Thermal conductivity of dense quark matter and cooling of stars

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The thermal conductivity of the color-flavor locked phase of dense quark matter is calculated. The dominant contribution to the conductivity comes from photons and Nambu-Goldstone bosons associated with the breaking of the baryon number which are trapped in the quark core. Because of their very large mean free path the conductivity is also very large. The cooling of the quark core arises mostly from the heat flux across the surface of direct contact with the nuclear matter. As the thermal conductivity of the neighboring layer is also high, the whole interior of the star should be nearly isothermal. Our results imply that the cooling time of compact stars with color-flavor locked quark cores is similar to that of ordinary neutron stars.

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I. INTRODUCTION

At sufficiently high baryon density the nucleons in nuclear matter should melt into quarks so that the system becomes a quark liquid. It should be weakly interacting due to asymptotic freedom [1], however, it cannot be described as a simple Fermi liquid. This is due to the nonvanishing attractive interaction in the color antitriplet quark-quark channel, provided by one-gluon exchange, which renders a highly degenerate Fermi surface unstable with respect to Cooper pairing. As a result the true ground state of dense quark matter is, in fact, a color superconductor [2].

Recent phenomenological [3] and microscopic studies [4–7] have confirmed that quark matter at a sufficiently high density undergoes a phase transition into a color superconducting state. Phenomenological studies are expected to be appropriate to intermediate baryon densities, while microscopic approaches are strictly applicable at asymptotic densities where perturbation theory can be used. It is remarkable that both approaches concur that the superconducting order parameter (which determines the gap Δ in the quark spectrum) lies between 10 and 100 MeV for baryon densities existing in the cores of compact stars. For a review of color superconducting quark matter see Ref. [8].

At realistic baryon densities only the three lightest quarks can participate in the pairing dynamics. The fact that the current mass of the strange quark is more than an order of magnitude larger than the masses of the up and down quarks suggests two limiting cases. The first extreme limit assumes that the strange quark is sufficiently massive that it does not participate in the pairing dynamics at all. The corresponding quark phase is the color superconducting phase involving two quark flavors (2SC). Since it was suggested in Ref. [9] that such a phase is absent in compact stars, we focus on the second extreme case. This applies when all three quark flavors have a mass which is small in comparison to the chemical potential so that they participate equally in the color condensation. The ground state is then the so-called color flavor locked (CFL) phase [10]. Here the original gauge symmetry $\mathrm{SU}(3)_c$ and the global chiral symmetry $\mathrm{SU}(3)_L \times SU(3)_R$ break down to a global diagonal "locked" $\mathrm{SU}(3)_{c+L+R}$ subgroup. Because of the Higgs mechanism the gluons become massive and decouple from the infrared dynamics. The quarks also decouple because large gaps develop in their energy spectra. The breaking of the chiral symmetry leads to the appearance of an octet of pseudo-Nambu-Goldstone (NG) bosons (π^0, $\pi^{\pm}$, $K^{\pm}$, K^0, $\overline{K^0}$, η). In addition an extra NG boson ϕ and a pseudo-NG boson η' appear in the low energy spectrum as a result of the breaking of global baryon number symmetry and approximate $\mathrm{U}(1)_A$ symmetry, respectively.

The low energy action for the NG bosons in the limit of asymptotically large densities was derived in Refs. [11,12]. By making use of an auxiliary "gauge" symmetry, it was suggested in Ref. [13] that the low energy action of Refs. [11,12] should be modified by adding a time-like covariant derivative to the action of the composite field. Under a favorable choice of parameters, the modified action predicted kaon condensation in the CFL phase. Some unusual properties of such a condensate were discussed in Refs. [14,15].

While the general structure of the low energy action in the CFL phase can be established by symmetry arguments alone [11], the values of the parameters in such an action can be rigorously derived only at asymptotically large baryon densities [12,13]. Thus, in the most interesting case of intermediate densities existing in the cores of compact stars, the details of the action are not well known. For the purposes of the present paper, however, it suffices to know that there are nine massive pseudo-NG bosons and one massless NG boson ϕ in the low energy spectrum. If kaons condense [13], an additional NG boson should appear.[1] These NG bosons should be relevant for the kinetic properties of dense quark matter.

It has been found [16] that neutrino and photon emission rates for the CFL phase are very small so that they would be

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[1]Strickly speaking the additional NG boson in a phase with kaon condensation has a small, but nonzero, mass as a result of weak interactions that break strangeness. The estimated value of the mass is of order 50 keV [15].

inefficient in cooling the core of a neutron star. The purpose of the present investigation is to determine quantitatively the thermal conductivity of the CFL phase of dense quark matter in order to see whether it can significantly impact the cooling rate. We shall argue that the temperature of the CFL core, as well as the neighboring neutron layer which is in contact with the core, falls quickly due to the very high thermal conductivities on both sides of the interface. In fact, to a good approximation, the interior of the star is isothermal. A noticeable gradient of the temperature appears only in a relatively thin surface layer of the star where a finite flux of energy is carried outwards by photon diffusion.

The paper is organized as follows. In the next section, we derive the microscopic expression for thermal conductivity of the (pseudo-)NG bosons in the CFL phase. In Sec. III we estimate the mean free path l of the NG boson in the CFL phase. Knowledge of this quantity is crucial to a reliable calculation of the thermal conductivity. The role of photons in affecting the thermal conductivity and specific heat of the quark core is discussed in Sec. IV. In Sec. V we discuss our findings and their implication for the cooling dynamics of a compact star with a quark core in the CFL phase. Our conclusions are summarized in Sec. VI. In the Appendix we present some useful formulas.

II. THERMAL CONDUCTIVITY

A detailed understanding of the cooling mechanism of a compact star with a quark core is not complete without a study of thermal conductivity effects in the color superconducting quark core. The conductivity, as well as the other kinetic properties of quark matter in the CFL phase, is dominated by the low energy degrees of freedom. It is clear then that at all temperatures of interest to us, $T \ll \Delta$, it is crucial to consider the contributions of the NG bosons. In addition, there may be an equally important contribution due to photons; this is discussed in Sec. IV. Note that, at such small temperatures, the gluon and the quark quasiparticles become completely irrelevant. For example, a typical quark contribution to a transport coefficient would be exponentially suppressed by the factor $\exp(-\Delta/T)$. Therefore, in the rest of this section, we concentrate exclusively on the contributions of the NG bosons.

Let us start from the general definition of the thermal conductivity as a characteristic of a system which is forced out of equilibrium by a temperature gradient. In response to such a gradient transport of heat is induced. Formally this is described by the following relation:

$$u_i = -\kappa \partial_i T, \tag{1}$$

where u_i is the heat current, and κ is the heat conductivity. As is clear from this relation, the heat flow would persist until a state of uniform temperature is reached. The higher the conductivity, the shorter the time for this relaxation.

In the linear response approximation, the thermal conductivity is given in terms of the heat current correlator by a Kubo-type formula. We derive the expression for the heat (energy) current carried by a single (pseudo-)NG boson field φ. The corresponding Lagrangian density reads

$$L = \tfrac{1}{2}\left(\partial_0\varphi\partial_0\varphi - v^2\partial_i\varphi\partial_i\varphi - m^2\varphi^2\right) + \cdots, \tag{2}$$

where the ellipsis stand for the self-interaction terms as well as interactions with other fields. Notice that we introduced explicitly the velocity parameter v. In microscopic studies of color superconducting phases, which are valid at very large densities, this velocity is equal to $1/\sqrt{3}$ for all (pseudo-)NG bosons. It is smaller than 1 because Lorentz symmetry is broken due to the finite value of the quark chemical potential. By making use of the above Lagrangian density, we derive the following expression for the heat current:

$$u_i = \frac{\partial L}{\partial(\partial^i\varphi)}\partial_0\varphi = v^2\partial_i\varphi\partial_0\varphi. \tag{3}$$

This definition leads to the expression [17] for the heat conductivity in terms of the corresponding correlator:

$$\kappa_{ij} = -\frac{i}{2T}\lim_{\Omega\to 0}\frac{1}{\Omega}\left[\Pi^R_{ij}(\Omega + i\epsilon) - \Pi^A_{ij}(\Omega - i\epsilon)\right], \tag{4}$$

where, in the Matsubara formalism,

$$\Pi_{ij}(i\Omega_m) = v^4 T\sum_n \int \frac{d^3k}{(2\pi)^3} k_i k_j i\Omega_n(i\Omega_n + i\Omega_{n-m}) \times S(i\Omega_n,\vec{k})S(i\Omega_{n-m},\vec{k}). \tag{5}$$

Here $\Omega_n \equiv 2\pi n T$ is the bosonic Matsubara frequency, and $S(i\Omega_n,\vec{k})$ is the propagator of the (pseudo-)NG boson. In general, the propagator should have the following form:

$$S(\omega,\vec{k}) = \frac{1}{(\omega + i\Gamma/2)^2 - v^2\vec{k}^2 - m^2}, \tag{6}$$

where the width parameter $\Gamma(\omega,\vec{k})$ is related to the inverse lifetime (as well as the mean free path) of the boson.

In our calculation, it is very convenient to utilize the spectral representation of the propagator,

$$S(i\Omega_n,\vec{k}) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{d\omega A(\omega,\vec{k})}{i\Omega_n - \omega}. \tag{7}$$

Then, the conductivity is expressed through the spectral function $A(\omega,\vec{k})$ as follows:

$$\kappa_{ij} = \frac{v^4}{2\pi T^2}\int_{-\infty}^{\infty}\frac{\omega^2 d\omega}{\sinh^2\omega/2T}\int\frac{d^3k}{(2\pi)^3}k_i k_j A^2(\omega,\vec{k}). \tag{8}$$

By making use of the explicit form of the propagator in Eq. (6), we see that the spectral function of the (pseudo-)NG boson is

$$A(\omega,\vec{k})=\frac{\omega\Gamma}{(\omega^2-e_k^2-\Gamma^2/4)^2+\omega^2\Gamma^2}$$

$$\equiv\frac{\Gamma}{4e_k}\left[\frac{1}{(\omega-e_k)^2+\Gamma^2/4}-(\omega\rightarrow-\omega)\right], \qquad (9)$$

where $e_k\equiv\sqrt{v^2k^2+m^2}$. By substituting this spectral function into the general formula (8), we notice that the off-diagonal components of the thermal conductivity are zero and all the diagonal components are equal. Both of these facts are a consequence of the rotational symmetry of the system. Thus the conductivity is characterized by a single scalar quantity κ which is introduced as follows: $\kappa_{ij}=\kappa\delta_{ij}$. The explicit expression for this scalar function reads

$$\kappa=\frac{1}{48\sqrt{2}\pi^2 v\Gamma T^2}\int_0^\infty\frac{\omega d\omega}{\sinh^2\omega/2T}(\sqrt{X^2+\omega^2\Gamma^2}+X)^{3/2}, \qquad (10)$$

where we introduced the notation $X\equiv\omega^2-m^2-\Gamma^2/4$. In the calculation leading to Eq. (10), we made use of the result [18]:

$$\int_0^\infty\frac{a^2x^4dx}{[(x^2+b)^2+a^2]^2}=\frac{\pi}{8\sqrt{2}|a|}(\sqrt{a^2+b^2}-b)^{3/2}. \qquad (11)$$

For our purposes it will be sufficient to consider the conductivity in the limit of a very small width. This is because the (pseudo-)NG bosons in the CFL quark matter are weakly interacting. In this case we derive the following approximate relation:

$$\kappa=\frac{1}{24\pi^2 v T^2\Gamma}\int_m^\infty\frac{d\omega\omega}{\sinh^2\omega/2T}(\omega^2-m^2)^{3/2}. \qquad (12)$$

It is interesting to notice that the same result can be easily derived from Eq. (8). Indeed, in the limit of a very small width, i.e., $\Gamma\rightarrow 0$, we can replace the square of the spectral function by a sum of two δ-functions:

$$A^2(\omega,\vec{k})\rightarrow\frac{\pi}{4\Gamma e_k^2}[\delta(\omega-e_k)+\delta(\omega+e_k)], \qquad (13)$$

where the coefficients in front of the δ-functions were determined unambiguously by using the following replacement rule:

$$\lim_{\Gamma\rightarrow 0}\frac{\Gamma^3}{(x^2+\Gamma^2/4)^2}\rightarrow 4\pi\delta(x). \qquad (14)$$

Now in the limit of small temperature, $T\ll m$, the result in Eq. (12) is given approximately by

$$\kappa=\frac{m^5}{24\pi^2 v\Gamma T^2}\int_0^\infty\frac{x^4dx}{\sinh^2 m\sqrt{1+x^2}/2T}$$

$$\simeq\frac{m^{5/2}\sqrt{T}}{2\sqrt{2}\pi^{3/2}v\Gamma}e^{-m/T}. \qquad (15)$$

This demonstrates clearly that the contributions of heavy pseudo-NG bosons to the thermal conductivity are strongly suppressed. The largest contribution comes from the massless NG boson ϕ for which the thermal conductivity is

$$\kappa_\phi=\frac{4T^3}{3\pi^2 v\Gamma_\phi}\int_0^\infty\frac{x^4dx}{\sinh^2 x}=\frac{2\pi^2T^3}{45v\Gamma_\phi}. \qquad (16)$$

In order to calculate κ_ϕ the width Γ_ϕ is required, or equivalently the mean free path l_ϕ since $l_\phi\equiv\overline{v}/\Gamma_\phi$, where $\overline{v}$ is the average thermal velocity of the particles responsible for heat transfer. This will be discussed in the next section. For the moment we note that expression (16) for the thermal conductivity is similar to the phonon conductivity often used in solid state physics [19]. It is also interesting to note that the result for the thermal conductivity of both massive and massless bosons in Eqs. (15) and (16) are consistent with a general relation from classical Boltzmann kinetic theory [20]:

$$\kappa=\frac{1}{3}\overline{v}c_v l, \qquad (17)$$

where c_v is the specific heat. In order to see that this relation holds, we note that the average thermal velocity, $\langle\partial e_k/\partial k\rangle$, of massive bosons is a factor $\sqrt{3T/m}$ smaller than the velocity of massless bosons $v=1/\sqrt{3}$. After taking this into account, as well as using the expressions for the specific heat in Eqs. (A8) and (A12) in the Appendix, we find that the relation (17) between the conductivity and the specific heat is indeed satisfied.

III. MEAN FREE PATH OF THE NG BOSON

In the preceding section we derived a general expression for the boson portion of the thermal conductivity. The result involved the decay width of the (pseudo-)NG boson or equivalently the mean free path, $l=\overline{v}/\Gamma$, for which we derive a simple estimate here. Using this we can understand the effect of the heat conductivity on the relaxation time of the temperature gradient inside the quark core of a star. This is crucial for understanding the cooling mechanism of compact stars with quark cores.

As we have remarked, the contribution of massive pseudo-NG bosons to the thermal conductivity is suppressed by the exponential factor $\exp(-m/T)$. In the CFL phase of quark matter, however, there is one truly massless NG boson ϕ which should therefore give the dominant contribution to the heat conductivity. The interactions of ϕ with the CFL matter leads to a finite value for its mean free path. Since this boson is a composite particle there is always a nonzero probability at finite temperature for its decay into a pair of quark quasiparticles. It is natural to expect that such a process is strongly suppressed at small temperatures, $T \ll \Delta$. This is confirmed by a direct microscopic calculation in the region of asymptotic densities which yields a decay width [21]:

$$\Gamma_{\phi \to qq}(k) \simeq \frac{5\sqrt{2}\pi v k}{4(21-8\ln 2)} \exp\left(-\sqrt{\frac{3}{2}}\frac{\Delta}{T}\right). \tag{18}$$

If this were the only contribution, then the order of magnitude of the mean free path of the NG boson would be

$$l_{\phi \to qq} \sim \frac{v}{T} \exp\left(\sqrt{\frac{3}{2}}\frac{\Delta}{T}\right). \tag{19}$$

This grows exponentially with decreasing the temperature. For example, if $T \lesssim \Delta/33$ the mean free path is 30 km or more (in deriving this estimate we set $\Delta \simeq 50$ MeV). This scale is a few times larger than the typical size of a compact star. The underlying physics in this calculation of the mean free path closely resembles the propagation of sound waves in superfluid helium. We recall that there are two types of excitations in superfluid helium: gapless phonons and finite gap rotons. The main contribution to the thermal conductivity comes from the phonons whose mean free path is mainly determined by scattering on rotons [22]. The value of the mean free path, as in the case of NG bosons, becomes exponentially large at small temperatures ($T \ll \Delta$).

Now, in the case of the NG bosons, the decay channel into quarks is not the only contribution to the mean free path since the NG bosons can also scatter on one another. The amplitude has been derived by Son [23] and is of order k^4/μ^4 which gives a cross section of $\sigma_{\phi\phi} \simeq T^6/\mu^8$. This yields the following contribution to the width of the NG boson:

$$\Gamma_{\phi\phi} = v \sigma_{\phi\phi} n_{\phi} \sim \frac{T^9}{\mu^8}, \tag{20}$$

where n_{ϕ} is the equilibrium number density of the NG bosons. The explicit expression for n_{ϕ} is given in Eq. (A11) in the Appendix. At small temperatures the scattering contribution in Eq. (20) dominates the width. This leads to a mean free path

$$l_{\phi\phi} \sim \frac{\mu^8}{T^9} \approx 8 \times 10^5 \frac{\mu_{500}^8}{T_{\mathrm{MeV}}^9} \mathrm{km}. \tag{21}$$

Here we defined the following dimensionless quantities: $\mu_{500} \equiv \mu/(500\ \mathrm{MeV})$ and $T_{\mathrm{MeV}} \equiv T/(1\ \mathrm{MeV})$. Both $l_{\phi\phi}$ and $l_{\phi \to qq}$ depend very strongly on temperature, however the salient point is that they are both larger than the size of a compact star for temperatures T_{MeV} of order 1.

We define $\tilde{T}$ to be the temperature at which the massive NG bosons decouple from the system. This is determined by the mass of the lightest pseudo-NG boson for which it is not presently possible to give a reliable value. Different model calculations [12,13,24] produce different values which can range as low as 10 MeV. Thus, conservatively, we choose $\tilde{T} \simeq 1$ MeV. Then the mean free path of the NG boson is comparable to or even larger than the size of a star for essentially all temperatures $T \lesssim \tilde{T}$ MeV. It is also important to note that the mean free path is very sensitive to temperature changes. In particular, at temperatures just a few times higher than $\tilde{T}$, the value of l may already become much smaller than the star size. This suggests that, during the first few seconds after the supernova explosion when the temperatures remain considerably higher than $\tilde{T}$, a noticeable temperature gradient may exist in the quark core. This should relax very quickly because of the combined effect of cooling (which is very efficient at $T \gg \tilde{T}$) and diffusion. After that almost the whole interior of the star would become isothermal.

Before concluding this section, we point out that the geometrical size of the quark phase limits the mean free path of the NG boson since the scattering with the boundary should also be taken into account. It is clear from simple geometry that $l \sim R_0$, where R_0 is the radius of the quark core. A similar situation is known to appear in high quality crystals at very low temperatures [20].

IV. PHOTON CONTRIBUTIONS

In this section, we discuss the role of photons in the CFL quark core. It was argued in Ref. [16] that the mean free path of photons is larger than the typical size of a compact star at all temperatures $T \lesssim \tilde{T}$. One might conclude, therefore, that the photons would leave a finite region of the core in a very short amount of time after the core becomes transparent. If this were so, the photons would be able to contribute neither to thermodynamic nor to kinetic properties of the quark core. However, the neighboring neutron matter has very good metallic properties due to the presence of a considerable number of electrons. As is known from plasma physics, low frequency electromagnetic waves cannot propagate inside a plasma. Moreover, an incoming electromagnetic wave is reflected from the surface of such a plasma [25]. In particular, if Ω_p is the value of the plasma frequency of the nuclear matter, then all photons with frequencies $\omega < \Omega_p$ are reflected from the boundary. This effect is similar to the well known reflection of radio waves from the Earth's ionosphere.

The plasma frequency is known to be proportional to the square root of the density of charge carriers and inversely proportional to the square root of their mass. It is clear, therefore, that the electrons, rather than the more massive protons, will lead to the largest value of the plasma frequency in nuclear matter. Our estimate for the value of this frequency is

$$\Omega_p = \sqrt{\frac{4\pi e^2 Y_e \rho}{m_e m_p}} \simeq 4.7 \times 10^2 \sqrt{\frac{\rho Y_e}{\rho_0}}\ \mathrm{MeV}, \tag{22}$$

where the electron density $n_e = Y_e \rho / m_p$ is given in terms of the nuclear matter density ρ and the proton mass m_p. Also m_e denotes the electron mass, $Y_e \simeq 0.1$ is the number of electrons per baryon, and $\rho_0 \approx 2.8 \times 10^{14}\ \mathrm{g\ cm^{-3}}$ is equilibrium nuclear matter density.

Since Ω_p is more than 100 MeV, electromagnetic waves of essentially all frequencies for temperatures appropriate to the CFL phase will be reflected back into the core region. Thus photons present very early in the life of the star, those produced by decays in the CFL matter as well as those produced at the nuclear interface, will be trapped in the core. In a way, the boundary of the core looks like a good quality mirror with some leakage, which will allow a thermal photon distribution to build up, even at the relatively high temperatures that existed during the first moments of stellar evolution.

Now, since photons are massless, they also give a sizable contribution to the thermal conductivity of the CFL phase. The corresponding contribution κ_γ will be similar to the contribution of the massless NG boson in Eq. (16). Since the photons move at approximately the speed of light [26] ($v \simeq 1$ in our notation) and they have two polarization states, we obtain

$$\kappa_\gamma = \frac{4\pi^2 T^3}{45\Gamma_\gamma}. \tag{23}$$

Since the thermal conductivity is additive, the total conductivity of dense quark matter in the CFL phase is given by the sum of the two contributions:

$$\kappa_{CFL} = \kappa_\phi + \kappa_\gamma \simeq \frac{2\pi^2}{9} T^3 R_0, \tag{24}$$

where for both a photon and a NG boson the mean free path $l \sim R_0$. This yields the value

$$\kappa_{CFL} \simeq 1.2\times 10^{32} T_{\mathrm{MeV}}^3 R_{0,km}\ \mathrm{erg\ cm^{-1} sec^{-1} K^{-1}}, \tag{25}$$

which is many orders of magnitude larger than the thermal conductivity of regular nuclear matter in a neutron star [27].

V. STELLAR COOLING

In discussing the cooling mechanism for a compact star we have to make some general assumptions about the structure of the star. We accept without proof that a quark core exists at the center of the star. The radius of such a core is denoted by R_0, while the radius of the whole star is denoted by R. We exclude the possibility that the star is made completely of CFL quark matter. Schematically, the internal structure of the star is shown in Fig. 1. The quark core stays in direct contact with the neighboring nuclear matter. From this nuclear contact layer outwards the structure of the star should essentially be the same as in an ordinary neutron star.

A detailed analysis of the interface between the quark core and the nuclear matter was made in Ref. [28]. A similar analysis might also be very useful for understanding the mechanism of heat transfer from one phase to the other here. For our purposes it is sufficient to know only that direct contact between the quark and nuclear phases is possible. In fact, the only assumption we need is that the temperature is continuous across the interface.

Having spelled out the basic assumptions, we can consider the physics that governs stellar cooling. Let us start from the moment when the star is formed in a supernova explosion. Right after the explosion a lot of high-energy neutrinos are trapped inside the star. After about 10 to 15 seconds most of them escape from the star by diffusion. The presence of the CFL quark core could slightly modify the rate of such diffusion [29–31]. By the end of the deleptonization process, the temperature of the star would rise to a few tens of MeV. Then, the star relatively quickly cools down to about $\tilde{T}$ by the efficient process of neutrino emission. It is unlikely that the quark core would greatly affect the time scale for this initial cooling stage. An ordinary neutron star would continue to cool mostly by neutrino emission for quite a long time even after that [32]. Here we need to discuss how the presence of the CFL quark core affects the cooling process of the star after the temperature drops below $\tilde{T}$.

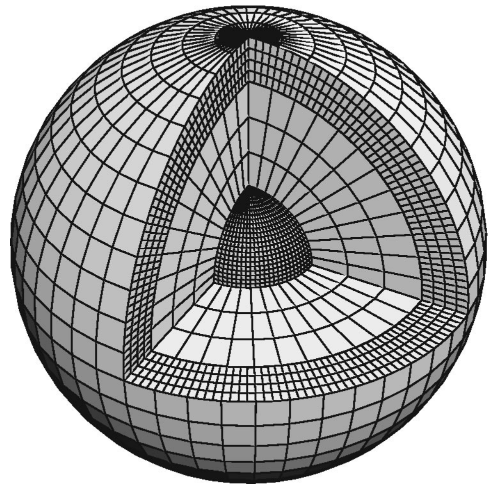

FIG. 1. The schematic structure of a compact star. At the center is a quark core which is surrounded by ordinary nuclear matter. On the outside is a crust of nuclei, neutrons and electrons.

Our result for the mean free path of the NG boson demonstrates that the heat conductivity of dense quark matter in the CFL phase is very high. For example, a temperature gradient of 1 MeV across a core of 1 km in size is washed away by heat conduction in a very short time interval of order $R_{0,\mathrm{km}}^2/v\,l(T) \simeq 6\times 10^{-4}$ s. In deriving this estimate, we took into account the fact that the specific heat and the heat conductivity in the CFL phase are dominated by photons and massless NG bosons and that $l \sim R_0$. In addition, we used the relation in Eq. (17). This estimate proves that, to a good approximation, the quark core is isothermal at all temperatures $T \lesssim \tilde{T}$.

The heat conductivity of the neighboring nuclear matter is also known to be very high because of the large contribution from degenerate electrons which have a very long mean free path. It is clear, then, that both the quark and the nuclear layers should be nearly isothermal with equal values of the temperature. When one of the layers cools down by any mechanism, the temperature of the other will adjust almost immediately due to the very efficient heat transfer on both sides of the interface. This observation has a very important consequence for the cooling rate of the star. It will eventually be determined by the combination of two effects: (i) neutrino emission from the nuclear part of the star, and (ii) diffusion of photons through the outer crust made of nuclei and nondegenerate electrons. Even if, for any reason, the intermediate nuclear layer happens to be abnormally thin photon diffusion will provide efficient cooling. The corresponding cooling time would be of order 10^7 years. This is many orders of magnitude shorter than the estimated time of cooling of the CFL matter by neutrino emission [16].

Now consider the order of magnitude of the cooling time for a star with a CFL quark core. One of the most important components of the calculation of the cooling time is the thermal energy of the star which is the amount of energy that is lost in cooling. There are contributions to the total thermal energy from both the quark and the nuclear parts of the star. The dominant amount of thermal energy in the CFL quark matter is stored in photons and massless NG bosons [see Eq. (A12) and the brief discussion in the Appendix]. It is given by the following expression:

$$E_{CFL}(T)=\frac{4\pi R_0^3}{3}\int_0^T c_v(T')dT' =\frac{6(1+2v^3)T}{5}\left(\frac{\pi T R_0}{3v}\right)^3, \qquad (26)$$

where, for simplicity, we use the Newtonian approximation. The specific heat here contains both ϕ and photon contributions ($c_v=c_v^{(\phi)}+c_v^{(\gamma)}$) which can be obtianed from Eq. (A12) using the appropriate degeneracies and velocities. Numerically the expression in Eq. (26) leads to the following result:

$$E_{CFL}(T)\simeq 2.1\times 10^{42} R_{0,\mathrm{km}}^3 T_{\mathrm{MeV}}^4 \ \mathrm{erg}. \qquad (27)$$

Here $R_{0,\mathrm{km}}$ is the quark core radius measured in kilometers. The thermal energy of the outer nuclear layer is provided mostly by degenerate neutrons. A numerical estimate is [33]

$$E_{NM}(T)\simeq 8.1\times 10^{49}\frac{M-M_0}{M_\odot}\left(\frac{\rho_0}{\rho}\right)^{2/3} T_{\mathrm{MeV}}^2 \ \mathrm{erg}, \qquad (28)$$

where M is the mass of the star, M_0 is the mass of the quark core and $M_\odot$ is the mass of the Sun. It is crucial to note that the thermal energy of the quark core is negligible in comparison to that of the nuclear layer. Moreover, as the star cools the ratio E_{CFL}/E_{NM} will further decrease.

The second important component that determines stellar cooling is the neutrino and/or photon luminosity which describes the rate of energy loss. Typically, the neutrino luminosity dominates the cooling of young stars when the temperatures are still higher than about 10 keV and after that the photon diffusion mechanism starts to dominate. As argued in Ref. [16], neutrino emission from the CFL quark phase is strongly suppressed at low temperatures, i.e., at $T\lesssim\widetilde{T}$ in our notation. The neighboring nuclear layer, on the other hand, emits neutrinos quite efficiently. As a result, it cools relatively fast in the same way as an ordinary neutron star. The nuclear layer should be able to emit not only its own thermal energy, but also that of the quark core which constantly arrives by the very efficient heat transfer process. The analysis of this cooling mechanism, however, is greatly simplified by the fact that the thermal energy of the quark core is negligible compared to the energy stored in the nuclear matter. By making use of the natural assumption that local neutrino emissivities from the nuclear matter are not affected by the presence of the quark core, we conclude that the cooling time of a star with a quark core is essentially the same as for an ordinary neutron star provided that the nuclear layer is not extremely thin.

VI. CONCLUSIONS

As is clear from our analysis, the thermal conductivity of the CFL color superconducting dense quark matter is very high for typical values of the temperature found in a newborn compact star. This is a direct consequence of the existence of the photon and the massless NG boson associated with the breaking of baryon number. (Note that the quark contribution to the thermal conductivity is strongly suppressed in the CFL phase, as discussed in Ref. [34].) The mean free path of the NG boson appears to be very large as a result of its weak self-interaction and its weak interaction with the rest of the matter in the CFL phase. The same is true for the photon. Our rough estimate shows that even at a relatively early stage of stellar cooling when the temperature is of order $\widetilde{T}$, say 1 MeV or so, the mean free paths are already larger than the size of the star. It should be noted, however, that the NG bosons cannot leave the CFL phase and escape from the core of the star carrying away their energy. By their nature, these NG bosons are collective excitations of the CFL phase and thus they are "confined" to the medium which made their existence possible. In this respect, they resemble phonons in solid state physics which also, by their nature, cannot escape from inside a crystal. Although for a different reason, thermal photons also cannot leave the finite region of the quark core. They are efficiently reflected from the electron plasma of the neighboring nuclear matter.

We mention that the (pseudo-)NG bosons and photons should also dominate other kinetic properties of dense quark matter in the CFL phase. For example, the shear viscosity should be mostly due to photons and the same massless NG bosons associated with the breaking of baryon number. The electrical conductivity, on the other hand, would be mostly due to the lightest *charged* pseudo-NG boson, i.e., the K^+. Thus, in the limit of small temperatures $T\to 0$ the electrical conductivity will be suppressed by a factor $\exp(-m_{K^+}/T)$.

Since the neutrino emissivity of the CFL core is strongly suppressed, the heat is transferred to the outer nuclear layer only through direct surface contact. While both the core and the outer layer contribute to the heat capacity of the star, it is only the outer layer which is capable of emitting this heat energy efficiently in the form of neutrinos. The combination of these two factors tends to extend the cooling time of a star. However, because of the very small thermal energy of the quark core compared to that of the nuclear matter, the time scale for cooling would only get noticeably shorter than that for an ordinary neutron star when the quark core radius is nearly the same as the stellar radius. Thus it appears that the cooling of stars with CFL quark cores will differ little from the cooling of standard neutron stars. A similar conclusion has been reached for stars with regular, non-CFL quark interiors [35].

In passing it is interesting to speculate about the possibility that a bare CFL quark star made entirely of dense quark matter could exist. If it were possible, it would look like a transparent dielectric [36]. Our present study suggests such a star would also have very unusual thermal properties. Indeed, if the star has a finite temperature $T \lesssim \tilde{T}$ after it was created, almost all of its thermal energy would be stored in the NG bosons. Notice that all the photons would leave the star very soon after transparency set in because the star is assumed to have no nuclear matter layer. The local interaction as well as the self-interaction of the NG bosons is very weak so that we argued in Sec. III that their mean free path would be limited only by the geometrical size of the star. This suggests that the only potential source of energy loss in the bare CFL star would be the interaction of the NG bosons at the star boundary, together with very strongly suppressed [16] photon and neutrino emission. It is likely, therefore, that such stars might be very dim and might even be good candidates for some of the dark matter in the Universe. A more detailed discussion of this issue is, however, beyond the scope of the present paper.

Finally, we would also like to note that the thermal conductivity of the 2SC color superconducting phase of two lightest quarks should also be very large, but for a different reason. In this case there are quarks which do not participate in color condensation and thus give rise to gapless quasiparticles. The corresponding contribution to the heat conductivity (which is similar to the contribution of degenerate electrons inside nuclear matter) should be very large. Therefore, if it were possible for the core of a compact star to be made of the 2SC quark phase, it would again be nearly isothermal. The results of Ref. [9] suggest, however, that the 2SC phase cannot appear in a compact star.

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APPENDIX: THERMODYNAMICAL QUANTITIES

In this appendix, for convenience we list expressions for the thermodynamic quantities that we use throughout the main text of the paper.

It is most convenient to start with the expression for the pressure. The other quantities (entropy, free energy, etc.) can then be easily derived [37]. In the case of a massive boson field, the pressure is

$$P^{(bos)} = -T \int \frac{d^3k}{(2\pi)^3} \ln(1 - e^{-e_k/T}), \tag{A1}$$

where $e_k = \sqrt{v^2 k^2 + m^2}$. The corresponding energy density, number density and the specific heat are

$$U^{(bos)} = \int \frac{d^3k}{(2\pi)^3} \frac{e_k}{e^{e_k/T} - 1}, \tag{A2}$$

$$n^{(bos)} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{e_k/T} - 1}, \tag{A3}$$

$$c_v^{(bos)} = \frac{1}{4T^2} \int \frac{d^3k}{(2\pi)^3} \frac{e_k^2}{\sinh^2 e_k/2T}. \tag{A4}$$

In the case of a small temperature (as compared to the value of the mass), these thermodynamic quantities approach the following asymptotic expressions [38]:

$$P^{(bos)} \simeq \frac{m^{3/2} T^{5/2}}{2\sqrt{2}\pi^{3/2} v^3} e^{-m/T}, \tag{A5}$$

$$U^{(bos)} \simeq \frac{m^{5/2} T^{3/2}}{2\sqrt{2}\pi^{3/2} v^3} e^{-m/T}, \tag{A6}$$

$$n^{(bos)} \simeq \frac{m^{3/2} T^{3/2}}{2\sqrt{2}\pi^{3/2} v^3} e^{-m/T}, \tag{A7}$$

$$c_v^{(bos)} \simeq \frac{m^{7/2}}{2\sqrt{2}\pi^{3/2} v^3 \sqrt{T}} e^{-m/T}. \tag{A8}$$

And, in the limit of massless particles, they are [38]

$$P^{(bos)} = \frac{\pi^2 T^4}{90 v^3}, \tag{A9}$$

$$U^{(bos)} = \frac{\pi^2 T^4}{30 v^3}, \tag{A10}$$

$$n^{(bos)} = \frac{\zeta(3) T^3}{\pi^2 v^3}, \tag{A11}$$

$$c_v^{(bos)} = \frac{2\pi^2 T^3}{15 v^3}, \tag{A12}$$

where the value of the Riemann function $\zeta(3) \approx 1.202$. For completeness, let us also present some thermodynamical properties of color superconducting quarks. In this case, it is very convenient to separate the zero and finite temperature contributions explicitly. For the pressure of a single quark with a gap Δ, we find

$$P_{qrk} = P^{(0)}_{qrk} + 2T \int \frac{d^3k}{(2\pi)^3} \ln(1 + e^{-E^{(-)}_k/T}) + 2T \int \frac{d^3k}{(2\pi)^3} \ln(1 + e^{-E^{(+)}_k/T}), \tag{A13}$$

where $E^{(\pm)}_k = \sqrt{(e_k \pm \mu)^2 + \Delta^2}$ are the quasiparticle energies and μ is the chemical potential. The zero temperature contribution is approximately given by

$$P^{(0)}_{qrk} = \int \frac{d^3k}{(2\pi)^3} (E^{(+)}_k + E^{(-)}_k) - \alpha\Delta^2. \tag{A14}$$

The last term is needed so that the gap equation is obtained when the pressure is maximized with respect to variations in Δ. Using the gap equation to eliminate α and renormalizing by removing the divergent term, the pressure becomes

$$P^{(0)}_{qrk} \simeq \frac{\mu^4}{12\pi^2} + \frac{\mu^2\Delta^2}{4\pi^2} + \cdots, \tag{A15}$$

in agreement with the results obtained using the framework of the Nambu-Jona-Lasinio model [36] and directly from quantum chromodynamics [39]. The ellipsis denotes subleading terms suppressed by powers of either $(m/\mu)^2$ or $(\Delta/\mu)^2$. As is easy to check, at small temperatures, i.e., $T \ll \Delta$, the finite temperature corrections in Eq. (A13) are suppressed by a factor $\exp(-\Delta/T)$. Thus, the above zero temperature contribution will dominate the pressure of the CFL phase. Similarly, the main contribution to the number density and the energy density of CFL matter at $T \ll \Delta$ comes from the zero temperature terms. Using Eq. (A15) the $T=0$ contribution to the number density is

$$n^{(0)}_{qrk} \simeq \frac{\mu^3}{3\pi^2} + \frac{\mu\Delta^2}{2\pi^2}\left(1 + \frac{\mu}{\Delta}\frac{\partial\Delta}{\partial\mu}\right). \tag{A16}$$

Note that the partial derivative term is needed since the gap equation, which specifies $\Delta(\mu)$, was used in arriving at Eq. (A15). This term contributes at the same order (unity) as the first term in parentheses. The energy density is then

$$U^{(0)}_{qrk} \simeq \frac{\mu^4}{4\pi^2} + \frac{\mu^2\Delta^2}{4\pi^2}\left(1 + \frac{2\mu}{\Delta}\frac{\partial\Delta}{\partial\mu}\right). \tag{A17}$$

Here it is appropriate to note that all thermodynamic functions which are defined through the temperature derivatives of either the pressure (e.g., entropy) or the energy (e.g., specific heat) are exponentially suppressed at $T \ll \Delta$. Therefore, the dominant contributions to this second type of function would come from (pseudo-)NG bosons in the CFL phase of dense quark matter.

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