10}_{\Lambda\Lambda}$Be).

Next, we discuss the second problem, the effect of the ΛN spin-spin interaction on $\Delta B_{\Lambda\Lambda}$ of Eq. (4.2). In Fig. 10, the calculated values of $\Delta B_{\Lambda\Lambda}$ are illustrated by the dashed bars. One notices clearly that $\Delta B_{\Lambda\Lambda}$ has a peculiar mass dependence, which suggests that some interesting mechanism is acting. It should be remarked here, however, that, as was already pointed out by Danysz *et al.* [2], the traditional definition of Eq. (4.2) is of simple meaning only when the nuclear core is spinless. On the other hand, in the case of a nuclear core with spin, the single-Λ binding energy B_{Λ} to be subtracted from $B_{\Lambda\Lambda}$ is distributed over the ground-state doublet of the corresponding single-Λ hypernucleus.

Here, we remark that the ΛN spin-spin interaction is not effective (canceled out) in the double-Λ hypernuclei having ΛΛ spin-singlet pairs. In the parent single-Λ hypernuclei, however, the spin-spin interaction plays an important role in giving rise to the energy splitting of the ground-state doublet. The typical example known experimentally is the spin doublet in $^{7}_{\Lambda}\mathrm{Li}$ with $J=\frac{1}{2}^{+}$ (ground; $B_\Lambda=5.58$ MeV) and $J=\frac{3}{2}^{+}$($E_{\mathrm{x}}=0.69$ MeV; $B_\Lambda=4.49$ MeV). One should use the spin-averaged value $\overline{B}_\Lambda(^{7}_{\Lambda}\mathrm{Li})=\frac{1}{3}B_\Lambda(\frac{1}{2}^{+}_{\mathrm{g.s.}})+\frac{2}{3}B_\Lambda(\frac{3}{2}^{+})$ instead of $B_\Lambda(\frac{1}{2}^{+}_{\mathrm{g.s.}})$ when one wants to deduce $\Delta B_{\Lambda\Lambda}$ from the $^{8}_{\Lambda\Lambda}\mathrm{Li}(1^{+})$ ground-state data, if any. If we adopt this prescription also for the adjacent systems, we should use

$$\overline{B}_\Lambda(^{6}_{\Lambda}\mathrm{He})=\frac{3}{8}B_\Lambda(1^{-}_{\mathrm{g.s.}})+\frac{5}{8}B_\Lambda(2^{-})=4.04\ \mathrm{MeV},$$

$$\overline{B}_\Lambda(^{6}_{\Lambda}\mathrm{Li})=\frac{3}{8}B_\Lambda(1^{-}_{\mathrm{g.s.}})+\frac{5}{8}B_\Lambda(2^{-})=4.35\ \mathrm{MeV},$$

$$\overline{B}_\Lambda(^{7}_{\Lambda}\mathrm{Li})=\frac{1}{3}B_\Lambda\left(\frac{1}{2}^{+}_{\mathrm{g.s.}}\right)+\frac{2}{3}B_\Lambda\left(\frac{3}{2}^{+}\right)=5.12\ \mathrm{MeV},$$

$$\overline{B}_\Lambda(^{8}_{\Lambda}\mathrm{Li})=\frac{3}{8}B_\Lambda(1^{-}_{\mathrm{g.s.}})+\frac{5}{8}B_\Lambda(2^{-})=6.61\ \mathrm{MeV},$$

$$\overline{B}_\Lambda(^{8}_{\Lambda}\mathrm{Be})=\frac{3}{8}B_\Lambda(1^{-}_{\mathrm{g.s.}})+\frac{5}{8}B_\Lambda(2^{-})=6.52\ \mathrm{MeV}.$$

Here, B_Λ of the excited states are taken from our calculation. In general, we have

$$\overline{B}_\Lambda(^{A-1}_{\Lambda}Z)=\frac{J_0}{2J_0+1}B_\Lambda(^{A-1}_{\Lambda}Z;J_1=J_0-\tfrac{1}{2})+\frac{J_0+1}{2J_0+1}B_\Lambda(^{A-1}_{\Lambda}Z;J_1=J_0+\tfrac{1}{2}),$$

where $J_1=J_0\pm\frac{1}{2}$ denote the doublet spins of the $\alpha+x+\Lambda$ system, J_0 being the ground-state spin of the $\alpha+x$ nuclear core. For the two spinless cases ($x=0$ and α), needless to say, $\overline{B}_\Lambda(^{5}_{\Lambda}\mathrm{He})=B_\Lambda(^{5}_{\Lambda}\mathrm{He};\frac{1}{2}^{+}_{\mathrm{g.s.}})$ and $\overline{B}_\Lambda(^{9}_{\Lambda}\mathrm{Be})=B_\Lambda(^{9}_{\Lambda}\mathrm{Be};\frac{1}{2}^{+}_{\mathrm{g.s.}})$.

Thus, replacing B_Λ with $\overline{B}_\Lambda$ in Eq. (4.2), we modify $\Delta B_{\Lambda\Lambda}$ by $\Delta\overline{B}_{\Lambda\Lambda}$ as

$$\Delta\overline{B}_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z)\equiv B_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z)-2\overline{B}_\Lambda(^{A-1}_{\Lambda}Z). \tag{4.4}$$

In Fig. 11, the solid bars illustrate $\Delta\overline{B}_{\Lambda\Lambda}$. Though $\Delta\overline{B}_{\Lambda\Lambda}$ is free from the effect of the ΛN spin-spin interaction, its magnitude for $A=7$–10 deviates significantly from $\Delta\overline{B}_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He})=1.01$ MeV. The deviation comes from the effect of the dynamical change in the core-nucleus structure (shrinkage in the $\alpha-x$ distance) due to the interaction of the Λ hyperons, and turns out to be a maximum in the case of $^{10}_{\Lambda\Lambda}\mathrm{Be}$. We emphasize that, even if one employs $\Delta\overline{B}_{\Lambda\Lambda}$, it is impossible to extract any consistent value of the $\Lambda\Lambda$ bond energy from Fig. 11 in which $\Delta\overline{B}_{\Lambda\Lambda}$ scatters in a range of a factor of 2.

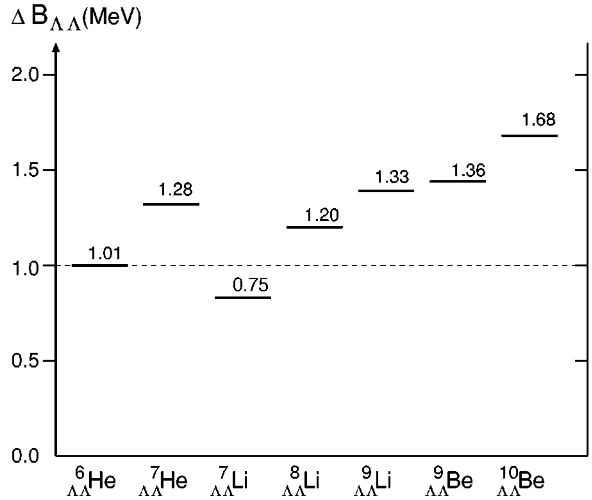

FIG. 11. Calculated values of $\Delta\overline{B}_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z)$ defined in Eq. (4.4).

As mentioned above, a consistent estimate of the $\Lambda\Lambda$ bond energy (0.9–1.0 MeV, nearly independent of the mass number, as seen in Table V) can be obtained by taking $\mathcal{V}^{\mathrm{bond}}_{\Lambda\Lambda}$ of Eq. (4.1) as the definition of that energy, though help of the theoretical calculation with $V_{\Lambda\Lambda}=0$ is necessary.

V. SUMMARY

We have carried out structure calculations for $^{6}_{\Lambda\Lambda}\mathrm{He}$, $^{7}_{\Lambda\Lambda}\mathrm{He}$, $^{7}_{\Lambda\Lambda}\mathrm{Li}$, $^{8}_{\Lambda\Lambda}\mathrm{Li}$, $^{9}_{\Lambda\Lambda}\mathrm{Li}$, $^{9}_{\Lambda\Lambda}\mathrm{Be}$, and $^{10}_{\Lambda\Lambda}\mathrm{Be}$ taking the framework of an $\alpha+x+\Lambda+\Lambda$ model with $x=0,n,p,d,t,{}^{3}\mathrm{He}$, and α, respectively. We determined the interactions between constituent particles so as to reproduce reasonably the observed low-energy properties of the $\alpha+x$ nuclei and the existing data of Λ-binding energies of the $x+\Lambda$ and $\alpha+x+\Lambda$ systems. The $\Lambda\Lambda$ interaction was constructed so as to reproduce $B_{\Lambda\Lambda}(^{6}_{\Lambda\Lambda}\mathrm{He})$ given by the NAGARA event within our $\alpha+\Lambda+\Lambda$ model, where the long-range part of our interaction was adjusted to simulate the behavior of the appropriate OBE model (NF). With no additional adjustable parameters, the four-body calculations for the $\alpha+x+\Lambda+\Lambda$ systems were performed accurately using the Jacobian-coordinate Gaussian-basis coupled-rearrangement-channel method. The obtained energy spectra of the double-Λ hypernuclei with $A=6$–10 are summarized in Fig. 12.

The major results to be emphasized are as follows.

(1) It is striking that the calculated $B_{\Lambda\Lambda}$ of the 2^{+} excited state in $^{10}_{\Lambda\Lambda}\mathrm{Be}$, 12.28 MeV, agrees with the experimental value $B^{\mathrm{exp}}_{\Lambda\Lambda}(^{10}_{\Lambda\Lambda}\mathrm{Be})=12.33\pm^{0.35}_{0.21}$ MeV in the *Demachi-Yanagi* event [34,35]. We therefore interpret this event as the observation of the 2^{+} excited state of $^{10}_{\Lambda\Lambda}\mathrm{Be}$. The agreement suggests that our systematic calculations are predictive for upcoming events expected to be found in the further analysis of the E373 data, etc.

(2) Together with the energy spectrum of each double-Λ hypernucleus, those of the corresponding core nucleus and single-Λ hypernucleus are exhibited side by side in Figs. 2–7 so one can see clearly that the injection of one and two Λ

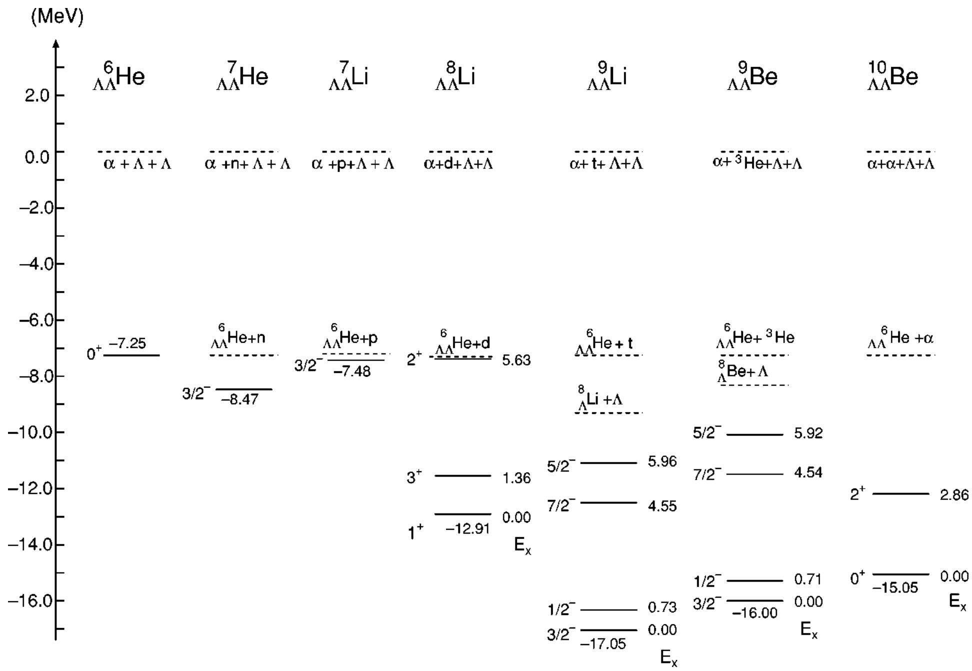

FIG. 12. Summary of the energy levels of the double-Λ hypernuclei ${}_{\Lambda\Lambda}^{6}$He, ${}_{\Lambda\Lambda}^{7}$He, ${}_{\Lambda\Lambda}^{7}$Li, ${}_{\Lambda\Lambda}^{8}$Li, ${}_{\Lambda\Lambda}^{9}$Li, ${}_{\Lambda\Lambda}^{9}$Be, and ${}_{\Lambda\Lambda}^{10}$Be calculated using the $\alpha+x+\Lambda+\Lambda$ model with $x=0,n,p,d,t,{}^{3}$He, and α, respectively.

particles leads to stronger binding of the whole system and the prediction of more bound states in most systems. In the bound states of any double-Λ hypernucleus, two Λ particles are dominantly coupled to $S=0$ and hence the spin and parity are the same as those of its nuclear core. Nevertheless the theoretical $B_{\Lambda\Lambda}$ values are of importance to guide the analysis of the emulsion experiments.

(3) The dynamical change of the $\alpha+x$ nuclear core due to the interaction of the Λ particles is seen in double-Λ hypernuclei; there occurs, on average, about an 8% shrinkage of the $\alpha-x$ rms distance compared with that in the single-Λ hypernucleus. This shrinkage comes about because of the large energy gain in the $\Lambda-\alpha$ and $\Lambda-x$ subsystems, which overcomes the energy loss in the $\alpha-x$ relative motion.

(4) We estimated the ΛΛ bond energy using our new definition $\mathcal{V}_{\Lambda\Lambda}^{\rm bond}=B_{\Lambda\Lambda}-B_{\Lambda\Lambda}(V_{\Lambda\Lambda}=0)$ and found it to be 0.88 MeV for ${}_{\Lambda\Lambda}^{6}$He and 0.93−0.98 MeV for the other double-Λ hypernuclei. We demonstrated that the quantity $\Delta B_{\Lambda\Lambda}=B_{\Lambda\Lambda}-2B_{\Lambda}$ is not a good measure of the ΛΛ bond energy, since $\Delta B_{\Lambda\Lambda}$ is contaminated by the contribution from the splitting of the ground-state doublet in the single-Λ hypernucleus and that of the structure change of the core nucleus. In fact, the value of $\Delta B_{\Lambda\Lambda}$ scatters from 0.28 to 1.68 MeV for the double-Λ hypernuclei with $A=6$–10. We then modified $\Delta B_{\Lambda\Lambda}$ by $\Delta\overline{B}_{\Lambda\Lambda}=B_{\Lambda\Lambda}-2\overline{B}_{\Lambda}$ with $\overline{B}_{\Lambda}$ being the spin average of B_{Λ}'s for the ground-state spin doublet. We found, however, that $\Delta\overline{B}_{\Lambda\Lambda}$ still ranges from 0.75 to 1.68 MeV due to the structure change of the core nucleus. Direct use of $B_{\Lambda\Lambda}$ rather than $\Delta B_{\Lambda\Lambda}$ or $\Delta\overline{B}_{\Lambda\Lambda}$ is recommended when experimental results and calculational results are compared to each other.

In conclusion, it is our intention that these extensive four-body cluster-model calculations should serve to motivate extensive spectroscopic studies of double-Λ hypernuclei.

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