Phenomenological Λ-nuclear interactions

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Variational Monte Carlo calculations for ${}^{4}_{\Lambda}$H (ground and excited states) and ${}^{5}_{\Lambda}$He are performed to decipher information on Λ-nuclear interactions. Appropriate operatorial nuclear and Λ-nuclear correlations have been incorporated to minimize the expectation values of the energies. We use the Argonne v_{18} two-body *NN* along with the Urbana IX three-body *NNN* interactions. The study demonstrates that a large part of the splitting energy in ${}^{4}_{\Lambda}$H (0^{+}-1^{+}) is due to the three-body ΛNN forces. ${}^{17}_{\Lambda}$O hypernucleus is analyzed using the *s*-shell results. Λ binding to nuclear matter is calculated within the variational framework using Fermi-hypernetted-chain technique. There is a need to correctly incorporate the three-body ΛNN correlations for Λ binding to nuclear matter.

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I. INTRODUCTION

The study of the response of a many-body system to a hyperon gives insight into the structure of baryon-baryon interactions. The binding energy data of light *s*-shell hypernuclei provide a unique opportunity to know more about the Λ-nuclear interactions, particularly on their spin dependence. In the past, basically two approaches have been followed. The first one involves Brueckner-Hartree calculations using Nijmegen *YN* potential with and without higher-order correction to single-particle energies [1,2]. This method uses the large $\Lambda N \rightarrow \Sigma N$ coupling that gives considerably lower binding energy for ${}^{5}_{\Lambda}$He. Any attempt to correct this leads to poor agreement with the scattering data. The second method is primarily based on reliable variational techniques, mostly using simplified *NN* interactions [3,4]. We follow this approach but use realistic Argonne v_{18} *NN* interaction [5] along with Urbana IX three-body *NNN* interaction [6,7]. The phenomenological approach we follow is consistent with meson-theoretic models as well as available low-energy scattering data. The $\Lambda N \rightarrow \Sigma N$ coupling is effectively taken care of by inclusion of the phenomenological ΛNN potential. The ΛNN potential consists of both the dispersive and two-pion-exchange (TPE) kind as employed in previous studies [4]. The hypernuclei considered in this work are ${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$H* (* on ${}^{4}_{\Lambda}$H refers to the excited 1^{+} state), and ${}^{5}_{\Lambda}$He.

The interaction parameters that we find based on our *s*-shell results are later used to make estimates of the binding energy of ${}^{17}_{\Lambda}$O. We also study the Λ binding to nuclear matter by using the Fermi-hypernetted-chain (FHNC) technique [4,8]. This study gives an indication of the implications of our *s*-shell results on heavier hypernuclear systems.

In Sec. II we describe the Hamiltonian used in this work. Section III gives the wave function and the approach. In Sec. IV we discuss the results and finally in Sec. V we give the conclusion and comments.

II. THE HAMILTONIAN

The complete hypernuclear Hamiltonian consists of the nuclear Hamiltonian H_N^{A-1} and the Λ Hamiltonian H_Λ. The nuclear Hamiltonian H_N^{A-1} is given by

$$H_N^{A-1} = -\sum_{i=1}^{A-1} \frac{\hbar^2}{2m_i}\nabla_i^2 + \sum_{i<j}^{A-1} V_{ij} + \sum_{i<j<k}^{A-1} V_{ijk}, \quad (2.1)$$

where V_{ij} and V_{ijk} are the two-nucleon *NN* and three-nucleon *NNN* potentials, respectively, and m_i is the mass of the nucleon.

The two-body *NN* interaction employed here is the Argonne v_{18} interaction [5]. The first 14 operator components of this model are charge-independent and are an updated version of the Argonne v_{14} potential [9]. Three additional charge-dependent and one charge-asymmetric operators are added along with a complete electromagnetic interaction, containing the Coulomb, Darwin-Foldy, vacuum polarization, and magnetic moment terms with finite-size effects. The potential has been fit directly to the Nijmegen *pp* and *np* scattering data base [10,11], low-energy *nn* scattering parameters, and deuteron binding energy. For the three-body *NNN* potential we use the Urbana model [6,7] consisting of the TPE part of Fujita and Miyazawa [12] and a repulsive phenomenological spin-isospin independent term. We have used

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the Urbana IX model [7] of the interaction where the values of the strength parameters are used in conjunction with the Argonne v_{18} interaction.

The Λ Hamiltonian H_Λ is given by

$$H_\Lambda = -\frac{\hbar^2}{2m_\Lambda}\nabla_\Lambda^2 + \sum_{i=1}^{A-1} V_{i\Lambda} + \sum_{i<j}^{A-1} V_{ij\Lambda}, \tag{2.2}$$

where $V_{i\Lambda}$ and $V_{ij\Lambda}$ are the two-body ΛN and three-body ΛNN potentials, respectively, and m_Λ is the mass of the Λ particle. The first terms of Eqs. (2.1) and (2.2) pertain to the total kinetic energy of the nucleons and Λ, respectively.

The two-body ΛN potential $V_{\Lambda N}$ includes a central potential [4] of the same form for the singlet and triplet spin states. These have a theoretically reasonable attractive tail due to the TPE in accord with Urbana-type potentials [13] with spin- and space-exchange terms:

$$V_{\Lambda N} = [\{V_c(r) - \overline{V}\}(1-\epsilon+\epsilon P_x) + \tfrac{1}{4}V_\sigma \vec{\sigma}_\Lambda \cdot \vec{\sigma}_N]T_\pi^2(r), \tag{2.3}$$

where $\overline{V}$ and V_σ are the spin-average and spin-dependent strengths, respectively. P_x is the Majorana space-exchange operator, ϵ is the corresponding exchange parameter, V_c is the Woods-Saxon repulsive core [4], and T_π is the one-pion-exchange (OPE) tensor potential shape modified with a cutoff. Further details can be found in Ref. [4].

In this study we consider potential parameters that are consistent with low-energy Λp scattering data that essentially determine the value of spin-average strength $\overline{V} = 6.15 \pm 0.05$ MeV [4].

For hypernuclei with zero-spin core nuclei, such as ${}^{5}_{\Lambda}$He, the major contribution arises from the spin-average strength $\overline{V}$ while the spin component contributes very little. The spin dependence V_σ is assumed to be positive, which is consistent with hypernuclear spins of mass 4 systems. We find that for the s-shell hypernuclei ($A \leqslant 5$) the s-state interaction is dominant but the higher partial-wave interactions, in particular, the p-state, also make a small but significant contribution contrary to earlier studies [14]. The importance of the p-state contribution becomes significant due to the Λ-nuclear correlations.

Studies on hypernuclei have shown that it is necessary to include a three-body ΛNN interaction in the Hamiltonian. We consider phenomenological ΛNN forces of the dispersive (spin-dependent and spin-independent) as well as the TPE kind [15], which arise from the suppression of Σ, Δ,... degrees of freedom by the medium, that is, the second nucleon.

The dispersive kind has a spin dependence that is given by

$$V_{\Lambda NN}^{DS}(r_{ij\Lambda}) = W_o T_\pi^2(r_{i\Lambda}) T_\pi^2(r_{j\Lambda})[1 + \tfrac{1}{6}\vec{\sigma}_\Lambda \cdot (\vec{\sigma}_i + \vec{\sigma}_j)]. \tag{2.4}$$

The TPE part of the interaction is given by [15]

$$W_p = -\tfrac{1}{6}C_p(\vec{\tau}_i \cdot \vec{\tau}_j)\{X_{i\Lambda}, X_{j\Lambda}\}Y(r_{i\Lambda})Y(r_{j\Lambda}), \tag{2.5}$$

where $X_{k\Lambda}$ is the OPE operator given by

$$X_{k\Lambda} = (\vec{\sigma}_k \cdot \vec{\sigma}_\Lambda) + S_{k\Lambda}(r_{k\Lambda})T_\pi(r_{k\Lambda}) \tag{2.6}$$

with

$$S_{k\Lambda}(r_{k\Lambda}) = \frac{3(\vec{\sigma}_k \cdot r_{k\Lambda})(\vec{\sigma}_\Lambda \cdot r_{k\Lambda})}{r_{k\Lambda}^2} - (\vec{\sigma}_k \cdot \vec{\sigma}_\Lambda). \tag{2.7}$$

In Eq. (2.5) {} represents the anticommutator term. $Y_\pi(r_{k\Lambda})$ and $T_\pi(r_{k\Lambda})$ are the usual Yukawa and tensor functions, respectively, with pion mass $\mu = 0.7$ fm^{-1}.

The Λ-nuclear interaction parameters, $\overline{V}$, V_σ, C_p, and W_o are considered as unknown. These are then fitted as a function of the B_Λ values that have been calculated using the s-shell results. Taking these values of $\overline{V}$, V_σ, C_p, and W_o we again perform variational calculations that give us the final results for ${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$H*, and ${}^{5}_{\Lambda}$He. These results are later used to analyze ${}^{17}_{\Lambda}$O and Λ binding to nuclear matter.

III. WAVE FUNCTION AND APPROACH

The trial variational wave function we adopt is of the following form:

$$|\Psi_v\rangle = \left[1 + \sum_{i<j<k} U_{ijk} + \sum_{i<j} U_{ij\Lambda} + \sum_{i<j} U_{ij}^{LS} + \sum_{i<j<k} U_{ijk}^{Tni}\right] \times \left[\prod_{i<j<k} f_{ijk}^c\right]|\Psi_p\rangle. \tag{3.1}$$

The pair wave function $|\Psi_p\rangle$ is a symmetrized product of two-body $(1+U_{ij})$ and $(1+U_{i\Lambda})$ correlation operators acting on a Jastrow trial function. This is written as

$$|\Psi_p\rangle = \left[S\prod_{i<j}^{A-1}(1+U_{ij})\right]\left[S\prod_{i=1}^{A-1}(1+U_{i\Lambda})\right]|\Psi_J\rangle. \tag{3.2}$$

U_{ij} in Eq. (3.2) is defined as

$$U_{ij} = \sum_{p=2,6}\left[\prod_{k\neq i,j} f_{ijk}^p(r_{ik}, r_{jk})\right]u_p(r_{ij})O_{ij}^P \tag{3.3}$$

with $O_{ij}^P = [1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}] \otimes [1, \vec{\tau}_i \cdot \vec{\tau}_j]$. S_{ij} is the tensor operator, $\vec{\sigma}$ and $\vec{\tau}$ are the spin and isospin operators, respectively. The factor f_{ijk}^p suppresses spin-isospin correlations between two nucleons in the presence of a third one.

In principle, the $U_{i\Lambda}$ correlation will consist of a spin and a Majorana space-exchange operator [16]:

$$U_{i\Lambda} = \alpha_\sigma u_\sigma(r_{i\Lambda})\vec{\sigma}_\Lambda \cdot \vec{\sigma}_i + \alpha_{px}u_{px}(r_{i\Lambda})P_x, \tag{3.4}$$

where α_σ and α_{px} are variational parameters and P_x is the space-exchange operator. In our calculations we have not included the space-exchange correlations since the calculations become complicated and time consuming. In any case the effect of these correlations is expected to be small for s-shell hypernuclei.

The spin-dependent correlation u_σ given in Eq. (3.4) is defined as

$$u_\sigma=\frac{(f_s^\Lambda-f_t^\Lambda)}{f_c^\Lambda}, \tag{3.5}$$

where f_c^Λ is the spin-average correlation function. f_s^Λ and f_t^Λ are the solutions of the quenched ΛN potential in singlet and triplet states, respectively, which are given by the following relation:

$$\left[-\frac{\hbar^2}{2\mu_{\Lambda N}}\nabla^2+\overline{V}_{s/t}(r_{\Lambda N})+V_{\Lambda N}^a\right]f_{s/t}^\Lambda=0, \tag{3.6}$$

where $\overline{V}_{s/t}$ is the quenched ΛN potential in singlet/triplet state, $\mu_{\Lambda N}$ is the reduced mass of the Λ-N pair, while $V_{\Lambda N}^a$ is an auxiliary potential.

The Jastrow wave function $|\Psi_J\rangle$ is given by

$$|\Psi_J\rangle=\left[\prod_{i=1}^{A-1} f_c^\Lambda(r_{i\Lambda})\prod_{i<j}^{A-1} f_c(r_{ij})\right]|\Phi\rangle. \tag{3.7}$$

Here $|\Phi\rangle$ is an antisymmetric product of single-particle wave function with the desired $(\mathbf{J},\mathbf{T})$. The initial uncorrelated state Φ has no coordinate dependence and is real. For example, consider the following Φ states for ${}^4_\Lambda$H and ${}^4_\Lambda$H* expressed in the spin-isospin basis with the appropriate $(\mathbf{J},\mathbf{T})$ states:

$$\begin{aligned}{}^4_\Lambda\mathrm{H}(J=0,\ T=\tfrac{1}{2})=&\,\mathbf{C}(\tfrac{1}{2},\tfrac{1}{2},0;\tfrac{1}{2},-\tfrac{1}{2},0)\,{}^3\mathrm{H}^{1/2}_{j=1/2}\Lambda^{-1/2}_{1/2}\\ &+\mathbf{C}(\tfrac{1}{2},\tfrac{1}{2},0;-\tfrac{1}{2},\tfrac{1}{2},0)\,{}^3\mathrm{H}^{-1/2}_{j=1/2}\Lambda^{1/2}_{1/2}\end{aligned} \tag{3.8}$$

and

$${}^4_\Lambda\mathrm{H}^*\ \ (J=1,T=\tfrac{1}{2})=\mathbf{C}(\tfrac{1}{2},\tfrac{1}{2},1;\tfrac{1}{2},\tfrac{1}{2},1)\,{}^3\mathrm{H}^{1/2}_{j=1/2}\Lambda^{1/2}_{1/2}, \tag{3.9}$$

where $\mathbf{C}$ represents the Clebsch-Gordon coefficients [17]. ${}^3\mathrm{H}^{1/2}_{j=1/2}$ and ${}^3\mathrm{H}^{-1/2}_{j=1/2}$ are the uncorrelated Φ's for triton, whereas $\Lambda^{-1/2}_{1/2}$ and $\Lambda^{1/2}_{1/2}$ are the spin-down and spin-up states, respectively, of the Λ particle.

The spin-orbit correlation U_{ij}^{LS} is given by

$$U_{ij}^{LS}=[u_{ls}(r_{ij})+u_{ls\tau}(r_{ij})\vec{\tau}_i\cdot\vec{\tau}_j](\mathbf{L}\cdot\mathbf{S})_{ij}. \tag{3.10}$$

The eight radial functions, $f_c(r_{ij})$, $u_{p=2,6}(r_{ij})$, $u_{ls}(r_{ij})$, and $u_{ls\tau}(r_{ij})$ are obtained from approximate two-body Euler-Lagrange equations with variational parameters [18].

U_{ijk}^{Tni} is a three-body correlation induced by the three-nucleon interaction, V_{ijk}. The other correlations incorporated in the wave function are a spatial three-body NNN correlation f_{ijk}^c, along with U_{ijk} that consists of a spin-orbit and an isospin three-body correlation. Further details can be found in Ref. [19].

The three-body ΛNN correlation $U_{ij\Lambda}$ has the following form:

$$U_{ij\Lambda}=\overline{V}_{ij\Lambda}(\hat{\delta}_1,\hat{\delta}_2), \tag{3.11}$$

where $\overline{V}_{ij\Lambda}$ differs from $V_{ij\Lambda}$ through the cutoff factor c of the usual Yukawa and tensor functions. $\hat{\delta}_1$ and $\hat{\delta}_2$ are variational parameters that multiply the ΛNN interaction parameters, C_p and W_o, respectively.

No attempt has been made to vary the two-body NN and three-body NNN correlation parameters [19] as their effect has been found to be small [20] and only the variational parameters of the wave function pertaining to Λ have been varied to obtain a minimum in the energy. The optimum values of these parameters which are used in our final calculations are given in Tables I and II.

We calculate energy expectation values using Monte Carlo (MC) integration [21,22]. The expectation values are sampled both in configuration space and in the order of operators in the wave function by following a Metropolis random walk [23]. The mathematical expressions used to evaluate the energy expectation values are given below.

The energy expectation value for the pure nucleus is given by

$$\langle E_N^{A-1}\rangle=\frac{\langle\Psi_N^{A-1}|H_N^{A-1}|\Psi_N^{A-1}\rangle}{\langle\Psi_N^{A-1}|\Psi_N^{A-1}\rangle}, \tag{3.12}$$

TABLE I. ΛN correlations parameters.

${}^A_\Lambda Z$	$\overline{V}$	$\kappa_{\Lambda N}$	$a_{\Lambda N}$	$C_{\Lambda N}$	$R_{\Lambda N}$	α_s	α_σ [a]
${}^5_\Lambda$He	6.20	0.117	0.50	2.0	1.0	0.965	0.80
	6.15	0.110	0.50	2.0	1.0	0.970	0.95
	6.10	0.095	0.50	2.0	1.0	0.940	0.95
${}^4_\Lambda$H	6.20	0.12	0.70	2.0	1.0	0.95	0.70
	6.15	0.10	0.70	2.0	1.0	0.95	1.20
	6.10	0.08	0.70	2.0	1.0	0.95	0.70
${}^4_\Lambda$H*	6.20	0.095	0.70	2.0	1.0	1.0	0.70
	6.15	0.065	0.70	2.0	1.0	1.0	0.70
	6.10	0.050	0.70	2.0	1.0	1.0	0.70

[a]Equation (3.4); for all other correlation parameters refer to Ref. [16].

TABLE II. ΛNN correlation parameters for ${}^5_\Lambda$He, ${}^4_\Lambda$H, and ${}^4_\Lambda$H*

Parameters	$\overline{V}=6.20$ MeV	$\overline{V}=6.15$ MeV	$\overline{V}=6.10$ MeV
$\hat{\delta}_1$ [a]	0.364 104	0.311 733	0.257 894
$\hat{\delta}_2$ [a]	0.006 096	0.004 845	0.003 766

[a]Equation (3.11).

TABLE III. Variational results for ^{3}H and ^{4}He. All energies are in MeV and radii in fm.

Components	^{3}H	^{4}He
Kinetic energy	50.76(4)	106.85(6)
NN potential energy	−57.95(4)	−129.30(6)
NNN potential energy	−1.13(3)	−5.27(7)
Total energy	**−8.32(2)**	**−27.71(6)**
rms (proton)	1.585(3)	1.478(2)
rms (neutron)	1.731(4)	1.478(2)
d-state probability	0.0933(1)	0.1512(2)

where Ψ_N^{A-1} is the wave function of the mass $(A-1)$ nucleus and H_N^{A-1} is the nuclear Hamiltonian.

The energy expectation value for the hypernucleus is given by

$$\langle E_H^A \rangle = \frac{\langle \Psi_H^A | H_H^A | \Psi_H^A \rangle}{\langle \Psi_H^A | \Psi_H^A \rangle}, \tag{3.13}$$

where Ψ_H^A is the wave function of the mass "A" hypernucleus and H_H^A is the hypernuclear Hamiltonian.

Therefore, binding energy of Λ to the hypernucleus is given by

$$-B_\Lambda = \langle E_H^A \rangle - \langle E_N^{A-1} \rangle. \tag{3.14}$$

The nuclear and hypernuclear wave functions, Ψ_N^{A-1} and Ψ_H^A are optimized with respect to the variational parameters to obtain the minimum in the energies.

The B_Λ value for each hypernucleus is calculated from the variational results using Eq. (3.14). The B_Λ value is thus written as a function of the adjustable parameters in the Λ Hamiltonian H_Λ, and is used to determine the set of parameters that are consistent with the experimental B_Λ values [26,27].

IV. RESULTS AND DISCUSSION

Table III gives the variational results for the nuclei, namely, ^{4}He and ^{3}H calculated using the two-body *NN* Argonne v_{18} interaction [5] and three-body *NNN* Urbana IX interaction [6,7] with relevant correlations. The numbers appearing in parentheses in all the tables in this work indicate the statistical error in the last digit. These calculations have been performed on similar lines as those by Wiringa and co-workers [5,19] and the results conform to theirs. These results also check very well with the recent calculations of Forest, Pandharipande, and Arriaga [24] who use a truncated version of the Argonne v_{18} interaction.

Next we calculate the energy expectation values for the *s*-shell hypernuclei, namely, $^4_\Lambda$H, $^4_\Lambda$H*, and $^5_\Lambda$He, using the two-body *NN* Argonne v_{18} and three-body *NNN* Urbana IX interactions along with the two-body ΛN and three-body ΛNN interactions with appropriate correlations incorporated in the wave function. Variational calculations have been performed for different values of spin-average potential strength $\bar{V}$ (6.10, 6.15, and 6.20 MeV). The different values of the space-exchange parameter ϵ used in this work are 0.24 ($\bar{V}=6.20$ MeV), 0.19 ($\bar{V}=6.15$ MeV), and 0.14 ($\bar{V}=6.10$ MeV) [25].

TABLE IV. Energy expectation values (calculated and fitted) for $^4_\Lambda$H (ground state) with $\bar{V}=6.20$ MeV. $\epsilon=0.24$, $\kappa_{\Lambda N}=0.12$, $\alpha_s=0.95$, $\alpha_\sigma=0.7$.

V_σ	W_o	C_p	$\langle E_{cal}\rangle$	$\langle E_{fit}\rangle$
0.17	0	0	10.58(05)	10.60
0.17	0	1.0	11.61(07)	11.63
0.17	0.01	0	9.97(05)	9.97
0.17	0.01	1.0	10.96(05)	10.90
0.17	0.02	0	9.25(04)	9.31
0.17	0.02	1.0	10.21(05)	10.13
0.17	0.02	2.0	12.28(07)	12.31
0.17	0.005	0	10.23(05)	10.29
0.17	0.005	1.0	11.25(05)	11.27
0.17	0.005	2.0	13.58(08)	13.60
0.17	0.015	1.0	10.50(05)	10.52
0.22	0	0	10.82(05)	10.71
0.22	0	1.0	11.66(05)	11.74
0.22	0.01	0	10.02(05)	10.09
0.22	0.01	1.0	10.90(05)	11.01
0.22	0.01	2.0	13.36(08)	13.30
0.22	0.02	0	9.43(05)	9.43
0.22	0.02	1.0	10.21(05)	10.25
0.22	0.02	2.0	12.35(06)	12.43
0.22	0.005	1.0	11.47(05)	11.38
0.22	0.015	1.0	10.68(05)	10.63
0.27	0	0	10.79(05)	10.83
0.27	0	1.0	11.79(06)	11.86
0.27	0	2.0	14.32(08)	14.24
0.27	0.01	0	10.26(05)	10.20
0.27	0.01	1.0	11.13(05)	11.12
0.27	0.01	2.0	13.41(08)	13.41
0.27	0.02	0	9.60(05)	9.55
0.27	0.02	1.0	10.39(05)	10.36
0.27	0.02	2.0	12.56(06)	12.54
0.27	0.005	0	10.50(05)	10.52
0.27	0.005	1.0	11.46(05)	11.49
0.27	0.015	1.0	10.81(05)	10.75

The bulk calculations consist of the energy expectation values for each $\bar{V}$ as a function of the interaction parameters, V_σ, C_p, and W_o. In Table IV we illustrate one such set of results for the ground state of $^4_\Lambda$H. Our results demonstrate that the B_Λ values for $^5_\Lambda$He, $^4_\Lambda$H, and $^4_\Lambda$H* show similar trends with the spin-average potential strength $\bar{V}$ and ΛNN interaction parameters, C_p and W_o. As expected, B_Λ increases with $\bar{V}$. B_Λ also increases significantly with the increase in C_p, while it decreases with W_o. As expected, the dependence on $\bar{V}$, C_p, and W_o is more pronounced for $A=5$ than for $A=4$ systems. This result is in accord with the

TABLE V. B_Λ as a function of coefficients y_{1-6} that include contribution due to Λ-nuclear correlations.

${}^A_\Lambda Z$	$\overline{V}$	y_1	y_2	y_3	y_4	y_5	y_6
${}^5_\Lambda$He	6.20	−0.92	−285.51	1.28	1701.78	1.50	−37.62
	6.15	−1.19	−280.03	0.99	1134.41	1.35	−7.44
	6.10	−0.20	−210.02	0.88	2997.79	1.19	−44.03
${}^4_\Lambda$H	6.20	2.27	−61.23	0.35	−141.55	0.68	−10.53
	6.15	2.21	−59.03	0.32	453.47	0.52	−8.14
	6.10	1.55	−43.48	0.23	298.23	0.37	−4.96
${}^4_\Lambda$H*	6.20	−0.89	−112.62	0.37	639.67	0.68	−8.51
	6.15	−0.98	−65.63	0.27	208.58	0.45	−6.68
	6.10	−0.81	−53.21	0.12	334.70	0.36	−5.69

earlier calculations where only simplified NN interactions have been used [4].

An important goal of the present study is to learn about the role of V_σ, C_p, and W_o through the 0^+-1^+ energy splitting in ${}^4_\Lambda$H and ${}^4_\Lambda$H*. We place limits on the values of these parameters, consistent with the following experimental B_Λ values:

$$B_\Lambda({}^4_\Lambda\mathrm{H})=2.22\pm0.04\ \ \mathrm{MeV},$$

$$B_\Lambda({}^4_\Lambda\mathrm{H}^*)=1.12\pm0.06\ \ \mathrm{MeV},$$

$$B_\Lambda({}^5_\Lambda\mathrm{He})=3.12\pm0.02\ \ \mathrm{MeV}. \tag{4.1}$$

Values of $B_\Lambda({}^4_\Lambda\mathrm{H})$ and $B_\Lambda({}^4_\Lambda\mathrm{H}^*)$ are averages for those of ${}^4_\Lambda$H and ${}^4_\Lambda$He. Limits on the parameters V_σ, C_p, and W_o are determined by the uncertainties in the experimental B_Λ values.

For a given value of $\overline{V}$ we did a χ^2 fit for the calculated energy expectation values according to the relation

$$B_\Lambda(V_\sigma,W_o,C_p)=y_1V_\sigma+y_2W_o+y_3C_p+y_4W_o^2+y_5C_p^2 +y_6W_oC_p+B_\Lambda^o, \tag{4.2}$$

where B_Λ^o is the corresponding value of B_Λ for $V_\sigma=C_p=W_o=0$ for each hypernuclear species. The coefficients y_{1-6} are varied to give a minimum in the χ^2 that is defined as

$$\chi^2=\frac{1}{N}\sum_{i=1}^{N}\left[\frac{B_\Lambda(V_\sigma,C_p,W_o)-B_\Lambda}{\Delta B_\Lambda}\right]_i^2, \tag{4.3}$$

Here N is the total number of energy calculations for a particular hypernucleus with different values of V_σ, C_p, and W_o. B_Λ is the calculated value of the Λ separation energy and ΔB_Λ is the corresponding Monte Carlo statistical error. The values of the coefficients y_{1-6}, as determined from this procedure, are displayed in Table V. In all the cases considered the χ^2 values are $\leqslant 1$ which demonstrates the goodness of the fit. There shall be correlated error bars on the coefficients y_{1-6} which would be reflected in the uncertainties in determining V_σ, C_p, and W_o. We find it more convenient to consider the uncertainties in the experimental values while placing limits on V_σ, C_p, and W_o. We hope to compensate some of the uncertainties associated with the y's by giving a generous allowance to the experimental ΔB_Λ values as well as by taking into account the Monte Carlo statistical errors in the calculation of the energies.

We use the coefficients y_{1-6} of Table V to obtain a fit with respect to the experimental B_Λ values, treating V_σ, C_p, and W_o as parameters to determine the best fit. We again construct a χ^2 fit using Eq. (4.3) but now "N" refers to the factor "3" for the three hypernuclear species and B_Λ refers to the experimental B_Λ values. The χ^2 fit is minimized with respect to C_p and W_o for a given value of V_σ. In Fig. 1 we plot χ^2, C_p, and W_o as a function of V_σ for $\overline{V}=6.15$ MeV. It is seen that both C_p and W_o decrease with increase in V_σ, the effect being more pronounced for C_p. Figure 2 displays the calculated values of B_Λ as a function of V_σ for ${}^4_\Lambda$H, ${}^4_\Lambda$H*, and ${}^5_\Lambda$He. Within the accuracy of the graphs the B_Λ values for ${}^4_\Lambda$H and ${}^5_\Lambda$He do not show any dependence on V_σ in the range 0.09–0.26 MeV. As one may expect, the B_Λ values for ${}^4_\Lambda$H* depend sensitively on V_σ, and thus in turn on the spin dependence of C_p and W_o.

The χ^2 stays very close to zero [which corresponds to almost an exact fit to B_Λ(exp) values] for $V_\sigma=0.176$

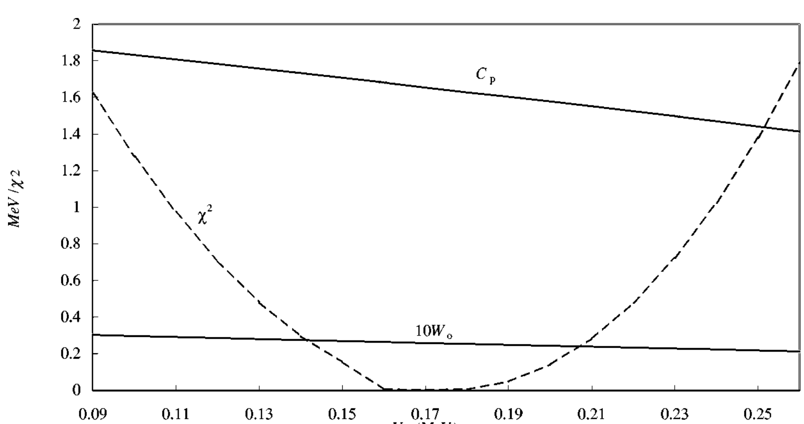

FIG. 1. χ^2, C_p, and W_o as a function of V_σ.

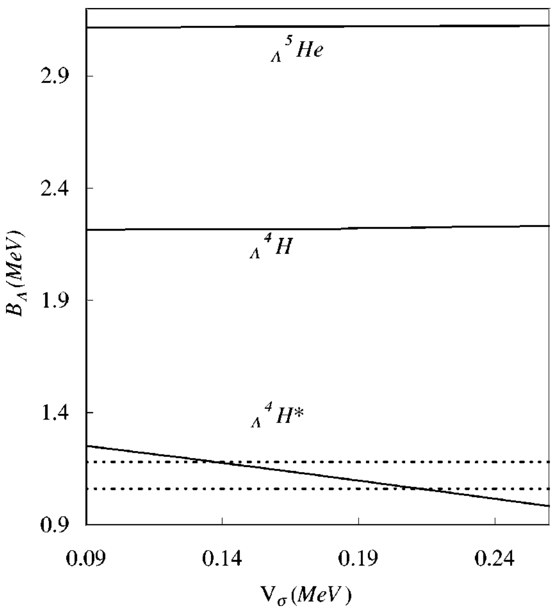

FIG. 2. B_Λ as a function of V_σ.

± 0.015, $C_p = 1.64 \mp 0.3$ and $W_o = 0.026 \mp 0.001$ MeV. Most of the deviation from zero of the χ^2 values in Fig. 1 arise from ${}^4_\Lambda$H*. The dotted horizontal lines of Fig. 2 display the limits on the experimental B_Λ value of ${}^4_\Lambda$H*, consistent with the experimental error bar of ± 0.06 MeV. This places limits on the values of V_σ, thus, in turn on C_p and W_o. However, the actual error bars on these parameters would be larger due to Monte Carlo statistical errors. To take this into account, we made a number of energy calculations for V_σ in the range of 0.10–0.24 MeV. The corresponding values of C_p and W_o have been taken from the calculations of Fig. 1. We could obtain acceptable fits to the energies for V_σ in the range of 0.12–0.23 MeV. This gives for $\overline{V} = 6.15$ MeV:

$$V_\sigma = 0.176 \pm 0.05, \quad C_p = 1.64 \mp 0.15, \quad W_o = 0.026 \mp 0.003. \tag{4.4}$$

A similar study for $\overline{V} = 6.20$ MeV gives

$$V_\sigma = 0.125 \pm 0.05, \quad C_p = 1.52 \mp 0.15, \quad W_o = 0.025 \mp 0.003. \tag{4.5}$$

We have been able to obtain good fits only for $\overline{V} = 6.15$ and 6.20 MeV. The value of $\overline{V} = 6.10$ MeV does not reproduce the correct binding energies of *s*-shell hypernuclei. Therefore, we have not carried out any error analysis for $\overline{V} = 6.10$ MeV. For the sake of completeness we mention the best parameter values for $\overline{V} = 6.10$ MeV:

$$V_\sigma = 0.193, \quad C_p = 1.84, \quad W_o = 0.027. \tag{4.6}$$

We can note from Table VI that the ΛN spin potential has a non zero contribution even in a closed-shell system such as ${}^5_\Lambda$He. This arises because of the ΛN spin-spin correlations incorporated in the wave function.

TABLE VI. Variational results for ${}^5_\Lambda$He, ${}^4_\Lambda$H, and ${}^4_\Lambda$H*.

Components	$\overline{V} = 6.20$	$\overline{V} = 6.15$	$\overline{V} = 6.10$
	${}^5_\Lambda$He		
Nuclear kinetic energy[a]	128.38(74)	128.16(71)	125.35(69)
NN potential energy[a]	−139.97(73)	−140.27(69)	−139.31(67)
NNN potential energy	−5.97(8)	−5.91(8)	−5.73(8)
Λ kinetic energy	11.64(15)	11.11(15)	9.13(13)
ΛN P.E (central)	−23.65(32)	−21.69(30)	−17.55(27)
ΛN P.E (spin)	−0.0138(1)	−0.0311(3)	−0.0281(3)
ΛN space exch. contribution	0.763(13)	0.578(10)	0.395(7)
ΛNN P.E (total)	−1.91(9)	−2.67(10)	−3.53(10)
ΛNN P.E (TPE)	−8.43(16)	−8.77(17)	−8.20(17)
ΛNN P.E (dispersive)	6.52(11)	6.10(11)	4.67(10)
Total energy	**−30.75(14)**	**−30.76(18)**	**−31.27(12)**
B_Λ	**3.03(15)**	**3.05(19)**	**3.56(13)**
rms radius (proton)	1.376(2)	1.379(2)	1.389(2)
rms radius (neutron)	1.377(2)	1.379(2)	1.389(2)
d state probability	0.1568(2)	0.1568(2)	0.1557(2)
	${}^4_\Lambda$H		
Nuclear kinetic energy[a]	65.76(47)	62.82(47)	60.34(46)
NN potential energy[a]	−66.91(47)	−65.10(46)	−63.60(46)
NNN potential energy	−1.33(3)	−1.25(3)	−1.19(3)
Λ kinetic energy	7.17(11)	5.99(9)	4.77(9)
ΛN P.E (central)	−14.00(22)	−11.55(20)	−8.84(18)
ΛN P.E (spin)	−0.291(3)	−0.363(4)	−0.300(4)
ΛN space exch. contribution	0.333(9)	0.221(6)	0.143(4)
ΛNN P.E (total)	−1.33(6)	−1.36(6)	−1.48(6)
ΛNN P.E (TPE)	−2.87(8)	−2.63(9)	−2.47(9)
ΛNN P.E (dispersive)	1.54(4)	1.27(4)	0.99(3)
Total energy	**−10.61(6)**	**−10.59(5)**	**−10.16(5)**
B_Λ	**2.28(6)**	**2.27(6)**	**1.83(6)**
rms radius (proton)	1.408(2)	1.435(3)	1.460(3)
rms radius (neutron)	1.516(3)	1.547(3)	1.577(3)
d-state probability	0.0984(1)	0.0960(1)	0.0962(1)
	${}^4_\Lambda$H*		
Nuclear kinetic energy[a]	63.78(49)	60.58(47)	57.96(46)
NN potential energy[a]	−65.71(48)	−63.64(46)	−62.36(45)
NNN potential energy	−1.34(3)	−1.27(3)	−1.20(3)
Λ kinetic energy	6.40(12)	4.78(9)	3.4516(8)
ΛN P.E (central)	−12.35(25)	−9.42(19)	−6.56(17)
ΛNN P.E (spin)	0.086(1)	0.083(1)	0.065(1)
ΛN space exch. contribution	0.311(8)	0.202(6)	0.110(4)
ΛNN P.E (total)	−0.57(6)	−0.62(5)	−0.73(5)
ΛNN P.E (TPE)	−3.11(9)	−2.32(8)	−1.98(8)
ΛNN P.E (dispersive)	2.54(7)	1.70(5)	1.24(5)
Total energy	**−9.40(7)**	**−9.31(5)**	**−9.26(5)**
B_Λ	**1.08(7)**	**0.99(5)**	**0.94(5)**
rms radius (proton)	1.431(3)	1.467(3)	1.494(3)
rms radius (neutron)	1.542(3)	1.585(3)	1.618(3)
d-state probability	0.0995(1)	0.0980(1)	0.0969(1)

[a]Includes contribution due to Λ-nuclear correlations.

TABLE VII. Breakup of the 0^+-1^+ splitting contributions. The first row gives the contribution to splitting from V_σ. The second row gives the contribution arising from $V_{\Lambda NN}$. The third row gives the total of V_σ and $V_{\Lambda NN}$. The last row gives the actual calculated energy difference between ${}^4_\Lambda$H and ${}^4_\Lambda$H*.

Contribution	$\overline{V}=6.20$	$\overline{V}=6.15$	$\overline{V}=6.10$
V_σ	0.377(3)	0.446(4)	0.365(4)
$V_{\Lambda NN}$	0.76(8)	0.74(8)	0.75(8)
Total	1.137(80)	1.186(80)	1.115(89)
Energy differences	1.21(9)	1.28(7)	0.90(7)

Comparing the results for the core nuclei (Table III) with the results for the hypernuclei (Table VI) we note, in general, a shrinking of the core nuclei by about 20% in all the hypernuclei due to the presence of the Λ particle. This decrease in radii of the core nuclei would imply that the Λ wave functions are closer for larger A. This also contributes to the fact that the dependence of B_Λ on $\overline{V}$, C_p, and W_o is more pronounced for the mass 5 than for the mass 4 hypernuclei. The change in the d-state probability is found to be small in all cases.

Table VII gives the breakup of the 0^+-1^+ splitting contributions in ${}^4_\Lambda$H and ${}^4_\Lambda$H* arising from the ΛN spin-dependent strength V_σ and the three-body ΛNN interaction $V_{\Lambda NN}$ for different values of spin-average strength $\overline{V}$. It can be noted from this table that the energy difference between ${}^4_\Lambda$H and ${}^4_\Lambda$H* is consistent with the total contribution from V_σ and $V_{\Lambda NN}$ within the error bars of the MC calculations. It can be seen that a large part ($\sim\frac{2}{3}$) of the splitting comes from the three-body ΛNN potential. The two-body contribution arising from V_σ is around $\sim\frac{1}{3}$ of the total splitting. This is in contrast to the earlier studies [4,28,29] wherein the 0^+-1^+ splitting has been thought to have arisen mainly from the spin dependence of the two-body ΛN potential. The present study clearly demonstrates that $V_{\Lambda NN}$ plays a significant role in explaining the splitting. This also results in a reduced V_σ as compared to the value of 0.23 ± 0.02 found in Ref. [4], though in our case the error bar on V_σ is much larger due to reasons discussed earlier. In the present study for $V_\sigma=0.23$, half of the splitting arises because of V_σ and the remaining half from the three-body ΛNN forces. For the extreme case, in particular, for $V_\sigma=0.12$, the three-body forces contribute nearly $\frac{3}{4}$ of the total splitting. It would be desirable to have an independent fix on V_σ, for example, from a more refined Λp scattering data. This can enlighten us further on the three-body ΛNN forces.

The Majorana space-exchange contribution for the various hypernuclei have been found to be small but significant in all s-shell hypernuclei. It is in the range 0.1–0.7 MeV for ε in the range 0.14–0.24 and as expected has a linear dependence with the Majorana exchange parameter ε.

A few variational calculations have been carried out with only the ΛNN potentials and no ΛNN correlations. We find that without the ΛNN correlations ${}^5_\Lambda$He is not bound. We also notice that the contributions from $V^{2\pi}_{\Lambda NN}$ and $V^{DS}_{\Lambda NN}$ become more repulsive without the correlations and the total contribution from the three-body ΛNN potentials become positive thereby decreasing B_Λ. We have performed a few energy calculations for ${}^5_\Lambda$He, ${}^4_\Lambda$H, and ${}^4_\Lambda$H* but with the three-body ΛNN part of the Hamiltonian completely switched off. We find in this case that ${}^5_\Lambda$He is overbound by about 2.34 MeV for $\overline{V}=6.20$ MeV and by about 1.39 MeV for $\overline{V}=6.15$ MeV. In general, the results for ${}^4_\Lambda$H show that it is underbound while ${}^4_\Lambda$H* is overbound without the three-body ΛNN interactions for all values of $\overline{V}$. Both these studies show the importance of the three-body ΛNN potentials and correlations in obtaining a consistent fit to the B_Λ values for all the s-shell hypernuclei considered in this study. In particular, we notice the importance of the ΛNN correlations $f^{2\pi}_{\Lambda NN}$ on the effect of TPE ΛNN forces $V^{2\pi}_{\Lambda NN}$. These correlations reduce reasonably the repulsive three-body contribution to an attractive contribution implying a strong nonlinear dependence on C_p.

Implications of s-shell results on ${}^{17}_\Lambda O$. We now examine the ${}^{17}_\Lambda$O hypernucleus in relation to the two- and three-body Λ-nuclear potential parameters that we find from our analysis of s-shell hypernuclei as described earlier. Usmani, Pieper, and Usmani (referred to as UPU) have carried out MC calculations for the ${}^{17}_\Lambda$O hypernucleus using the v_6 part of the older Argonne v_{14} potential [9]. For the three-nucleon potential they use the same form (Urbana model) as in the present study but with different strength parameters ($A_o=-0.0333$ and $U_o=0.0038$). Their trial wave function consists of pair and triplet operators acting on a single-particle determinant. In many respects, these calculations are similar to our present calculations. The difference lies in the treatment of noncentral correlations for which they use the cluster Monte Carlo method with up to four-baryon clusters. The central correlations are treated exactly. We carry out this study in the hope of analyzing further our estimated s-shell Λ-nuclear parameters. UPU have given the following empirical relation for C_p in the range 0–1 MeV and W_o in the range 0–0.02 MeV:

$$B_\Lambda=27.3-8.9C_p+11.2C_p^2+870.0W_o. \tag{4.7}$$

This equation relates the B_Λ of ${}^{17}_\Lambda$O with C_p and W_o. In order to test the consistency of our results with $B_\Lambda({}^{17}_\Lambda\mathrm{O})$, we assume that relation (4.7) holds for our values of C_p and W_o. UPU have done calculations for spin-average strength $\overline{V}=6.16$ MeV with space-exchange parameter $\varepsilon=0.3$. Our values of $\overline{V}=6.15$ MeV with $\varepsilon=0.17$ are closest to theirs. We thus need to modify Eq. (4.7) for our values of $\overline{V}$ and ε. Unfortunately, there is no simple method to scale relation (4.7) for $\overline{V}=6.15$ MeV, as the scaling can be considerably nonlinear. However, since the two values of $\overline{V}$ are very close, we assume that this will not affect the results much. The correction for ε is simple, since in the absence of space-exchange correlation the space-exchange energies are expected to be linear with ε. Thus, relation (4.7) can be modified as

TABLE VIII. Results for nuclear matter calculations. All energies are in MeV. ρ_o is the normal nuclear matter density in fm^{-3}. The third column gives the well depth D. The fourth column gives the value of D without the ΛNN forces and the space-exchange contribution, i.e., for $\epsilon=0$. The fifth column gives the reduction in the contributions to D due to the space-exchange part (Spc. exch.) of the ΛN potential. The last column gives the contribution due to the three-body ΛNN forces.

$\overline{V}$	ρ_o	$-D$	$\langle T_\Lambda + V_{\Lambda N}\rangle$	Spc. exch.	$V_{\Lambda NN}$
6.20	0.162	−21.525	−77.617	8.561	47.530
6.15	0.162	−17.819	−72.628	6.644	48.164
6.10	0.162	−11.727	−67.729	4.798	51.205

$$B_\Lambda = 27.3 + (\varepsilon - \varepsilon_o)\langle (1-P_x)V_{\Lambda N}\rangle - 8.9C_p + 11.2C_p^2 + 870.0W_o, \tag{4.8}$$

where $\varepsilon_o = 0.3$ as taken by UPU, P_x is the space-exchange operator and $\langle V_{\Lambda N}\rangle$ is the energy expectation value of the ΛN potential.

Using the entries for $v_o(r)(1-\varepsilon)$ and $v_o(r)\varepsilon P_x$ from Table II of Ref. [16] and using our values of $C_p = 1.6407$ and $W_o = 0.0255$ for $\overline{V} = 6.15$ MeV we obtain

$$B_\Lambda = 23.3 \pm 1.6 \text{ MeV}.$$

This is considerably larger than the empirical estimate [16] $\sim 13.0 \pm 0.4$ MeV. Thus the results of the present study are incompatible with those of Ref. [16]. The reason for this incompatibility may largely lie in the use of v_6 part of the v_{18} hamiltonian for ${}^{17}_{\Lambda}$O. Another reason can be attributed to the use of relations (4.7) and (4.8) for large values of C_p and W_o for which these relations may not be adequate. This discrepancy can probably be resolved by carrying out calculations for ${}^{17}_{\Lambda}$O with Argonne v_{18} Hamiltonian, which, at the moment is an extremely challenging task.

Implications of s-shell results on Λ binding to nuclear matter. The presence of a Λ particle inside nuclear matter can reveal information on the Λ-nuclear interactions. The well depth D is identified with the separation energy for a Λ in nuclear matter. It is an important parameter which can help to distinguish between different ΛN potentials and also throw light on the ΛNN interaction. Λ-binding to nuclear matter can put further constraints on the potential parameters, namely $\overline{V}$, V_σ, W_o, and C_p. With this aim in mind we have performed calculations for D using the FHNC technique [4] to calculate the energy expectation values.

We have calculated the well depth D variationally using the same underlying principle as for our s-shell hypernuclei. Our discussion on D is based on the results given in Table VIII. The empirical value of D is now fairly well established at 29 ± 1 MeV [30,31] at the normal nuclear matter density of $\rho_o \approx 0.16$ fm^{-3}. These results clearly indicate that Λ is underbound at the normal nuclear matter density, $\rho_o \approx 0.16$ fm^{-3}. This indeed is a disturbing feature. Bodmer and Usmani [4] have found that it is possible to obtain a consistent phenomenology with hypernuclear interactions which include the s-shell and the medium and heavy hypernuclei as well as Λ binding to nuclear matter. Our results indicate that with the present available techniques of treating the nuclear matter this is not possible. The resolution of this paradox perhaps lies in the proper handling of the three-body correlations, particularly the ΛNN correlations for nuclear matter. It may be noted that the contribution from the TPE ΛNN forces $V^{2\pi}_{\Lambda NN}$ for nuclear matter is always positive [4]. On the other hand, $V^{2\pi}_{\Lambda NN}$ for s-shell hypernuclei and ${}^{17}_{\Lambda}$O is always negative and substantial. The three-body ΛNN correlations that are taken in nuclear matter calculations always pertain to the s shell [4]. The reason for adopting this correlation lies in its simplicity. At present the techniques for incorporating the realistic three-body ΛNN correlations for nuclear matter are not sufficiently developed as compared to those for the s-shell hypernuclei incorporated in this work. These affect the contribution from $V^{2\pi}_{\Lambda NN}$ quite substantially, even to the extent of reversing its sign in the presence of the ΛNN correlations as can be seen in the present as well as in the ${}^{17}_{\Lambda}$O studies. The correct incorporation of the three-body correlations in nuclear matter may affect the results to quite an extent, particularly those at high densities [32]. The incorporation of these correlations is indeed a challenging task and is very much needed for the present work as well as for other related studies.

V. CONCLUSION AND COMMENTS

From the results discussed in the previous section we note that the values of the spin-average strength $\overline{V} = 6.20$ and 6.15 MeV give a reasonable description of the s-shell hypernuclei. We have been able to provide a consistent account of the B_Λ values of ${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$H*, and ${}^{5}_{\Lambda}$He using the realistic Argonne v_{18} *NN* interaction and Urbana *IX NNN* interaction along with Λ-nuclear interactions with appropriate correlations. Our results for B_Λ show very similar trends with the spin-average strength $\overline{V}$ of the ΛN interaction, and the ΛNN interaction parameters, C_p and W_o for all the s-shell hypernuclei considered.

An important conclusion of our study is that ~25–50 % of the 0^+-1^+ splitting energy between the ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$H* comes from the ΛN spin-dependent strength V_σ. The earlier studies [4,28,29] attribute a larger part of the splitting to the spin dependence of the two-body ΛN interactions. In contrast, our study indicates that the major part (~50–75 %) of the splitting is generated by the three-body ΛNN forces.

Our study on Λ binding to nuclear matter shows that Λ is underbound. This indicates the fact that there is a need to include the three-body correlations while treating nuclear matter. This would require a different technique altogether and is a challenging problem in itself. Our analysis on ${}^{17}_{\Lambda}$O also indicate the importance of the noncentral correlations. It is possible that the inconsistency between the results of Ref. [16] and our s-shell results is due to their neglecting terms with higher than four-baryon clusters and thereby neglecting the contributions from the non-central clusters. Moreover, our values of the ΛNN interaction parameters, C_p and W_o

are higher than those of Ref. [16] and which, in turn, would induce stronger ΛNN correlations.

Contrary to the findings by Bando and Shimodaya [33] and Shinmura, Akaishi, and Tanaka [34] regarding the effect of tensor forces on the overbinding problem, we find that the tensor forces do not play a significant role. Further, separate studies by Hiyama *et al.* [29] and Carlson [28] on four- and five-body hypernuclei have shown that the binding energies and the splitting energies are not reproduced correctly using the Nijmegen interactions that have strong tensor terms. However, the small suppression effects expected from ΛN tensor forces are already implicitly included in our phenomenological dispersive ΛNN force. Moreover, the Argonne v_{18} potential used in our study has a weak tensor part and this in fact, provides a much better binding to nuclei [5].

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[1] Dean Halderson, Phys. Rev. C **61**, 034001 (2000).
[2] Y. Yamamoto and H. Bando, Prog. Theor. Phys. Supp. **81**, 9 (1985).
[3] A. R. Bodmer, Q. N. Usmani, and J. Carlson, Phys. Rev. C **29**, 684 (1984).
[4] A. R. Bodmer and Q. N. Usmani, Nucl. Phys. **A477**, 621 (1988).
[5] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
[6] J. Carlson, V. R. Pandharipande, and R. B. Wiringa, Nucl. Phys. **A401**, 59 (1983).
[7] B. S. Pudliner, V. R. Pandharipande, J. Carlson, and R. B. Wiringa, Phys. Rev. Lett. **74**, 4396 (1995).
[8] Q. N. Usmani, Nucl. Phys. **A340**, 397 (1980).
[9] R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, Phys. Rev. C **29**, 1207 (1984).
[10] J. R. Bergervoet, P. C. van Campen, R. A. M. Klomp, J. L. de Kok, T. A. Rijken, V. G. J. Stoks, and J. J. de Swart, Phys. Rev. C **41**, 1435 (1990).
[11] V. G. J. Stoks, R. A. M. Klomp, M. C. M. Rentmeester, and J. J. de Swart, Phys. Rev. C **48**, 792 (1993).
[12] J. Fujita and H. Miyazawa, Prog. Theor. Phys. **17**, 360 (1957).
[13] I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. **A359**, 331 (1981).
[14] Thomas Schimert, D. J. Stubeda, M. Lemere, and Y. C. Tang, Nucl. Phys. **A343**, 429 (1980); Wang Xi-chang, H. Takaki, and H. Bando, Prog. Theor. Phys. **76**, 865 (1986).
[15] R. K. Bhaduri, B. A. Loiseau, and Y. Nogami, Ann. Phys. (N.Y.) **44**, 57 (1967).
[16] A. A. Usmani, S. C. Pieper, and Q. N. Usmani, Phys. Rev. C **51**, 2347 (1995).
[17] M. K. Pal, *Theory of Nuclear Structure* (Affiliated East-West, New Delhi), Appendix A, p. 606.
[18] R. B. Wiringa, Phys. Rev. C **43**, 1585 (1991).
[19] A. Arriaga, V. R. Pandharipande, and R. B. Wiringa, Phys. Rev. C **52**, 2362 (1995).
[20] S. Murali, Ph.D. thesis, Jamia Millia Islamia, New Delhi, India, 1995.
[21] J. Lomnitz-Adler, V. R. Pandharipande, and R. A. Smith, Nucl. Phys. **A361**, 399 (1981).
[22] J. Carlson and R. B. Wiringa, in *Computational Nuclear Physics*, edited by S. E. Koonin, K. Langanke, J. A. Maruhn, and M. R. Zirnbauer (Springer-Verlag, Berlin, 1991).
[23] N. Metropolis, A. W. Rosenbluth, M. N. Rosenbluth, A. H. Teller, and E. Teller, J. Chem. Phys. **21**, 1087 (1953).
[24] J. L. Forest, V. R. Pandharipande, and A. Arriaga, Phys. Rev. C **60**, 014002 (1999).
[25] Q. N. Usmani and A. R. Bodmer (private communication).
[26] J. Pniewski and D. Zieminska, Proceedings of the Conference on Kaon-nuclear Interactions and Hypernuclei, Zvenigorod, 1977, p. 33.
[27] M. Bedjidian, E. Descroix, J. Y. Grossiord, A. Guichard, M. Gusakow, M. Jacquin, M. J. Kudla, H. Piekarz, J. Piekarz, J. R. Pizzi, and J. Pniewski, Phys. Lett. **83B**, 252 (1979), and references therein.
[28] J. Carlson, in *LAMPF Workshop on (π,K) Physics*, edited by Benjamin F. Gibson, William R. Gibbs, and Mikkel B. Johnson, AIP Conf. Proc. No. 224 (AIP, New York, 1991).
[29] E. Hiyama, M. Kamimura, T. Motoba, T. Yamada, and Y. Yamamoto, Nucl. Phys. **A639**, 169c (1998).
[30] Q. N. Usmani and A. R. Bodmer, Nucl. Phys. **A639**, 147c (1998).
[31] Q. N. Usmani and A. R. Bodmer, Phys. Rev. C **60**, 055215 (1999).
[32] A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C **58**, 1804 (1998), and references therein.
[33] H. Bando and I. Shimodaya, Prog. Theor. Phys. **63**, 1812 (1986).
[34] S. Shinmura, Y. Akaishi, and H. Tanaka, Prog. Theor. Phys. **65**, 1290 (1981).